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Abstract

Two policies toward payments-system risk are common, but superficially appear to be contradictory. One policy is to restrict the exposure to risk generated by one participant to other participants who are, by one measure or another, directly concerned with the risky participant. The other policy is to provide a ″safety net,″ typically provided by government and funded by taxes collected from all participants and even from non-participants, to share losses due to ″systemic risk." In this paper, we provide a model in which both of these policies can be constituents of an economically efficient regime of payments-risk management.

Key words: settlement system, risk sharing, private information, mechanism
design, core

JEL classification: D82, G18

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1 Introduction

Large-value payments are typically made through continuing, multi-party, contractual, clearing and settlement arrangements. During the past several decades, there has been progressively increasing awareness of the importance of risk management in such arrangements. Because a very large loss can potentially be incurred if settlement of a payment fails, how such a loss would be shared should be a matter of substantial concern for the participants in an arrangement. Moreover, to the extent that complete contingent-claims markets do not exist for insurance against settement failures and that there are political pressures for governments or central banks to assume losses from such failures, management of settlement risk is also a public policy issue.

Some specific questions regarding risk management in a settlement arrangement are the following. If there is some risk of failure to settle a payment from one party to another, should the payment be settled through that arrangement? (For example, in a net-settlement arrangement, what is the level of risk at which a payment ought to be made instead through an alternative, real-time-gross-settlement, arrangement?) If so, then what considerations are relevant to determing whether third parties ought to share that risk? Are there conditions under which the general public or the central bank (in the case of a private arrangement) ought to bear some risk and, if so, what level of compensation would it be appropriate for them to receive? If a third party possesses private information that would be of value in determining how best to settle a payment, how does the exposure of that party to the settlement risk affect the quality of information that the party chooses to provide? In this paper, we address these questions by analyzing a schematic, formal, model of a settlement arrangement.

Settlement-arrangement designers, managers, and policy makers are well aware that the rules governing an arrangement can affect users' decisions about which transactions to make through the arrangement. Thus, to set the rules of an arrangement is implicitly to decide which payments will be settled through it, and which payments people will decide to settle in alternative ways. (In fact, rules governing an arrangement that lacks stringent risk controls are sometimes designed deliberately to make the arrangement infeasible or unattractive for use in making very large-value payments.) By modelling the cooperative setting of rules by participants in a settlement arrangement, and by participants in the economy as a whole, from this perspective, we are able to analyze welfare questions in a conceptually satisfactory way. Rather than taking that approach of specifying transactions exogenously as previous researchers have typically done, what we take to be exogenous are traders' utility functions, which we specify in a way that provides scope for welfare-improving transactions among some of the traders to occur. We also specify a settlement technology that imputes risks and costs to those potential transactions. Having specified the model in these terms, we are able to characterize the patterns of transactions that the traders would cooperatively choose to make.

This approach provides answers, for the class of model economies that we study, to the questions posed above. Not surprisingly, risk considerations play a role in determining which payments ought to be made. Under some conditions, even the general public (that is, traders who would not have transactions with the members of the settlement arrangement if risk were not present) ought to share settlement risk, as can happen in practice when a central bank serves as guarantor of a settlement arrangement. Private information

regarding risk, even when it is possessed by a third party rather than by a direct party to a payment, is likely to be untruthfully reported unless the settlement arrangement is deliberately designed to elicit the truth. While these results about a schematic model economy are far from constituting definitive advice regarding actual settlement arrangements, we hope that this analysis may at least provide a helpful framework within which to think in an organized way about the issues involved in practical cases.

2 Modelling a transaction

Our first task is to formulate a model of a transaction that involves a risky asset transfer. The model should be rich enough to describe such a transaction recognizably, but simple enough to be analytically tractable.

Consider what sort of model could satisfy both the requirements of richness and simplicity. A transaction is a related set of asset transfers between traders. The assets involved might be either commodities or financial assets. An asset transfer involves two traders, the donor and the recipient, but a transaction can generally involve more than two traders. Therefore, at the very least, a model of a transaction involving a risky transfer should include three traders, so that a distinction can be drawn between a participant in the broad transaction and a participant (that is, the donor or the recipient) in the specific transfer where the risk occurs. in order for the third-party participant in the transaction—that is, the participant who is neither the donor nor the recipient of the risky transfer—to be essential to making a mutually beneficial transaction, there should be no "double coincidence of wants" between the donor and the receiver. This consideration suggests modelling the three participants as a "Wicksell triangle."

