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Financing Constraints, and Financial Intermediation  
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**The Joint and Several Effects of Liquidity Constraints,  
Financing Constraints, and Financial Intermediation  
on the Welfare Cost of Inflation**

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Abstract

This paper examines two features of modern economies that are often overlooked when formally considering the welfare costs of inflation. The first is the short-term financing requirements of firms, and the second is the joint roles played by banks in providing valued liquidity services to households and in acting as financial intermediaries. Measured welfare losses of moderate inflation are seen to become quite large when firms finance their working capital expenses by issuing short-term debt, with estimates of those losses ranging to over 450% higher than is the case when these financing requirements are ignored. Banks are seen to mitigate substantially the welfare costs of inflation by lessening the distortions in household decisions, and by intermediating a larger share of short-term loans to firms as inflation increases.

Key words: inflation taxes, financing constraints,  
financial intermediation

JEL classification: E31, E51, E44

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## I. Introduction.

In 1997, *per capita* holdings of M1 assets in Japan totaled 1.5 million yen. Of this total, 369 thousand yen consists of currency in circulation, with the balance representing demand deposits.<sup>1</sup> These assets are held principally for the liquidity services that they provide in facilitating transactions. While bank deposits do provide some interest income, the rate of interest is generally very low. For example, from 1971-1996 the average interest rate paid on ordinary deposits was 1.43 percent, while the inflation rate averaged 4.3 percent, implying a negative real return. Consequently, transactions that are carried out with these liquid monetary assets are subjected to inflation taxes. Given the high volume of *per capita* holdings of M1 assets in Japan, the avoidance of inflation taxes could result in a misallocation of resources sufficient to induce significant welfare losses.

Inflation may also adversely affect credit conditions when debt obligations are denominated in currency units. The inflation premium in nominal market interest rates raises the cost of borrowing, and deters activities that these funds are used to finance. Most firms finance a significant portion of their working capital expenses with short-term debt. High inflation raises their financing costs, thereby requiring higher productivities both from their marginal unit of capital, thus retarding investment, and from their marginal worker, thereby reducing employment. As a consequence, overall economic activity declines, and this can result in a substantial increase in the welfare costs of inflation.

Commercial banks that raise funds by offering valued liquidity services in the form of demand deposit accounts can mitigate these welfare costs somewhat by intermediating the loans from households to firms, where the latter are used to finance working capital expenses. However, the extent to which banks can intermediate these loans is limited by the demand for their deposit offerings. For this reason, bank-intermediated loans can

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<sup>1</sup> Humphrey, Pulley, and Versala (1996) document the high usage of currency in the payments system in Japan relative to the United States and Europe. By way of comparison, *per capita* currency holdings in the United States in 1996 totaled \$1524, or 191 thousand yen (using a 125 yen/dollar exchange rate). However, approximately two-thirds of U.S. currency is estimated by Porter and Judson (1995) to be held outside of the United States, suggesting that this difference in currency usage is even more pronounced than these numbers imply.

coexist with direct placement of private paper in the capital markets, even in the absence of private information that would induce scale economies in monitoring as described by Douglas Diamond (1984).

This paper examines the interaction between liquidity constraints associated with household transactions and financing constraints associated with working capital requirements of firms. The models are calibrated to the Japanese economy. It is found that in the absence of financing constraints, the welfare costs of moderate inflation are significant, but not large, much in line with the results obtained by Cooley and Hansen (1989,1991) for the U.S. economy. However, in the absence of bank intermediation, those costs effectively double when the firms' wage bills are financed by bond issues, more than triple when gross investment is financed by bond issues, and increase by more than four-fold when all working capital expenses are financed with bonds, rendering those costs quite large. The introduction of a bank that provides deposits to households and uses the proceeds to intermediate a portion of the household loans to firms by purchasing a share of the firms' bond issues is then seen to mitigate these losses significantly. In the case where firms fully finance their working capital expenses with bonds, bank intermediation lowers the measured welfare costs by 15-20 percent. These results suggest that banks can play an important role in alleviating the adverse economic effects that fully anticipated inflation has on the economy.

In the next section, a general model is developed that can be parameterized to obtain all of the cases described above. These models are calibrated in section III, and the results presented in Section IV. The final section contains conclusions.

## **II. The theoretical model.**

This section develops a general equilibrium, representative agent model in which banks provide liquidity services through deposit account offerings, and after meeting reserve requirements, use the proceeds to purchase a portion of the short-term bond issues by firms. The remaining bonds are purchased by households, who use the deposit accounts to purchase a subset of their consumption goods. Firms' working capital expenses are financed with the revenues raised from issuing bonds. For ease of exposition, the model

is structured such that parameterizations can be selected that produce any one of the following five versions of the general model: (1) a simple cash-in-advance model with no bank and no financing constraints on firms; (2) model (1), where firms issue one-period bonds to finance their wage bill; (3) model (1), where firms issue one-period bonds to finance gross investment; (4) model (1), where firms finance all of their working capital, i.e., their wage bill and gross investment, by issuing bonds; and (5) the general case described above, which is model (4), with banks providing deposit accounts and intermediating a portion of the loans from households to firms.

### II.1 Household sector.

The representative household takes its consumption/savings decision by optimally selecting a consumption bundle and a short-term asset portfolio allocation between money, bank deposits, and bonds. It is assumed to be the residual claimant to *per capita* shares of period profits for firms and banks.<sup>2</sup> The household's consumption purchases are subject to liquidity constraints, where a portion of the consumption goods is purchased with money, and the remainder is purchased with bank deposits. It also makes a labor/leisure choice.

The household maximizes lifetime utility, with period utility,  $u$ , derived from leisure,  $l_t$ , and two consumption goods, where the latter are distinguished by the means of payment needed to acquire them. Money is used to purchase  $c_{1t}$ , referred to as the “cash good,” and bank deposits are used to acquire  $c_{2t}$ , referred to as the “deposit good.”

$$\max_{\{c_{1t}, c_{2t}, n_t^s, l_t, M_{t+1}^d, X_{t+1}^d, B_{t+1}^d\}} \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t; \phi_1), \quad \beta(0, 1), \quad \phi_1 \in \{0, 1\} \quad (1)$$

The household's choice set includes optimal sequences for the consumption goods, leisure, labor supply,  $n_t^s$ , and next period's holdings of money,  $M_{t+1}^d$ , deposits,  $X_{t+1}^d$ , and bonds,  $B_{t+1}^d$ . The household's subjective discount factor is given by  $\beta$ , and  $\phi_1$  is an indicator variable that is one in model (5), where households have a positive demand for bank deposits, and zero otherwise, when consumption of the “deposit good” is also zero.

