IMES DISCUSSION PAPER SERIES

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Discussion Paper No. 2017-E-6

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

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CHUO-KU, TOKYO 103-8660
JAPAN

IMES Discussion Paper Series 2017-E-6
July 2017

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Abstract

This paper investigates optimal currency choice, particularly the choice between paper and digital currencies, when currency is utilized solely as a medium of exchange. The Baumol-Tobin model of transactions demand for money is extended to derive conditions under which digital currency is preferred to paper currency, taking into consideration the network externality in the choice of currencies. The model is applied to explain potential variations in currency preferences across countries, especially between advanced and developing economies. Also discussed is how the introduction of negative interest rates, currency taxes, and central bank digital currency affect optimal currency choice.

Keywords: Digital currency; Money demand; Network externality; Negative interest rate; Currency tax
JEL classification: E41, E58, E20, P44

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The author would like to thank Hiroshi Fujiki and participants in the IMES seminar at the Bank of Japan for their helpful comments. The opinions expressed here as well as any remaining errors are those of the author and should not be ascribed to the Institute for Monetary and Economic Studies or the Bank of Japan.

1. INTRODUCTION

Digital currency is a digital unit of value and can be used as a medium of exchange, store of value, or speculative asset. In other words, it is not made of physical materials, but performs precisely the same roles as paper currency.[1] Of the various existing digital currencies, the best known is bitcoin. Designed from research by Satoshi Nakamoto (2008) and launched in January 2009, bitcoin has already been used for payments in virtual shops on the internet and even in real shops on the street.[2] Noted for its technological innovations, particular attention has been paid to bitcoin's core technologies, the distributed ledger and block-chain, whose broad applicability to business has spurred a new wave of financial technology, or *FinTech*.

This paper focuses on another notable feature of digital currency, namely its potential as a medium of exchange.[3] Some say that due to their low transfer fees, bitcoin and its ilk will take over existing payment systems, which are heavily reliant on the banking sector, and that they will come to replace paper currency as a medium of exchange and store of value.[4] Statistics indicate, however, that the total quantity of digital currencies in circulation does not compare to the equivalent figure for paper currencies issued by monetary authorities. In addition, there seem to be numerous obstacles to be overcome, including security and legal issues, before digital currencies become a major transaction medium. On balance, it will take some time for digital currency to be accepted as a natural substitute for paper currency.

The theoretical core of this paper is the Baumol-Tobin model (Baumol, 1952; Tobin, 1956), which describes succinctly how much cash individuals hold on average to

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[1] Here we use "paper currency" to refer to all physical currency, including coins, plastic notes as issued in Australia and some other countries, etc.

[2] See, e.g., Ali et al. (2014a) for a brief introduction to bitcoin.

[3] It is said that there are three functions of money: as a medium of exchange, a store of value, and a unit of account. This paper deals with the first function and postpones the analysis of the latter two for future research.

[4] Digital currencies that are alternatives to bitcoin are known as *altcoins*. A lot of altcoins have already been issued, including *Ripple*, *Litecoin*, and *Auroracoin*.

satisfy their transactions demand. This paper extends their model and applies it to the question of individuals' optimal currency choice, particularly whether to hold paper or digital currency. According to Keynes (1936), people are motivated to hold money for transactional, precautionary, and speculative reasons.[5] This paper focuses exclusively on the first of these motivations. The model allows us to specify conditions under which individuals use digital currency rather than paper currency in transactions and to explain why preferences for digital currency are stronger in some countries than others. We also discuss how negative interest rates and taxes on currency holdings affect the choice between paper and digital currencies, and how competitive central bank digital currency will be in a situation when non-governmental digital currency is also in circulation.

The extended Baumol-Tobin model allows us to investigate the effects of the actions of money suppliers on the behavior of money holders. Clearly, the acceptability of a currency depends on who issues it. In advanced countries, central banks typically have solid reputations as reliable providers of money. Paper currencies issued by central banks are rarely refused in exchange for goods and services. Central bank digital currencies, if issued, will be accepted with a comparable degree of confidence. Private suppliers of new digital currencies, on the other hand, have to build such reputations from scratch. To make their currencies attractive, they must introduce advanced technologies, advertise effectively, and so on. Without such efforts, pushing their digital currencies into circulation would be well-nigh impossible.

In this paper, we take as given a fully functioning banking system and the existence of paper money already in circulation. We do not ask why they have emerged. Among the several strands of the literature devoted to monetary theory, the search-theoretic approach to money, started by Kiyotaki and Wright (1989, 1993), examines the birth of money or the economic mechanisms whereby certain goods emerge endogenously as media of exchange. The question of optimal currency choice

[5] In reality, transactions demand is not the main motive for holding money in Japan. Most existing Bank of Japan notes, particularly those in large denominations, are hoarded for the purpose of saving.

could be investigated in that direction.[6] We, however, consider the extended Baumol-Tobin model to provide sufficient insight to address many of the practical issues faced by central bankers, as shown below.

The remainder of this paper is constructed as follows. Section 2 reviews the Baumol-Tobin model and extends it to deal with the optimal choice of currency for transactional purposes. Section 3 discusses several issues related to the circulation of digital currency: varying currency preference across countries; negative interest rates and taxes on currencies; the case where digital currency is issued by a central bank; and the long-run implications of the model. Section 4 concludes.