There is a distinction between two types of third party (or potential third party) that a good model ought to capture. A third party to risky transfer in a Wicksell triangle might be intrinsically necessary in the sense that the donor and recipient of the risky transfer would have no double coincidence of wants, even if the transfer did not involve risk (that is, if the recipient would receive the expected value of the transfer with certainty). Alternatively, the riskiness of the transfer might impair a double coincidence of wants that would exist under certainty between the donor and the recipient, and the third party might be needed solely to restore that double coincidence by serving as a guarantor or insuror of the transfer. For characterizing the differences between the roles of these two types of third parties, a four-trader model (including both an intrinsic third party and a trader whose only involvement would be to share risk) can be useful. On the basis of these considerations, we will specify the set of traders to be $\{1, 2, \ldots, N\}$, where either $N = 3$ or $N = 4$. In either case, we will assume that trader 1 is essential to a mutually beneficial transaction but that trader 2 is the donor and trader 3 is the recipient of the risky transfer. When a four-trader economy is considered, the attributes of trader 4 will be specified in such a way that trader 4 can only participate in a risk-sharing capacity.

The risky transfer will be formalized in terms of a probability space of events on which a probability measure $\Pr$ is defined.

There is a distinguished event S, with $0 < \Pr(S) < 1$. Assume that the risky transfer from trader 2 to trader 3 succeeds in S, and that it fails in the complementary event F. When we say that the transfer succeeds, we mean that trader 3 receives the entire quantity of the asset that is transferred. When we say that the transfer fails, we mean

that the quantity of the asset that was intended to be transferred disappears irretrievably from the economy.[1]

Later, to analyze incentive issues, we will specify that trader 1 privately observes an event that is statistically relevant to the outcome of that risky transfer.

Assume that each trader i has an endowment consisting solely a type of commodity that only he possesses. We denote that type of commodity also by i. Intuitively, trader i is endowed with one unit of commodity of type i with certainty.

In general, a commodity bundle provides a random amount of each of the N types of commodity in the economy. That is, each trader can acquire commodities by transactions with others, and randomness is introduced by the riskiness of the transactions technology. We use the letter γ to denote such a random commodity bundle.

Each trader's preference between commodity bundles conforms to expected utility. Trader i has a von Neumann-Morgenstern utility function $U^i \colon \mathbb{R}^N_+ \to \mathbb{R} \cup \{-\infty\}$. Trader i's expected utility of consuming a commodity bundle γ is the expectation the random variable $U^i(\vec{\gamma})$.

The sequence of economic activities in this economy is as follows.

Initially, before knowing whether the actual state of nature is in S or F, traders make an agreement for transfers of goods among them. The agreement among the traders is binding.

With one exception, the transfers are safe. That is, everything sent out reaches its intended recipient in its entirety and with certainty. The exception is the transfer of trader 2's endowment to trader 3. Recall that this transfer reaches trader 3 in its entirety in event S, but is completely and irretrievably lost in event F.

The traders also agree *ex ante* on a second round of transfers, to be made after the first transfers have been completed and the result of the risky transfer has become known. Thus the transfer to be made in the second round can be made contingent on which of the events S and F has occurred.[2]

All second-round transfers, including the one from trader 2 to trader 3, are nonstochastic. However, second-round transfers are costly. Only a proportion $\rho < 1$ of the goods that a trader sends in the second round are received.[3]

Traders consume their stocks of goods after these two rounds of transfers have been completed. To simplify the characterization of traders' consumption resulting from settlement, we make two assumptions: that a trader is able to transfer only his own endowment

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[1]Failure of an actual transfer seldom involves such an irretrievable loss, although there are some contemporary examples and many historical examples of that type of failure.