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<sup>2</sup> Since this paper is not concerned with asset pricing *per se*, the equity markets are not modeled.

The optimization in equation (1) takes initial asset holdings as given:  $M_0^d, X_0^d, B_0^d$ , and is subject to four constraints. The first is the budget constraint:

$$P_t(c_{1t} + \phi_1 c_{2t}) + M_{t+1}^d + \phi_1 X_{t+1}^d + \phi_2 B_{t+1}^{dh} \leq$$

$$W_t n_t^s + M_t^d + (1 - \phi_1) J_t + (1 + r_{xt}) \phi_1 X_t^d + (1 + r_{bt}) \phi_2 B_t^{dh} + \Pi_t^f + \phi_1 \Pi_t^{cb}, \quad \phi_1, \phi_2 \in \{0, 1\} \quad (2)$$

where  $P_t$  is the money price of goods,  $W_t$ , is the money wage,  $J_t$  is a lump-sum, *per capita* monetary transfer from the government, and  $r_{xt}$  and  $r_{bt}$  are deposit and bond rates, respectively. Note that in models (1)-(4), there is no bank (and no bank profits), and monetary injections are direct transfers to households rather than reserves injections to the banks, as is the case in model (5). This implies that the deposit good and deposit balances are zero, and  $J_t > 0$ . In model 1, there are no financing constraints, implying  $B^{dh}$  is zero. This is modeled by setting  $\phi_2$  to zero for model 1, and to one otherwise. The household therefore allocates its nominal labor income,  $W_t n_t^s$ , beginning of period post-transfer nominal money balances,  $M_t^d + (1 - \phi_1) J_t$ , initial deposit balances and interest income on deposits,  $(1 + r_{xt}) \phi_1 X_t^d$ , and the principal plus interest on previous bond investments,  $(1 + r_{bt}) \phi_2 B_t^{dh}$  to nominal consumption  $P_t(c_{1t} + \phi_1 c_{2t})$  and its asset portfolio positions,  $M_{t+1}^d, \phi_1 X_{t+1}^d, \phi_2 B_{t+1}^{dh}$ , which are carried over to next period.

The household faces two liquidity constraints. The first is that nominal consumption purchases of the cash good are constrained by the household's initial money balances, which include the monetary transfer in models (1)-(4).

$$P_t c_{1t} \leq M_t^d + (1 - \phi_1) J_t, \quad \phi_1 \in \{0, 1\} \quad (3)$$

The second liquidity constraint limits nominal consumption purchases of the deposit good by the stock of the household's deposit balances carried over from the previous period.<sup>3</sup>

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<sup>3</sup> While the cash-in-advance constraint, equation (3), is well entrenched in the literature, the "deposit-in-advance" constraint, equation (4), is less common. Recent examples of its use are Edwards and Vegh (1997) and Hartley (1998).

$$P_t c_{2t} \leq X_t^d \quad (4)$$

where equation (4) is binding only in model (5).

Lastly, the household faces a time constraint on its labor/leisure decision.

$$n_t^s + l_t \leq 1 \quad (5)$$

## II.2 Recursive representation of the household's optimization problem.

To ensure stationarity, nominal variables in the model will be normalized throughout by the nominal money supply,  $M_t$ . Define  $p_t = P_t/M_t$ ,  $m_t^d = M_t^d/M_t$ ,  $x_t^d = X_t^d/M_t$ ,  $b_t^{dh} = B_t^{dh}/M_t^d$ ,  $w_t = W_t/M_t$ ,  $j_t = J_t/M_t$ ,  $\pi_t^f = \Pi_t^f/M_t$ , and  $\pi_t^{cb} = \Pi_t^{cb}/M_t$ . Then, the household problem becomes:

$$\max_{\{c_{1t}, c_{2t}, n_t^s, l_t, m_{t+1}^d, x_{t+1}^d, b_{t+1}^{dh}\}} \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, l_t; \phi_1) \quad (6)$$

given:  $m_0^d$ ,  $x_0^d$ , and  $b_0^{dh}$ , subject to:

$$p_t(c_{1t} + \phi_1 c_{2t}) + G_{t+1}(m_{t+1}^d + \phi_1 x_{t+1}^d + \phi_2 b_{t+1}^{dh}) \leq$$

$$w_t n_t^s + m_t^d + (1 - \phi_1) j_t + (1 + r_{xt}) \phi_1 x_t^d + (1 + r_{bt}) \phi_2 b_t^{dh} + \pi_t^f + \phi_1 \pi_t^{cb} \quad (7)$$

$$p_t c_{1t} \leq m_t^d + (1 - \phi_1) j_t \quad (8)$$

$$p_t c_{2t} \leq x_t^d \quad (9)$$

$$n_t^s + l_t \leq 1 \quad (10)$$

The household's problem can now be set up as a dynamic programming problem. Dropping the time subscripts and using the prime ( $'$ ) notation to denote next's period's



values, define the value function as  $v(\mathbf{s}^h)$ , where the household's state vector is given by  $\mathbf{s}^h = [m^d, x^d, b^{dh}, \mathbf{S}]$ , where  $\mathbf{S}$  is the aggregate state vector, defined below. Given the initial state,  $\mathbf{s}_0^h$ , the Bellman equation for this problem becomes:

$$v(\mathbf{s}^h) = \sup_{\lambda^h(\mathbf{s}^h) \in \Gamma^h(\mathbf{s}^h)} [u(c_1, c_2, l; \phi_1) + \beta v(\mathbf{s}^{h'})], \quad (11)$$

where:  $\lambda^h(\mathbf{s}^h) = [c_1(\mathbf{s}^h), c_2(\mathbf{s}^h), n^s(\mathbf{s}^h), l(\mathbf{s}^h), m^{d'}(\mathbf{s}^h), x^{d'}(\mathbf{s}^h), b^{dh'}(\mathbf{s}^h)]$  is the vector of household decision rules drawn from the feasible set of correspondences,  $\Gamma^h(\mathbf{s}^h)$  defined by equations (7)-(10).