2. THE MODEL

2.1 A quick review of the Baumol-Tobin model

Consider a steady state economy, where an individual receives income, y, which is paid into his bank account at the beginning of each period. He spends the income evenly over the period. To buy goods and services, he needs cash. Every time he goes to the bank and withdraws some cash, he incurs costs, which are called *shoe leather costs* below.[7] Clearly, it is not a good idea for him to go to the bank every time he goes shopping. Conversely, if he withdraws all deposits at once, he forgoes all the interest which otherwise would accrue to his account. Thus, his question is how many times he should go to the bank and withdraw cash from his bank account during the period.

Denote the individual's shoe leather cost by a and the number of trips to the bank by n. The total cost of these trips is then given by an, which is shown as a upward-sloping line in Figure 1. Assume that he withdraws the same amount of cash,

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[6] Saito (2015) extended Trejos and Wright (1995) and examined the properties of bitcoin, taking a search-theoretic approach.

[7] Economists often use the words "shoe leather costs" in a specific sense, namely to describe the "costs of inflation." As the inflation rate rises, individuals visit banks more often and incur higher shoe leather costs. Note, however, that they incur shoe leather costs even when inflation is zero percent, as long as they still visit banks.

y/n, every time he visits the bank. Then, his average cash holding is given by $y/(2n)$, as is clear in Figure 2. The interest rate on deposits is r. Thus, his foregone interest earnings are given by $ry/(2n)$, as depicted in the downward-sloping curve in Figure 1. The total cost he incurs over the period is given by

$$c = an + \frac{ry}{2n}, \tag{1}$$

tracing out the U-shaped curve in Figure 1. The optimal number of trips, n^*, is found at the lowest point of this curve, and is obtained by minimizing equation (1) with respect to n. The solution is[8]

$$n^* = \sqrt{\frac{ry}{2a}}. \tag{2}$$

The resulting minimum total cost is given by

$$c^* = \sqrt{2ary}\,. \tag{3}$$

2.2 Incorporating costs of storing currency in the model

This paper extends the Baumol-Tobin model by incorporating a new ingredient in it, namely, a storage cost for cash. There are two components of this cost that require accounting for in the model. The first is a fixed cost component, regardless of the size of the individual's cash holding. This captures, for instance, the cost incurred from needing to keep a big safe at home for storing cash. Assume that a storage cost of z is incurred every period. Then, equation (1) is modified to

$$c = an + \frac{ry}{2n} + z\,. \tag{4}$$

Since z is constant, it does not affect the individual's cost minimization problem. Thus,

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[8] More precisely, n must be an integer greater than zero. Thus, n^* in equation (2) is viewed as an approximation of the exact solution, which is given by the integer n satisfying the inequalities $(n-1)n \leq ry/2a \leq n(n+1)$. If r and y are sufficiently small and a is sufficiently large, the exact solution turns out to be 1.

the optimal number of trips is given by equation (2) as before. The minimum total cost is modified, however, to

$$c^* = \sqrt{2ary} + z\,, \tag{5}$$

which, for the sake of explanatory simplicity, is referred to below as the total cost of using paper currency.

Second, the storage cost may vary with the size of cash holding, reflecting the idea that individuals carrying larger amounts of cash may incur larger costs. Denote such a varying storage cost by v per dollar. Then, equation (4) is modified further to become

$$c = an + \frac{(r+v)y}{2n} + z\,. \tag{6}$$

Note that the variable storage cost is added on the interest rate in the equation. Thus, the minimum total cost is simply given by

$$c^* = \sqrt{2a(r+v)y} + z\,. \tag{7}$$

2.3 The transactions demand for digital currency

There are various ways to obtain digital currency. This paper considers an individual buying it from a broker at a currency exchange. To simplify the argument, we assume that the individual expects the exchange rate between paper and digital currencies to be fixed forever. Under this assumption, the exchange rate does not feature in his calculations at all. We discuss how the exchange rate is determined later.

The shoe leather cost of digital currency, $\hat{a}$, includes all costs incurred by a digital currency user in order to exchange bank deposits for digital currency. No trip to the bank is required; instead, each individual simply spends a short time completing the necessary transactions on his computer or smartphone. Thus, there are virtually no shoe leather costs in the sense there were for paper currency. There are, however, costs in the form of the brokerage fee that must be paid to the currency exchange, as well as the transfer fee paid to the bank for transferring the requisite funds from his bank account to the broker's. These comprise the shoe leather costs incurred by the individual to obtain some digital currency in his "wallet," the virtual purse downloaded for free on his computer or smartphone.

No storage cost is incurred for holding digital currency. An electronic device, such as a computer or a smartphone, is required to use digital currency. Most digital currency users, however, already possess at least one of those devices, which they bought for other purposes. Consequently, there are virtually no additional storage-related costs when they begin shopping in digital currency.

However, the use of digital currency involves individuals in various other kinds of costs. We summarize those costs in x, termed the digital payment cost below, which comprises a number of components. First, using the example of bitcoin, when an individual pays digital currency for goods and services, he pays a remittance fee to the "miner" who wins the "proof of work" race.[9] We assume that the remittance fee is fixed regardless of how much the individual spends per purchase; also that he does not change the frequency of purchases, although he increases the expenditure per purchase in proportion to his income. This implies that the total remittance fee he pays every period is constant.

