The Determination of Monetary Aggregates and Interest Rates

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This paper is concerned with a perfectly competitive equilibrium model of the money market under the assumption of increasing marginal costs for both deposits and loans. This model illustrates a determination mechanism of monetary aggregates and interest rates. As this novel assumption of increasing marginal costs is not widely adopted in the literature, this paper discusses, at some length, how marginal costs can be considered to increase with the size of deposits by employing the results from the queuing theory.

I. Purpose and Summary

The system of regulated interest rates, which has been in place in Japan since the end of World War II, is expected to be abolished in the near future. Given this prospect, this paper reexamines several major issues related to the determination of monetary aggregates and interest rates in the money market.

Most analytical studies of Japan's money market have assumed regulated interest rates, particularly with respect to deposit rates. Few studies have addressed the mechanism of the money market when deposit rates are deregulated. In traditional models, although deposit rates appear as an exogenous variable determining the overall conditions of the money market, they do not play an important role in determining more specific variables, such as the shares of individual banks in the deposit market. These variables are usually explained either by non-price competition such as the quality of banking services or by an institutional factor such as the regulations governing the establishment of new branches.

This traditional approach may be appropriate so long as deposit rates are regulated. When we look ahead to the deregulation of deposit rates, however, it is useful to adopt an alternative approach to analyze the money market — namely, an approach that treats deposit rates as an endogenous variable of the model. Such an approach is better suited to studying the implications of interest rate deregulation as well as the effectiveness of monetary policy instruments in a deregulated money market. The present paper adopts this approach and models the money market under perfect competition.

The structure and main conclusions of this paper are summarized as follows:

Section II describes the money market (which is defined as comprising the deposit,
loan, and interbank credit markets) in terms of a model of perfectly competitive equilibrium, consisting of the central bank, the banking sector and the non-bank private sector.

This model differs from the standard model traditionally used to analyze the Japanese money market, in that it assumes increasing marginal costs for both deposits and loans. The traditional models have assumed increasing marginal costs for loans but not for deposits. As noted earlier, this is a reasonable assumption if deposit rates are regulated and there is little need to specify their function as part of the price mechanism.

If deposit rates are deregulated, however, the model must specify an explicit cost function for banks' deposit taking activities. In analyzing the impact of deposit rate deregulation, this paper will specify increasing marginal cost functions for both loans and deposits of individual banks. With this approach, we hope to present a model of perfectly competitive equilibrium, which will be useful in discussion of deposit rate deregulation.

An assumption of increasing marginal costs for deposits is not widely adopted in relevant literature. To strengthen the case for this assumption, Section III discusses the conditions leading to an increasing marginal cost of deposits relative to scale. It shows how the queuing theory can be used to characterize the increasing administrative cost of deposit taking activities, thus deriving a marginal cost function that increases with the size of deposits. Based on this analysis, the section also discusses whether bank deposit-taking operations can be profitable after deposit rates are deregulated, and identifies some problems associated with the possible introduction of deposit related fees.

Section IV discusses how the instruments of monetary policy operate in the money market. Specifically, it models the money market in terms of a system of equations and shows that: (1) interest rate policy and quantitative adjustment of central bank credit can both be treated as changes in exogenous variables; (2) adjustment of reserve requirement ratios can also be treated as a parameter change; and (3) window guidance can be treated as an additional constraint in the system of equations.

Hereby, as far as market equilibrium is concerned, the regulation of deposit rates will have the same qualitative effects on the money market as the control of interbank interest rates. It also helps to clarify the characteristics of our model with specific examples.

Section V suggests some issues that will be important in the discussion of deposit rate deregulation. It concludes that deposit rate deregulation is a necessary, but not always a sufficient condition, for restoring the equity and efficiency of the money market, and that many issues still remain to be studied in detail including the nature of banks' cost function.

Finally, Appendix uses the Petri-Net system of graphic representation to address the question of whether the amount of credit creation in the banking system can be controlled by the quantitative adjustment of the supply of reserves. It shows that direct control over credit creation may be feasible in a system that accumulates "contemporaneously" or "in advance." However, no such direct control is possible in the current
Japanese reserve requirement system of "lagged" reserve accumulation.

II. Basic Structure of the Model

In this section, we will construct a model in order to analyze how monetary aggregates and interest rates are determined in the money market. For simplicity, we ignore the government as well as external factors, and assume the credit flow described in the following table comprising the central bank, the banking sector which is made up of \( n \) banks, and the non-bank private sector.\(^1\) In this table, a positive sign indicates that the item is on the assets side of the balance sheet, and a negative sign that it is on the liabilities/capital side.

<table>
<thead>
<tr>
<th></th>
<th>Central Bank</th>
<th>Banking Sector (( n ) Banks)</th>
<th>Non-Banking Sector</th>
<th>Interest Rates</th>
</tr>
</thead>
</table>
| High-Powered Mony Credit | \(-Z\)       | \(
+k\)th Bank \( R_k \)       | \( M \)            | \( i_s = 0 \)   |
| Central Bank Credit      | \( X \)      | \(-X_k\)                        | \( 0 \)            | \( i_s \)       |
| Deposits                | \( 0 \)      | \(-D_k\)                        | \( D \)            | \( i_d \)       |
|Loans                    | \( 0 \)      | \( F_k \)                       | \(-B\)             | \( i_f \)       |
|Interbank Credit         | \( 0 \)      | \(-W_k\)                        | \( 0 \)            | \( i_w \)       |

By setting the horizontal sum of each row at zero, we obtain five market equilibrium conditions. By ignoring the equities of the central bank and the banking sector, and by setting the vertical sum of each column at zero, we obtain \((n+1)\) balance sheet constraints for the central bank and the banking sector. If we further assume that cash and deposits held by the non-bank private sector represent borrowings from the banking sector, the vertical sum of the non-bank private sector also becomes zero.