Using the envelope conditions, the first-order conditions yield the following set of Euler equations.

$$\beta(u'_{c_1}/p') = G'(u_l/w) \quad (12)$$

$$\beta[r'_x(u'_l/w') + u'_{c_2}] = G'(u_l/w) \quad (13)$$

$$\beta(1 + r'_b)(u'_l/w') = G'(u_l/w) \quad (14)$$

where  $G' = M'/M$ . Equations (12)-(14) have the interpretation of equating the marginal cost of foregoing a unit of leisure in exchange for the marginal benefits of a unit of labor, where the additional labor income is saved as money in equation (12), deposits in equation (13), and bonds in equation (14). Note that in model (1), firms do not issue bonds, implying that there is no bond market, and that equation (14) is therefore dropped from the model. Also note that in models (1)-(4), there is no bank, and hence deposits are zero. In this case, equation (13) is dropped from the model.

### *II.3 Firm sector.*

The firm sector is assumed to be perfectly competitive, and comprised of a large

number of identical firms, which for simplicity is set equal to the number of households.<sup>4</sup> Ignoring agency costs, the representative firm is assumed to act in the interest of its owners. Its objective is to maximize the present discounted value of the stream of dividends, or period profits.

$$\max_{\{k_{t+1}, n_t^d, B_{t+1}^s\}} \sum_{t=0}^{\infty} \beta^{t+1} (u_{c_{1t+1}}/P_{t+1}) \Pi_t^f \quad (15)$$

where  $k_t$  is the firm's capital stock,  $B_t^s$  is the nominal stock of the firm's one-period bonds maturing at date  $t$ , and with period profits given by:

$$\Pi_t^f = P_t F(k_t, n_t^d) - \phi_2 (1 + r_{bt}) B_t^s - \phi_3 P_t [k_{t+1} - (1 - \delta) k_t] - \phi_4 W_t n_t^d, \quad \phi_2, \phi_3, \phi_4 \in \{0, 1\} \quad (16)$$

The firm's revenues equal nominal sales, which are represented by  $P_t F(k_t, n_t^d)$ , with output given by the production function,  $F : \mathfrak{R}_{++}^2 \rightarrow \mathfrak{R}_+$ , which is continuous and twice-differentiable, with  $F_1, F_2, F_{12} > 0$  and  $F_{11}, F_{22} < 0$ . The firm's expenses depend on the financing requirements.

$$(1 - \phi_3) P_t [k_{t+1} - (1 - \delta) k_t] + (1 - \phi_4) W_t n_t^d \leq \phi_2 B_{t+1}^s, \quad (17)$$

In model (1), there is no financing constraint,  $\phi_2 = 0$ , and both gross investment and the wage bill are financed out of current revenues, which corresponds to setting the indicator variables  $\phi_3 = \phi_4 = 1$ . In model (2), the wage bill is financed with bond issues, and gross investment is financed with current period revenues, or  $\phi_2 = \phi_3 = 1$  and  $\phi_4 = 0$ . In model (3), gross investment is financed with bonds, and the wage bill is financed out of current revenues, or  $\phi_2 = \phi_4 = 1$  and  $\phi_3 = 0$ . In models (4) and (5), the firm's entire working capital expenses, i.e., both its gross investment and the wage bill, are financed with bonds, implying  $\phi_2 = 1$  and  $\phi_3 = \phi_4 = 0$ . In models (2)-(5), the firm must retire the bonds issued in the previous period by using current period revenues to pay the principal and interest, based on the current period interest rate,  $r_{bt}$ .

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<sup>4</sup> The reason for not modeling the firm sector by a single large firm is to enable a cleaner presentation of the dynamic programming problem with separate firm and aggregate state variables.

Note that nominal profits,  $\Pi_t^f$ , are paid out as dividends each period in currency that households must hold one period before spending them, say, for cash goods, due to the liquidity constraint, equation (3). In the interim, prices may change such that each unit of currency received in period  $t$  buys  $1/P_{t+1}$  units of consumption goods in period  $t+1$ , which are, in turn, valued at date  $t+1$  at the marginal utility  $u_{c_{1t+1}}$ , and the total is discounted back one period at the rate given by the discount factor,  $\beta$ , to determine its present value.

Given the firm's stock of capital,  $k_0$ , and the outstanding stock of one-period bonds,  $B_0^s$ , the firm chooses its demand for labor, or the sequence  $\{n_t^d\}$ , makes its gross investment decision by choosing next period's capital stock, or the sequence  $\{k_{t+1}\}$ , and the quantity of bonds needed for financing its working capital, given by the sequence  $\{B_{t+1}^s\}$ .

#### II.4 Recursive representation of the firm's optimization problem.

To normalize the nominal variables by  $M_t$ , define  $b_t^s = B_t^s/M_t$ , and the firm's problem becomes:

$$\max_{\{k_{t+1}, n_t^d, b_{t+1}^s\}} \sum_{t=0}^{\infty} \beta^{t+1} (u_{c_{1t+1}}/p_{t+1})(1/G_{t+1})\pi_t^f \quad (18)$$

given:  $k_0$  and  $b_0^s$ , with the firm's normalized period profits given by:

$$\pi_t^f = p_t F(k_t, n_t^d) - \phi_2(1 + r_{bt})b_t^s - \phi_3 p_t [k_{t+1} - (1 - \delta)k_t] - \phi_4 w_t n_t^d \quad (19)$$

subject to the financing constraint:

$$(1 - \phi_3)p_t [k_{t+1} - (1 - \delta)k_t] + (1 - \phi_4)w_t n_t^d \leq \phi_2 b_{t+1}^s, \quad (20)$$

For a recursive representation of the firm's problem, using the prime ( $'$ ) notation, define the firm's value function as  $V(\mathbf{s}^f)$ , where the firm's state vector is given by  $\mathbf{s}^f = [k, b^s, \mathbf{S}]$ . Bellman's equation becomes:

$$V(\mathbf{s}^f) = \sup_{\lambda^f(\mathbf{s}^f) \in \Gamma^f(\mathbf{s}^f)} [R + \beta V(\mathbf{s}^{f'})], \quad (21)$$

where the one-period return function is  $R = \beta(u'_{c1}/p')(1/G')\pi_t^f$  and the vector of the firm's decision rules, denoted  $\lambda^f(\mathbf{s}^f) = [k'(\mathbf{s}^f), n^d(\mathbf{s}^f), b^{s'}(\mathbf{s}^f)]$ , is drawn from the feasible set of correspondences,  $\lambda^f(\mathbf{s}^f)$  defined by the financing constraint equation (20).