Second and most importantly, there is a psychological component that should be included in the digital payment cost x. Individuals tend to feel more stress in paying by digital currency than paper currency. Such psychological costs are not insignificant and may indeed be greater than the remittance fee for some people. This is more likely to be the case for individuals unfamiliar with digital currency.

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[9] The winner of a bitcoin mining race, a competition to solve resource-consuming mathematical puzzles to verify each block of transactions, receives two kinds of rewards for his proof-of-work: newly created bitcoins and existing bitcoins received as remittance fees by senders of funds. Note that the senders are not generally obliged to pay remittance fees, but tend to pay them voluntarily to give miners an incentive to work on their transactions. There are a few transactions for which remittance fees are mandatory (see Vigna and Casey, 2015). One is for transfers involving only tiny amounts of money, which is considered as an effective way to protect the system against distributed denial of service (DDOS) attacks. The other is for transfers containing a huge amount of data.

Lastly, the inconvenience of digital currency due to its limited circulation should be included in x. Clearly, not many high street shops are yet ready to accept digital currency. Therefore, individuals tend to experience considerable inconvenience turning their digital currency into consumption goods. This constitutes a digital payment cost.

The total cost the individual incurs in using digital currency is given by

$$\hat{c} = \hat{a}\hat{n} + \frac{ry}{2\hat{n}} + x \, . \tag{8}$$

The individual's cost minimization problem is not affected by x in equation (8). Therefore, the same solution as for the Baumol-Tobin model applies. The optimal number of digital currency purchases is given by

$$\hat{n}^* = \sqrt{\frac{ry}{2\hat{a}}}, \tag{9}$$

and the resulting minimum total cost is given by

$$\hat{c}^* = \sqrt{2\hat{a}ry} + x \, . \tag{10}$$

2.4 A rational choice between paper and digital currencies

Each individual determines which of the alternative currencies to use, paper or digital, by comparing the costs he incurs. In what follows, $A \succ B$ indicates that the individual prefers currency A to B; $A \backsim B$ means that he is indifferent between currencies A and B. Comparison of equations (5) and (10) immediately yields the following set of necessary and sufficient conditions regarding these relative costs:

$$paper\ currency \begin{Bmatrix} \succ \\ \backsim \\ \prec \end{Bmatrix} digital\ currency$$

$$\Leftrightarrow \qquad \sqrt{2ary} + z \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \sqrt{2\hat{a}ry} + x \, . \tag{11}$$

Expression (11) provides each individual with a criterion for choosing his optimum currency for transactions purposes.[10] Note that individuals, whatever their preferences, are assumed to use one or other of the two currencies exclusively. This begs the question of whether there may be some individuals who prefer to use both currencies rather than restricting themselves to using only one of them. This issue is dealt with in Appendix A, where it is shown that, at least in the current model setting, there is no such individual.[11]

It is convenient to show expression (11) graphically. To do so, we define a *currency border,* on which an individual is indifferent between paper and digital currencies. From expression (11), the currency border is defined as

$$x = \sqrt{2r}\left(\sqrt{a} - \sqrt{\hat{a}}\right)\sqrt{y} + z\,. \tag{12}$$

In general, the values of each parameter in equation (12) vary across individuals. To see the model's implications clearly, however, we assume that individuals are heterogeneous with regard to x and y, but homogeneous with regard to the other parameters. This is a good approximation of reality, at least within a given country. Under these assumptions, equation (12) becomes a linear function of $\sqrt{y}$ and x. Its slope is given by $\sqrt{2r}\left(\sqrt{a} - \sqrt{\hat{a}}\right)$ and the intercept is determined by z. Under normal circumstances, the shoe leather cost of digital currency is lower than that of paper

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[10] There is an argument that bitcoin is not and will not be used as a major medium of exchange, due to its large exchange rate fluctuation against traditional currencies such as US dollars. This argument, however, loses its power to convince if the shoe leather cost of digital currency is small. In that case, average digital currency holdings are also small, so consumers do not suffer much from exchange rate volatility. Consumers can also minimize their exposure to devaluation risk by spending their digital currency as soon as possible after buying it. Furthermore, as Iwamura et al. (2014) demonstrate, it is possible to propose rules to reduce the volatility of the value of bitcoin without a central bank.

[11] The current model does not take into consideration the size of payment. In reality, however, the choice of payment medium is affected by payment size. There is much empirical literature reporting that low-value payments tend to be made in cash, while high-value payments are made with a credit card (e.g., Fujiki and Tanaka, 2016). Comparable payment customs may influence the choice between paper and digital currencies. Extending the model to incorporate such payment customs would be an interesting topic of future research.

currency, i.e., $a > \hat{a}$. Thus, the currency border is drawn as an upward-sloping line in Figure 3. If an individual's $(\sqrt{y}, x)$ locate him below the border, he uses digital currency; otherwise he uses paper currency.