[2]Strictly speaking, this sentence describes a different information structure from the preceding one. If traders can only distinguish between events S and F on the basis of observing the success or failure of a transfer, then they can not make any distinction unless a (non-zero) transfer has been attempted. To assume that they can make a state-contingent transfer in the second round even if no first-round transfer from 2 to 3 has been attempted neglects this limitation of their opportunity for inference. In the case where there is no private information, this ambiguity is harmless because risk-averse traders would not cooperatively choose to make a state-contingent transfer in the second round unless they had exposed themselves to settlement risk in the first round. How the ambiguity is resolved is important in the private-information case, though, and we will discuss this issue further when we analyze private information.

[3]This assumption, sometimes called "iceberg cost," can be viewed as a crude way of reflecting various intuitive considerations including time preference and exposure to business loss due to delayed availability of transferred funds.

good, and that only a few of the possible flows of those goods are feasible. Specifically, trader 1 can make a transfer to 2, 2 to 3, and 3 to 1. (Traders 1,2, and 3 together will constitute the Wicksell triangle to which reference was made earlier.) In addition, in the version of the model where there is a fourth trader, traders 3 and 4 can each transfer their endowment good to the other.

As described above, either all, a proportion ρ, or none of the goods sent may be received. A *transaction* is a sequence $\tau = (\tau^1, \tau^S, \tau^F)$ of rounds of transfers. The elements τ^1, τ^S, and τ^F specify the initial round of transfers, the round of transfers in event S, and the round of transfers in event F, respectively.

Figure 1: Round of Transfers
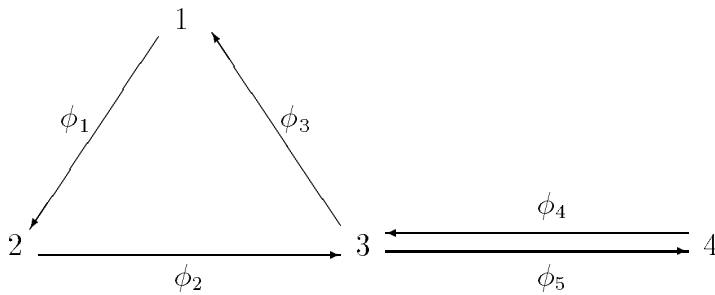
A transaction is feasible if no trader is ever required to send a cumulative amount that would exceed his endowment. That is, transaction τ is feasible if

$$\forall i \quad \tau_i^1 + \max\{\tau_i^S, \tau_i^F\} \leq 1. \tag{1}$$

Let $\mathcal{T}$ denote the set of feasible transactions.[4]

Now we provide an explicit definition of traders' consumptions resulting from a transaction. To begin, informally let τ^χ be the random net trade that results from transaction τ, which depends on whether event S or F instead occurs. By adding the positions of a particular trader in all such net trades in which that trader is involved to the trader's endowment, the random consumption of the trader is determined. Specifically the consumption vector $c^i(\tau)$ that trader i receives as a consequence of transaction τ is as follows.

---

[4]As noted in the footnote above, the informational constraint that, if $\tau_1^2 = 0$, then $\tau^S = \tau^F$, may or not be added to the definition of feasibility for a transaction. If all traders are risk averse, then the constraint is never binding when traders have common information.

$$
\begin{aligned}
c^1(\tau) &= (1 - (\tau_1^1 + \tau_1^\chi)z^1 + (\tau_3^1 + \rho\tau_3^\chi)z^3 \\
c^2(\tau) &= (1 - (\tau_2^1 + \tau_2^\chi)z^2 + (\tau_1^1 + \rho\tau_1^\chi)z^1 \\
c^3(\tau) &= (1 - (\tau_3^1 + \tau_3^\chi)z^3 - (\tau_5^1 + \tau_5^\chi)z^5) \\
&\quad + (\chi_S\tau_2^1 + \rho\tau_2^\chi)z^2 + (\tau_4^1 + \rho\tau_4^\chi)z^4 \\
c^4(\tau) &= (1 - (\tau_4^1 + \tau_4^\chi)z^4 + (\tau_5^1 + \rho\tau_5^\chi)z^5
\end{aligned}
\tag{2}
$$

3 The core

We modify the core of an exchange economy to serve as the solution concept to characterize the set of mechanisms to which the traders might agree. A core allocation is one that can be obtained (according to (2)) by a feasible transaction, and such that no coalition of traders can implement another allocation that its members unanimously prefer–with at least one of them having a strict preference–by using an alternative transaction that is feasible for its members. Define a core transaction to be a feasible transaction from which a core allocation is obtained via (2).