The Euler equations for the firm's problem depend on the method of financing its working capital. For model 1, all working capital is financed out of current revenues,  $\phi_2 = 0$  and  $\phi_3 = \phi_4 = 1$ , and the Euler equations become:

$$\beta^2(u''_{c1}/p'')(1/G'')p'[F'_k + (1 - \delta)] = \beta(u'_{c1}/p')(1/G')p \quad (22)$$

$$pF_n = w \quad (23)$$

Equations (22) and (23) are the efficiency conditions on the capital investment and labor decisions of the firm. The, say, utility loss to the household of foregoing current period profits sufficient to purchase one unit of capital at the normalized price of  $p$ , is given by (after discounting) the right-hand side of (22). The left-hand side is the net benefit to the household of this investment, which consists of two terms. The first is the discounted value of additional output made possible in the next period, and the second is the discounted value of the additional units of undepreciated capital stock remaining (or the amount by which the firm could reduce output next period and shift revenues toward increasing its dividend payout). At the margin, the firm is indifferent between making the investment and raising dividends, implying that equation (22) holds with equality. Equation (23) has a similar interpretation, albeit this is not a dynamic choice and both costs and benefits of employing the marginal unit of labor have the same discount factor that appears on both sides of (23), and thereby cancels. The firm equates the marginal revenue product of labor in the current period, given on the left-hand side, to the marginal factor cost, or wage rate on the right-hand side.

For model 2, the firm finances its gross investment out of current revenues (retained earnings),  $\phi_3 = 1$ , and finances its wage bill by issuing one-period bonds,  $\phi_2 = 1$  and  $\phi_4 = 0$ . In this case, the Euler equation on the capital investment decision, (22), remains unchanged; however, the employment decision now reflects the costs of financing, and

becomes intertemporal. Equation (23) is thus replaced by equation (24).

$$\beta(u'_{c1}/p')(1/G')pF_n = \beta^2(u''_{c1}/p'')(1/G'')(1+r'_b)w \quad (24)$$

The right-hand side of (24) reflects the present discounted value of the cost of employing one unit of labor, where  $w$  is the (normalized) price of labor, each unit of which is financed by bonds that mature one period hence. Note that the value of the claim against the firm that is represented by the bond incorporates a two-period discount, since the bond must be held one period, then the currency received by the household cannot be used for another period due to the liquidity constraint. The left-hand side of (24) is the present discounted value of the additional dividends that the firm can pay out in the current period due to the increase in revenues from the higher output associated with the increase in employment. The costs and benefits must be equal at the margin, implying that equation (24) holds with equality.

For model 3, the firm finances its wage bill out of current revenues,  $\phi_4 = 1$ , and finances its gross investment from funds raised by issuing one-period bonds,  $\phi_2 = 1$  and  $\phi_3 = 0$ . In this case, the optimal employment decision is identical to model 1, and given by the Euler equation (23). The gross investment decision must reflect the financing costs, and appears as equation (25):

$$\beta^2(u''_{c1}/p'')(1/G'')p'F'_k + \beta^3(u'''_{c1}/p''')(1/G''')(1+r''_b)p'(1-\delta) = \beta^2(u''_{c1}/p'')(1/G'')(1+r'_b)p \quad (25)$$

The right-hand side of (25) has an interpretation that is analogous to that given for the right-hand side of (24), where the firm is financing the marginal unit of capital investment at the normalized price of  $p$ . The left-hand side of (25) gives the total present value of the benefits to the marginal investment unit, with the first term reflecting the increase in potential dividend payout next period due to the greater output and revenues. The second term corresponds to the value of the additional undepreciated capital stock, that enables the firm in the next period to reduce its investment, along with the financing costs. The three period discount reflects the fact that the additional capital stock reduces next

period's financing requirements.

In models 4 and 5, both the wage bill and gross investment are financed by issuing one-period bonds. In these cases, the appropriate Euler equations for capital investment and employment must reflect these financing costs, and are thus given by equations (24) and (25) respectively.

### II.5 Commercial banking sector.

In model 5, the commercial bank, standing in for a perfectly competitive industry, is introduced to provide valued liquidity services in the form of demand deposit accounts, and to intermediate loans between households and firms. Like firms, the bank is owned by households and, in the absence of agency costs, its objective is to maximize the present discounted value of the stream of dividends, or nominal period profits *per capita*,  $\Pi_t^{cb}$ .

$$\max_{\{Z_{t+1}, B_{t+1}^{db}, X_{t+1}\}} \sum_{t=0}^{\infty} \beta^{t+1} (u_{c_{1t+1}}/P_{t+1}) \Pi_t^{cb} \quad (26)$$

where  $Z_t$  is the bank's *per capita* reserves,  $B_t^{db}$  is the *per capita* stock of bonds purchased from the firm by the bank, and  $X_t$  is its *per capita* stock of deposits. The bank profits are given by its net cash flow:

$$\Pi_t^{cb} = (1 + r_{bt})B_t^{bd} + Z_t - (1 + r_{xt})X_t - \xi X_t, \quad \xi > 0 \quad (27)$$

with cash inflow equal to the principal plus interest received on bonds,  $(1 + r_{bt})B_t^{bd}$ , plus its reserve holdings  $Z_t$  less the principal and interest paid on deposits,  $(1 + r_{xt})X_t$ , less its cost of servicing deposits  $\xi X_t$ , with  $\xi$  the cost per currency unit of deposits.

The bank takes the initial balance sheet as given, or  $Z_0, B_0^{db}, X_0$ , and performs the maximization in (26) by choosing optimal sequences  $\{Z_{t+1}\}, \{B_{t+1}^{db}\}, \{X_{t+1}\}$  subject to its reserve requirements and balance sheet constraints:

$$Z_t = \zeta X_t, \quad \zeta \in (0, 1) \quad (28)$$

$$Z_t + B_t^{db} \leq X_t \quad (29)$$

where  $\zeta$  is the reserve ratio applied to deposits.