Of course, one of these equations is not independent because it is a sum of the other equations. This means that one equation is redundant in characterizing market equilibrium conditions for the money market as a whole (Walras' law). In the following discussion of market equilibrium, we will therefore eliminate the vertical sum of the non-banking sector \((M+D-B=0)\) from equilibrium conditions. Thus, we begin our analysis with the following equations (1) – (7):

\[
\begin{align*}
\text{equilibrium in high-powered money:} & \quad \Sigma R_k + M - Z = 0 \\
\text{equilibrium in central bank credit:} & \quad X - \Sigma X_k = 0 \\
\text{equilibrium in deposits:} & \quad D - \Sigma D_k = 0
\end{align*}
\]

\(^1\)This table is based on the model of Horiuchi (1980), a framework for general equilibrium analysis of the asset market along the lines first proposed by Tobin (1969).
equilibrium in loans: \( \Sigma F_k - B = 0 \)  \hspace{1cm} (4)
equilibrium in interbank credit: \( \Sigma W_k = 0 \)  \hspace{1cm} (5)
budget constraint of the central bank: \( X - Z = 0 \)  \hspace{1cm} (6)
budget constraints of banks: \( F_k + R_k - X_k - D_k - W_k = 0 \)
\((k=1, 2, \ldots n)\)  \hspace{1cm} (7)

Next, we will integrate equations (1) – (7) into a system of equations by imposing the following assumptions:

1. We assume that the central bank determines the aggregate supply of high-powered money \( (Z) \) for the money market before determining the amount of discretionary lending to individual banks. That is, the central bank determines the amount of credit to an individual bank \( (X_k) \) according to an allocation decision function \( (\Phi_k) \). With this assumption, equation (6) can be replaced by the following \( n \) equations,

\[
X_k = \Phi_k (Z) \quad (k=1, 2, \ldots n)
\]

2. We assume that the amount of reserves held by a bank \( (R_k) \) is a constant fraction of the amount of deposits \( (D_k) \),

\[
R_k = rD_k \quad (0<r<1)
\]

3. We assume that non-bank private sector’s cash-holdings \( (M) \) consist of two components: the variable portion which has an arbitrage relationship with deposit holdings \( (D) \) through the deposit rate \( (i_d) \), and the fixed portion \( (L) \). That is, \( M \) can be specified by the following equation.

\[
M = f (i_d) D + L
\]
where \( f (i_d)>0, f' (i_d) = \partial f (i_d) / \partial i_d<0, \) and \( L \) is a positive constant.

4. Assumptions (2) and (3) will yield the following constraint on the relationships between \( \Sigma D_k, i_d \) and \( Z \) in equations (1) and (3):

\[
\Sigma D_k = (Z - L) / \{r + f (i_d)\}
\]
The derivative of the right side of this equation with respect to \( i_d \) gives \(-f' (i_d) \cdot (Z-L) / \{r+f(i_d)\}^2 > 0.\) Thus, we know that the aggregate amount of deposits \( (\Sigma D_k) \) is an increasing function of the deposit rate \( (i_d) \).

5. By assuming that the demand for loans \( (B) \) is a function of the lending rate \( (i_l) \), we can express equation (4) as

\[
\Sigma F_k = B (i_l)
\]
where \( \partial B / \partial i_l < 0.\)

6. We assume perfect competition in the banking industry, and that the operational
costs of the kth bank \((E_k)\) are a function of the amount of deposits \((D_k)\) and amount of loans \((F_k)\):

\[
E_k = E_k(F_k, D_k)
\]

The bank’s behavior is determined by maximizing the following profit function:

\[
P_k(F_k, D_k, W_k) = i_F F_k - i_d D_k - i_w W_k - i_x X_k - E_k(F_k, D_k)
\]
given the market interest rate, and subject to the following balance sheet constraint:

\[
\theta(F_k, D_k, W_k) = F_k - (1-r) D_k - W_k - X_k = 0
\]

Thus, the bank’s behavior can be expressed by the following set of equations:

\[
\begin{align*}
\frac{\partial P_k}{\partial F_k} - \lambda \frac{\partial \theta}{\partial F_k} &= i_F - \frac{\partial E_k}{\partial F_k} - \lambda = 0 \\
\frac{\partial P_k}{\partial D_k} - \lambda \frac{\partial \theta}{\partial D_k} &= -i_d - \frac{\partial E_k}{\partial D_k} + \lambda(1-r) = 0 \\
\frac{\partial P_k}{\partial W_k} - \lambda \frac{\partial \theta}{\partial W_k} &= i_w + \lambda = 0
\end{align*}
\]

where \(\lambda\) is the Lagrangean multiplier.

Finally, eliminating the Lagrangean multiplier and rearranging the equations, we obtain the following system of equations as the equilibrium conditions of the money market:

\[
\begin{align*}
F_k - (1-r)D_k - W_k - X_k &= 0 \quad (k=1, 2, \ldots n) \quad (8) \\
i_F - i_w &= \frac{\partial E_k}{\partial F_k} \quad (k=1, 2, \ldots n) \quad (9) \\
(1-r)i_w - i_d &= \frac{\partial E_k}{\partial D_k} \quad (k=1, 2, \ldots n) \quad (10) \\
\Sigma F_k &= B(i_F) \quad (11) \\
\Sigma D_k &= (Z - L)/\{r + f(i_d)\} \quad (12) \\
\Sigma W_k &= 0 \quad (13) \\
X_k &= \Phi_k(Z) \quad (k=1, 2, \ldots n) \quad (14)
\end{align*}
\]

\(^3\)We assume the following properties for the cost function:

\[
E_k(0, D_k) > 0, \frac{\partial E_k}{\partial F_k} > 0, \frac{\partial^2 E_k}{\partial F_k^2} > 0 \\
E_k(F_k, 0) > 0, \frac{\partial E_k}{\partial D_k} > 0, \frac{\partial^2 E_k}{\partial D_k^2} > 0
\]
The direct implication of these equilibrium conditions is that, as in the model for an ordinary manufacturing industry, the concept of marginal cost pricing can be applied to analyzing money market equilibrium. That is, we can think of banking as an industry that purchases credit in raw materials (deposits), and sells it either as an intermediate product in the interbank credit market or as a finished product in the bank loans market. Thus, the market reaches equilibrium when the price of each product (revenue) equals each bank’s marginal cost (equations (9) and (10)).

Of course, this in itself is nothing new; it is a standard description found in many financial economics textbooks. The distinguishing characteristic of our model is the assumption of the increasing marginal cost for both loans and deposits. In our model, the conditions for equilibrium in the money market are seen as a problem of constraints on profit maximization, including the deposit market.\(^3\)

Is it appropriate to assume increasing marginal costs for banks’ deposit taking activities to allow this type of simplification? Of course, this is, after all, an empirical question, one that cannot be resolved by theoretical arguments alone. However, to the extent that our analysis is based on a new and different assumption about the cost function of banks, it may be necessary to present some supporting arguments. This is the focus of the next section.

**III. The Marginal Cost of Deposits**

In this section, we will provide justification for the increasing marginal cost of deposit taking activities, which was essential in characterizing equilibrium in the money market in terms of marginal cost pricing, both in the deposit and loan markets. More specifically, we will use the queuing theory to characterize the increasing administrative costs of banks’ deposit taking activities. We can thus clarify and organize some of the issues related to bank cost for deposit taking activities, as well as probe some problems that may arise in a financial system with deregulated deposit rates. Because the following analysis is based on an individual bank, we will omit the subscript \(k\) (meaning the \(k\)th bank) in this section.