To formalize the notion of unanimous preference within a coalition, for each nonempty $C \subseteq \{1, \ldots, N\}$, define $\theta \in \mathcal{T}$ to C-*dominate* $\tau \in \mathcal{T}$ if

$$
\begin{aligned}
\forall i \in C \quad &\mathsf{E}[U^i(c^i(\tau))] \le \mathsf{E}[U^i(c^i(\theta))] \quad \text{and} \\
\exists i \in C \quad &\mathsf{E}[U^i(c^i(\tau))] < \mathsf{E}[U^i(c^i(\theta))].
\end{aligned}
\tag{3}
$$

Also define $\theta \in \mathcal{T}$ to be *feasible for* C if, with certainty,

$$
\forall i \notin C \quad c^i(\theta) = z^i \qquad \text{(No participation of other traders is required).} \tag{4}
$$

Finally, define $\tau \in \mathcal{T}$ to be a *core transaction* if there exist no $C \subseteq N$ and $\theta \in \mathcal{T}$ such that θ is feasible for C and θ C-dominates τ.

Let us say that transaction τ is *individually rational* if it is weakly preferred to autarky by every $i \in N$, and that τ is *Pareto-undominated* if it is undominated for N.

Proposition 1 *Let each trader's utility function be locally nonsatiated, at all points, in his own endowment good. Then a feasible transaction τ is a core transaction if and only if the following conditions hold: τ is individually rational, Pareto-undominated, and not either $\{1, 2, 3\}$-dominated or $\{3, 4\}$-dominated.*

4 Analysis of a public information environment

It will be useful to carry through our analysis using specific utility functions to show why the preference and private information do matter in the settlement system.

To this end, we study core transactions in some parametric versions of the economic environment defined above. We begin with a simple environment, where $N = \{1, 2, 3, 4\}$ and there is no private information. 1, 2, and 3 are the essential parties and 4 is the stand-by party to transactions. We specify the traders' utilities as follows.

$$
\begin{aligned}
U^1(c) &= \ln(c_1 + \beta c_3) \\
U^2(c) &= \ln(c_2 + \beta c_1) \\
U^3(c) &= \ln(c_3 + \beta c_2 + \psi c_4) \\
U^4(c) &= \ln(c_4 + \varphi c_5) \\
&\quad \text{with } \beta > \max\{\sigma^{-1}, \rho^{-1}\}, 0 < \varphi\psi < 1.
\end{aligned}
\tag{5}
$$

Here, goods received in trade are "better" substitutes for endowment goods for essential participants 1,2,3. Trader 4 considers trader 3's good to be a "worse" substitute for his own endowment good, and trader 3 considers 4's good to be a "worse" substitute for trader 2's good or even for his own endowment good. We assume that the transfer technology to satisfy $0 < \rho \leq \sigma < 1$ and $\sigma > 1/2$, and we assume that $\beta\rho \geq \sqrt[3]{2}$.[5]

Analysis of this model shows that a core allocation exists, and that it has the following characteristics.

Proposition 2 *A core transaction τ always specifies state contingent transfers. A typical core transaction τ specifies transfers such that:*

All essential traders send first-round transfers within the Wicksell triangle. In addition, trader 3 sends a first-round transfer (which might be considered to be an insurance premium against settlement failure) to the stand-by party, trader 4.
The first-round transfer sent from 2 to 3 does not supply all of trader 3's consumption of good 2. Rather, even in event S, 2 also sends a second-round transfer to 3. Thus, despite its costliness, trader 2 uses the safe transfer technology to minimize exposure to the risk of settlement failure in the first round. However, except for this risk-mitigating transfer, no other second-round transfers are made in event S.
In event F, both traders 2 and 4 make second-round transfers to 3. The transfer from 2 to 3 in this event is larger than the second-round transfer in event S. Thus, settlement risk in round 1 implies consumption risk for trader 2 as well as for trader 3. Trader 1 shares this risk by making a second-round transfer to 2. Moreover, trader 3 acts to minimize this induced consumption risk for trader 1 by making a second-round transfer. (Although 3 has suffered a loss himself, it is efficient for him to help 1 because of the difference in their marginal rates of substitution between goods 1 and 3.) Thus settlement failure in round 1 triggers economy-wide transactions in round 2.