The simplifying assumption that the bank pays out its entire net cash flow each period as dividends renders the bank's problem a static one-period optimization. This problem can be written in prime ( $'$ ) notation with normalized variables by defining  $z = Z/M$ ,  $b^{db} = B^{db}/M$ , and  $x = X/M$ .

$$\max_{z', b^{db'}, x'} \pi^{cb'} \quad (30)$$

where:

$$\pi^{cb'} = (1 + r_b')b^{bd'} + z' - (1 + r_x')x' - \xi x', \quad (31)$$

subject to:

$$z' = \zeta x' \quad (32)$$

$$z' + b^{db'} \leq x' \quad (33)$$

The first-order condition to this problem is:

$$(1 + r_x') = (1 - \zeta)(1 + r_b') + \zeta - \xi \quad (34)$$

which establishes the spread between the loan (or bond) rate and the deposit rate that ensures bank profits are dissipated through competition.

### III. Equilibrium

This section defines the equilibrium and identifies the set of equations that must be solved to obtain the steady-state values for each of the five models described above.

#### III.1 Defining equilibrium.

Define the aggregate state vector as  $\mathbf{S} = [K, b, z, x]$ , where  $K$  and  $b$  are the aggregate *per capita* stocks of capital and bonds. Then, aggregate decision rules corresponding to the decision rules of the representative household and representative firm can

be defined as  $\Lambda^h(\mathbf{S}) = [C_1(\mathbf{S}), C_2(\mathbf{S}), N^s(\mathbf{S}), L(\mathbf{S}), \hat{m}^{d'}(\mathbf{S}), \hat{x}^{d'}(\mathbf{S}), b^{\hat{d}h'}(\mathbf{S})]$  and  $\Lambda^f(\mathbf{S}) = [K'(\mathbf{S}), N(\mathbf{S}), b'(\mathbf{S})]$  respectively, where  $C_1, C_2, N^s, L, \hat{m}^{d'}, \hat{x}^{d'}, b^{\hat{d}h'}, K', N,$  and  $b'$  are aggregate *per capita* variables. To construct the equilibrium, a state dependent monetary policy rule needs to be specified. Since this paper is interested in the effects of steady-state inflation on welfare under various financing constraints on firms' working capital expenses, the gross monetary growth rate is taken as a constant across states, or  $G'(\mathbf{S}) = G > \beta$ . The inequality ensures that money will be valued in equilibrium. Note that for model 5, where monetary policy is conducted through reserves injections into the banking system, the monetary rule implies that  $G$  also equals the gross growth rate of bank reserves.

A *recursive competitive equilibrium* is defined by the value functions  $v(\mathbf{s}^h)$  and  $V(\mathbf{s}^f)$ , the household decision rules,  $\lambda^h(\mathbf{s}^h)$ , the firm's decision rules,  $\lambda^f(\mathbf{s}^f)$ , the corresponding aggregate decision rules,  $\Lambda^h(\mathbf{S})$  and  $\Lambda^f(\mathbf{S})$ , the pricing functions,  $p(\mathbf{S}), w(\mathbf{S}), r_b(\mathbf{S}),$  and  $r_x(\mathbf{S})$ , and the policy rule,  $G'(\mathbf{S})$ , that satisfy:

- (i) household optimization, equations (12)-(14);
- (ii) firm optimization, equations (22)-(23) for model 1, (22) and (24) for model 2, (23) and (25) for model 3, and (24)-(25) for models 4 and 5;
- (iii) commercial bank optimization, equation (34) in model (5);
- (iv) liquidity constraints, equation (8) and in model 5, equation (9);
- (v) the time resource constraint, equation (10);
- (vi) the firm's financing constraint, equation (20) in models (2)-(5);
- (vii) the bank's reserve requirements, equation (32) in model (5);
- (viii) the bank's balance sheet constraint, equation (33) in model (5);
- (ix) aggregate consistency conditions, or  $\lambda^h(\mathbf{s}^h) = \Lambda^h(\mathbf{S})$  and  $\lambda^f(\mathbf{s}^f) = \Lambda^f(\mathbf{S}), \forall \mathbf{S}$ ; and
- (x) equilibrium conditions in the goods, labor, money, bond [in models (2)-(5)], and deposit [in model (5)] markets:  $C_1(\mathbf{S}) + C_2(\mathbf{S}) + K'(\mathbf{S}) - (1 - \delta)K = F(K, N(\mathbf{S})), N^s(\mathbf{S}) = N(\mathbf{S}), \hat{m}^d = 1, b^{\hat{d}h} + b^{db} = b,$  and  $\hat{x}^d = x$ .



### *III.2 Steady-state equilibria.*

Each of the models described above has been rendered stationary by normalizing the nominal variables by the nominal money supply. This implies that the steady-state equilibria will be characterized by constants for the consumption bundle, labor, leisure, and normalized asset stocks. The equilibria are then found as the solutions to the following sets of equations, when modified by appropriately toggling the indicator variables,  $\phi_1 - \phi_4$ .

From the household sector:

$$pC_1 = 1 \quad (35)$$

$$pC_2 = x \quad [\text{for model (5)}] \quad (36)$$

$$N + L = 1 \quad (37)$$

$$\beta(u_{C1}/p) = G(u_L/w) \quad (38)$$

$$\beta[r_x(u_L/w) + u_{C2}] = G(u_L/w) \quad [\text{for model (5)}] \quad (39)$$

$$\beta(1 + r_b) = G \quad [\text{for models (2)-(5)}] \quad (40)$$

From the firm sector:

$$wN = b \quad [\text{for model (2)}] \quad (41)$$

$$p\delta K = b \quad [\text{for model (3)}] \quad (42)$$

$$p\delta K + wN = b \quad [\text{for models (4)-(5)}] \quad (43)$$

$$\beta[F_K + (1 - \delta)] = 1 \quad [\text{for models (1) and (2)}] \quad (44)$$

$$pF_N = w \quad [\text{for models (1) and (3)}] \quad (45)$$

$$pF_N = \beta(1 + r_b)w \quad [\text{for models (2), (4), and (5)}] \quad (46)$$

$$F_K + \beta(1 + r_b)(1 - \delta) = (1 + r_b) \quad [\text{for models (3), (4), and (5)}] \quad (47)$$

