Expected Inflation Rates and the Term Structure of Interest Rates

A Theoretical Model and Empirical Analysis Using Yields on Japanese Government Bonds in the Secondary Market

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I. Summary

This paper presents a model of the relationship between the term structure of interest rates and the expected inflation rate, and also an empirical analysis using data on yields on Japanese government bonds in the secondary market.

Concerning the mechanism for determining the yields on Japanese government bonds in the secondary market, Kuroda and Ōkubo [2], [3] have shown that the pure expectations theory satisfactorily explains the determination of yields in the negotiable government bond market and that the expected inflation rate is significantly reflected in the nominal interest rate (the "Fisher Effect"). Empirical investigation of (i) the "Liquidity Premium Hypothesis" (Hicks [12]), (ii) the "Preferred Habitat Hypothesis", and (iii) the "Coupon Oriented Hypothesis" (Kuroda and Okubo [2]), all of which emphasize the segmentation of markets, was conducted. Results indicated that (i) and (ii) could clearly be rejected while (iii) had some explanatory power.

This paper builds upon empirical results of Kuroda–Ōkubo [2], [3]. Here, I first present a model which explicitly incorporates the "Fisher Effect" in relation

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to the term structure of interest rates. Forecast values of the "time series model" are used as expected rates of inflation, and these values, in turn, are used to estimate the magnitude of the "Fisher Effect" by term to maturity of government bonds. Next, the three hypotheses listed previously which emphasize the existence of market segmentation, are investigated for significance in explaining the yield on Japanese government bonds in the secondary market on a real interest rate basis after adjusting for the expected inflation rate.

A summary of the empirical results is as follows.

1) The magnitude of the "Fisher Effect" for the market yield on Japanese government bonds in the secondary market averaged about 0.3–0.4 for the sample period extending from June 1977 to December 1980 (quarterly data was used). In other words, expected inflation rates are not totally reflected in the yields on Japanese government bonds. Rather, only 30 – 40% of expected inflation rates are reflected in these yields.

2) Looking at government bonds in the secondary market by term to maturity, the longer the term to maturity is, the larger the "Fisher Effect" is. In other words, the "Fisher Effect" is larger for long term interest rates than for short term interest rates.

3) The "Liquidity Premium Hypothesis", the "Preferred Habitat Hypothesis" and the "Coupon Oriented Hypothesis" are all rejected as explanations of the determination of yields on negotiable government bonds on a real interest rate basis.

To date, theory and empirical analyses of the term structure of interest rates have left vague the relationship between the nominal rate of interest and the expected inflation rate (for example, Fisher [10] and Yohe's and Karnosky's distributed lag model [22]). However, this paper, through the explicit modeling of this relationship, clarifies the magnitude of the "Fisher Effect" and the mechanism for determining the term structure of interest rates on a real interest rate basis.

It should be pointed out that two problems in the empirical analysis remain. The first is the question of how accurately expected inflation rates introduced through the "time series model" reflect people's "actual expectations" toward future inflation rates. Also, there is a question about the appropriateness of the "assumption of a constant equilibrium real short term interest rate" which was adopted to make the empirical analysis feasible. These two questions cast some doubt on the validity of the empirical results, and hence these results should be interpreted with great care.

II. A Model of Expected Inflation Rates and the Term Structure of Interest Rates.

A. Model under Conditions of Certainty

As a starting point for the analysis of expected inflation rates and the term structure of interest rates, a certain world fulfilling the following three assumptions is assumed.[1]

1) Investors have perfect knowledge of future short term real interest rates and inflation rates.
2) There is no risk of default for any bond in the market.
3) There are no obstacles to arbitrage between two different bonds.

In this certain world, in the current and each future period equilibrium is realized through the equality of the short term nominal interest rate (r_{t+j} ; $j = 0, 1, \ldots, n-1$) with the sum of the short term real interest rate (ρ_{t+j}) and the inflation rate (π_{t+j+1})[2]. In other words, the "Fisher Effect" is complete.

$$r_{t+j} = \rho_{t+j} + \pi_{t+j+1} \tag{2-1}$$

Also, the following equation based on the pure expectations theory relates the realized compound yield ($R_t^{(n)}$) in the present period (time t) of an n period long term bond to the present and future short term nominal interest rates (Assuming for the sake of convenience of calculation, discount bonds)[3].

1. See R. Shiller [20], page 19. In order to simplify the model below, transaction costs and taxes are assumed to be zero.
2. The relationship between the current (time t) nominal short term interest rate (r_t) and the bond price (v_t) is defined as follows

$$r_t = \frac{v_{t+1} - v_t}{v_t} \tag{A}$$

In comparison with this, the relationship between the inflation rate at time t (π_t) and the price level (p_t) is defined as follows.

$$\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}} \tag{B}$$

Consequently, the inflation rate corresponding to r_t is not the realized inflation rate from the last period to this period (π_t) but instead is the expected value of the inflation rate from this period to the next (π_{t+1}).

3. In the case of coupon bonds, the equation between the realized compound yields ($R_t^{(n)}$) and the present and future nominal short term interest rates is, instead of equation (2-3), now given by the following.

$$(1+R_t^{(n)})^n = (1+r_t)(1+r_{t+1}) \cdots (1+r_{t+n-1}) \tag{2-2}$$

Taking logs of both sides of equation (2-2), the following linear approximation is obtained.

$$R_t^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1} r_{t+j} \tag{2-3}$$

Now, substituting (2-1) into (2-3), equation (2-4) is obtained

$$R_t^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1}(\rho_{t+j}+\pi_{t+j+1}) = \frac{1}{n}\sum_{j=0}^{n-1}\rho_{t+j} + \frac{1}{n}\sum_{j=0}^{n-1}\pi_{t+j+1} \tag{2-4}$$

Hence, in a certain world, the return on an n period long term bond is equal to the sum of (i) the average value of the real short term interest rate from the present period (t) to n – 1 period in the future (period t + n – 1) plus (ii) the average value of the realized inflation rate from the next period (t + 1) to n period in the future (period t + n).

B. Model under Conditions of Uncertainty

In a real economy, the three conditions listed above of a certain world are not fulfilled. Equation (2-4), which shows the relationship between the return on long term bonds and the real short term interest rate and the inflation rate, needs to be revised in at least the following three ways.

1) It is necessary to incorporate expected inflation rates. In the model under certainty, future inflation rates π_{t+j+1} ; j = 1, . . . , n – 1 which are known need to be replaced by expected inflation rates (${}_{t+j+1}\hat{\pi}_t$; the expected value of inflation in the j-th period at time t). In other words, it is necessary to specify the formation process of future expected inflation rates.

$$R_t^{(n)} = \frac{nC+Mv}{A}\cdot r_t + \frac{(n-1)C+Mv}{A}\cdot r_{t+1} + \cdots + \frac{C+Mv}{A}\cdot r_{t+n-1}$$

[C : coupon rate
M_V: final period redemption value
A : $\frac{n(n+1)C}{2} + nM_v$]

In other words, in the case of coupon bonds, $R_t^{(n)}$ is equivalent to a weighted average of the present nominal interest rate and future rates to the period n - 1, with the weights decreasing as time passes (Kuroda and Ōkubo [3], page 10).
The difference in discount bonds and coupon bonds in the pure expectations theory formula is only found in the different pattern of weights. If this difference is ignored, the following discussion for discount bonds is directly applicable to coupon bonds.

Furthermore, in a model under uncertainty, it is difficult to decide how to treat the future real short term interest rates which were assumed known in the model under certainty. This paper will adopt for convenience the hypothesis of a constant equilibrium real short term interest rate which has traditionally been used in analyzing the relationship between the expected inflation rate and the nominal rate of interest. Concretely, this hypothesis is expressed in the equilibrium conditions given by equation (2-1) as

$$\rho_{t+j} = \rho \,(j=0, \cdots, n-1) \tag{2-5}$$

(For discussion concerning the hypothesis of a constant equilibrium real short term interest rate, see the Appendix.)

2) The second revision concerns the possibility of a partial "Fisher Effect". In the model under conditions of certainty, the average value of present and future inflation rates ($\frac{1}{n}\sum_{j=0}^{n-1} \pi_{t+j+1}$) is completely reflected in the yield on long term bonds. However, in the model under conditions of uncertainty, because of imperfect arbitrage between financial assets and real assets, etc., there is the possibility that even if expected inflation rates are correctly anticipated, these rates will not be completely reflected in the yield on long term bonds. Consequently, it is necessary to formulate a model which incorporates the possibility of a partial "Fisher Effect."

3) Thirdly, the possible inadequacy of the pure expectations theory must be considered. In the model of a certain world, the pure expectations theory formula (2-3) holds between the yield on long term bonds and the present and future short term nominal interest rates. However, in the model of an uncertain world, it is necessary to consider the possibility of the yield on long term bonds being influenced by factors other than expectations concerning future short term interest rates.[4]

The Formulation of Expected Inflation Rates

Concerning the formation of "expectations" of future inflation rates, consider a general autoregressive model. In other words, assume these expectations are "the optimal forecasts of a linear model in which the inflation rate between this period

4. As factors other than "expectations" which influence the determination of the yield on long term bonds, there are the "Liquidity Premium Hypothesis" of Hicks [12], the "Preferred Habitat Hypothesis" of Modigliani and Sutch [18], and the "Coupon Oriented Hypothesis" popular in Japan. (See Kuroda and Ōkubo [2] for reference.)

and the next (${}_{t+1}\hat{\pi}_t$) is based on the actual values of past inflation rates.[5]"

$$ {}_{t+1}\hat{\pi}_t = \sum_{i=0}^{m} \lambda_i^{(0)} \cdot \pi_{t-i} + C \qquad (2\text{-}6) $$

($\lambda_i^{(j)}$ here and below is the lagged weight which is assigned to the actual past inflation rate in the formation of the expected inflation rate between the j-th period and the (j + 1)-th period. C is a constant.)