Suppose that individuals' incomes increase, i.e., $\sqrt{y}$ increases. Figure 3 indicates that some individuals will switch from paper to digital currency. The intuition is simple. Consider an individual who uses paper currency initially. Suppose that his income grows. He then goes to the bank more frequently, which means more time and money spent withdrawing paper currency. In the previous period, being unfamiliar with digital currency and so experiencing psychological stress from its use, he chose to use paper currency. Now, however, the stress from using digital currency is outweighed by the increased inconvenience from using paper currency, and he therefore switches from paper to digital currency. It is also easy to see in Figure 3 that technological progress in the security of digital currency, i.e., a decline in x, encourages him to use digital currency.[12]

2.5 Network externality effects arising from currency choice

The digital payment cost includes both the psychological stress people feel in using an unfamiliar currency and the inconvenience arising from its limited acceptance by shopkeepers. However, these costs will decline as the population of digital currency users and the number of shops which accept digital currency increase. Moreover, since a reduction in the digital payment cost increases the population of digital currency users, it will also cause the number of shops which accept digital currency to increase, which leads to a further reduction in the digital payment cost. The currency choice thus exhibits a network externality, which turns out to be highly significant particularly when we discuss the long run implications of the model.

In Section 2.4 we identified which individuals choose to use digital currency. Given the joint distribution of x and y, we can calculate the proportion of digital

[12] It is also likely that competition among firms offering digital currency exchange puts downward pressure on the brokerage fee, a component of $\hat{a}$. This steepens the currency border line. Some of the paper currency users are encouraged to switch to digital currency.

currency users, θ. To simplify the exposition, we assume that θ is a function of only $\bar{x}$ and $\bar{y}$, the means of x and y, respectively. Appendix B demonstrates the existence of such a distribution and provides a formal exposition of the discussion here. Suppose that $\bar{x}$ goes to zero. This implies that all the x's go to zero, since they take non-negative values. Clearly, all individuals prefer digital currency to paper currency in this case. Thus, θ goes to unity. Contrast the opposite case where $\bar{x}$ goes to infinity. This would normally imply that many of the x's must take the value of infinity. Many individuals prefer paper currency to digital currency. Therefore, θ approaches zero. These observations suggest that there is a negative relation between θ and $\bar{x}$. Below, we call this the digital payment population reaction function (PRF for short), which is shown as a downward-sloping curve, PRF, in Figure 4 (1).

Let us incorporate the network externality from the currency choice into the model. Three cases are considered below: the no-externality, weak-externality, and strong-externality cases. First, in the no-externality case, $\bar{x}$ is fixed irrespective of θ. We call the response of $\bar{x}$ to a change in θ the digital payment cost reaction function (CRF for short). The CRF is shown as a vertical line, CRF_1, in Figure 4 (1). Suppose that there is a shock which lowers $\bar{x}$ from $\bar{x}_0$ to $\bar{x}_1$. Then, the equilibrium shifts from E_0 to E_1 along the PRF. The proportion of digital currency users increases from θ_0 to θ_1.

Second, in the weak-externality case, $\bar{x}$ responds modestly to a change in θ, as illustrated by the downward-sloping line, CRF_2, in Figure 4 (2). Suppose that $\bar{x}$ is hit by the same magnitude of shock as above, which lowers $\bar{x}$ from $\bar{x}_0$ to $\bar{x}_1$. The equilibrium shifts from E_0 to E_2 along the PRF. The proportion of digital currency users increases from θ_0 to θ_2. Note that due to the network externality effect, the digital payment cost is driven down endogenously from $\bar{x}_1$ to $\bar{x}_2$, thus further expanding the proportion of digital currency users.

Third, in the strong-externality case, $\bar{x}$ is highly responsive to a change in θ, as illustrated by the gradient of the downward-sloping curve, CRF_3, in Figure 4 (3). Assume that an economy was initially at E_0. If $\bar{x}$ is hit by the same magnitude of shock as above, the equilibrium shifts from E_0 to H at the top left of the figure, rather than E_3, since H is dynamically stable, but E_3 is not.

An interesting case is obtained with a non-linear CRF. Three such CRFs are depicted in Figure 5. When θ is small, the network externality is weak. However, as θ increases, the network effect strengthens. Suppose that the initial CRF is given by CRF_0. The equilibrium is achieved at the intersection of CRF_0 and PRF, i.e., at E_0. The share of digital currency is θ_0. Suppose that technological progress occurs and shifts the CRF to the left, say, to CRF_1. Now the equilibrium is achieved at E_1.[13] The share of digital currency rises to θ_1, but the increase is very small. Now suppose that further technological progress pushes the CRF to CRF_2, which is tangent to PRF at E_2. Note that the share of digital currency, θ_2, has a special meaning. It can be called the *critical mass* of innovation diffusion. In its infancy, an innovation is accepted by only a few people, and the diffusion rate rises only slowly. However, once the diffusion rate reaches a certain level, the innovation becomes progressively more acceptable. This special level is termed critical mass in the literature (see, e.g., Rogers, 2003). In the current case, once the economy deviates from E_2, it converges to a new equilibrium point, E_3, rapidly. The share of digital currencies rises quickly from θ_2 to θ_3.