From the banking sector:

$$z = \zeta x \quad [\text{for model (5)}] \quad (48)$$

$$z + b^{db} = x \quad [\text{for model (5)}] \quad (49)$$

$$(1 + r_x) = (1 - \zeta)(1 + r_b) + \zeta - \xi \quad [\text{for model (5)}] \quad (50)$$

From equilibrium conditions:

$$C_1 + \delta K = F(K, N) \quad [\text{for models (1)-(4)}] \quad (51)$$

$$C_1 + C_2 + \delta K = F(K, N) \quad [\text{for model (5)}] \quad (52)$$

$$b^{\hat{d}h} + b^{db} = b \quad [\text{for model (5)}] \quad (53)$$

Note that in all models, the cash-in-advance constraint, equation (35), coupled with the definition of  $G$  as the gross growth rate of the money supply, implies that the steady-state inflation rate is also equal to  $G$ . That is, from (35),  $p = 1/C = \text{constant}$  in the steady-state. This implies that the money price of goods and the nominal money supply are growing at the same rate, i.e., at the gross inflation rate,  $P'/P = G$ .

#### IV. Calibration and computation of the steady-state.

To perform the calibration, the economies studied are specialized with common preferences and technology. The utility function is assumed to be loglinear, with the indicator variable  $\phi_1 = 1$  for model 5, and zero otherwise.

$$u(C_1, C_2, L; \phi_1) = \ln C_1 + \eta \phi_1 \ln C_2 + \gamma \ln L, \quad \eta, \phi > 0, \quad \phi \in \{0, 1\} \quad (54)$$

The marginal utilities of consumption of the cash and deposit goods and of leisure are thus given by  $u_{C_1} = 1/C_1$ ,  $u_{C_2} = \eta \phi_1 / C_2$ , and  $u_L = \gamma / L$ , respectively.

A Cobb-Douglas production function is assumed for all models.

$$F(K, N) = K^\alpha N^{1-\alpha}, \quad \alpha \in (0, 1) \quad (55)$$

The marginal products of capital and labor are respectively  $F_K = \alpha K^{\alpha-1} N^{1-\alpha}$  and  $F_N = (1 - \alpha) K^\alpha N^{-\alpha}$ .

In each of the models (1)-(4), there are four parameters to be determined:  $\beta, \gamma, \delta$ , and  $\alpha$ , along with the exogenously set policy variable,  $G$ . To determine these values, five conditions must be imposed from the data. First,  $\alpha$  is taken as capital's share of income, and computed from the Japanese National Income and Product Accounts as the sample (quarterly) average over the period 1970:1 to 1996:1 to be 0.3916.<sup>5</sup> Second, the investment-output ratio is fixed at 0.2114, which is the quarterly average over the same sample period, where investment is defined as gross private domestic investment. Third, the steady-state value of  $N$  is set to 0.336, which corresponds to a 40 hour workweek. Fourth, the policy parameter  $G$  is set equal to 1.010752, which is the average quarterly gross inflation rate over the sample period.<sup>6</sup> Finally, for consistency, equation (40) is used

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<sup>5</sup> Following Cooley and Prescott (1996), a share of income to the self-employed attributed to labor is imputed from the data to be identical to that for publicly-held firms. The following equation is used to determine  $\alpha$  in each period based on the items in the NIPAs. These values are then averaged over the sample. Compensation of employees [item 3.3] +  $(1 - \alpha)$ (Entrepreneurial Income [item 3] + Indirect taxes and subsidies [item 5] + Income of public enterprises [item 3.2]) =  $(1 - \alpha)$  GNP.

<sup>6</sup> This inflation rate is based on the monthly Consumer Price Index. Source: *Monthly*

to determine a value for the discount factor  $\beta$  that was used throughout all five models. Given  $G$ , it is found by setting  $r_b$  to 0.01174. This value is obtained by computing the average quarterly interest rate paid on ordinary deposits over the (monthly) sample period 1971:02-1996:01, and adding to that figure, the average spread of the (backward three-month moving-average) of the 3-month commercial paper rate (primary market) over the deposit rate for the period 1989:01-1996:11, which is the only period for which these data are available.<sup>7</sup>

For model 5, there are three additional parameters:  $\eta$ ,  $\zeta$ , and  $\xi$ . The three additional restrictions from the data that are used in the calibration of the model are: (1) the average deposit rate,  $r_x$ , is computed directly as described above; (2) the average currency-deposit ratio over the sample period, where deposits are taken as the non-currency components of M1, is set equal to  $M/X$  or equivalently  $1/x$ ; and (3) the average ratio of bank reserves to total deposits (as defined above) over the (monthly) sample period 1970:01-1997:08, is set equal to the reserve ratio, or  $\zeta = 0.041063$ .<sup>8</sup>

The above data restrictions enable the models to be solved in the steady-state under the “benchmark” inflation policy,  $G$ , which corresponds to the average policy over the 1970:01-1997:08 period. Model 1 consists of the six equations (35), (37)-(38), (44)-(45), and (51), and six endogenous variables:  $C_1, N, L, K, p$  and  $w$ . Model 2 is comprised of the eight equations (35), (37)-(38), (40)-(41), (44), (46), and (51), and in addition to the six endogenous variables of model 1:  $b$  and  $r_b$ . Model 3 also consists of eight equations (35), (37)-(38), (40), (42), (45), (47), and (51), and the same set of endogenous variables as in model 2. Model 4 is made up of the eight equations (35), (37)-(38), (40), (43), (46)-(47), and (51) in the same set of eight endogenous variables as in models 2 and 3. Model 5 is comprised of the 14 equations (35)-(40), (43), (46)-(50), and (52)-(53), with the following six endogenous variables added to the list from models (2), (3), and (4):  $C_2, b^{\hat{d}h}, b^{db}, z, x,$

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*Report on the Consumer Price Index*, Statistics Bureau, Management and Coordination Agency.

<sup>7</sup> Source: *Economic Statistics Monthly*, Research and Statistics Department, Bank of Japan.

<sup>8</sup> Source: *Economic Statistics Monthly*, Research and Statistics Department, Bank of Japan.

and  $r_x$ . The steady-state values for each of these models are displayed in Table 1.

[Insert Table 1.]