Next, in regards to expectations of the inflation rate after the next period, the chain rule[6] can be applied. Using the actual values of past inflation rates and the next period's inflation rate (${}_{t+1}\hat{\pi}_t$) from equation (2-6), the following is obtained

$$ {}_{t+2}\hat{\pi}_t = \lambda_0^{(0)} \cdot {}_{t+1}\hat{\pi}_t + \sum_{i=1}^{m} \lambda_i^{(0)} \cdot \pi_{t-i+1} + C \qquad (2\text{-}7) $$

$$ = \sum_{i=0}^{m} \lambda_i^{(1)} \cdot \pi_{t-i} + C \qquad (2\text{-}7)' $$

Through repetition of this same procedure, the expected inflation rate from period j to period j + 1 is given by

$$ {}_{t+j+1}\hat{\pi}_t = \sum_{i=0}^{m} \lambda_i^{(j)} \cdot \pi_{t-i} + C \qquad (2\text{-}8) $$

By substituting equation (2-8), which gives a generalized form for expected inflation rates into equation (2-4) which was derived from the model under conditions of certainty, the yield on an n period long term bond is expressed as the equilibrium short term interest rate and actual values of past inflation rates (A constant equilibrium real short term interest rate is assumed).

$$ R_t^{(n)} = \rho + \frac{1}{n}\sum_{j=0}^{n-1} {}_{t+j+1}\hat{\pi}_t = \rho + \frac{1}{n}\sum_{j=0}^{n-1}\left[\sum_{i=0}^{m} \lambda_i^{(j)} \cdot \pi_{t-i} + C\right] \qquad (2\text{-}9) $$

$$ = \rho + \left[C + \frac{1}{n}\sum_{j=0}^{n-1}\sum_{i=0}^{m} \lambda_i^{(j)} \cdot \pi_{t-i}\right] $$

$$ = \rho + \left[C + \sum_{i=0}^{m} \lambda_i^{(n)} \cdot \pi_{t-i}\right] (2-9)' \qquad \left\{ \text{where } \lambda_i^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1} \lambda_i^{(j)} \right\} \qquad (2\text{-}9)' $$

The Formulation of a Partial "Fisher Effect"

To what extent the expected inflation rate is reflected in the yield on long

5. For comments on linear optimal forecasts in general autoregressive models, see R. Shiller [20].
6. See Kuroda and Ōkubo [2] concerning the chain rule.

term bonds depends on the degree of arbitrage between these bonds and real assets (or financial assets in which price rises are perfectly indexed). Here, let the extent to which the expected inflation rate is reflected in the yield on long term bonds be expressed generally by $\theta_t^{(n)}(0 \leq \theta_t^{(n)} \leq 1)$. If $\theta_t^{(n)} = 1$, the "Fisher Effect" is complete and if $\theta_t^{(n)} = 0$, the "Fisher Effect" is also zero and complete money illusion exists.

If the "Fisher Effect" is partial, equation (2-9) is transformed in the following fashion.[7]

$$R_t^{(n)} = \rho + \theta_t^{(n)} \left[\frac{1}{n} \sum_{j=0}^{n-1} {}_{t+j+1}\hat{\pi}_t \right] \tag{2-10}$$

$$= \rho + \left[\theta_t^{(n)} \cdot C + \theta_t^{(n)} \cdot \sum_{i=0}^{m} \lambda_i^{(n)} \cdot \pi_{t-i} \right] \tag{2-10)'$$

The Formulation When the Pure Expectations Theory Does Not Hold

In order to guarantee the realization of the pure expectations theory formula (2-3) between the return on long term bonds and the present and future nominal short term interest rates, there must be perfect reciprocal arbitrage between financial assets (bonds) of differing terms to maturity. However, in the real economy, this arbitrage is not always perfect, and consequently it is necessary to consider the possibility of the pure expectations theory not holding. In other words, the yield on long term bonds is influenced by other factors aside from

7. Concerning the specification of the "Fisher Effect", it is possible to consider to what extent the expected inflation rate is reflected in the nominal interest rate in each individual time period. In other words, formulate this as

$${}_{t+j}\hat{r}_t = \rho + \theta_t^{(j)} \cdot {}_{t+j+1}\hat{\pi}_t \tag{A}$$

[$\theta_t^{(j)}$ is the magnitude of the "Fisher Effect" in the j-th period fron now. $0 \leq \theta_t^{(j)} \leq 1$]

The forward short term interest rate is calculated as follows from this long term yield used in place of the expected value of the short term interest rate in the j-th period from now.

$${}_{t+j}i_t = \frac{(1 + R_t^{(j+1)})^{j+1}}{(1 + R_t^{(j)})^j} - 1 \tag{B}$$

Through use of this forward interest rate, it is possible to test the "Fisher Effect" for each period. However, there is the problem that the calculation of the forward short term interest rate in equation (B) is meaningless unless limited to the "case of groups of common bonds with yields determined by exactly the same factors, but with possibly different terms to maturity." Because of this problem, this specification was not adopted. (concerning this point, see Kuroda and Okubo [3].)

expectations of future nominal interest rates as assumed by the pure expectations theory. Consequently, equation (2-3) becomes

$$R_t^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} {}_{t+j}\hat{r}_t + XR_t^{(n)} \tag{2-11}$$

($XR_t^{(n)}$ represents factors other than expectations affecting the yield on long term bonds.)

Revising equation (2-10) like equation (2-11), the following is obtained.

$$R_t^{(n)} = \rho + \theta_t^{(n)} \cdot \left[\frac{1}{n} \sum_{j=0}^{n-1} {}_{t+j+1}\hat{\pi}_t \right] + XR_t^{(n)} \tag{2-12}$$

$$= \rho + \left[\theta_t^{(n)} \cdot C + \theta_t^{(n)} \cdot \sum_{i=0}^{m} \lambda_i^{(n)} \cdot \pi_{t-i} \right] + XR_t^{(n)} \tag{2-12'}$$

Equation (2-12)′ is the final form of the model under conditions of uncertainty. In the next section, problems concerning the theory and empirical analyses to date of the relationship between the expected inflation rate and the nominal interest rate will be examined in comparison to the model under uncertainty presented above.

C. A Critical Examination of Distributed Lag Models

The traditional method of analysis of the relationship between nominal interest rates and expected rates of inflation is the distributed lag model which was first developed by I. Fisher [10] and later revised by D. Meiselman [16], T. Sargent [19], W.P. Yohe and D.S. Karnosky [22], and others.

The fundamental concepts of these distributed lag models can be explained in the following two points.

1) The nominal interest rate is composed of the real interest rate (assumed constant) and expected inflation rates (with either all or part of these expected inflation rates being reflected in the nominal interest rate).

2) The expected inflation rate is constructed by means of an autoregressive model of past realized values of the inflation rate.

Based on these two points, the nominal interest rate is expressed as a constant plus a distributed lag of past realized values of the inflation rate. In other words, the distributed lag model is expressed generally as

$$R_t = \alpha + \sum_{i=0}^{m} w_i \cdot \pi_{t-i} \tag{2-13}$$

assuming R_t is the nominal interest rate and π_t is the inflation rate.

Comparing this distributed lag model with the model under conditions of uncertainty presented in section B, several problems with the former become apparent. Rewriting (2-12)′ in a form corresponding to (2-13),

$$R_t^{(n)} = (\rho + \theta_t^{(n)} \cdot C + XR_t^{(n)}) + \theta_t^{(n)} \cdot \sum_{i=0}^{m} \lambda_i^{(n)} \cdot \pi_{t-i} \tag{2-14}$$

is obtained. By contrasting (2-13) with (2-14), problems concerning the concepts and interpretation of the traditional distributed lag model will be pointed out.

1) Misinterpretation of the Magnitude of the "Fisher Effect"

According to the interpretation of the traditional distributed lag model, the summed coefficient value ($\sum_{i=0}^{m} w_i$) on the distributed lag inflation term is thought to directly show the magnitude of the "Fisher Effect." However, as is clearly shown by equation (2-14), the summed coefficient value on the distributed lag inflation term does not directly give the degree of the "Fisher Effect." Only if $\sum_{i=0}^{m} \lambda_i^{(n)} = 1$, can this term $\theta_t^{(n)} \cdot \sum_{i=0}^{m} \lambda_i^{(n)} = \theta_t^{(n)}$ be interpreted as showing the magnitude of the "Fisher Effect."

Also, according to the interpretation of the distributed lag model, the coefficient of determination, R^2, of the regression is interpreted as showing how well changes in the nominal rate of interest are explained by changes in the expected inflation rate. However, the R^2 of the regression strictly depends on the sample used. Even if the R^2 is large, whether or not changes in the post sample expected inflation rate explain well changes in the post sample nominal interest rate is a totally different problem.[8]

2) The Problem of Viewing the Constant Term as the Real Rate of Interest

In the distributed lag model, the constant term (α) in equation (2-13) is frequently interpreted as the real interest rate. However, as is clearly shown in equation (2-14), the constant term of the regression consists of (i) the equilibrium value of the real short term interest rate (ρ), (ii) the term obtained by multiplying the degree of the "Fisher Effect" ($\theta_t^{(n)}$) by the constant term (C) from the estimating equation for the expected inflation rate, and (iii) factors other than "expectations" influencing the yield on long term bonds ($XR_t^{(n)}$). Because this constant term consists of three elements, it is a mistake to regard this term as being, in general, the real interest rate.

8. Similar comments can be made concerning the interest rate "term structure equation" which explains yields on long term bonds by a distributed lag of present and past short term interest rates (Kuroda and Ōkubo [3]).

3) Problems Concerning the Estimation of the Coefficients of the Distributed Lag Model

In estimating the distributed lag model, Fisher [10] used a linear lag while Yohe and Karnosky [22] used an Almon lag. However, in both of these cases, the equation is estimated under restrictions of an a priori, ad hoc lag pattern. In comparison with the general "time series model" now being widely used in contemporary economic analysis, these approaches do not use all available information in estimation and consequently the efficiency of the values of the estimated coefficients is not high.[9]

Moreover, because the distributed lag model is estimated under the assumptions that the real interest rate and factors other than expectations influencing the yield on bonds are both constant, it is highly probable that the estimated coefficients on the distributed lag inflation rate term will be biased if either of these assumptions is violated.

4) Deficiencies of the Theory of the Term Structure of Interest Rates

In empirical analyses based on the distributed lag model, not only short term interest rates but frequently long term interest rates are used as dependent variables. However, not enough attention is paid to the term structure of interest rates when specifying models, and how far into the future nominal interest rates are influenced by the corresponding expected inflation rate is not well explained. Consequently, differences in the lag pattern of inflation rates and the constant term arising from using long term versus short term interest rates as a dependent variable can only be interpreted in an ad hoc fashion.

In contrast to this, in equation (2-14), the relationship between the expected inflation rate and the term structure of interest rates is clearly formulated. Hence, this equation clarifies the dependence between term to maturity of the interest rate chosen as the dependent variable and the magnitude of the constant term coefficient.

III. Empirical Analysis Using Yields on Japanese Government Bonds in the Secondary Market

A. Method of Empirical Analysis

Here, empirical analysis is conducted based on the model presented in Part II, Section B using data on yields on Japanese government bonds in the secondary

9. See Oritani [1] for criticism of the methodology of estimating distributed lag models from the point of view of the general "time series model".

market. The purposes of this empirical analysis are twofold. The first purpose is to measure for the yield on such bonds the magnitude of the "Fisher Effect" by each term to maturity. The second purpose is to investigate, on a real interest rate basis after eliminating expected inflation rates, factors determining the term structure of yields on Japanese government bonds in the secondary market. First, however, the methodology used for the empirical analysis will be explained.