2.6 The exchange rate between paper and digital currencies

So far we have not explicitly mentioned the exchange rate between paper and digital currencies. To close the model, we discuss how the equilibrium exchange rate is determined. We indicate individual i's parameters by subscript $i \in \Omega$, where Ω is the set of all residents in the country.[14] From equation (9), we see that his optimal average currency holding is given by $y_i/(2\hat{n}_i^*) = \sqrt{\hat{a}_i y_i/(2r_i)}$. Denote the set of all digital currency users by $\hat{\Omega} \subseteq \Omega$. Then, in general, the aggregate transactional demand for digital currency is given by

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[13] In Figure 5, CRF_1 intersects PRF three times at E_1, E_1', and E_1''. The equilibrium is achieved most likely at E_1, since it is stable and closest to the initial point.

[14] This paper focuses on a closed economy. It is easy, however, to extend the current model to an open economy setting.

$$\widehat{D} = \sum_{i \in \widehat{\Omega}} \sqrt{\frac{\hat{a}_i y_i}{2r_i}}. \tag{13}$$

Let us continue the assumptions made in Section 2.4, where parameters r, a, $\hat{a}$, and z are common to all individuals. Then, $\widehat{\Omega}$ is given by a set of i such that $(\sqrt{y_i}, x_i)$ is located below the currency border in Figure 3. Equation (13) is simplified to

$$\widehat{D} = \sqrt{\hat{a}/(2r)} \sum_{i \in \widehat{\Omega}} \sqrt{y_i}\,. \tag{14}$$

The equilibrium exchange rate is such that the aggregate demand for digital currency is equal to the supply. Denote the exchange rate between paper and digital currencies by e, which is the dollar price of a unit of digital currency. Suppose that s units of digital currency have already been supplied. Then the dollar-denominated supply of digital currency is given by $e\hat{s}$. Note that the aggregate transactional demand above, $\widehat{D}$, is also expressed in terms of dollars. Thus, the equilibrium exchange rate is given by

$$e^* = \frac{\widehat{D} + \widehat{O}}{\hat{s}}, \tag{15}$$

where $\widehat{O}$ denotes digital currency demand for non-transactions purposes, i.e., demand arising from precautionary and speculative motives. Whether the equilibrium exchange rate will rise or fall depends not only on the supply of digital currency, but also on aggregate transactional demand $\widehat{D}$ as well as non-transactional demand $\widehat{O}$. The supply of bitcoins will grow at a predetermined pace until around 2140, and stay flat at 21 billion bitcoins (BTCs) from then on. This does not mean that the value of each bitcoin will be fixed, but rather that its value will increase as demand for the currency grows over time.[15]

[15] Bitcoin demand for transaction purposes will change over the course of the business cycle or due to seasonality, as well as in line with trend economic growth. Large fluctuations in the bitcoin price are said to be mostly due to demand for speculative purposes, reflected in the volatility of $\widehat{O}$. When combined with its inflexible supply scheme, this explains why bitcoin is subject to significant exchange rate volatility against traditional currencies, such as US dollars. See Ali et al. (2014b) for similar arguments.

Suppose that incomes grow and that digital payment becomes less stressful due to technological progress in the security of digital currency.[16] Then the aggregate demand for digital currency increases as follows.[17] First, the demand of each existing digital currency user increases as y_i grows. Second, the number of digital currency users increases, that is, $\hat{\Omega}$ expands, thanks not only to increased income per capita but also the reduced anxiety over digital currency security. Third, the network externality causes the digital payment cost to decline further, increasing the population of digital currency users and giving an added boost to the demand for digital currency. Figure 6 represents this process visually. E is the initial point. Income growth pushes up the PRF. The increase in the share of digital currency users lowers the digital payment cost. The equilibrium point then shifts from E to E' along the CRF as paper currency users are encouraged to switch to digital currency, enlarging the share of digital currency users further.

3. APPLICATIONS

3.1 Varying currency preference across countries

Some bitcoin enthusiasts claim that bitcoin's most important mission is to provide financial services to the huge number of unbanked on the planet (see, e.g., Vigna and Casey, 2015). As witnessed in the rapid expansion of *M-Pesa* in Kenya, there is great potential for digital currency to succeed in developing countries.[18] In contrast, digital currencies have so far played only negligible roles in transactions in advanced

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[16] The stress people feel in using digital currency is highly sensitive to technological progress in improving security, but also to the strengthening of institutions – for example, laws to ban the kinds of misconduct that resulted in the collapse of Mt. Gox, the first bitcoin exchange, in 2014. Legal costs from such events are counted among the costs included in x in equation (8).

[17] As the demand for digital currency increases, the demand for paper currency may decrease. Excess paper currency is returned to banks and ultimately to the central bank.

[18] M-Pesa is a money transfer system using mobile phones and not a digital currency in its precise sense.

economies. This is particularly the case in Germany and Japan. Here we explore some background causes of the variation in currency preference across countries, based on the framework built in the previous section.

Income per capita, y, is higher in advanced countries than in developing countries; meanwhile, although there is relatively little variation in the digital payment cost, x, across the world, it may be higher in developing countries than in advanced countries. Figure 7 illustrates these stylized facts in $(\sqrt{y}, x)$ space (as in Figure 3). Here, the majority of advanced countries are located in the area labeled "Advanced," while developing countries are found predominantly in the area labeled "Developing."

Suppose that all the preference parameter values other than x and y are common to advanced and developing economies. This would imply that advanced and developing economies share the same currency border. Three typical borders are drawn in Figure 7. First, let us assume that the border is given by $B_1 - B_1$. Then both advanced and developing countries use digital currency. Second, if the border is given by $B_2 - B_2$, advanced countries use digital currency, while developing countries use paper currency. Third, if the border is given by $B_3 - B_3$, all countries use paper currency. It is immediately clear that none of these cases is able to explain the empirical reality described in the opening paragraph above.