From Table 1, it is noteworthy that the steady-state values for the five models are nearly identical, with the exception of the size of the bond market. Comparing the quantity of real bonds that the firm must issue to meet its financing requirements in models (2)-(4), it is evident that under these calibrations, roughly 3/4 of the firm's working capital expenses is comprised of its wage bill. Also note that the steady-state equilibria of models (4) and (5) are identical in all respects other than the fact that in model (5) the household finances only 30% of the firm's working capital requirements directly. The bank provides the balance of the firm's financing with funds that it raises by attracting household savings into its bank deposit offerings.

## **V. Quantifying the welfare cost of inflation.**

In this section, the welfare costs of moderate inflation rates are quantified for each of the five models. The procedure follows Cooley and Hansen (1989), where the deep parameters of the model determined in the calibration are held fixed, and the inflation policy is altered over a range of values for the steady-state gross inflation rate,  $G$ . The new steady-states are determined under the alternative policies, and welfare losses are measured by comparing lifetime utility under an alternative policy with that under the benchmark steady-state. Specifically, the welfare loss associated with an increase in  $G$  above the benchmark inflation is measured as the percent increase in period consumption that the household would require under the higher inflation regime to be indifferent between it and the benchmark policy.

The purposes of conducting these exercises are twofold. First, the exercises demonstrate how important financing constraints can be in determining the welfare costs of inflation. As discussed below, their effect is very pronounced. Second, comparisons between models (4) and (5) illustrate how significant reductions in the welfare costs of inflation can be realized when banks intermediate the loans between households and firms while providing valued liquidity services to households in the form of demand deposit accounts.

### *V.1 Financing constraints and the welfare costs of inflation.*

Model 1 is the standard cash-in-advance economy that has been analyzed extensively in the literature. One feature of the model is that fully anticipated inflation acts as a tax on consumption purchases. As the inflation tax increases, households seek to reduce their consumption expenditures, and they partially offset the accompanying utility loss by increasing leisure time. This additional leisure comes at the expense of labor, and employment and hence output fall. The magnitude of these effects has been documented by Cooley and Hansen (1989) in a model calibrated to fit the U.S. post-War economy.<sup>9</sup> They find the measured welfare costs of this inflation tax distortion to be significant, but not excessive for moderate inflations. For example, increasing the inflation rate from 0 to 10% resulted in a welfare loss of a 0.376% reduction in period consumption. Similar relatively low costs are found for model 1 when calibrated to the Japanese economy. As seen in Table 2, column 1, an increase in the inflation rate from the benchmark value of 4.3% to 10% results in an estimated loss of welfare that corresponds to a 0.226% reduction in period consumption.<sup>10</sup> From Table 3, column 1, this welfare loss corresponds to a reduction of 0.92% in investment (from 2.0380 to 2.0191) and a decline of 0.93% in both employment (from 0.3360 to 0.3329) and output (from 9.6406 to 9.5512).

These welfare costs of inflation increase considerably when financing constraints are imposed on the working capital expenses of the firm. As previously discussed, higher inflation results in a higher bond rate that raises the firm's financing costs. This further retards investment and reduces employment, as output declines. The magnitude of these losses varies with the incidence of the "tax." This is illustrated in Table 2, by the figures reported in the bottom row of columns 3, 4, and 5. When the firm finances its wage bill with one-period bonds and its gross investment out of current period sales revenues, model

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<sup>9</sup> Their model differs from model 1, in that they include labor indivisibilities as described in Rogerson (1988) and Hansen (1985).

<sup>10</sup> The welfare loss is computed by obtaining steady-state values for consumption, and leisure under the benchmark case, denoted  $\bar{c}_1, \bar{c}_2, \bar{l}$ , and again under the alternative inflation policy, denoted  $\tilde{c}_1, \tilde{c}_2, \tilde{l}$ , and then solving the following equation for  $\phi_{ch}$ :  $\sum_{t=0}^{\infty} u(\bar{c}_1, \bar{c}_2, \bar{l}) = \sum_{t=0}^{\infty} u[(1 + \phi_{ch})\tilde{c}_1, (1 + \phi_{ch})\tilde{c}_2, \tilde{l}]$ . The welfare costs reported in Table 2 are then given by  $100\% \times \phi_{ch}$ .

2 (column 2), an increase in the inflation rate from the benchmark 4.3% to 10% results in a more than doubling of the welfare losses relatively to the case of no financing constraints, model 1 (column 1), i.e., from 0.226% to 0.477%. The corresponding percentage declines in investment (from 2.0380 to 2.0003), employment (from 0.3360 to 0.3298), and output (from 9.6406 to 9.4622) are also seen essentially to double to 1.85%. Alternatively, when gross investment is financed with bond issues, while the wage bill is financed out of current sales revenues, model 3, the welfare losses increase to a 0.814 % reduction in period consumption, which is more than 3-1/2 times that of model 1. The additional distortion in the firm's factor employment decisions is reflected quantitatively in Table 3, where production is seen to become more labor intensive, with investment declining by 3.42 % (from 2.0076 to 1.9389), while employment falls by 1.16 % (from 0.336 to 0.3298), and output drops by 2.06% (from 9.4965 to 9.4622). In this case, even though investment represents only about one-quarter of the firm's working capital expenses, the effective tax on capital has a more adverse effect on welfare than when this effective tax applies only to labor. As expected, applying the financing constraint to both the wage bill and gross investment, model 4, further increases the welfare losses. They are seen in column 4 to rise to the sizable figure of 1.068 % loss in period consumption, which is nearly 4-3/4 times the losses computed for model 1, when no financing constraints apply. Investment falls by 4.32 %, employment declines by 2.08 %, and output is reduced by 3.19 %.

[Insert Tables 2 and 3.]

## *V.2 Financial intermediation and the welfare costs of financing working capital.*

When the firm's financing requirements expand, the size of the bond market grows accordingly. Again noting that roughly 3/4 of the firm's working capital expenses go to pay the wage bill, it is evident that the bond market would be about three times larger in model 2 than in model 3, and about four times larger in model 4 than in model 3. These figures are borne out in Table 3. In each of these cases, there was no financial intermediary, that is, lending was direct from households to firms. In model 5, a bank is introduced through which a portion of these loans is intermediated. The extent to which banks are able to

provide financing to firms is limited by the demand by households for its demand deposit account offerings. This demand for bank deposits arises out of both the pecuniary return and the liquidity services that they yield. Because the liquidity services are valued by households, the deposit rate can be held below the bond rate, even when these liquidity services are costly to the bank to provide. In this case, households can respond to an increase in the inflation rate by relying more on interest-bearing bank deposits for their transactions and less on cash, thereby reducing the distortionary effects of the inflation tax, and the welfare losses are lower than they would otherwise be in the absence of the bank.