Method for Estimating the "Fisher Effect"

Estimation of the "Fisher Effect" in the yield on negotiable government bonds proceeded as follows, using quarterly data from June 1977 to December 1980 on realized compound yields in the government bond market.[10]

1) The wholesale price index was used for the inflation rate (seasonally adjusted six month changes at a yearly rate).[11] The WPI was assumed to be realized values of a stationary stochastic process of an ARMA model. For expected inflation rates, optimal linear forecasts of the ARMA model were used. Expected inflation rates were forecast for each quarter from June, 1977 to December, 1980, and for each point of forecast, the expected inflation rate was calculated up to twenty future terms (10 years) $({}_{t+j+1}\hat{\pi}_t : j = 0, \cdots, 19)$.[12]

2) The expected inflation rate ($\hat{\pi}_t^{(n)}$) was calculated using values forecast by the ARMA model corresponding to each quarterly market yield on government bonds ($R_t^{(n)}$). $\hat{\pi}_t^{(n)}$ was calculated as follows:[13]

$$\hat{\pi}_t^{(n)} = \frac{nC + Mv}{A} \cdot {}_{t+1}\hat{\pi}_t + \frac{(n-1)C + Mv}{A} \cdot {}_{t+2}\hat{\pi}_t + \cdots + \frac{C + Mv}{A} \cdot {}_{t+n}\hat{\pi}_t \qquad (3\text{-}1)$$

C: coupon rate $\quad M_V$: redemption value at the time of maturity

$$A = \frac{n(n+1)C}{2} + n \cdot M_V$$

3) Through the regression of the yield on government bonds ($R_t^{(n)}$) on the cor-

10. The data used in analyzing the yields on long term government bonds, and the period of time of the analysis is exactly the same as Kuroda and Ōkubo [3].

11. It is also possible to use the CPI as the inflation rate, but in reference to the empirical results of K. Kama [13], the WPI was chosen because it is thought that the "Fisher Effect" is relatively more significant when the WPI is used.

12. Concerning the methodology of estimating the ARMA model and the methodology of forecasting using the ARMA model, see Kuroda and Ōkubo [3].

13. See Part II (footnote 3).

responding expected inflation rates, the magnitude of the "Fisher Effect" was calculated.[14] In other words, a_1 in the following equation gives the magnitude of the "Fisher Effect."

$$R_t^{(n)} = a_0 + a_1 \hat{\pi}_t^{(n)} + u_t \tag{3-2}$$

[u_t: random error term]

Concerning the yield on government bonds and expected inflation rates, equation (3-2) was estimated using (i) time series data by term to maturity of government bonds, (ii) cross section data by period of estimation, and (iii) pooled cross section, time series data.[15]

In comparison to previous methods of estimating distributed lag models, analytical methods using the "time series model" provide a plain and simple method of estimating the "Fisher Effect." The "Fisher Effect" depends, more than anything else, on the calculation of expected inflation rates corresponding to the yields on long term government bonds. The coefficient on the expected inflation rate, a_1, in equation (3-2), measures only the "Fisher Effect" and avoids the problems of the aggregate coefficient on the distributed lag inflation rate term in the distributed lag models.

Methods Used to Investigate Factors Determining the Term Structure on a Real Interest Rate Basis

Next, the return on government bonds on a real interest rate basis ($RR_t^{(n)}$) was calculated by subtracting from these yields on government bonds ($R_t^{(n)}$) the corresponding expected inflation rates ($\hat{\pi}_t^{(n)}$). Then, factors determining the term structure of the yields on government bonds were investigated.

According to the model under conditions of uncertainty presented in Part II, Section B, the yield on government bonds ($R_t^{(n)}$) can be generally expressed as follows.

$$R_t^{(n)} = \rho + \theta_t^{(n)} \cdot \hat{\pi}_t^{(n)} + XR_t^{(n)} \tag{3-3}$$

14. Here the "Fisher Effect" is estimated assuming a "constant equilibrium short term real interest rate" and also assuming constant factors other than "expectations" which influence the yields.

15. Feldstein and Summers [9] and Oritani [1] have conducted similar analyses estimating the "Fisher Effect" using forecast values based on an ARMA model. However, in these papers, attention is not paid to the term structure of interest rates nor are factors other than "expectations" which determine the yields on bonds considered.

Consequently, on a real interest rate basis this becomes

$$RR_t^{(n)} = R_t^{(n)} - \hat{\pi}_t^{(n)} \tag{3-4}$$

$$= \rho - (1 - \theta_t^{(n)})\, \hat{\pi}_t^{(n)} + XR_t^{(n)} \tag{3-4)'$$

As factors other than expectations influencing the yield on government bonds, (i) the "Liquidity Premium Hypothesis" of Hicks [12], (ii) the "Preferred Habitat Hypothesis" of Modigliani and Sutch [18] and (iii) the "Coupon Oriented Hypothesis", which is widely popular in Japan, were investigated in the following forms.[16]

1) Investigation of the "Liquidity Premium Hypothesis"

$$RR_t^{(n)} = b_0 + b_1 \hat{\pi}_t^{(n)} + b_2 Y + u_{2t} \tag{3-5}$$

$$= b'_0 + b'_1 \hat{\pi}_t^{(n)} + b'_2 \ln Y + u'_{2t} \tag{3-5)'$$

[Y: term to maturity (year)
u_{2t}, u'_{2t} : random error terms]

These two equations were regressed and the coefficients b_2, b'_2 were checked for significance. According to the "Liquidity Premium Hypothesis", the longer the term to maturity is, the greater should be the yield on government bonds. Consequently, b_2 and b'_2 should be positive.

2) Investigation of the "Preferred Habitat Hypothesis"

$$RR_t^{(n)} = c_0 + c_1 \hat{\pi}_t^{(n)} + c_2 W + u_{3t} \tag{3-6}$$

[W : proportional composition of outstanding government bonds by coupon rate and by term to maturity.
u_{3t} : random error term]

This equation was regressed and the coefficient c_2 was checked for significance. By the "Preferred Habitat Hypothesis", the greater the proportion of outstanding government bonds (W) by coupon rate and by term to maturity is, the higher should be the yield on government bonds. Hence, c_2 is expected to be positive.

3) Investigation of the "Coupon Oriented Hypothesis"

16. Concerning the specification of each hypothesis for estimating purposes and the variables Y, W, and CP, see Kuroda and Ōkubo [2].

$$RR_t^{(n)} = d_0 + d_1 \hat{\pi}_t^{(n)} + d_2 CP + u_{4t} \tag{3-7}$$

[CP : coupon rate level
u_{4t} : random error term]

Again, this equation was regressed and the coefficient d_2 was checked for significance. Because the "Coupon Oriented Hypothesis" implies that the higher the coupon rate is, the lower should be the yield on government bonds, d_2 is expected to be negative.

Also the coefficients b_1, b'_1, c_1, and d_1 on the expected inflation rate terms in equations (3-5), (3-5)′, (3-6), and (3-7) reflect the magnitude of the "Fisher Effect" when factors other than "expectations" affecting the yield ($XR_t^{(n)}$) are not constant. If the magnitude of the "Fisher Effect" is θ, then coefficients b_1, b'_1, c_1, d_1 correspond to $\theta - 1$.

B. Results of Empirical Analysis

Calculation of the Expected Inflation Rate

Time series data on the wholesale price index (seasonally adjusted six month changes at a yearly rate) is presented in Graph 1.[17] The time series data on wholesale prices is considered to be realized values of a stationary stochastic process.[18] Under this assumption, the ARMA model was estimated and forecasts using this estimated model were made. Using as a starting point the third quarter of 1953, data until the second quarter of 1977 (sample = 96) were used for estimation. Forecasts of wholesale prices were then calculated from the second quarter of 1977 for each of the 20 future terms (in other words, 10 years future). Next, the final period was extended by 1 until the third quarter of 1977, and the model was again estimated along with calculations of extrapolated values for up to 20 future terms. This procedure was repeated in a similar fashion 15 times until the fourth quarter of 1980. Forecast values of wholesale prices necessary for the calculation of expected inflation rates corresponding to each market yield on government bonds

17. The time series data on wholesale prices actually used began with the third quarter of 1953, but Graph 1 shows wholesale price series data beginning with the first quarter of 1960.

18. When viewing the time series data as fulfilling a stationary stochastic process, there is a problem with the high inflationary period of 1973–74. The forecast results of the expected inflation rates are subject to this same problem and the empirical results of this paper must be interpreted with caution.

Graph 1. Wholesale Price Index (Seasonally adjusted six month changes at a yearly rate)

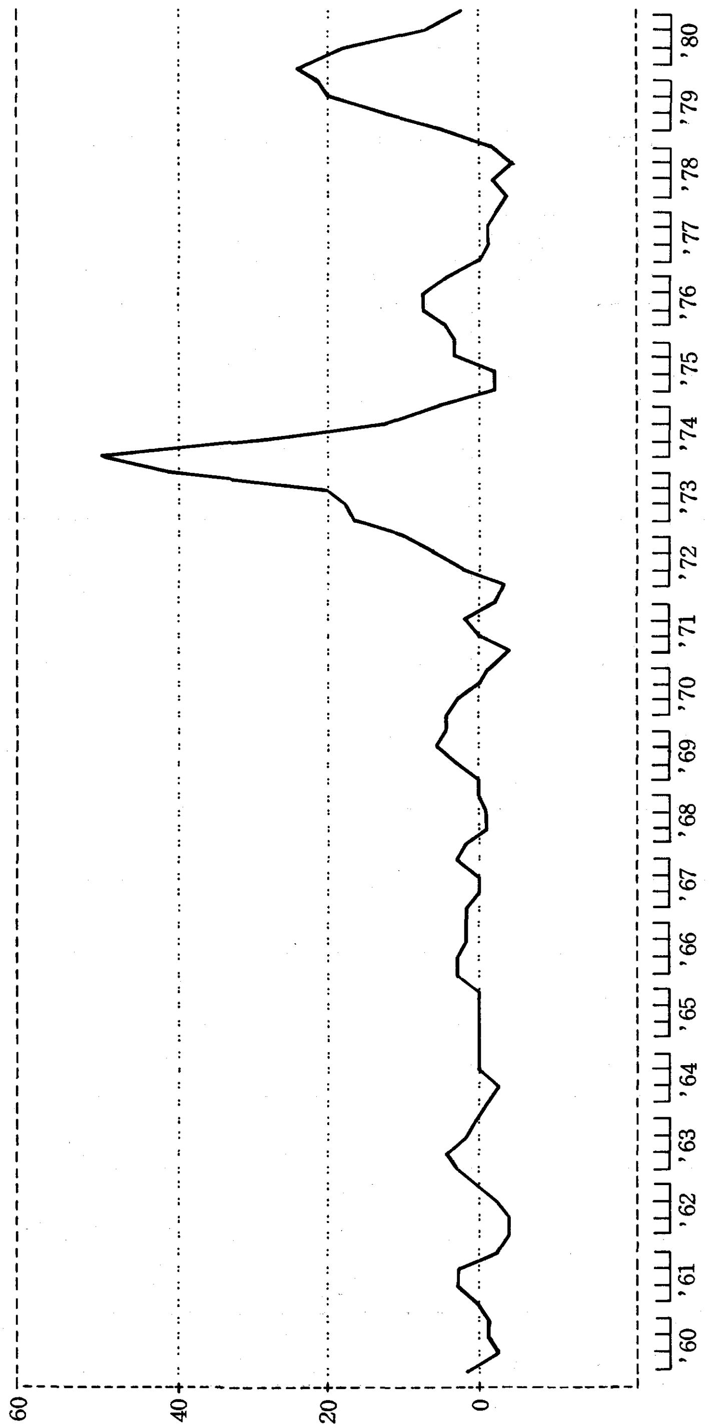


Table 1. ARMA Model Forecasts of Wholesale Price Index

$$\pi_t = \sum_{i=1}^{m} \alpha_i \cdot \pi_{t-i} + \sum_{i=1}^{n} \beta_i \cdot u_{t-i} + u_t$$