5 Preliminary analysis of a private-information environment

In the general discussion above, we have contemplated that a third party within the coalition might have some private information, not possessed by either the payor or the payee, about the level of risk. In such a case, the information is potentially relevant to

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[5] The condition for a small transfer at the endowment allocation, using the safe technology, to increase the sum of utilities of the two traders is that that $\beta\rho > 1$. Thus, the intuitive meaning of the latter assumption is that traders would have clear willingness to use the safe technology if it were the only transfer technology available.

how the transaction should be conducted and even to how large a transaction ought to be undertaken.

If the privately informed third party were involved solely as the reporter of that information to the coalition, then there would be no problem about ensuring the truthfulness of the report. In particular, if compensation were required to induce reporting, that compensation could be made in the form of a flat fee. If the possessor of information functions in the payments process as an agent for one of the principals in the transaction, though, then there will generally be an issue of whether there is incentive for truthful reporting.[6] One might think, for example, that efficiency would generally require a payments coalition would have to penalize an information provider when a payment would fail without a warning of particularly risky circumstances having been given.

In this section we will show that there is indeed an incentive-compatability issue for the payments coalition to resolve, but that there is no simple generalization about how to resolve it. The incentive for truthful revelation of information depends on the pattern of risk sharing within the payments coalition, the differences in risk attitudes among coalition members, and the distribution of rents that is to be achieved by a *core transaction mechanism*, which generalizes the notion of a core transaction to a private-information environment. In fact, for the parametric environment that we study, some core mechanisms involve a binding incentive-compatibility constraint for truthful revelation that a transfer is likely to fail (that is, revelation of event L), while other core mechanisms for the same environment involve a binding constraint for truthful revelation that failure is unlikely (event H). As a practical matter, then, an implication of using the core transaction mechanisms as an equilibrium concept for payment arrangements is that supervisory authorities ought to accord substantial discretion to the governing body of a payments coalition to establish rules aimed at eliciting accurate information from members.

5.1 Generalizing the model to encompass private information

To model private information, suppose that an event that is statistically relevant to the outcome of that risky transfer will be privately observed by trader 1, who is not directly involved in the risky transfer but who is an essential participant in a mutually beneficial transaction among the traders. To consider the simplest case of nontrivial private information, suppose that trader 1 observes a signal that takes the value either 'H' or 'L'. Suppose that 'H' and 'L' satisfy

$$\Pr(S|L) < \Pr(S|H). \tag{6}$$

The agreement among traders regarding the structure of the transaction, described in section 2, is an *ex ante* agreement, made before trader 1 has received any information.

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[6]This idea, that a dual role of privately informed members of a payments coalition is critical for understanding how the institutional design of a payment arrangement is related to the attainment of economic efficiency, has previously been studied by Rochet and Tirole. In their model, unlike the present one, traders' information can only be revealed through their trades, and not by making explicit report. In many actual payments networks, the limited opportunities for traders to make explicit reports seem to fall between the absence of opportunity modelled by Rochet and Tirole and the completely adequate opportunity modelled here. When traders are required to set prior limits (which will not necessarily ever be binding in equilibrium) on their bilateral exposure to counterparties, for instance, their choices of which limits to set can be regarded as partially informative reports of their private information about those counterparties' riskiness.

However, trader 1 will observe H or L before the first round of transfers takes place. Thus it is natural for the agreement to specify that trader 1 will report what he observes, and that his report will determine which transaction to make. That is, the agreement among the traders specifies a *transaction mechanism* rather than a single transaction. Formally, a transaction mechanism is a mapping μ: $\{$'H', 'L'$\} \to \mathcal{T}$.