Refer to Table 2, columns 4 and 5. By introducing the bank into the model, the welfare losses associated with an increase in the inflation rate from the benchmark 4.3% to 10% are seen to fall by 16.87% (from 1.0683 % loss in period consumption to 0.8881 %). This sharp decline is accompanied by smaller reductions in investment of 3.67% (from 2.0076 to 1.9340) versus 4.32%, in employment of 1.43% (from 0.336 to 0.331) versus 2.08%, and in output of 2.31 % (from 9.4965 to 9.2775) versus 3.19%. Note that the financing requirements of the firm are higher under the 10% inflation regime when banks intermediate a portion of the loans. That is, with an increase in the inflation rate to 10% from 4.3%, the size of the bond market (in real terms) declines by 3.67 % in model 5, versus 4.32% in model 4. This result is simply due to the fact that output and hence the working capital expenses of the firm are not as adversely affected by inflation in model 5. Moreover, note that the shift in the short-term liquid asset holdings of households lowers the currency-deposit ratio, and that this portfolio adjustment of households results in an increase in the share of loans to firms that are intermediated by the bank, which rises from 70.66% to 72.15%, or by about 1.5%. Therefore, a by-product of inflation is a tendency for the size of the banking sector to expand.<sup>11</sup>

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<sup>11</sup> This result is consistent with those of Ireland (1994) and Marquis and Reffett (1994), whereby an increase in inflation induces a shift of resources into the financial services sector, and generates welfare losses.



## VI. Conclusions.

*Per capita* holdings of highly liquid monetary assets that carry a negative real return are very high in Japan. Inflation lowers this return and serves to tax transactions for which these assets are used. While individuals can avoid these taxes by actively managing their asset portfolios, the economy as a whole cannot. This suggests that high inflation may distort resource allocations sufficiently to induce significant welfare losses. The results in the previous section suggest that these welfare losses can become quite large for even moderate rates of inflation, when firms use short-term debt instruments to finance their working capital expenses.

However, when banks offer valued liquidity services in the form of interest-bearing demand deposit accounts, households are able to shield themselves somewhat from inflation, and the accompanying distortions in resource allocations are reduced. As inflation increases, households rely more on bank deposits and less on currency for transaction purposes, and the banking sector as a share of output expands, as banks intermediate a larger share of working capital loans to firms. These results underscore the important role that financial intermediation can play in affecting the welfare consequences of inflation, and they suggest that extensions of this analysis to private information economies, in which banks perform roles as delegated monitors of risky short-term debt, or as efficient players in risk-transferring financial markets with high participation costs, would be a fruitful avenue for future research.

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**Table 1: Benchmark settings**

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benchmark gross inflation rate

$G = 1.10752$

parameters:

$\beta = 0.999021613$ , for all models

$\alpha = 0.3916$ , for all models

$$\delta = \begin{cases} 0.0011489, & \text{for models (1) and (2)} \\ 0.0017610, & \text{for models (3), (4), and (5)} \end{cases}$$

$$\gamma = \begin{cases} 1.50692, & \text{for models (1) and (3)} \\ 1.49089, & \text{for models (2) and (4)} \\ 6.21135, & \text{for model (5)} \end{cases}$$

$\eta = 3.15503$ , for model (5)

$\zeta = 0.041063$ , for model (5)

$\xi = 0.007693$ , for model (5)

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steady-state values

variables	model 1	model 2	model 3	model 4	model 5
output, $y$	9.641	9.641	9.497	9.497	9.497
consumption, $C$	7.603	7.603	7.489	7.489	7.489
consumption, $C_1$					1.798
consumption, $C_2$					5.691
investment	2.038	2.038	2.008	2.008	2.008
capital, $K$	1773.869	1773.869	1706.971	1706.971	1706.971
employment, $N$	0.336	0.336	0.336	0.336	0.336
real bonds, $b/p$		5.803	2.008	7.724	7.724
household share of bonds, $b^{\hat{d}h}/b$					0.293
bank share of bonds, $b^{db}/b$					0.707

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**Table 2: Welfare costs of inflation**

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welfare losses measured as percent reduction in period consumption

inflation rate [ $100\% \times (G - 1)$ ]	model 1	model 2	model 3	model 4	model 5
3.00	-0.0505	-0.1039	-0.1850	-0.2389	-0.2008
4.30	0	0	0	0	0
5.00	0.0273	0.0566	0.0996	0.1292	0.1082
6.00	0.0666	0.1385	0.2421	0.3149	0.2634
7.00	0.1061	0.2214	0.3848	0.5017	0.4190
8.00	0.1458	0.3054	0.5277	0.6895	0.5750
9.00	0.1858	0.3905	0.6707	0.8784	0.7314
10.00	0.2259	0.4766	0.8138	1.0683	0.8881

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**Table 3: Steady-state effects of inflation**

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steady-state values at the benchmark 4.3% and 10% inflation

variables	model 1	model 2	model 3	model 4	model 5
<hr/>					
<u>output</u>					
4.30	9.6406	9.6406	9.4965	9.4965	9.4965
10.00	9.5512	9.4622	9.3009	9.2142	9.2775
<u>consumption</u>					
4.30	7.6025	7.6182	7.4890	7.4890	7.4890
10.00	7.5320	7.4619	7.3621	7.2934	7.3434
<u>consumption, cash good</u>					
4.30					1.7976
10.00					1.7447
<u>consumption, deposit good</u>					
4.30					5.6914
10.00					5.5987
<u>investment</u>					
4.30	2.0380	2.0380	2.0076	2.0076	2.0076
10.00	2.0191	2.0003	1.9389	1.9208	1.9340
<u>employment</u>					
4.30	0.3360	0.3360	0.3360	0.3360	0.3360
10.00	0.3329	0.3298	0.3321	0.3290	0.3312
<u>real bonds</u>					
4.30		5.8029	2.0076	7.7238	7.7238
10.00		5.6164	1.9389	7.3900	7.4407
<u>household share of bonds</u>					
4.30					0.2934
10.00					0.2785
<u>bank's share of bonds</u>					
4.30					0.7066
10.00					0.7215

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