[π_t : Wholesale price index, seasonally adjusted six month changes at a yearly rate
u_t : Noise]

Period of estimation	Order of Lag		α_i (t satistic)		β_i (t statistic)	
	m	n	i = 1	i = 2	i = 1	i = 2
1953/III – 77/II	1	2	-0.5846 (-5.2675)		1.1344 (8.8142)	0.3164 (2.4999)
III	1	2	-0.5987 (-5.4586)		1.1037 (8.4965)	0.2890 (2.2628)
IV	1	2	-0.6052 (-5.4473)		1.0284 (7.7039)	0.2517 (1.9274)
78/I	1	2	-0.5963 (-5.3963)		1.0642 (8.1174)	0.2731 (2.1269)
II	1	2	-0.5924 (-5.4498)		1.0985 (8.6089)	0.2984 (2.3814)
III	1	1	-0.7260 (-10.1467)		0.7536 (11.0207)	
IV	1	2	-0.6159 (-5.8213)		1.0896 (8.4596)	0.2517 (1.9818)
79/I	1	2	-0.6183 (-5.7785)		1.0416 (7.9329)	0.2249 (1.7421)
II	1	2	-0.5891 (-5.0784)		0.8848 (6.4583)	0.2239 (1.7026)
III	1	1	-0.6892 (-8.8951)		0.6018 (7.0468)	
IV	1	1	-0.7022 (-9.4636)		0.6630 (8.5000)	
80/I	2	1	-0.9718 (-5.6108)	0.2589 (1.6242)	0.3589 (2.1211)	
II	2	1	-0.8718 (-7.7700)	0.1670 (1.5018)	0.7986 (11.4906)	
III	2	1	-0.9202 (-8.5759)	0.2064 (1.9343)	0.8189 (12.6985)	
IV	1	2	-0.6189 (-6.3153)		1.1067 (9.4187)	0.3097 (2.6767)

were thus obtained. Akaike's information criterion was used for the optimal identification of the AR part and the MA part in estimating the ARMA model. Results of estimating the ARMA model are presented in Table 1.

Using the forecast values of wholesale prices from the ARMA model expected inflation rates were calculated corresponding to the market yield on government bonds at each forecast point.[19] ($R_t^{(n)}$, realized compound yield, see Graph 2 for reference.)

Graph 3 shows expected inflation rates by forecast period and by term to maturity. As is clearly shown in this graph, the expected inflation rate fluctuates rather a lot. Consequently, it is easily recognized that the term structure of yields on government bonds on a nominal interest rate basis and the term structure of yields on a real interest basis greatly differ. Also, comparing fluctuations in the expected inflation rate according to term to maturity, when the term to maturity is short, fluctuations are extremely high, but when the term to maturity is long, the expected inflation rate is relatively stable. This can be viewed as showing that the formation of people's expectations for the distant future are based on the regular, heretofore experienced average rate of inflation.

The Magnitude of the "Fisher Effect"

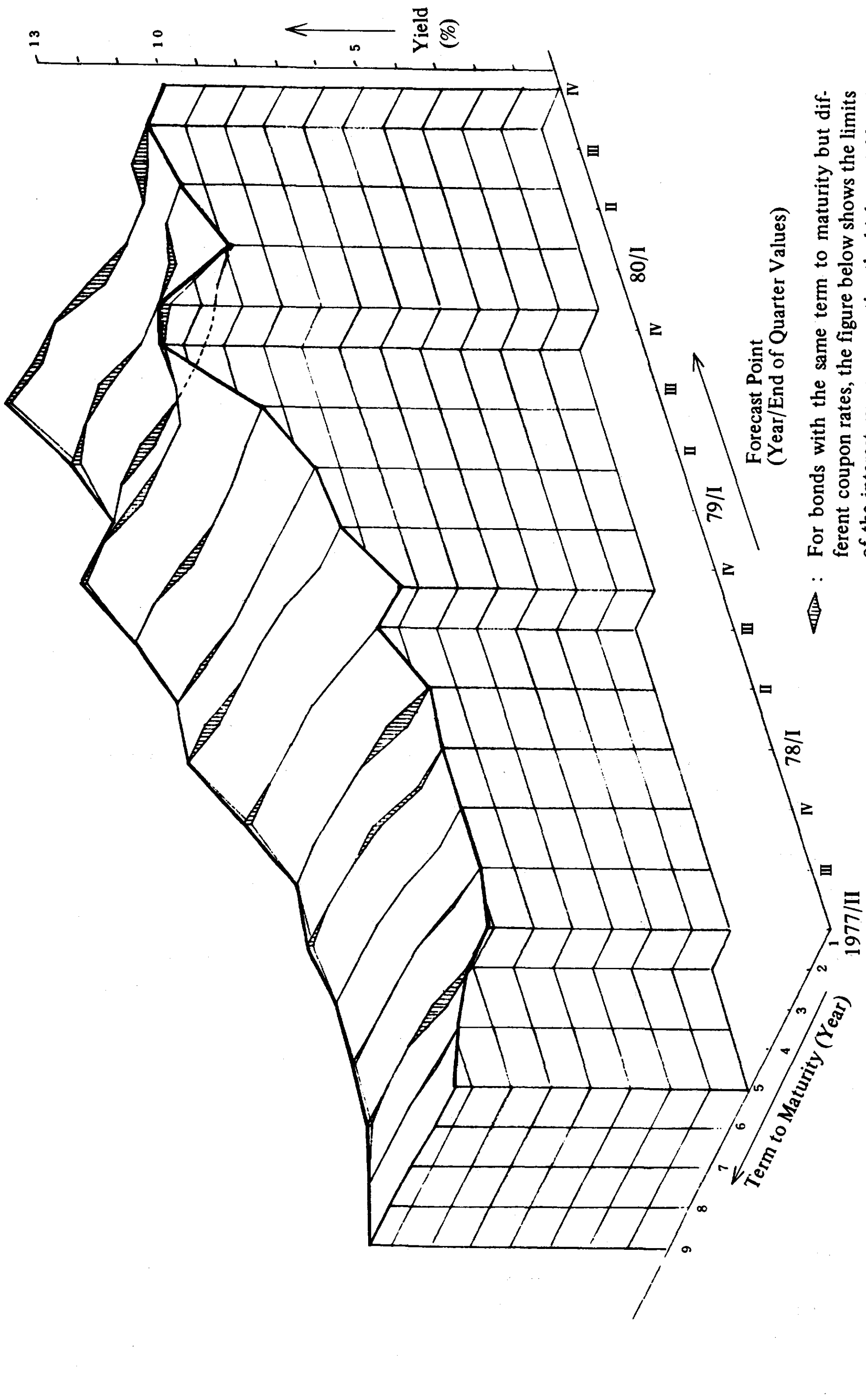
Using quarterly data from the second quarter of 1977 until the fourth quarter of 1980, the magnitude of the "Fisher Effect" was estimated by comparing the expected inflation rate calculated from the forecast values of the ARMA model ($\hat{\pi}_t^{(n)}$) with the market yield on government bonds ($R_t^{(n)}$). A summary of the estimated results is as follows. (See Tables 2, 3 for reference.)

1) In the estimation of the pooled data, the coefficient a_1, which shows the magnitude of the "Fisher Effect", is clearly significant at the usual significance level of 5% (Hereafter, the word significance implies significance at the 5% level). However, the size of the coefficient is 0.3946, which is greatly less than 1. Consequently, this result reconfirms the work of others (Yoshinori Shimizu [4] and K. Kama [13]) that the "Fisher Effect" is partial for Japanese short and long term interest rates.

2) According to the estimation of the time series data by term to maturity, the coefficient a_1 does not have the expected sign for bonds having two and three

19. In calculating expected inflation rates, the term to maturity was simplified to n years ($n = 1, 2, \ldots, 9$) for the purpose of simplifying estimation. Also, in the estimation of the weights in equation (3-1), coupon payments were set at once a year.

Graph 2. Observed Values of Yields on Government Bonds ($R_t^{(n)}$)



: For bonds with the same term to maturity but different coupon rates, the figure below shows the limits of the interest curve connecting the highest yield and the lowest yield.