Evidently, there must be at least one preference parameter for which advanced and developing countries have different values. There are several possibilities. First, the storage cost of paper currency, z, may vary across countries. Thefts and crimes are more frequent in developing countries than in advanced countries. Thus, the storage cost would typically be much higher in developing countries than advanced countries. This would have the effect of shifting the developing country currency border upwards, so that it is located above that of advanced countries, as shown in Figure 8.

Second, the shoe leather cost of paper currency, a, may vary across countries. In advanced economies, bank branches and ATM machines are readily found on the street or in stores. These are harder to find in developing countries, where carrying cash on the street may also be quite risky. Thus, the shoe leather cost is likely to be much higher in developing countries than in advanced countries. A higher shoe leather cost would imply a steeper currency border, as shown in Figure 9.[19]

Third, the interest rate on deposits, r, may vary across countries. Interest rates in advanced countries have been quite low in the aftermath of the global financial crisis. In particular, to support fragile economies, the European Central Bank and the Bank of Japan introduced negative interest policies in 2014 and in 2016, respectively. Thus, the interest rate tends to be higher in developing countries than in advanced countries. As with the shoe leather cost, this would have the effect of steepening the currency border (see Figure 9 again).[20, 21]

3.2 Negative interest rates and a currency holding tax

As mentioned above, some central banks adopt a negative interest rate policy to solve the problem of the zero lower bound on interest rates (e.g., Haldane, 2015). As long as rates do not fall too deeply into negative territory, most deposits stay in bank accounts. Very high negative interest rates, however, may drive people to withdraw deposits and hold cash. Such currency hoarding weakens the effects of monetary easing significantly. One idea to deal with this problem is to impose taxes on cash holdings. We do not discuss the technological feasibility of taxing cash holdings here.[22] We examine the

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[19] These assumptions align well with what has happened since the introduction of M-pesa in Kenya, where a is much higher than $\hat{a}$ and z is much higher than x. M-pesa is viewed as a hybrid case of Figures 8 and 9. A caveat is that most M-pesa users are unbanked (people who have no banking account). Thus, the situation is slightly different from that assumed in the Baumol-Tobin model.

[20] Another possible but less plausible source of variation is the shoe leather cost of digital currency, $\hat{a}$, differing across countries.

[21] Beside the differences in costs incurred by currency users, one of the most important reasons for a developing country to prefer digital currency is the huge initial cost of setting up and the subsequent expense of maintaining a properly functioning nationwide bank-based payment system.

[22] Several schemes have been proposed for taxing cash holdings: Gesell's (1916) idea of a stamp tax on each bill was updated by Goodfriend (2000) in the form of a magnetic strip in the bill to record tax owed; Eisler's (1932) twin currency system was similarly revisited by Agarwal and Kimball (2015), who discussed the implementation of an exchange rate

impact of negative interest rates and the effects of taxing cash holdings on the optimal choice of transactions currency, taking it as given that the central bank or government is able to impose such a tax.

Let us begin with the situation where no tax is levied on currency holdings:[23] That is, interest rates on currencies are zero percent as before. In theory, faced with a negative rate on deposits, everybody would withdraw their full balance at once, i.e., $n^* = \hat{n}^* = 1$. The total cost is given by $c^* = a + ry/2 + z$ for using paper currency and by $\hat{c}^* = \hat{a} + ry/2 + x$ for using digital currency. The optimal currency choice problem is reduced to a comparison between $a + z$ and $\hat{a} + x$. Given z, a, and $\hat{a}$, individuals whose x is low use digital currency, while those whose x is high continue to use paper currency. Note that y has no effect on the choice of currency in the negative interest rate regime.

Next, we consider the situation where the government imposes a tax of t percent on holdings of currencies, both paper and digital. The currency holdings tax, t, is similar to the variable storage cost of cash, v, and thus added on the deposit interest rate, r, as in Section 2.2. Set t large enough for the adjusted interest rate, $r' \equiv r + t$, to be positive. This brings us back to the situation where the interest rate on deposits is positive. Recall that with positive interest rates on deposits, individuals are motivated to maintain deposits for the sake of the interest. With positive taxes on cash, they reduce cash holdings so as to save on tax payments. Expression (11) with r replaced with r' provides individuals with a criterion for choosing their optimal currency for

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between cash and reserves in a central bank account; and Mankiw (2009) has playfully suggested a lottery to invalidate particular bills. It is also worth noting that *de facto* negative rates can be introduced by restricting the use of paper currency in various ways, for instance, by setting an upper limit on cash payments, abolishing high value bills, and so forth (see Agarwal and Kimball (2016) for a discussion of related issues). A concise summary of these schemes and discussion of their relative merits is provided by Rogoff (2016). Engaging with this debate on the feasibility of such schemes is beyond the scope of the current paper.

[23] Taxes on cash holdings are not science fiction. After the Civil War, the Federal Government levied taxes on paper money issued by private banks to prompt circulation of government-issued paper money (Act of Mar. 3, 1865, Ch. 78, Sec. 6 Stat. 484).

transactions purposes.