1. We assume that demand for administrative transactions associated with a bank’s deposit taking operations follows a Poisson process with parameter \((\mu)\), which takes a real number proportionate to the volume of deposits \((D)\). That is, the expected value \((\mu)\) of the number of the bank’s transactions associated with deposit taking per unit of time

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\(^3\)As noted earlier, the standard model does not assume that the marginal cost of deposit taking is increasing with respect to volume. This means that previous analyses have posited the following properties for banks’ cost function:

\[
E_A(0, D) > 0, \frac{\partial E_A}{\partial F_k} > 0, \frac{\partial^2 E_A}{\partial F_k^2} > 0 \]

\[
E_A(F_k, 0) > 0, \frac{\partial E_A}{\partial D_k} \equiv 0, \frac{\partial^2 E_A}{\partial D_k^2} = 0
\]
(x) increases in proportion to the volume of deposits, according to

\[ \mu = sD \]

2. The cost function \((E)\) of the bank’s deposit taking takes the following form:

\[ E = C + ax + \max\{c, bx\}, \]

where \(x\) is the number of transactions per unit of time;

\(C\) is the portion of the cost that is independent of the number of transactions or the time it takes to execute them; it is the fixed cost exclusive of \(c\) to be defined below;

\(a\) is the cost coefficient that is proportionate to the number of transactions, such as the cost of paper;

\(c\) represents the minimum fixed costs associated with deposit taking, such as the salaries of full-time personnel;

\(b\) is the cost coefficient proportionate to the time it takes to execute a transaction associated with deposit taking, from the arrival of a customer at the window to his departure, such as the hourly wage rate or overtime wage rate for employees engaged in deposit taking.

\(q\) is the time it takes to execute a transaction associated with deposit taking.

In the above equation, the first term represents the cost independent of the volume of deposits in the short run, and the second term represents the cost proportional to the volume of deposits. The meaning of the third term is perhaps less obvious and will be explained below.

When demand for administrative transactions increases, a certain type of inefficiency (an increase in \(q\)) will result as the processing capacity of the bank’s administrative system (i.e. the process of executing the entire chain of administrative steps at the window) approaches its limit. We can think of this inefficiency as a slowdown in the response of a computer terminal or as a longer wait for a supervisor’s approval.

When this type of inefficiency arises, the cost of executing an administrative transaction increases. When the computer system is congested, for example, the resulting delay may require additional salaries (in terms of opportunity cost) to employees who may have to placate customers, or the additional cost of gifts (such as tissue paper) to customers.

We assume that this cost is proportional to \(qx\) (which represents the waiting time of the customer before the transaction is executed) with the factor of proportionality \(b\). In other words, the second argument for the third term in the equation (that is, \(bx\)) can be thought of as representing the cost of administration associated with deposit taking based on the relationship between the capacity of the system and the frequency of transactions.

In practice, however, the administrative cost based on the frequency of transactions is not usually realized in the bank’s actual expenditures. The bank employs human and non-human resources over the long term according to the projected frequency of transac-
tions. Thus, the administrative cost based on the frequency of transactions will be calculated only when it exceeds the fixed cost $c$. This is why the third term takes the form of $\max\{c, bxq\}$.

3. Next, we will elaborate on $q$. Our basic assumption is that the time it takes the bank to execute a transaction follows an exponential distribution with parameter $\kappa$, which represents the efficiency of the bank’s administrative system. In effect, this implies that, although the actual time required to execute a transaction may be shorter or longer, its average time is $1/\kappa$, and that it can execute $\kappa$ transactions at full capacity per unit of time.

With this assumption, $q$ also becomes a stochastic variable that follows an exponential distribution with parameter $\kappa - \mu$. According to the queuing theory, its expected value is given by

$$\frac{1}{\kappa - \mu}$$

Consequently, the expected value of the bank’s cost associated with deposit taking can be expressed as follows:

$$C + a\mu + \max\{c, b\mu/(\kappa - \mu)\}$$

4. Here, $\mu/(\kappa - \mu)$, is a typical equation that frequently appears in queue analysis. It takes a relatively small value when the frequency of transactions $\mu$ is sufficiently low relative to administrative capacity $\kappa$. However, it increases rapidly as it approaches $\kappa$, and explodes when $\kappa = \mu$. This means that there is the maximum volume of deposits for each bank, which can be determined from the given capacity of the system (considered fixed in the short run):

$$\Delta = \kappa/S$$

Thus, with the normalization of $a$ and $b$, we can derive an individual bank’s expenditure ($E$) as a function of the volume of deposits ($D$) as follows:

$$E = C + aD + \max\{c, bD/\Delta\}$$

5. With the bank’s expenditure ($E$) expressed as a function of the volume of deposits ($D$), we use this relationship to examine the relationship between $E$ and $D$. Here, we define $D$ as the critical value (that makes the following equality to hold) that satisfies the following equation,

$$c = bD/\Delta$$.
Given the bank's resource endowment in the short run, $D$ is treated as fixed.

**Average cost:**
\[
\frac{E}{D} = a + \frac{(C + c)}{D} \quad \text{(if } D \leq D) \\
= a + \frac{C}{D} + \frac{b}{(\Delta - D)} \quad \text{(if } D > D)
\]

First derivative, i.e., marginal cost:
\[
\frac{\partial E}{\partial D} = a \quad \text{(if } D \leq D) \\
= a + \frac{b\Delta}{(\Delta - D)^2} \quad \text{(if } D > D)
\]

Second derivative:
\[
\frac{\partial^2 E}{\partial D^2} = 0 \quad \text{(if } D \leq D) \\
= 2b\Delta/(\Delta - D)^3 \quad \text{(if } D > D)
\]

6. The inferences we draw from these specifications are as follows (see Figure 1 for graphical representation):

a. Although there exists a marginal cost that ensures competitive equilibrium in the money market, its value must lie in the range $D > D$. This suggests that, if the bank contracts the resources that are excessive according to the capacity ($\kappa$) of the administrative system for deposit taking, the actual volume of deposits in the money market may not reach $D > D$. As a result, the banks may continue their efforts to compete for shares in a deposit market even after deposit rate deregulation.

b. Even if the actual volume of deposits lies within the range $D > D$ for individual banks, there is no guarantee that the spread (equilibrium price) will exceed the average administrative cost of deposit taking and generate net profits in deposit taking operations (i.e. the profit margin of the difference between the interest rate in the interbank credit market and the deposit rate may exceed the cost of deposit taking). This is because whether deposit taking operations generate net profits or not hinges on the following boundary condition:

Figure 1
\[ \frac{C}{D} + \frac{b}{(\Delta - D)} = b\Delta/(\Delta - D)^2 \]

This suggests that, for individual banks, the size of \( C \) becomes an important strategic variable. If \( C \) is too high, it becomes difficult to produce a profit. On the other hand, if \( C \) is too low, the share will be small because processing capacity (\( \kappa \)) is insufficient.