Graph 3. Expected Inflation Rate ($\hat{\pi}_t^{(n)}$)

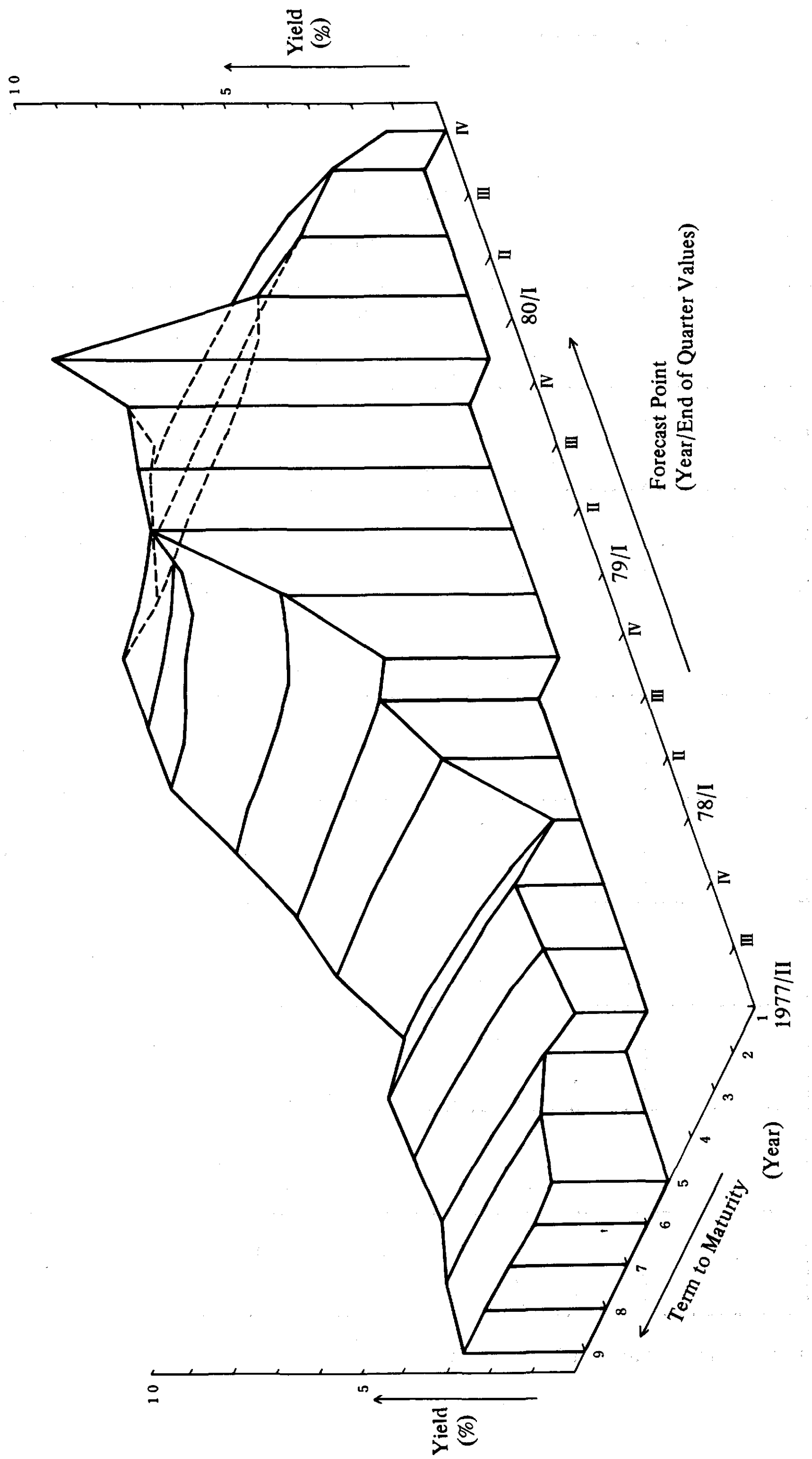


Table 2. Estimates of the Fisher Effect (time series data)

$$R_t^{(n)} = a_0 + a_1 \hat{\pi}_t^{(n)}$$

$R_t^{(n)}$: The yield on an n period government bond at time t

$\hat{\pi}_t^{(n)}$: The inflation rate corresponding to $R_t^{(n)}$

		a_0 (t-statistic)	a_1 (t-statistic)	$\bar{R}2$ / S.E.	D.W. / D.F.
Pooled data		6.2356	* 0.3946	0.3268	0.4272
		(40.7395)	(10.7055)	0.9939	233
Term to Maturity (year)	n = 2	8.9363	0.1697	0.3145	0.9529
		(17.8248)	(2.0521)	0.7134	6
	3	8.9193	-0.0408	-	0.4551
		(10.9052)	(-0.3009)	1.2435	16
	4	5.7775	* 0.3832	0.3807	0.2140
		(13.2072)	(4.1212)	1.0492	25
	5	5.6164	* 0.4585	0.4755	0.2567
		(16.9144)	(5.3948)	0.8344	30
	6	6.0411	* 0.4207	0.3664	0.2580
		(16.8387)	(4.4162)	0.7842	31
	7	5.9780	* 0.4939	0.3827	0.2142
		(14.6319)	(4.7647)	0.8371	34
	8	5.6823	* 0.5978	0.4140	0.2212
		(13.1377)	(5.0717)	0.8331	34
	9	5.0156	* 0.7741	0.6620	0.3649
		(16.6695)	(9.0166)	0.5977	40

* Indicates significance at the 5% level.

Table 3. Estimates of the Fisher Effect (cross section data)

$$R_t^{(n)} = a_0 + a_1 \hat{\pi}_t^{(n)}$$

Forecast Point (End of Quarter Values)		a_0 (t statistic)	a_1 (t statistic)	$\bar{R}^2$ / S.E.	D.W. / D.F.
Pooled data		6.2356 (40.7395)	* 0.3946 (10.7055)	0.3268 / 0.9939	/ 233
	t = 1977/II	5.9220 (54.8319)	* 0.5589 (14.0317)	0.9469 / 0.0464	/ 10
	III	6.2237 (59.1609)	* 0.2987 (7.2691)	0.8122 / 0.0666	/ 11
	IV	4.3412 (11.1652)	* 0.9868 (5.4405)	0.7223 / 0.1246	/ 10
	78/I	3.3997 (11.8462)	* 1.2933 (10.1910)	0.8878 / 0.0831	/ 12
	II	1.5598 (3.3800)	* 1.9643 (10.1408)	0.8791 / 0.0958	/ 13
	III	5.2147 (36.9807)	* 0.8454 (8.1657)	0.8348 / 0.1416	/ 12
	IV	65.4125 (8.0485)	*-20.8801 (-7.2773)	0.8125 / 0.1421	/ 11
	79/I	10.8190 (24.2805)	* -1.0757 (-8.6382)	0.8402 / 0.1267	/ 13
	II	10.1599 (43.1685)	* -0.4650 (-9.6402)	0.8518 / 0.1280	/ 15
	III	8.3218 (46.7706)	* -0.0881 (-3.2286)	0.3864 / 0.1207	/ 14
	IV	8.6543 (29.5766)	-0.0428 (-0.9215)	- / 0.1877	/ 14
	80/I	7.1134 (22.0866)	* 0.3765 (7.8848)	0.7726 / 0.3200	/ 17
	II	6.2002 (14.5650)	* 0.5188 (5.1516)	0.5735 / 0.1371	/ 18
	III	53.5257 (7.1075)	*-12.5332 (-5.9207)	0.6545 / 0.2167	/ 17
	IV	10.8440 (20.4983)	* -0.5021 (-2.7774)	0.2611 / 0.3695	/ 18

* Indicates significance at the 5% level.

years to maturity, nor are these coefficients significant.[20] Hence the existence of the "Fisher Effect" can be rejected for these cases. However, for bonds having a term to maturity of over three years, the coefficient a_1 has the expected sign and is significant. Thus, for data by length of term to maturity, the existence of the "Fisher Effect" is generally proved.

What must be pointed out here is that with a term to maturity of four years, a_1 equals 0.3832 and with a term to maturity of nine years, a_1 equals 0.7741. This can be interpreted as implying that as the term to maturity increases, the magnitude of the coefficient a_1 increases. In other words, this result indicates that under the assumption of a constant equilibrium real short term interest rate, the "Fisher Effect" tends to be larger for long term interest rates than for short term interest rates.[21] In past empirical analyses by Yohe and Karnosky, and Shimizu [4] and Kama [13], it was emphasized that the "Fisher Effect" is larger for short term rather than long term interest rates. Coupled with the problems of the distributed lag model pointed out in Part II, Section C, this raises doubt concerning the empirical results of the distributed lag models.

The "Fisher Effect", estimated on the basis of a constant equilibrium real short term interest rate, is larger in the long term than in the short term. Hence, it is necessary to consider the possibility that this is caused by the equilibrium real short term interest rate greatly fluctuating in reality. Also, concerning the relationship between the expected inflation rate and the nominal rate of interest, Fisher [10] himself thought that it would take a long period of time for the expected inflation rate to be reflected in the nominal interest rate. Fisher [10] conducted his empirical research thinking that this effect would largely appear in the long term interest rate. The results of the empirical research in this paper suggest that Fisher's idea has some relevance.

3) In the estimation using cross section data for each forecast point, the coefficient a_1 had the wrong sign for 7 of the 15 forecast periods. Also, the largest absolute value for a_1 was 20.8801 (1978, IV) which deviates greatly from the 0 to 1 range predicted by the theoretical model in Part II. In addition, fluctuations in the value of the coefficient a_1 for each period are extremely large. In other

20. The "Fisher Effect" may not have been significant for bonds with the term to maturity of 2 – 3 years because the sample size was not sufficient.

21. In estimating the "Fisher Effect" for the "gensaki" rate (rate on bonds with repurchase agreement, in the case below with repurchase after three months), Oritani [1] used expected inflation rates based on the same ARMA model as this paper. Results indicate that the magnitude of the "Fisher Effect" was 0.2437 (T statistic of 8.5658). This result conforms with the results of this paper. Oritani used as his estimation period February, 1967 to December, 1983 and wholesale price data (monthly rate of increase on a yearly basis) for expected inflation rates.

words, looking at the individual estimating periods, a stable relationship between the expected inflation rate and the nominal interest rate in which the former is reflected in the latter is not observed. These cross section results indicate that a significant "Fisher Effect" is only observed over a long period and is not instantly observed at each cross section point.

The Term Structure of Interest Rates on a Real Interest Rate Basis

The previous estimation of the "Fisher Effect" was based on the two assumptions of (i) a constant equilibrium real short term interest rate, and (ii) factors other than "expectations" which determine the bond yields being constant as well. The following analysis eliminates this latter assumption and examines the three hypotheses concerning factors other than "expectations" which determine the yield on government bonds. In addition, the magnitude of the "Fisher Effect" is again estimated.