Difficulties in tax collection may differ between paper and digital currencies. Thus, even if the same face tax rate is imposed, the effective tax rates may differ between the two currencies. This difference affects the individual's optimal choice of currency. Denote the effective tax rate on paper currency by τ and that on digital currency by $\hat{\tau}$. Set τ and $\hat{\tau}$ large enough for the tax-adjusted interest rates, $\rho \equiv r + \tau$ and $\hat{\rho} \equiv r + \hat{\tau}$, to be positive. Then, the criterion governing currency choice, expression (11), is adjusted so that r on the left hand side is replaced with ρ and r on the right hand side with $\hat{\rho}$. The currency border then becomes

$$x = \sqrt{2\rho}\left(\sqrt{a} - \sqrt{\hat{\rho}/\rho}\sqrt{\hat{a}}\right)\sqrt{y} + z\,. \tag{16}$$

Individuals located above the currency border use paper currency, while those located below the border use digital currency.

Let us assume that the same face tax rate is imposed on paper and digital currencies. If taxes are equally difficult to collect for both currencies (the taxation difficulty is the same), the tax-adjusted interest rates are also the same, i.e., $\hat{\rho} = \rho\ (\geq 0)$. Under the assumption that $a > \hat{a}$, the currency border can be drawn as an upward-sloping line, as in Figure 10, which is almost the same as equation (12), except that r is replaced with ρ. Alternatively, if digital currency taxation is more difficult than paper currency taxation, the tax-adjusted interest rate on paper currency is higher than that on digital currency: $\rho > \hat{\rho}\ (\geq 0)$. This has the effect of rotating the currency border counterclockwise, as shown in the figure. Digital currency users increase in number, while paper currency users decrease. Lastly, if taxation on paper currency is more difficult than on digital currency, the tax-adjusted interest rate on digital currency is higher than that on paper currency: $\hat{\rho} > \rho\ (\geq 0)$. In this case, the currency border is rotated clockwise. Paper currency users increase in number, while digital currency users decrease. Appendix B shows that similar results are obtained even in the cases where $\rho < 0$ and/or $\hat{\rho} < 0$. That is, if a higher effective tax rate is imposed on paper currency, the currency border turns counterclockwise; otherwise, it turns clockwise.

3.3 Central bank digital currency

Now that negative interest rate policy has become a standard item on the monetary policy agenda, some central bank officials have started talking enthusiastically about the introduction of a central bank digital currency.[24] As discussed in Section 3.2, the efficacy of negative interest rate policy can be shored up by the imposition of a large enough tax on cash holdings. Since the taxation of paper currency is difficult in practice, however, central bankers are naturally drawn to the idea of issuing a digital currency, which would be much more readily taxable. Yet they face the problem that, if taxes are imposed only on the central bank currency, users may well switch to a non-governmental digital currency option. To avoid such an outcome, the government has to device an effective way to collect taxes on non-governmental digital currency.

Below, we consider individuals' optimal choice between the central bank digital currency and non-governmental digital currency. The latter is the digital currency we have examined above. The criterion governing optimal currency choice is given as follows.

$$central\ bank\ digital\ currency \begin{Bmatrix} \succ \\ \backsim \\ \prec \end{Bmatrix} non\ governmental\ digital\ currency$$

$$\Leftrightarrow \qquad \sqrt{2\hat{a}_c\hat{\rho}_c y} + x_c \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \sqrt{2\hat{a}_n\hat{\rho}_n y} + x_n\,, \tag{17}$$

where $\hat{\rho}_c \equiv r + \hat{\tau}_c\ (\geq 0)$ and $\hat{\rho}_n \equiv r + \hat{\tau}_n\ (\geq 0)$. Subscripts c and n stand for central bank digital currency and non-governmental digital currency, respectively.

As a benchmark, let us assume that the shoe leather cost for the central bank digital currency is the same as that for the non-governmental digital currency: $\hat{a}_c = \hat{a}_n = \hat{a}$. Given x_c and $\sqrt{\hat{\rho}_c}$, the currency border between the central bank and non-governmental digital currencies is given by

$$x_n = \sqrt{2\hat{a}}\left(\sqrt{\hat{\rho}_c} - \sqrt{\hat{\rho}_n}\right)\sqrt{y} + x_c. \tag{18}$$

[24] Danezis and Meiklejohn (2016) present a prototype system for issuing a central bank digital currency.

Suppose that the face tax rates are the same between paper and digital currencies and also that the two currencies are equivalent in terms of taxation difficulty. This implies that they will also share both the same effective tax rate and the same tax-adjusted interest rate: i.e., $\hat{\rho}_c = \hat{\rho}_n(\geq 0)$. In this case, the currency border takes the shape of a horizontal line, as depicted in Figure 11.

As mentioned above, however, we typically expect taxation on central bank digital currency to be easier than on non-governmental currency. Thus, given the same face tax rate, the effective tax rate on central bank digital currency should be higher than that on non-governmental digital currency. This implies a tax-adjusted interest rate for central bank digital currency which is higher than that for non-governmental digital currency: i.e., $\hat{\rho}_c > \hat{\rho}_n(\geq 0)$. The currency border for this case is depicted by the upward-sloping line in Figure 11. Individuals located above the border prefer central bank digital currency, while those located below the border prefer non-governmental digital currency.