Transaction mechanism μ will elicit truthful reporting from trader 1 if the following incentive-compatibility condition is satisfied.

$$\forall A \in \mathcal{P} \quad \forall B \in \mathcal{P} \quad \mathsf{E}[U^1(c^1(\mu(B)))|A] \leq \mathsf{E}[U^1(c^1(\mu(A)))|A]. \tag{7}$$

Let $\mathcal{M}$ denote the set of incentive-compatible transaction mechanisms. We restrict attention to incentive-compatible mechanisms, as is justified by the revelation principle.[7] If $\mu \in \mathcal{M}$, then the resulting transaction τ and the consumption, ^{i} for each trader $i \in N$ is defined as follows (with χ_A denoting the characteristic function of an event A).

$$,^i(\mu) = \sum_{A \in \mathcal{P}} \chi_A c^i(\mu(A)). \tag{8}$$

A core transaction mechanism can be defined in a way that is straightforwardly analogous to the definition of a core transaction.[8] Specifically, to formalize the notion of unanimous preference within a coalition, for each $C \subseteq N$, define $\nu \in \mathcal{M}$ to C-*dominate* $\mu \in \mathcal{M}$ if

$$\forall i \in C \quad \mathsf{E}[U^i(,^i(\mu))] \leq \mathsf{E}[U^i(,^i(\nu))] \quad \text{and}$$
$$\exists i \in C \quad \mathsf{E}[U^i(,^i(\mu))] < \mathsf{E}[U^i(,^i(\nu))]. \tag{9}$$

Also define $\nu \in \mathcal{M}$ to be *feasible for* C if, with certainty,

$$\forall i \notin C \quad ,^i(\nu) = z^i \quad \text{(No participation of other traders is required)}$$
$$\text{(7) holds if} \quad 1 \in C \quad \text{(Incentive compatability)} \tag{10}$$

Finally, define $\mu \in \mathcal{M}$ to be a *core transaction mechanism* if there exist no $C \subseteq N$ and ν such that ν is feasible for C and ν C-dominates μ.

With the core of a transaction mechanism so defined, proposition 1 has the following, straightforward generalization.

Proposition 3 *Let each trader's utility function be locally nonsatiated, at all points, in his own endowment goods. Then $\mu \in \mathcal{M}$ is a core transaction mechanism if and only if the following conditions hold: μ is individually rational, Pareto-undominated, and optimal for payments-system participants, and μ is not $\{3,4\}$-dominated.*

5.2 A parametric environment with accurate private information

Consider the three-trader environment in which trader 1 receives a private signal about the success or failure of a transfer from trader 2 to trader 3. To simplify this preliminary

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[7]Myerson (1991) provides an exposition of incentive compatability and the revelation principle.

[8]Allen (undated) has previously used incentive-compatible mechanisms in this way to define a core equilibrium concept for environments with private information.

analysis, we assume that success and failure have equal probability, and that trader 1's signal is perfectly accurate, That is, we assume that $\Pr(H) = \Pr(L) = 1/2$, $\Pr(S|H) = 1$, and $\Pr(S|L) = 0$.

In this section, we work with piecewise-linear utility functions for the traders. Their utilities will be defined in terms of parameters δ and ϵ, which are assumed to satisfy $0 < \epsilon < \delta < 1/4$. Utility functions are defined in terms of the following functions on the nonnegative real numbers.

$$
\begin{aligned}
V(x) &= \min\{1, x\} + \epsilon \max\{0, x - 1\}; \\
W(x) &= \min\{1/2, x\} + \epsilon \max\{0, x - 1/2\}.
\end{aligned}
\tag{11}
$$

Define the agents' utilities as

$$
\begin{aligned}
U^1(c) &= c_1 + c_3; \\
U^2(c) &= V(c_1 + c_2); \\
U^3(c) &= W(c_2) + \delta c_3.
\end{aligned}
\tag{12}
$$

Because trader 1's utility function is linear and information is perfectly accurate, the incentive-compatability constraint reduces to the following two equations. The first and second equations state that trader 1 has incentive to report truthfully in event H and L, respectively.