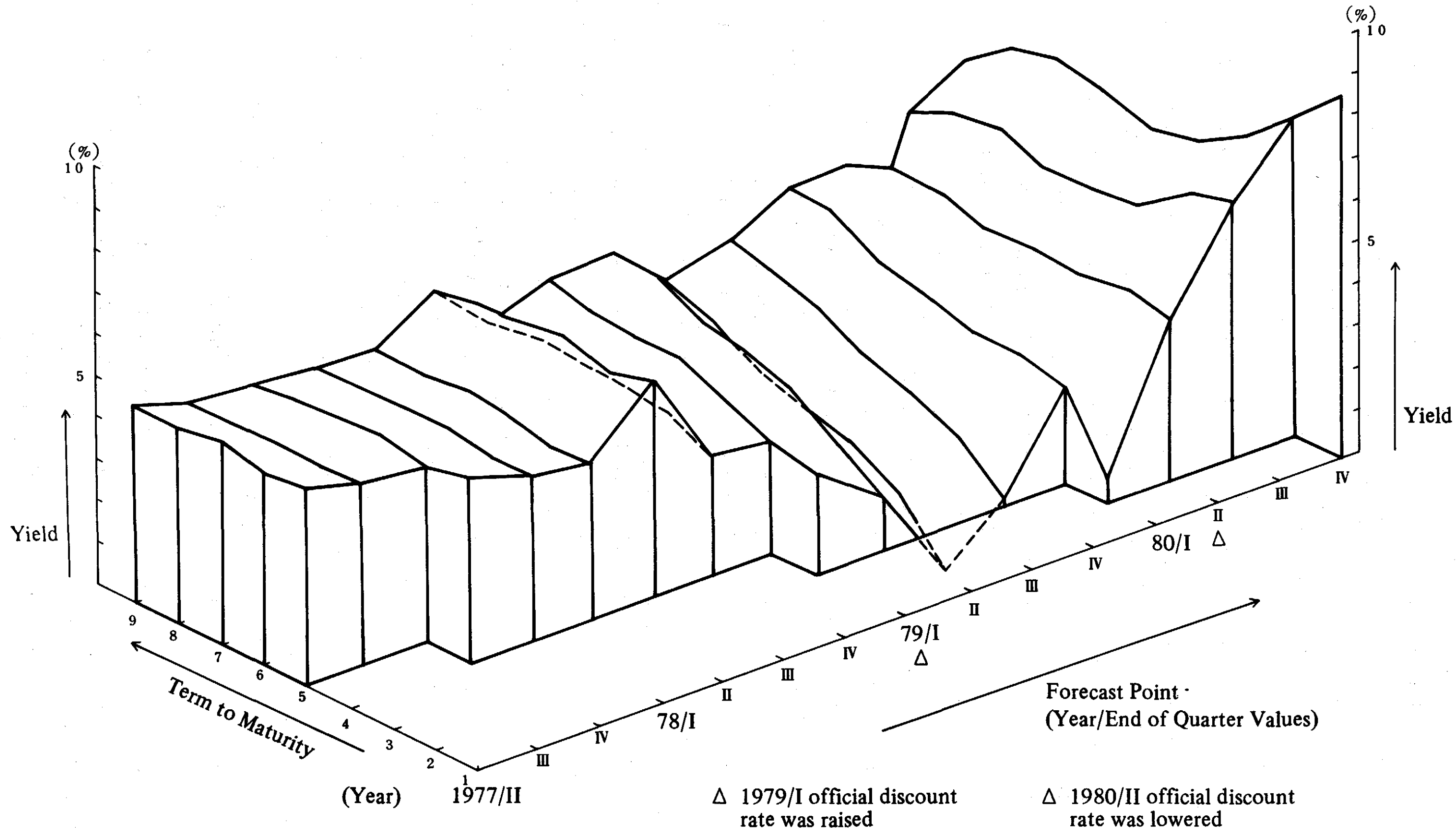
Graph 4 shows the yield on government bonds on a real interest rate basis after subtracting the corresponding expected inflation rates ($\hat{\pi}_t^{(n)}$) from the market yield on government bonds ($R_t^{(n)}$). (Hence, $RR_t^{(n)} = R_t^{(n)} - \hat{\pi}_t^{(n)}$) As can be seen clearly by comparing Graph 2 with Graph 4, the term structure of interest rates on a real interest rate basis and on a nominal interest rate basis greatly differ. In other words, (i) overall fluctuations in the real interest rate and the nominal interest rate (the yield curve) are quite different. For example, in April of 1979, after the rise in the official discount rate, the interest rate level in nominal terms rose strongly and reached a peak in August of 1980 when the official discount rate was decreased. On a real interest rate basis, however, low interest rates continued until the third quarter of 1979 and did not exhibit a sharp rise until after the third quarter of 1980. Also, (ii) examining the term structure of short and long term interest rates in each period (the shape of the yield curve), there are periods when this term structure greatly differs depending on whether it is looked at in real or nominal terms. When the term structure of inflation rates is not horizontal, this result is natural, and this suggests the necessity of conducting separate empirical analyses for theories of the term structure of interest rates on both a real and a nominal interest rate basis.

The results of examining the three different hypotheses concerning factors other than "expectations" which determine the yield on government bonds are as follows.

1) Investigation of the "Liquidity Premium Hypothesis" (Tables 4 and 5)

i) In the estimation based on the pooled data, the coefficient b_2 (or b'_2) on the variable showing the term to maturity Y (or ln Y) did not have the expected sign, and the "Liquidity Premium Hypothesis" was rejected.

Graph 4. Expected Real Interest Rate (Observed yield on government bonds ($R_t^{(n)}$) minus the expected inflation rate ($\hat{\pi}_t^{(n)}$))



ii) For the cross section data, the coefficients b_2 (b'_2) for each of the fifteen periods had the expected sign but were significant for only six of the periods. (five periods)

2) Investigation of the "Preferred Habitat Hypothesis" (Table 6)

i) In the estimation of the pooled data, the coefficient c_2 on the composition ratio by term to maturity and by coupon rate (W) did not have the expected sign, and hence this hypothesis was rejected.

ii) In the estimation of the cross section data, c_2 generally had the expected sign but was significant in only three cases. Also, the greatest absolute value of c_2 was 0.02. Thus, even if the composition ratio by coupon rate and by term to maturity influences the real yield on government bonds, this influence is extremely small.

3) Investigation of the "Coupon Oriented Hypothesis" (Tables 7 and 8)

i) The coefficient d_2 on the coupon rate CP had the expected sign for the pooled data but was not significant at the 5% level. Hence, this Hypothesis was also rejected for the pooled data. (However, it was significant at the 20% level.)

ii) For the time series data by term to maturity, d_2 had the expected sign but was only significant when the terms to maturity were seven and eight years.

iii) For the cross section data by forecast period, d_2 had the expected sign and was significant for six periods (1979, III, IV, 1980, I, II, III, IV). Hence, after the middle of 1979, the "Coupon Oriented Hypothesis" possesses significant explanatory power. This roughly corresponds with the empirical results obtained by Kuroda–Ōkubo [3] using nominal interest rates.[22]

Concerning the term structure of long term yields on government bonds on a real interest rate basis, the three hypotheses concerning factors other than expectations influencing these yields were all rejected. After the middle of 1979, however, out of the three, only the "Coupon Oriented Hypothesis" had good explanatory power.

When including changes in factors other than expectations which influence the yield on government bonds, how does the magnitude of the "Fisher Effect" change? Calculation of the magnitude of the "Fisher Effect" (θ) based on estimation results of the pooled data from Tables 4 – 8 are as follows:

1) The "Liquidity Premium Hypothesis"

$$b_1 = -0.6097 \rightarrow \theta = 0.3903$$

$$b'_1 = -0.6181 \rightarrow \theta = 0.3819$$

22. See Kuroda and Ōkubo [3]. page 68.

Table 4. Investigation of the "Liquidity Premium Theory" (cross section data)

$$R_t^{(n)} - \hat{\pi}_t^{(n)} = b_0 + b_1 \hat{\pi}_t^{(n)} + b_2 Y_t^{(n)}$$

[$Y_t^{(n)}$: Period to maturity (year) corresponding to $R_t^{(n)}$]

		b_0	b_1	b_2	$\bar{R}^2$	D.W.
					S.E.	D.F.
Pooled data		6.3349	*- 0.6097	-0.0139	0.5311	
		(23.0520)	(-16.0230)	(-0.4369)	0.9976	232
Forecast Point (End of Quarter Values)	t = 1977/II	6.1445	*- 0.5907	* 0.0280	0.9561	
		(57.9813)	(-10.5738)	(3.1333)	0.0338	9
	III	6.3876	*- 0.8886	* 0.0466	0.9884	
		(98.6080)	(-21.1885)	(5.2655)	0.0360	10
	IV	4.6712	- 0.2703	0.0316	-	
		(4.2214)	(- 0.3282)	(0.3208)	0.1306	9
	78/I	1.8694	* 1.2659	-0.0966	0.3882	
		(2.2369)	(2.4448)	(-1.9264)	0.0751	11
	II	1.7671	0.8547	0.0079	0.5988	
		(0.9791)	(0.9077)	(0.1192)	0.0998	12
	III	5.2360	0.1119	-0.0565	0.0284	
		(34.6177)	(0.2111)	(-0.5134)	0.1460	11
	IV	126.7528	*-43.1204	-0.1778	0.8635	
		(3.9754)	(- 3.9015)	(-1.9789)	0.1260	10
	79/I	8.5991	*- 1.5724	0.0691	0.9535	
		(4.5700)	(- 3.6349)	(1.2130)	0.1246	12
	II	7.8294	*- 1.1267	* 0.1135	0.9867	
		(7.5500)	(- 7.3412)	(2.2938)	0.1129	14
	III	5.3269	*- 0.7846	* 0.1741	0.9968	
		(9.1491)	(-13.0281)	(5.2295)	0.0711	13
	IV	4.1283	*- 0.0653	* 0.2490	0.9881	
		(4.1220)	(- 5.2359)	(4.6002)	0.1201	13
	80/I	4.2536	*- 0.3600	* 0.1935	0.9315	
		(4.0234)	(- 3.5168)	(2.7993)	0.2702	16
	II	4.5427	- 0.1519	0.0472	0.5592	
		(3.6336)	(- 0.5980)	(1.4053)	0.1335	17
	III	60.9712	*-15.6642	0.0241	0.6760	
		(3.7477)	(- 3.3774)	(0.5188)	0.2212	16
	IV	11.1173	*- 1.6638	0.0346	0.7725	
		(14.7010)	(- 4.5788)	(0.5163)	0.3772	17

* Indicates significance at the 5% level.

Table 5. Investigation of the "Liquidity Premium Theory" (cross section data)

$$R_t^{(n)} - \hat{\pi}_t^{(n)} = b_0' + b_1' \hat{\pi}_t^{(n)} + b_2' \ln Y_t^{(n)}$$

		b_0'	b_1'	b_2'	$\bar{R}^2$ / S.E.	D.W. / D.F.
Pooled data		6.7170	*- 0.6181	-0.2497	0.5361	
		(20.1695)	(-16.4211)	(-1.6309)	0.9924	232
Forecast Point (End of Quarter Values)	t = 1977/II	6.6524	*- 0.8930	* 0.2768	0.9680	
		(34.7742)	(- 7.8651)	(4.0783)	0.0289	9
	III	6.7785	*- 1.2389	* 0.4476	0.9904	
		(63.2455)	(-13.3051)	(5.9160)	0.0328	10
	IV	5.0510	- 1.0994	0.8455	-	
		(4.6656)	(- 0.7087)	(0.7053)	0.1278	9
	78/I	1.6895	1.8629	-0.9693	0.3127	
		(1.3924)	(1.7068)	(-1.4472)	0.0796	11
	II	3.1492	- 0.0717	0.4684	0.6086	
		(1.0943)	(- 0.0385)	(0.5600)	0.0985	12
	III	5.1795	- 0.1864	0.0417	0.0054	
		(3.9818)	(- 0.1592)	(0.0272)	0.1477	11
	IV	152.7987	*-51.7390	-1.5157	0.8386	
		(2.4177)	(- 2.3946)	(-1.3954)	0.1370	10
	79/I	3.8364	- 0.6377	1.0559	0.9581	
		(0.9379)	(- 0.7537)	(1.7160)	0.1183	12
	II	4.1527	*- 0.7144	* 1.3576	0.9879	
		(1.8453)	(- 2.5240)	(2.6798)	0.1077	14
	III	1.3549	*- 0.4960	* 1.8276	0.9953	
		(0.7459)	(- 3.1962)	(3.8449)	0.0857	13
	IV	-2.7024	- 0.0310	* 2.8568	0.9852	
		(-0.8975)	(- 0.1151)	(3.7810)	0.1344	13
	80/I	1.7564	- 0.1811	1.4703	0.9194	
		(0.6724)	(- 0.8281)	(2.0641)	0.2931	16
	II	2.9791	0.1282	0.3974	0.5489	
		(1.1349)	(0.2562)	(1.2431)	0.1350	17
	III	53.0585	*-13.3986	-0.0071	0.6720	
		(2.5641)	(- 2.2560)	(-0.0247)	0.2226	16
	IV	11.3093	*- 1.7894	0.2305	0.7710	
		(8.6849)	(- 2.3739)	(0.3931)	0.3784	17

* Indicates significance at the 5% level.