c. Some argued that banks should charge appropriate fees to depositors as a way to bring stability to banking operations in the face of the increasing cost of automation. According to the conditions postulated in the model, however, the fees corresponding to the number of deposit related transactions would not prove useful for this purpose. This is clear when we express bank expenditure (\( E \)) exclusive of such fee based revenue as

\[ E = C + (a - \alpha)D + \max \{ c, \frac{bD}{(\Delta - D)} \} \]

where \( \alpha \) is the fee corresponding to the volume of deposits converted from the fee corresponding to the number of transactions. Then, we obtain,

average cost:
\[ \frac{E}{D} = (a - \alpha) + \frac{(C + c)}{D} \]
\[ = (a - \alpha) + \frac{C}{D} + \frac{b}{(\Delta - D)} \]
\[ \text{(if } D \leq D) \]
\[ \text{(if } D > D) \]

marginal cost:
\[ \frac{\partial E}{\partial D} = (a - \alpha) \]
\[ = (a - \alpha) + \frac{b\Delta}{(\Delta - D)^2} \]
\[ \text{(if } D \leq D) \]
\[ \text{(if } D > D) \]

The introduction of this type of fee does not change the boundary condition for realizing profits in deposit taking operations:

\[ \frac{C}{D} + \frac{b}{(\Delta - D)} = \Delta \frac{b}{(\Delta - D)^2} \]

which is identical to the condition described in Paragraph b. above.

That is, although the introduction of fixed fees appears to have a positive effect on the bank’s profits because it lowers the average cost (adjusted for the fees) of deposit taking, it lowers marginal cost by the same amount. As long as the deposit market is competitive, therefore, the profit margin \( (i_w - i_d) \) is similarly reduced, so the bank’s profits are unchanged.

d. The above argument also suggests an economic explanation for the difference between interest rates on demand deposits and savings deposits. The expected frequency of transactions per volume (\( \mu \)) should be greater for demand deposits than savings deposits, so that the marginal cost of demand deposits should be higher. Therefore, even if we ignore the term structure of interest rates, one can argue that interest rates on demand deposits should be set lower than those on savings deposits.
IV. Instruments of Monetary Policy

Having justified the assumption of increasing marginal cost in banks' deposit taking activities, we use the model to identify the transmissions mechanisms of monetary instruments.

A. Quantitative Adjustment of Central Bank Credit and Interest Rate Policy

First, we must focus on the relationship between the number of variables and the number of equations in the equilibrium conditions specified in Section II. The equilibrium conditions are specified as a system of $4n+3$ equations with $4n+4$ variables ($Z, X_1, X_2, \ldots, X_n, W_1, W_2, \ldots, W_n, F_1, F_2, \ldots, F_n, D_1, D_2, \ldots, D_n, i_p, i_w, i_d$). Consequently, we can solve them for equilibrium solutions, once we specify any one of the variables as a predetermined variable.

Figure 2 depicts the determination of the supply of credit and the interest rate in the money market when the total supply of high-powered money $Z (= \Sigma X_k)$ is given exogenously. In this case, $D$ (the trace of the total amount of deposits under an arbitrary deposit rate $i_d$) is shown as an upward sloping curve in accordance with the derivatives obtained in Section II (4), and $F$ (the trace of the total amount of loans under an arbitrary $D$) is also depicted as an upward sloping curve. On the other hand, $B$ (the trace of the demand for loans at an arbitrary lending rate $i_f$) is depicted as a downward sloping curve.

Under these conditions, any arbitrary point $P$ on the $D$-curve would give the amount of deposits $D$ under an arbitrary deposit rate $i_d$. The intersection ($Q$) of a horizontal line
extending from $P$ and the $F$-curve would give the total amount of bank loans $F$. Consequently, the intersection $(R)$ of a vertical line extending from $Q$ and the $B$-curve would give the lending rates $i_r$. That is, any arbitrary combination of $P$, $Q$, and $R$ would give all the possible combinations of the equilibrium interest rate and the equilibrium supply of credit in the money market. However, considering the conditions given by equations (9) and (10), actual market equilibrium would be given by the combination of $P$, $Q$, and $R$, so the following condition holds for all banks:

$$i_f - \frac{\partial E_k}{\partial F_k} = \frac{1}{1-r} (i_d + \frac{\partial E_k}{\partial D_k}) = i_w$$

This explanation is based on the assumption that the central bank initially determines the amount of high-powered money $Z$. However, predetermining the supply of high-powered money $Z$ is not the only way to characterize equilibrium in the money market. Given the fact that equilibrium conditions are specified by a model in which there is one more variable than equation, any action that exogenously determines one of the variables can serve as a tool of monetary policy.

For example, we can posit a market equilibrium in which the predetermined variable is the interbank interest rate $(i_w)$ or the deposit rate $(i_d)$, instead of the supply of high-powered money $(Z)$. The resulting interest rates and supply of loans or deposits would be the same.

Policy makers are often tempted to think that they can obtain different policy results by selecting different variables as an instrument of monetary policy, or that they can fine tune the effects by manipulating the combination of variables or the method of allocating central bank credit. However, the present model suggests a theoretical problem with such an assumption. From the view point of comparative statics analysis of this model, it would be difficult to pursue such a policy as “tight credit and low interest rates.”

These equilibrium conditions do not include the discount rate of Bank of Japan $(i_c)$. The reason is that, because the level of the discount rate has no effect on money market equilibrium, it is inappropriate as an instrument of monetary policy. As long as the supply of central bank credit is left to the discretion of the central bank, it follows from the model that adjusting the discount rate, which lacks any direct effect on credit supply and demand conditions, has no bearing on money market equilibrium.

It should be noted, however, that although the present model ignores the possible indirect effects of the discount rate on market equilibrium, we do not deny the possibility that the discount rate may influence market equilibrium through its effects on the other variables in equilibrium conditions. In the Japan’s post war system of regulated interest rates, deposit rates were in practice linked to the discount rate. Under this system, the discount rate could work as a useful instrument of monetary policy. Moreover, even with deregulated interest rates, the discount rate could influence market equilibrium through its effects on market expectations about subsequent movements of other instruments of
monetary policy.\footnote{A different conclusion will emerge if we change the model's assumption that the central bank allocates credit at its own discretion. For example, if we assume that, although each bank is free to choose its allocation of central bank credit, the marginal cost of obtaining such credit increases relative to volume apart from the discount rate (see, for example, Furukawa (1985) and Goodfriend (1983) for this type of assumption), then the discount rate can be specified as an exogenous variable such determines the equilibrium conditions. More precisely, we specify bank behavior as maximizing the following objective function:

\[ U_k = U_k \{ P_k (F_k, D_k, W_k, X_k), X_k \} \]

subject to the balance sheet constraint

\[ \theta (F_k, D_k, W_k, X_k) = F_k - (1-r) D_k - W_k - X_k = 0 \]

where \( P_k \) is the profit of the \( k \)th bank, and \( X_k \) is its use of central bank credit during the current period. Then, the solution is the previously stated system of equilibrium conditions, except that

\[ X_k = \Phi_k (Z) \quad (k=1, 2, \ldots n) \]

is replaced by

\[ i_w - i_s = \partial E_k / \partial X_k - (\partial U_k / \partial X_k) / (\partial U_k / \partial P_k) \]

\[ \Sigma X_k = Z \]

It must be clear that an exogenous change in the discount rate (\( i_s \)) in this system can determine the equilibrium conditions in the money market.