$$
\begin{aligned}
(\mu_3^1(H) + \mu_3^S(H)) - (\mu_1^1(H) + \mu_1^S(H)) &\geq (\mu_3^1(L) + \mu_3^S(L)) - (\mu_1^1(L) + \mu_1^S(L)) \\
(\mu_3^1(L) + \mu_3^F(L)) - (\mu_1^1(L) + \mu_1^F(L)) &\geq (\mu_3^1(H) + \mu_3^F(H)) - (\mu_1^1(H) + \mu_1^F(H))
\end{aligned}
\tag{13}
$$

5.3   IC constraint can bind in eith H or L

It has been proved that, in the three-trader environment, a transfer mechanism is in the core if and only if it implements a Pareto efficient (subject to both technological and incentive constraints) allocation that is individually rational for each trader. An allocation that maximizes a weighted sum of traders' expected utilities, with all weights strictly positive is Pareto efficient. Therefore, for $\alpha \in \mathbb{R}_{++}^3$, consider

$$
U(\mu, \alpha) = \sum_{i=1}^{3} \alpha_i \mathsf{E} U^i(, {}_i(\mu)).
\tag{14}
$$

Regarding the transfer technology, suppose that

$$
\rho = 1/2.
\tag{15}
$$

In this environment, it can be shown that the incentive-compatability constraint can bind in either event H or L, depending on which value of the utility-weight vector α is used for maximization of U.

6   Conclusion

The rules in a settlement system must encourage the participants to take optimal degrees of risk in accordance with their attitude towards risk. If some participants have socially useful private information, the rules in a settlement system must be constructed that it does not give participant adverse incentives to mask their information. Policy makers can achieve this objective if they think about the rules in a settlement system as a mechanism design problem. If policy makers ignore those points and introduce new rules into a settlement system, the equilibrium allocation of goods might be distorted.

If we regard the crucial issue in the settlement system as the efficient risk sharing among the participants in the presence of private information, this view allows policy makers to consider the rules governing a settlement system as a kind of social safety net. The efficient risk sharing under the default of banks might involve the transfer of resources from the agents who normally are not directly involved with the settlement network. The central bank plays such a role as a lender of last resort, by transferring the resources of general public into the banking sector during a period of financial panic.

Appendix: Understanding central banks as a risk-sharing device

The discussion in this paper is based on the view that standard microeconomic theory could be useful to analyze the settlement network. We try to understand the flow of funds between the parties as an endogenous phenomenon. We emphasis that both private information and preference could play independent roles to determine the optimal risk sharing in equilibrium. In principle, the approach could be useful to design regulations that avoid unintended distortions by giving due consideration to the incentives and private information of participants.

This strategy is consistent with the other recent contributions to the microeconomics of banking that emphasize the anlysis of formal models (See Dewatripont and Tirole (1994), Freixas and Rochet (1997)). In this appendix, we justify the practical relevance of this approach by examining several heuristic examples that support our argument.

We will discuss, in turn, (i) the role of central bank as a lender of last resort, and (ii) historical examples of barter trade that exactly match our model.

The role of central bank

Throughout this paper, we argue that the fourth trader who works as if a lender of last resort, can be viewed as a central bank only if there is no gains from trade with this agent in the absence of risk. We argue that it should be just treated as a general public if there is gains from trade without risk.

More formally, trader 4, who obtains goods at the usual time and sends goods to trader 3, looks like an central bank if $\beta\gamma < 1$. Note that the fact this inequality does not hold implies that an unit of exchange of goods between trader 3 and trader 4 is Pareto improving, hence there is no wonder that trader 4 transfer his own goods to trader 3. The implication of our model is that since the inequality does not hold ex- ante, there is

no gain from trade in the absence of risk. The fact that the marginal utility of trader 3, which becomes substantially high given the shipment failure from trader 2, induces trader 4 to work as if a lender of last resort, because trader 4 sends goods to trader 3 only under the situation of settlement failure. The fact that trader 3 sends some goods to trader 4 without shipment failure could be understood that trader 3 pays some fee in order to obtain the insurance under the situation of shipment failure.