Table 6. Investigation of the "Preferred Habitat Theory" (cross section data)

$$R_t^{(n)} - \hat{\pi}_t^{(n)} = c_0 + c_1 \hat{\pi}_t^{(n)} + c_2 W_t^{(n)}$$

[$W_t^{(n)}$: Proportional composition of outstanding government bonds by coupon rate and term to maturity corresponding to $R_t^{(n)}$]

		c_0	c_1	c_2	$\bar{R}^2$ / S.E.	D.W. / D.F.
Pooled data		6.8839	*- 0.6857	*-0.0275	0.5648	
		(32.3433)	(-16.9960)	(-4.2586)	0.9612	232
Forecast Point (End of Quarter Values)	t = 1977/II	5.9282	*- 0.4609	0.0018	0.9243	
		(57.3545)	(-11.2973)	(1.3534)	0.0444	9
	III	6.2201	*- 0.7521	* 0.0043	0.9865	
		(101.3047)	(-28.5930)	(4.7101)	0.0389	10
	IV	4.5110	- 0.1580	0.0049	0.0659	
		(12.0970)	(- 0.8377)	(1.6637)	0.1149	9
	78/I	3.4771	0.2429	0.0015	0.1946	
		(9.9143)	(1.3598)	(0.4166)	0.0862	11
	II	2.0555	* 0.7008	0.0084	0.6918	
		(4.1552)	(3.1254)	(1.9081)	0.0874	12
	III	5.1571	*- 0.3165	* 0.0195	0.3956	
		(44.1750)	(- 3.0485)	(2.6667)	0.1152	11
	IV	51.2615	*-16.9830	0.0180	0.8416	
		(4.2512)	(- 4.0363)	(1.5212)	0.1358	10
	79/I	9.9779	*- 1.8644	0.0096	0.9573	
		(15.0221)	(-10.6788)	(1.6329)	0.1194	12
	II	10.0660	*- 1.4488	0.0023	0.9819	
		(25.2121)	(-19.5961)	(0.2963)	0.1320	14
	III	7.6588	*- 1.0066	* 0.0193	0.9935	
		(26.7304)	(-26.6783)	(2.7013)	0.1003	13
	IV	8.3451	*- 1.0049	0.0093	0.9699	
		(15.3314)	(-13.7415)	(0.6794)	0.1913	13
	80/I	6.7823	*- 0.5891	0.0155	0.9017	
		(12.6845)	(- 9.0199)	(0.7811)	0.3237	16
	II	5.6607	*- 0.3674	0.0098	0.5335	
		(8.0356)	(- 2.3622)	(0.9621)	0.1373	17
	III	55.2025	*-14.0100	0.0033	0.6729	
		(4.5138)	(- 4.0469)	(0.1761)	0.2223	16
	IV	10.8276	*- 1.4931	-0.0018	0.7690	
		(17.3986)	(- 5.9914)	(-0.0544)	0.3801	17

* Indicates significance at the 5% level.

Table 7. Investigation of the "Coupon Oriented Hypothesis" (time series data)

$$R_t^{(n)} - \hat{\pi}_t^{(n)} = d_0 + d_1 \hat{\pi}_t^{(n)} + d_2 CP_t^{(n)}$$

[$CP_t^{(n)}$: Coupon rate corresponding to $R_t^{(n)}$]

		d_0	d_1	d_2	$\bar{R}^2$ / S.E.	D.W. / D.F.
Pooled data		7.3057	*-0.6108	-0.1443	0.5351	
		(9.8319)	(-16.4718)	(-1.4728)	0.9934	232
Term to Maturity (year)	n = 2	15.4148	*-0.7636	-1.0211	0.9254	0.9861
		(1.2359)	(-4.9003)	(-0.5199)	0.7612	5
	3	11.5504	*-1.0796	-0.3719	0.7593	0.4759
		(1.3085)	(-5.7618)	(-0.3107)	1.2801	15
	4	-5.2379	*-0.6986	* 1.6012	0.9114	1.4192
		(-4.2504)	(-15.1937)	(9.0740)	0.5087	24
	5	-1.3123	*-0.8413	* 1.0887	0.7035	0.4009
		(-0.7354)	(-8.1335)	(3.9288)	0.6856	29
	6	5.8425	*-0.5832	0.0276	0.5137	0.2561
		(2.6081)	(-5.4892)	(0.0898)	0.7970	30
	7	17.9591	*-0.5430	*-1.5402	0.8555	1.4370
		(15.4461)	(-10.6961)	(-10.4599)	0.4090	33
	8	15.0192	*-0.8349	*-1.0788	0.7090	0.8972
		(11,8336)	(-9.0135)	(-7.5241)	0.5131	33
	9	1.4281	-0.0653	* 0.4254	0.2603	0.6470
		(1.1162)	(-0.6746)	(2.8719)	0.5499	39

* Indicates significance at the 5% level.

Table 8. Investigation of the "Coupon Oriented Hypothesis" (cross section data)

$$R_t^{(n)} - \hat{\pi}_t^{(n)} = d_0 + d_1 \hat{\pi}_t^{(n)} + d_2\, CP_t^{(n)}$$

		d_0	d_1	d_2	$\bar{R}^2$ / S.E.	D.W. / D.F.
Pooled data		7.3057	*- 0.6108	-0.1443	0.5351	
		(9.8319)	(-16.4718)	(-1.4728)	0.9934	232
Forecast Point (End of Quarter Value)	t = 1977/II	5.6022	*- 0.4991	* 0.0653	0.9813	
		(75.2281)	(-23.4036)	(5.9318)	0.0221	9
	III	5.6447	*- 0.7793	* 0.1053	0.9998	
		(425.9839)	(-253.3726)	(50.9673)	0.0043	10
	IV	3.5613	*- 0.6091	* 0.2731	0.8511	
		(20.5904)	(- 6.1057)	(8.0433)	0.0459	9
	78/I	2.9678	- 0.0574	* 0.1626	0.7394	
		(15.5144)	(- 0.5519)	(4.8506)	0.0490	11
	II	1.4436	* 0.5628	* 0.1454	0.8259	
		(4.5434)	(3.3684)	(3.9606)	0.0657	12
	III	4.0983	*- 0.2990	* 0.1793	0.5649	
		(13.1144)	(- 3.6855)	(3.7594)	0.0977	11
	IV	56.0235	*-19.0031	* 0.1673	0.9164	
		(9.0572)	(- 8.8909)	(3.6535)	0.0986	10
	79/I	10.1750	*- 2.0369	0.0689	0.9548	
		(15.9243)	(-16.4383)	(1.3663)	0.1228	12
	II	10.3933	*- 1.4713	-0.0281	0.9822	
		(22.4713)	(-29.1780)	(-0.5906)	0.1309	14
	III	9.5005	*- 1.1011	*-0.1535	0.9989	
		(73.0133)	(-116.2269)	(-10.2656)	0.0415	13
	IV	10.1698	*- 1.0330	*-0.2192	0.9956	
		(49.4107)	(-56.7763)	(-8.8585)	0.0734	13
	80/I	10.1203	*- 0.6371	*-0.4022	0.9810	
		(26.1053)	(-29.8673)	(-8.3486)	0.1425	16
	II	7.9915	*- 0.5834	*-0.1879	0.8977	
		(26.7167)	(-11.9305)	(-8.0412)	0.0643	17
	III	47.5564	*-11.3152	*-0.2669	0.8975	
		(10.7135)	(- 8.9003)	(-5.9521)	0.1244	16
	IV	14.0392	*- 1.3894	*-0.4842	0.9824	
		(52.2605)	(-26.7390)	(-14.3504)	0.1050	17

* Indicates significance at the 5% level.

2) The "Preferred Habitat Hypothesis"

$c_1 = -0.6857 \rightarrow \theta = 0.3143$

3) The "Coupon Oriented Hypothesis"

$d_1 = -0.6108 \rightarrow \theta = 0.3892$

In other words, when including the variable W, estimates of the "Fisher Effect" change somewhat, but in the other cases, it is stable with $\theta = 0.38 \sim 0.39$. This is only slightly different from the value $\theta = 0.3946$ obtained from estimates assuming that factors other than expectations influencing the yield are constant. It can be reasonably concluded that the magnitude of the "Fisher Effect" for the yield on government bonds is between 30% and 40%.[23]

Finally, there remains the problem of how the wide range of fluctuations concerning the term structure of yields on government bonds on a real interest basis shown in Graph 4 should be interpreted. If, as is shown in the model in Part II, (i) there is a constant equilibrium real short term interest rate, (ii) the "Fisher Effect" is complete, and (iii) factors determining the yield on government bonds other than expectations are constant, then the term structure of interest rates on a real basis graphed against the term to maturity and the estimation period should produce a flat surface. However, in actuality, an undulating surface was observed. There are three possible reasons for this.