\footnote{To focus our thinking, let us take the reserve requirement system with strictly lagged reserve accumulation (i.e., no overlap between accounting period and accumulation period). Then, we can specify equilibrium conditions as follows:

\[ F_k + r' D_{k(-1)} - (1-r') D_k - W_k - X_k = 0 \]

\[ i_f - i_w = \partial E_k / \partial F_k \]

\[ (1 - r') i_w - i_d = \partial E_k / \partial D_k \]

\[ \Sigma F_k = B (i_f) \]

\[ \Sigma D_k = (Z - r' \Sigma D_{k(-1)} - L) / (r' + f (i_d)) \]

\[ \Sigma W_k = 0 \]

\[ X_k = \Phi_k (Z) \]

where \( D_{k(-1)} \) is a constant that denotes the amount of the \( k \)th bank's deposits in the previous period, \( r' \) is the ratio of cash reserves to current deposits, \( r' \) is the legal reserve ratio of the previous period's deposits, and \( R_k = r' D_k + r' D_{k(-1)} \).}
In fact, there was a lot of discussions in the United States during the 1970s about how the relationship (between the period in which reserves are calculated and the period in which they are accumulated) in the reserve requirement system might affect the controllability of the money market. It is outside the scope of this paper to discuss this issue in great detail. We simply note that, under some conditions, the supply of high-powered money Z could not be controlled exogenously under the reserve requirement system with lagged reserve accumulation (See Appendix.).

B. Changes in the Reserve Requirement Ratios

The equilibrium conditions of the money market include the reserve ratio (r) as a parameter that can be used as an instrument of monetary policy. The effects of a change in the reserve ratio r show up as a quantitative effect in equations (12) and (8), and as an interest rate effect (or cost effect) in equation (10). In terms of Figure 2, the former causes the D-curve and F-curve to shift horizontally (i.e. a fall in the reserve ratio would cause the D-curve to shift to the right, and the F-curve to shift to the left). The latter changes the margin between the interbank interest rate i_w and the deposit rate i_d (i.e. a fall in the reserve ratio reduces the margin).

Since changing the reserve ratio has two effects, one may hope that policy makers could pursue a policy of, for example, tight credit and low interest rates. This could be implemented by a policy mix, where the central bank changes the reserve requirement ratio and simultaneously controls some exogenous variables that would offset its quantitative or interest rate effect. It turns out, however, that this type of control is successful only in extremely limited sense.

Figure 3 depicts the two cases (denoted by superscripts * and **) that would result from a change in the reserve ratio, while the interbank interest rate i_w is constant. In these cases, the lending rate i_f and money supply M+D (=F) are controlled by the interbank interest rate i_w, and the effect of the change in the reserve ratio shows up only as a change in the spread between the interbank interest rate i_w and the deposit rate i_d. That is, changing the reserve ratio affects only the deposit rate. It will have no effect on the overall conditions of the money market, except for the distribution of income between the banking sector and the non-banking (unless legal reserves pay interest, the

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We further assume that (1) the interest elasticity of cash holding in the non-banking sector is zero (f(i_d)=0), that is M=L; and (2) the cash reserves of the bank always take the constant value of R^k, that is R_k=R^k+r' \\[ \Sigma D_{k(-1)} \]. Again, we find that the properties of the equilibrium model are unchanged in the sense that the number of variables is still one higher than number of equations, except that R_k and M are now both constants. In this situation, therefore, the supply of high-powered money (Z), which is already a constant, cannot be used as a tool of monetary policy.

*In the United States, Poole (1976) proposed a system with contemporaneous reserve accumulation. Concerned with quantitative control over money supply, Laurent (1979) argued for the efficiency of a system with advance reserve accumulation in which the reserve maintenance period precedes the reserve accounting period.
distribution gain accruing to the banking sector would be claimed entirely by the central bank).

This type of analysis raises doubt about the usefulness of the reserve ratio as an instrument of monetary policy. This result is not surprising, however, in view of the comparatively static nature of this model, which gives no explicit consideration to the effects of the central bank's policy signals on market expectations.

Rather, the point to be emphasized in the present analysis is that, under a certain framework, other policy instruments can substitute for the reserve ratio, and that the reserve requirement system should be evaluated from this point of view. For example, if distribution gains from changing the reserve ratio accrue to the central bank when required reserves pay no interest, we can certainly think of the reserve requirement system as a way of charging fees for the central bank's services. Just as the holders of non-interest bearing central bank notes are implicitly paying for the use of the central bank's "off-line" services, it is reasonable to interpret the distribution effect of the reserve requirement system as a way of charging fees for the central bank's on-line electronic network services.7

C. Window Guidance8

We will now turn to the effect of introducing an additional constraint on variables in equilibrium condition in the money market. We will consider window guidance as an

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7 Needless to say, another method is to pay interest on reserves and to charge separate fees for central bank services.
8 Note that window guidance was officially abolished in July, 1991.
example. Although we could think of other examples such as the BIS capital adequacy requirements, their effects, on equilibrium should be analogous.

First, we will consider window guidance is applied to one of the $n$ banks. More specifically, we will consider window guidance to set the amount of one bank’s loans $F_1$ equal to $F_1^*$.\footnote{Although the exact meaning of window guidance is $F_1 \leq F_1^*$, and not $F_1 = F_1^*$, we make this simpler assumption to highlight the effects of window guidance.}

\[ F_1 = F_1^* \] (15)

Equilibrium conditions in the money market will then be obtained by replacing equation (9), $i_p - i_w = \partial E_1 / \partial F_1$, with equation (15), $F_1 = F_1^*$, for the profit maximization of that bank. Consequently, even with the introduction of window guidance, the number of variables in the system of equations is still one greater than the number of equations.

The conclusion that there is one more variable than the number of equations, both before and after the introduction of window guidance, is a proposition that holds independently of the number of banks that are subject to window guidance. Consequently, although window guidance affects the shares of individual banks in the loan market, it does not affect the overall conditions of the money market, which depends on whether the supply of high-powered money ($Z$) or the interbank credit interest rate ($i_w$) is treated as an exogenous variable. This is the reason why window guidance is often called “an insufficient or complementary monetary policy instrument.”