We stress the view that the forth trader plays the role of central bank, but it is not different from the usual traders intrinsically, and that there is no particular reason to believe there are gains from trade between the central bank and others without risk consideration. We further argue that if there were a risk in the settlement network, without central bank, there would be a private institution that works for that sake. Our view is based on the U.S. history of banking.

For example, take agent 2 as a New York bank, agent 3 as a Boston bank, and agent 1 as a Philadelphia bank. Consider a shipment failure of a bank as a default. Safer shipments take time because an inspection by banks requires checking the quality of bank notes. Note that in this setting it would be natural to expect the fourth trader to act as a clearing house (of New York) supervising banks with membership fees. It thus leads to an intuition that whenever there is no central bank by one reason or other, a private arrangement would emerge to create a quasi-central bank. (See Gorton and Mullineaux (1987)).

In this sense we might be able to argue that our model captures some of the important aspects of the free banking area documented by King (1983) and the emergence of a central bank. Moreover, we might argue further that our model is even consistent with the recent view with regard to the role of Federal Reserve as the supplier of settlement service among private netting arrangements once we regard trader 1,2 and 3 as the private clearing networks (See Summers and Gilbert (1996)).

Note that Green (1997) shows that by allowing traders to issue so called novation securities, in equilibrium, both the initial securities and the novation securities will trade at face value in a monetary economy a la Freeman (1996a, b). It means the risk induced by trading-opportunity uncertainty will be fully insured, and efficiency will be attained. The novation securities introduced below bear striking resemblance to the clearinghouse loan certificates that were issued during those episodes in the absence of a central bank. Those certificates, and the central banking role played by U.S. clearinghouses at that time generally, are described by Timberlake (1984).

Loss sharing rule in the historical economies

Strictly speaking, we have discussed the implication of triangular and barter trades with risky delivery of goods.

Given these limitations, one way of viewing our model is as a somewhat realistic model of shipping insurance and other loss-sharing arrangements in historical economies. Early modern Europe and feudal Japan provide such examples. Regarding Europe, Lopez and Raymond (1955, p.259) reprint Genoese documents of 1191 and 1192, in which a merchant pays a premium to a shipper who puts up security for the successful delivery of the merchant's goods. Moreover, some examples of insurance contracts involving third-party underwriters can be found as early as the fourteenth century. In such a contract, the

underwriters are supposed to purchase for a certain price a certain amount of goods from a merchant, but that the contract is to be void if the goods arrive safely at a certain port.

Feudal Japanese sea law, dating back to at least 723 AC, also pertained to an economy that exemplified the general features of our model.[9] Most sea transportation was commissioned by government. Specifically, local government officials (owners of the cargo) hired sailors to ship goods from their regions to the central government as feudal tax payments.[10] Sometimes bad weather forced the sailors to jettison the cargo in order to stay afloat. In that event, the central government asked the sailors and the local government to pay 40% and 60% of the damage of the cargo respectively. (That is, the central government tried to induce the sailors not to abandon too much cargo intentionally by introducing this rule.) However, if the ship sank or more than half of the sailors were drowned despite having jettisoned the cargo, then central government did not ask them (that is, either surviving sailors or drowned sailors' heirs) to pay indemnity. These provisions were evidently designed to induce sailors to take appropriate actions contingent on the severity of the weather at sea—a situation regarding which they possessed private information relative to the senders and receivers of their cargo.[11]

As well as serving as a fairly realistic model of such historical arrangements, it also serves as a more schematic model of loss-sharing arrangements adopted by various payments systems today. It is noteworthy that the model explains these arrangements in terms of standard concepts of insurance theory, without having to invoke unproved assertions about a special, ill-defined, kind of "systemic" risk.

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[9] The following discussion is based on Takeda (1992) and Toyoda and Kota (1970).

[10] Even after taxes became payable in money, goods continued to be transported to the capital by sea in order to be sold to raise the tax money. The common law of sea transportation did not change very much from the one in the eighth century, judging from the one of the oldest written sea laws, called "Kaisen-Shikimoku," which dates from 1223.

[11] Presumably the provisions also served to prevent sailors from engaging in intentional fraud—a phenomenon from which our model abstracts—by surreptitiously stopping en route to sell the cargo and then claiming to have jettisoned it in a storm.

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