The first is that the "Fisher Effect" is partial. If the relationship at each point of estimation between the expected inflation rate and the nominal interest rate is very unstable, the term structure on a real interest basis, calculated by subtracting the expected inflation rate from the nominal interest rate, shows a great deal of fluctuation. It can be thought that fluctuations in the term structure of the yield

23. For reference, the three hypotheses concerning factors other than "expectations" which influence the yields were examined as nested hypotheses and the "Fisher Effect" was estimated. The following results were obtained.

$$RR_t^{(n)} = \underset{(9.5517)}{6.9383} - \underset{(-17.1219)}{0.6882\,\hat{\pi}_t^{(n)}} - \underset{(-0.5717)}{0.0558\,CP_t^{(n)}} - \underset{(-4.5170)}{0.0345\,W_t^{(n)}} + \underset{(2.0609)}{0.0727\,Y_t^{(n)}}$$

$\bar{R}^2 = 0.5693$ D. F. = 230

S. E. = 0.9061

$$RR_t^{(n)} = \underset{(9.6827)}{7.1327} - \underset{(-16.9082)}{0.6856\,\hat{\pi}_t^{(n)}} - \underset{(-0.5056)}{0.0503\,CP_t^{(n)}} - \underset{(-3.7768)}{0.0281\,W_t^{(n)}} + \underset{(0.4200)}{0.0715\,\ln Y_t^{(n)}}$$

$\bar{R}^2 = 0.5617$ D. F. = 230

S. E. = 0.9645

on Japanese government bonds are primarily caused by a short term unstable "Fisher Effect."

The second reason is the large, unignorable yield differentials in coupons. These coupons reflect the "Coupon Oriented Hypothesis" concerning factors other than expectations which determine the yield on government bonds. (In contrast, the influence of other factors as suggested by the "Liquidity Premium Hypothesis" and the "Preferred Habitat Hypothesis" are very small.)

The final reason is the possibility that the equilibrium real short term interest rate itself is actually changing. The estimating equations which incorporate the first and second reasons listed above (Tables 4 – 8) had R^2's of 0.5311 – 0.5698 and standard errors of 0.9612 – 0.9976. The explanatory power of these regressions for the real interest rate was not particularly high. In other words, the equations (5-3) to (5-5) concerning the mechanism of determining the yield on government bonds on a real interest basis are not necessarily satisfactorily specified. The possible reasons for this inadequate specification are twofold. First, there might be factors affecting the yield on government bonds which have been overlooked. Secondly, the equilibrium real short term interest rate might be changing. The question of which of these reasons is correct must be left for later empirical research. However, as is shown in the Appendix, it is not an easy task to conduct empirical research which does not depend on the assumption of a constant equilibrium short term interest rate. The clarification of this point will be a primary topic of future research.

APPENDIX. Discussion of the Theory of a "Constant Equilibrium Real Short Term Rate of Interest"

In the model in Part II, for convenience in specifying the relationship between the term structure of interest rates and expected inflation rates, the assumption of a constant equilibrium real short term interest rate was adopted. Here, through investigation of the mechanisms determining the equilibrium real short term interest rate, to what extent the assumption of constancy is appropriate in actual empirical analyses will be investigated. Also, why differences must be made between the ex post real short term interest rate calculated from the nominal interest rate and the corresponding value of realized value of the inflation rate, and the ex ante concept of the equilibrium real short term inflation rate will be explained.

A. Classical Theory of Equilibrium Real Interest Rate Determination[1]

The classical school stated that the real interest rate is determined by the equilibrium in the real sector between investment (I) and savings (S). Following I. Fisher [10], the classical theory can be outlined as follows. Of the two factors that determine the interest rate, the first is the rate of time preference. This refers to the degree of subjective impatience of people in curtailing consumption this period and trying to save for what should become consumption in future periods (human impatience). The second factor is the rate of return over cost to investment. This refers to objective investment opportunity, or how much income in the future exceeds the investment principal of an investment made this period.

The classical model, if simplified, is expressed in the following two equations.
[Money market equilibrium]

$$M = k P y \tag{1}$$

[Equilibrium equality between savings and investment]

$$\frac{I}{P}(\rho) = \frac{S}{P}(\rho, y) \tag{2}$$

M: money supply (exogenous)
k : Marshallian k
P : Price level
I : investment
y : real income
S : savings
ρ : real interest rate

In the classical model, because the labor market is always in equilibrium through changes in the real wage rate (the assumption of full employment), real income (y) is fixed at the production level (y_f) corresponding to full employment. Consequently, the real interest rate is determined by equality between savings and investment in the real sector, while the price level is decided through equilibrium in the money market. In other words, there is a dichotomy between the real and financial sectors in the classical model.

In a classical model of the kind above, the appropriateness of the hypothesis of a constant real interest rate depends upon how long the two factors outlined above,

1. In classical models, because the concept of time does not clearly enter, the word equilibrium real short term interest rate was replaced by the word equilibrium real interest rate.

(i) people's subjective rate of time preference and (ii) the rate of return on investment decided by economically objective investment opportunities, are stable.

Concerning this point, the economy painted by the classical school is static and it was generally thought that in the long run, the economy converged to a stable equilibrium. This way of thinking is in the background of the adoption of the assumption of a constant real interest rate first by Fisher and others in traditional methods of analyzing the expected inflation rate and the nominal interest rate.

B. Theory of Real Interest Rate Determination in a Keynesian Model[2]

In the naive Keynesian model, the price level was fixed. In this case, the theory of the determination of the real interest rate is the same as that for the nominal interest rate.

[Money market equilibrium equation]

$$M = L(r, Y) \tag{3}$$

[Equilibrium equality between savings and investment]

$$I(r) = S(r, Y) \tag{4}$$

L: liquidity preference function
r : nominal interest rate
Y: nominal income

In this model, through the simultaneous equilibrium of equations (3) and (4), the nominal interest rate (r) and nominal income (Y) are determined. As Keynes [15] himself thought, in a situation when the interest elasticity of savings is close to zero, nominal income is determined through the equilibrium of investment and savings. Through the equilibrium of the money market given this level of nominal income, the nominal interest rate is determined. In other words, the theory of interest rate determination of Keynes [15] was fundamentally a monetary theory. This is in sharp contrast to the classical theory of interest rate determination in the real sector.

However, the theory of interest rate determination with a fixed price level is

2. Here, the word equilibrium real short term interest rate is inappropriate. In Keynesian models of the theory of determination of the real interest rate, just as in classical models, the length of time is not clearly specified. Also, in Keynesian models, it is assumed there is disequilibrium in one sector of the economy (for instance, the labor market). This assumption of disequilibrium greatly affects the determination of the real interest rate.

not appropriate in present day economies which have experienced serious inflation and large fluctuations in expected inflation rates. Consequently, the mechanism determining real interest rates will be investigated below in a more general Keynesian model which includes endogenous determination of the price level.
[Money market equilibrium equation]

$$\frac{M}{P} = L(\rho, y) \tag{5}$$

[Equilibrium equality between savings and investment]

$$\frac{I}{P}(\rho) = \frac{S}{P}(\rho, y) \tag{6}$$

[Production function]

$$y = f(N) \tag{7}$$

N: number of workers employed

[Labor demand function]

$$N^d = g\left(\frac{W}{P}\right) \tag{8}$$

N^d: labor demand, W: nominal wages

[Determination of nominal wages]

$$W = \bar{W} \text{ (fixed)} \tag{9}$$

The mechanism determining the real interest rate in this model is as follows.
1) The real interest rate is, in general, determined by the simultaneous equilibrium of the money market and the goods market. It is not, as thought by the classicists, determined only in the real sector.
2) In the special situation when the interest elasticity of savings and investment is zero, the real interest rate is determined solely by equilibrium in the money market. (Through the equilibrium of the real sector, y and P are decided. Thus ρ is determined as the value which satisfies equation (5) given the values y and P.)

As a general conclusion of the Keynesian model, the real interest rate is determined not only by real sector factors, but also by supply and demand in the money market as well. Also, because supply and demand in the money market changes rather a great deal, this casts doubt on the validity of the assumption of a constant real interest rate.

In order to avoid criticism from a Keynesian perspective concerning the "assumption of a constant real interest rate", there have been attempts to clearly introduce supply and demand factors of the money market in analyzing the relationship between nominal interest rates and expected inflation rates. For instance, T. Sargent [19], in analyzing the "Fisher Effect" for the yield on American government bonds (10 year and 1 year bonds), estimated the following equation.

$$R_t^{(n)} = f(y_t, \Delta y_t) + g\left(\frac{M_t^* - M_{t-1}^*}{M_{t-1}^*}\right) + \sum_{i=0}^{m} \lambda_i \cdot \pi_{t-i} \qquad (10)$$

$$(M_t^* = M_t / P_t)$$

Here, the first term on the right hand side represents supply and demand factors influencing the equality between savings and investment, while the second term expresses supply and demand factors of the money market. However, when, for instance, the long term yield on a 10 year bond is used as the dependent variable, the real interest rate on the right hand side should be the real interest rate from the present to ten years in the future. Nevertheless, in Sargent's specification, supply and demand factors in the present period only are used to explain changes in the real interest rate.

C. The Equilibrium Real Short Term Interest Rate and the Ex Post Real Short Term Interest Rate

In evaluating the appropriateness of the hypothesis of a constant equilibrium real short term interest rate, it is important to distinguish between the equilibrium real short term interest rate (ρ_t) and the ex post real short term interest rate (ρ_t^{exp}). The ex post real short term interest rate is calculated by subtracting from the nominal short term interest rate the corresponding value of the realized inflation rate. First, in a model under conditions of uncertainty, the nominal short term interest rate is given by

$$r_t = \rho_t + \theta_t^{(1)} \cdot {}_{t+1}\hat{\pi}_t + XR_t^{(1)} \qquad (11)$$

Hence, the ex post real short term interest rate is given by

$$\rho_t^{exp} = r_t - \pi_{t+1} \qquad (12)$$

$$= \rho_t + [\theta_t^{(1)} \cdot {}_{t+1}\hat{\pi}_t - \pi_{t+1}] + XR_t^{(1)} \qquad (12)'$$

Consequently, separating the equilibrium and ex post real short term interest rates, the following is obtained

$$\rho_t - \rho_t^{exp} = (\pi_{t+1} - {}_{t+1}\hat{\pi}_t) + (1 - \theta_t^{(1)})\, {}_{t+1}\hat{\pi}_t + XR_t^{(1)} \tag{13}$$

As can be seen in equation (13), differences between these two rates are caused by (i) the discrepancy between the expected and the realized inflation rates, (ii) the portion of the expected inflation rate not reflected in the nominal short term interest rate because of a partial "Fisher Effect," and (iii) the existence of factors other than expectations which influence the yield on government bonds. Clearly, even if the ex post real short term interest rate fluctuates a great deal, this is no reason to reject the hypothesis of a constant equilibrium real short term interest rate.

D. Summary of the Hypothesis of a Constant Equilibrium Real Short Term Interest Rate

The previous discussion can be summarized as follows.

1) The hypothesis of a constant equilibrium real short term interest rate has its origins in the classical theory of real sector interest rate determination. Criticisms of this theory arise from Keynesian theories of interest rate determination. However, in attempts to empirically analyze the yield on long term bonds, incorporating Keynesian criticism in the method of analysis is difficult. For convenience, it is necessary to adopt the hypothesis of a constant equilibrium real short term interest rate.

2) The hypothesis of a constant equilibrium real short term interest rate cannot be rejected just because of large fluctuations in ex post real short term interest rates.

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