Of course, this problem of window guidance does not imply that window guidance as an additional constraint does not affect market equilibrium. Rather, if such a constraint is added to a competitive market, market equilibrium will be affected in the way as some other market inefficiency is introduced. In particular window guidance could raise the spread between the lending rate (the sale price of banking services) and the interbank interest rate (the purchase price of raw materials).\footnote{Strictly speaking, this could be better stated as follows. Let us posit the condition that will leave the amount of loans and the amount of deposits unchanged, when window guidance is introduced in a money market with initial equilibrium to reduce the amount of loans of the first bank from $F_1$ to $F_1^*$. If the aggregate amount of loans from the first bank through the $n$th bank ($\Sigma F_k$) is to be unchanged by this kind of window guidance, the amounts of loans extended by $(n-1)$ banks, numbered 2 through $n$, must increase. Thus, these banks’ marginal cost of extending loans ($E_k / \partial F_k$, $k=2, \ldots n$) must also increase. If $\Sigma F_k$ is unchanged, however, $i_p$ must also be unchanged, so the increased marginal cost of loans will reduce $i_w$. Conversely, if $i_w$ is not reduced, $i_p$ must increase and $\Sigma F_k$ must decrease.} Thus, the question of window guidance boils down to whether such a function should be considered as an appropriate effect of window guidance.

We can further argue about the effectiveness of window guidance if it is possible to implement window guidance with respect to all banks. In this type of window guidance, $n$ equations (9) will simply be replaced by $n$ equations (15), with no change in the number of variables and the number of equations. Because $F_1$, $\ldots$, $F_n$ would predetermined the
total amount of loans $F = \Sigma F_k$ and the lending rate $i_f$ (equation (11)), manipulating any other policy instruments (e.g. the interbank interest rate $i_w$) in these conditions would have little overall effect in the money market. In other words, if effective window guidance could be implemented with respect to all banks, it would completely determine the overall equilibrium conditions of the money market in roughly the same sense as the previous conclusion that “the interbank interest rate determines money market equilibrium.” A more fundamental question concerning the effectiveness of window guidance is, then, whether it is realistic to assume that window guidance can be implemented with respect to all banks.

V. Conclusion: The Implications of Deposit Rate Deregulation

The analysis of this paper shows that money market equilibrium can be effectively controlled through either the deposit rate $i_d$ or the interbank interest rate $i_w$. So long as Japan maintains both deposit rate regulation and interbank interest rate control, the problem of deposit rate regulation would arise due to the overdetermination of market equilibrium by multiple policy variables.

Figure 4 depicts the situation where the central bank controls interbank rate $i_w$ while regulating deposit rate $i_d$. Figure 4a shows, by superscripts * and **, two situations that could result from changing the interbank rate while the deposit rate is held constant; similarly, Figure 4b indicates two situations that could result from changing the deposit rate when the interbank rate is held constant.
As is evident from the figures, it is the interbank rate that determines the lending rate and money supply \((M+D=B=F)\) — the macroeconomic variables that are important to monetary policy. On the other hand, the main effect of the deposit rate is on the distribution of income between the banking sector and the non-bank private sector. Of course, the deposit rate may indirectly influence the money market if a choice of the deposit rate affects the market expectations about the interbank rate. However, the figures suggest that the direct control of the interbank rate would be a more effective means to influence the market.

What is the basic problem of the coexistence of both deposit rate regulation and interbank interest rate control? A typical answer is that it would distort the allocation of resources in the economy by creating excess profits in the banking sector, or that it would inhibit the efficient operation of the banking industry through some secondary regulations that would restrict competition. On these issues, our analysis suggests the following:

(1) If excess profits are defined as "profits in excess of the profits under marginal cost pricing," they will depend on the specific levels of the deposit rate and the interbank rate. Consequently, the coexistence of deposit rate regulation and interbank rate control does not by itself imply the existence of excess profits. Nevertheless, the central bank's control of the interbank rate would not have been possible to maintain unless it led to pricing above the marginal cost. Moreover, Japanese banks have consistently sought quantitative expansion during the post-war period. In view of this, it may be safe to say that these banks have probably enjoyed excess profits.

(2) On the other hand, the problem of secondary regulations can be discussed in terms of money market equilibrium conditions. That is, if deposit rate regulation and interbank rate control coexist, equation (10) will drop from the equilibrium conditions. Thus, unless additional constraints are substituted for equation (10), non-price competition could easily arise.\(^{11}\) Under this situation, consensus could easily emerge on the introduction of regulation in the money market to moderate non-price competition, or even directly on the shares of banks in the deposit market: Such anticompetition measures include restrictions on the number of branches and regulations on advertising. Thus, we may conclude that interest rate regulation has provided the rationale for the introduction of secondary regulations and anticompetitive measures.

These considerations naturally suggest what kind of benefits the deregulation of deposit rates would bring. The problems of deposit rate regulation basically arise from the fact that money market equilibrium is overdetermined in the presence of deposit rate regulation; that is, there are two exogenous policy variables when the number of vari-

\(^{11}\)However, it is not obvious that all non-price competition under regulated interest rates was undesirable, because such it may have contributed to improving the quality of banking services. On this point, see Nambu (1978).
bles in the system is only one greater than the number of equations. As a result of this overdetermination, inefficiency would emerge in the market. Therefore, deposit rate regulation needs to be abolished in order to restore efficiency and equity in the market. The remaining question is whether the deregulation of deposit rates would be sufficient to restore efficiency and equity in the market.

There are at least two issues that need be addressed in this respect. First, there is a question of whether the marginal cost principle, which should prevail in the money market under deregulated deposit rates, would function appropriately and thus strengthen bank management. Although this paper suggests one possible solution, it also shows that the resulting market equilibrium may not necessarily guarantee the profitability of individual banks’ deposit taking activities. To answer this question, we need further theoretical and empirical analyses on the cost function of the bank’s deposit taking activities.

Second, we need to recognize that deposit rate deregulation will not automatically bring about marginal cost pricing. If, for example, secondary regulations remain in place even after deposit rate deregulation, market equilibrium will be distorted. There is also a possibility that bank management will continue to pursue quantitative expansion, at least in the short run, even after the deregulation of deposit rates, leading to non-marginal cost pricing. This, as is often the case, abolishing distortionary regulations is a necessary condition for an equitable and efficient market, but it is not by itself a sufficient condition.

Appendix. The Reserve Requirement System and the Theory of Multiple Credit Creation

There has been much discussion on whether the central bank can control the amount of credit creation in the banking sector through quantitative control over reserves. To clarify the issues involved it is necessary to describe an explicit institutional setting that allows control over credit creation. Using a network graph to describe the mechanism of

\[ U_k = U_k \{P_k, R_k, D_k, W_k, X_k\}, D_k \]

the market equilibrium conditions will be given by the system of equations (8) through (14), except that equation (10),

\[ (1 - r) i_w - i_d = \frac{\partial E_k}{\partial D_k} \]

is replaced by,

\[ (1 - r) i_w - i_d = \frac{\partial E_k}{\partial D_k} - \frac{\partial U_k}{\partial D_k} \cdot \frac{\partial D_k}{\partial P_k} \]

Consequently, if bank managers have a propensity for quantitative expansion, insufficient profits may result, corresponding to the second term of the right side (the marginal rate of substitution between deposits and profits).
credit creation, we will examine whether the multiplier mechanism, which is implicit in
the theory of multiple credit creation, depends on the institution or the structure of the
credit creation process in the banking sector.

The network graph is the Petri-Net system of description and analysis, which is based
on the dissertation of C. A. Petri submitted in 1962. It is frequently used in computer
engineering to analyze the characteristics of a system made up of several parallel pro-
cesses. It is well suited to providing a concise answer to such questions as “whether and
under what conditions a process (provision of reserves) can control a parallel process
(credit creation)”.

Let me briefly explain the structure of the Petri-Net system:
(1) Place: a “place” that expresses the condition of the system;
(2) Token: a “counter” that expresses the state of the place;
(3) Transition: an “activity” that changes the number of tokens in the place;
(4) Arc: an “arrow” that defines the relationship between the place and the transition;
the transition takes tokens out of the origin of the arrow, and puts tokens into the
destination of the arrow, with the number of tokens equal to the predetermined
“multiplicity” of each arc.

These are all the components of the Petri-Net system. For a simplified example,
think of a factory that produces wooden toys, as in Figure A-1. Here, the three places
correspond to the inventory of wood as a raw material \( P_1 \), the inventory of paint as a
raw material \( P_2 \), and the inventory of finished products \( P_3 \). The only transition is \( T_1 \),
which depicts the manufacturing process. (To simplify presentation, a place is denoted by
a single line box, and a transition by a double line box.)

Assume that it takes 3 units of wood and 2 units of paint to produce 1 unit of toys,
with each of these numbers (written in parentheses next to each arc) the multiplicity of
the arc. Thus, the multiplicity of \( P_1 \rightarrow T_1 \) is 3, that of \( P_2 \rightarrow T_1 \) is 2, and that of \( T_1 \rightarrow P_3 \) is 1.
In this case, \( T_1 \) continues to feed tokens into \( P_3 \) as long as tokens are left in \( P_1 \) and \( P_2 \). If
we assume that the initial number of tokens in \( P_1 \) is \( N_1 \) and that in \( P_2 \) it is \( N_2 \), the level of
activity of \( T_1 \) is limited to the smaller of \( (N_1/3) \) and \( (N_2/2) \).

Therefore, if the factory has a parent company that is the sole supplier of paint, the
parent company can control the activity level by controlling the supply of paint without
visiting the factory floor. Clearly, the transition at the destination of the arrow can be
controlled by the number of tokens at the origin of the arrow, but the reverse does not
hold. This relationship is not altered even if places and transitions are connected in a
multiplying mode. We can use this framework to express the relationship between the
reserve requirement system and the process of credit creation.

13 Along with offering a graphic presentation, the Petri-Net is capable of multiple set presentation, and can be
used to examine the controllability of a complex system or the compatibility of seemingly different systems. The
use of the Petri-Net here is only a very simplified application. Because it has been modified to fit the analysis at
hand, those who are interested in the method itself should refer to the textbooks listed in the bibliography.
Figure A-1

Figure A-2 depicts the relationship between reserve holding behavior and credit creation under the reserve requirement system with "lagged" reserve accumulation. For simplicity, it is assumed that there are only two banks named A and B, that the loans of each (its credit creation) are all settled through respective deposit accounts, and that no cash is withdrawn from or deposited to accounts. Because the question here is whether the central bank can control the credit creation of the banks \((T_A-1, T_B-1)\) through its control over the supply of reserves (the number of tokens in \(P_x\)), the number of tokens in \(P_0\) is set at infinity, with the assumption that each bank's behavior reflects "infinite" demand for funds.\(^{14}\) By defining \(n\) as the inverse of the reserve ratio, Bank A feeds \(n\) tokens into \(P_{A-1}\), \(n\) tokens into \(P_{A-2}\), and one token into \(P_{A-3}\) (as a result of feeding \(n\) tokens into \(P_{A-2}\)), each time it extends a loan of \(n\) yen. Thus, the behavior of credit creation, settlement and reserve holding can be expressed as in Figure A-2. For simplicity, however, the arcs here are described only on the left side of the graph (corresponding principally to the transitions of Bank A), and are omitted on the right side. Moreover, note that the multiplicity of an arc expressed in parentheses is meaningful only in their

\(^{14}\) In practice, the demand for funds is subject to the influence of the interest rate, so that the initial number of tokens in \(P_0\) becomes \(B (i_f)\) according to the notation of the text. However, because the question here is whether the "multiplier constraint" is binding in the absence of an interest rate based adjustment mechanism (i.e. whether it is binding even if \(i_f\) is zero), for simplicity we set the initial number of tokens in \(P_0\) at infinity. If \(i_f\) is given exogenously, the initial number of tokens in \(P_0\) \((B (i_f))\) will be finite and control the activity level of the system. This is the situation when monetary policy works through control over interest rates.
relative relationship in achieving a transition. Its absolute level has no significance in this analysis, to the extent that the transitions can be activated many times as long as tokens are left in the place.

Looked at in this way, it is easy to see why there is no mechanism for multiple credit creation in a reserve requirement system with lagged reserve accumulation. As long as one follows the direction of the arc, it is impossible to reach the “transition” of credit creation \((T_A-1, T_B-1)\) from the supply of reserves \((P_A)\) because the “places” along the way \((P_A-3, P_B-3)\) are lagged by one period. The concept of contemporaneous reserve accumulation can be thought of as a way of correcting this problem by eliminating the time lag in the “places” along the way.

In fact, however, the simple elimination of the time lag does not solve the problem by itself. This is explained in Figure A-3. Contemporaneous accumulation of reserves cannot result in a mechanism for multiple credit creation, as long as banks maintain their current practice of allowing their branches or divisions to make loan decisions independently of their reserve holding behavior.

Can “contemporaneous accumulation” of reserves generate a mechanism of multiple credit creation? One possibility is depicted in Figure A-4. Here, to limit the “transition” of credit creation by the amount of reserves held, the definition of “place” has been changed to make the arc extend to the “transition” of credit creation. The network is constructed by defining \(P_A-3\) as

\[
\begin{align*}
\text{(Excess reserves)} &= \text{(Amount of reserves held)} - \text{(Required reserves)}
\end{align*}
\]

This suggests that, if banks follow the practice of constantly monitoring their excess reserves and create credit while controlling the available amount of reserves (the number of tokens), a multiplier based mechanism of credit creation could function. The practical problem with this system is the difficulty that banks would have in controlling their settlement function in the same area \((P_A-3)\) as the buffer needed for controlling credit creation. Banks cannot refuse to perform settlement just because excess reserves are unavailable. If this system is to be implemented, therefore, the bank’s settlement behavior must change radically from the current practices. Thus, even if it is possible, the cost of adopting such a system would be considerable.\(^{15}\)

We can also conceive of a reserve requirement system with advance reserve accumulation in which the central bank allows a bank to extend credit on the basis of the actual amount of reserves it held during the previous period. It is as if the present system were reversed. If this system were based on a typical reserve requirement ratio, there

\(^{15}\text{This problem will be minimized if the system encourages banks to hold excess reserves. Although the U.S. system, in which excess reserves can be carried over, can be deemed an example, it does not seem to be working adequately as far as we can judge from the movements of the Federal Funds rate at the end of reserve maintenance periods.}\)
would be many problems, as shown in Figure A-5. As in the system depicted in Figure A-
4, the bank must control its settlement in the same area \( P_{A-3} \) as the buffer for control-
ling credit creation, when it cannot reasonably refuse to perform settlement just because
excess reserves are unavailable. When we recognize the current impossibility of changing
the number of tokens in the \( P_{A-3} \) buffer, we must conclude that such a system is
dangerous.

To correct this problem, we can conceive of a system with reserves accumulated in
advance as a fixed percentage of future loans, as depicted in Figure A-6. With this
modification, it is logically possible to implement a system with "the aggregate amount of
credit creation mechanically controlled as a certain multiple of the reserve supply," as
assumed in the theory of multiple credit creation. However, there may be a sense of
uneasiness arising from the historical awareness that the reserve requirement system
originated as a way to maintain reserves against unforeseen payment. There may also be
such problems as the difficulty in determining the accounts to which this system should be
applied as well as a possible increase in the volatility of market interest rates. In the face
of these problems, there could be little argument for introducing such a system.\(^{16}\)
Although it can provide an interesting theoretical insight into the reserve requirement
system, it probably cannot go beyond the realm of thought experiments.\(^{17}\)

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\(^{16}\) If this system were to be introduced, we must be able to say that it is a better tool of monetary policy than
any other (in the sense that it overcomes these problems). If the central bank can effectively control interbank
interest rates, however, it would be difficult to argue that the control of credit creation based on such a
mechanical reserve requirement system is a better tool of monetary policy than interest rate control. We can
make a similar argument for the reserve requirement system with contemporary reserve accumulation, as
shown in Figure A-4.

\(^{17}\) A comparison of Figure A-5 and Figure A-6 also suggests that, the amount of required reserves should be
calculated against "loans" rather than against "deposits," if the reserve requirement system were to serve as a
multiplier constraint on the aggregate amount of credit creation, as postulated in the theory of credit creation.
This is apparent if we think of the credit control implicit in the theory of multiple credit creation as an effort to
constraint the process of credit creation. Thus, the system must function better when reserve requirements are
imposed on loans, which are the "process" of credit creation, rather than on deposits, which are the "result" of
credit creation.
Assessment: Because there is not an arc from $P_n$ to $T_{n-1}$, the state of activity in $T_{n-1}$ cannot be controlled by the number of tokens in $P_n$. With this structure, the mechanism implicit in the theory of multiple credit creation does not function as a binding constraint.
Figure A-3  Simple Contemporaneous Reserve Accumulation

\[
\begin{align*}
  P_{n-1} & \quad \text{Outstanding Loans of Bank A} \\
  P_{n} & \quad \text{Demand for Funds} \\
  \text{Number of Initial Token} = \infty \\
  T_{n-1} & \quad \text{Loan Production of Bank A} \\
  T_{n-2} & \quad \text{Loan Collection of Bank A} \\
  T_{n-3} & \quad \text{Reserve Requirement of Bank A for Current Period} \\
  T_{n-4} & \quad \text{Reserve Holding of Bank A for Current Period} \\
  T_{n-5} & \quad \text{Bank A Borrowing from the Central Bank} \\
  T_{n} & \quad \text{Settlement from Bank A to Bank B} \\
  T_{n-1} & \quad \text{Loan Production of Bank B} \\
  T_{n-2} & \quad \text{Loan Collection of Bank B} \\
  T_{n-3} & \quad \text{Reserve Requirement of Bank B for Current Period} \\
  T_{n-4} & \quad \text{Reserve Holding of Bank B for Current Period} \\
  T_{n-5} & \quad \text{Bank B Borrowing from the Central Bank} \\
  T_{n} & \quad \text{Interbank Loan from Bank A to Bank B} \\
  P_{n} & \quad \text{Supply of Reserves} \\
  \text{Number of Initial Token} = N
\end{align*}
\]

Assessment: Because there is not an arc from \( P_{n} \) to \( T_{n-1} \), the state of activity in \( T_{n-1} \) cannot be controlled by the number of tokens in \( P_{n} \). With this structure, the mechanism implicit in the theory of multiple credit creation does not function as a binding constraint, as in the reserve requirement system with lagged reserve accumulation.
Figure A-4  Contemporaneous Reserve Accumulation Based on Excess Reserves

Assessment:

1. Because there is an arc from $P_x$ to $T_{A-1}$ (i.e., $P_x \rightarrow T_A-5 \rightarrow P_A-3 \rightarrow T_A-1$), the state of activity in $T_A-1$ can be controlled by the number of tokens in $P_x$. With this structure, the mechanism implicit in the theory of multiple credit creation functions as a binding constraint.

2. However, the number of tokens in $P_A-3$, which functions as a buffer to control $T_A-1$, is difficult for Bank A to monitor, because it is also subject to the influence of transitions ($T_A-3$ and $T_B-3$) that are difficult to control.
Assessment:

1. Because there is an arc from the previous period to the current period, the state of activity in current Tn-1 (or next Tn-1) can be controlled by the number of tokens in previous Pn (or current Pn). That is, the mechanism implicit in the theory of multiple credit creation functions as a binding constraint across time periods.

2. However, this is a "dangerous" structure because the transitions that are difficult for Bank A to control (Tn-3 and Tn-3) are constrained by the number of tokens in Pa-3, which is predetermined in the previous period.
Figure A-6  Advance Reserve Accumulation Based on Future Loans

1. Because there is an arc from the previous period to the current period, the state of activity in current $T_{n-1}$ (or next $T_{n-1}$) can be controlled by the number of tokens in previous $P_n$ (or current $P_n$). That is, the mechanism implicit in the theory of multiple credit creation functions as a binding constraint across time periods.
2. This is a safer structure than the system of advance reserve accumulation based on future deposits because $P_{n-3}$, which is a buffer to control $T_{n-1}$, is independent of transitions that are difficult to control ($T_{n-3}$ and $T_{n-5}$).
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