By contrast, if the government were able to impose a high enough face tax rate on non-governmental digital currency, the effective tax rate on non-governmental digital currency could exceed that on central bank digital currency, $\hat{\tau}_n > \hat{\tau}_c$, implying $\hat{\rho}_n > \hat{\rho}_c(\geq 0)$. In this case, the currency border would be downward-sloping, expanding the area in which the central bank digital currency is preferred. However, such a discriminatory tax policy would be politically unpopular and likely to encounter considerable resistance. The implementation of such a policy would therefore involve prohibitively high administrative costs.

A central bank has access to a variety of policy levers to make its digital currency more attractive. To lower the digital payment cost, x_c, the central bank can implement the following policies. First, it can charge a lower remittance fee on each transfer of central bank digital currency. Costs may instead be defrayed by imposing a lump sum tax on all residents, regardless of whether they are central bank currency users or non-governmental currency users.[25] Second, the central bank can use its

[25] Remittance fees for bitcoin are cheap at present, but, will become more expensive in the future. The costs of verifying a transaction's validity, i.e., the proof-of-work, are currently

budget to hire expert staff to improve the security of the central bank digital currency. Third, a central bank which has succeeded in maintaining a stably valued paper currency can leverage its trustworthiness, reputation, and credibility to bolster confidence in its digital currency. All of these measures would contribute effectively to expanding the area of central bank digital currency preference in the figure.

The above discussion suggests, at least mathematically, that the government and the central bank could potentially get together to drive out non-government digital currency completely. In Figure 12, first the central bank would shift the currency border downward by setting $x_c = 0$, after which the government would rotate the border clockwise by setting $\hat{\rho}_c < \hat{\rho}_n$.[26] This policy mix, however, would scarcely be implementable in practice. This is firstly because individuals tend to wish to preserve a minimum level of privacy, and this psychological factor will make it impossible to force x_c all the way down to zero. Secondly, as mentioned above, the administrative costs of imposing discriminatory taxes on non-governmental digital currency would be extremely high. These facts point to the existence of an upper limit on $\hat{\rho}_n$, and one which may conceivably be lower than $\hat{\rho}_c$.

In addition, there has been disagreement about whether the government or the central bank should have a monopoly on issuing currency. Proponents argue that

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incurred by miners and mostly paid for by newly created bitcoins (see, e.g., Vance and Stone (2014) for details on the mining business). The provision of new bitcoins, however, will be reduced every four years and will cease entirely by around 2140. Thereafter, the costs of proof-of-work will likely be paid by service users. Remittance fees will need to be raised to compensate for those costs. Aste (2016) reports estimated costs exceeding $5 per transaction. This means that the digital payment cost of bitcoin has a positive lower bound. Note also that the costs of proof-of-work should be kept high enough to protect the system against attackers.

[26] Another way for a central bank to rotate the currency border clockwise is to reduce (potentially to zero) the cost of exchanging bank deposits for central bank digital currency, i.e., the shoe leather cost of central bank digital currency, $\hat{a}_c$. In other words, the central bank would impose a minimal or indeed no charge for providing central bank digital currency. Substituting, for example, $\hat{a}_c = 0$ in equation (17), we see the currency border between central bank and non-governmental digital currencies taking on a negative gradient as in Figure 12.

central bank digital currency, if introduced with an enforceable currency tax, would improve the efficacy of negative interest rate policy. They also point out that discouraging the use of non-governmental digital currency will contribute to containing the underground economy. In contrast, opponents such as Hayek (1976) focus attention on the various problems that have accompanied currency monopolization by the government. They insist that currency issuers, including the government and the central bank, should compete with each other so that *good money drives out bad* in a process of natural selection. Competition is seen as saving the government and the central bank from the misuse of money.

3.4 Concurrence of multiple currencies

The model is not limited to the choice between just two alternative currencies, but is equally applicable to cases where three or more currencies circulate simultaneously. Let us consider the case of three currencies: paper currency, central bank digital currency, and non-governmental digital currency. We assume positive interest rates on deposits and no taxes on the currencies here.

The total costs are given by

$$\sqrt{2ary} + z \qquad \textit{for paper currency;} \tag{19a}$$

$$\sqrt{2\hat{a}ry} + x_c \qquad \textit{for central bank digital currency;} \tag{19b}$$

$$\sqrt{2\hat{a}ry} + x_n \qquad \textit{for non-governmental digital currency,} \tag{19c}$$

where the same shoe leather cost is applied to central bank and non-governmental digital currency as in the previous section. The three currency borders are given by

$$\sqrt{y} = \frac{x_c - z}{\sqrt{2r}(\sqrt{a} - \sqrt{\hat{a}})} \qquad \textit{between paper and central bank digital currencies;} \tag{20a}$$

$$x_n = x_c \qquad \textit{between central bank and non-governmental digital currencies;} \tag{20b}$$

$$x_n = \sqrt{2r}\left(\sqrt{a} - \sqrt{\hat{a}}\right)\sqrt{y} + z \qquad \textit{between paper and non-governmental digital currencies.} \tag{20c}$$

In Figure 13, Case 1 assumes $x_c \geq z$. Given an individual's $\sqrt{y}$ and x_n, the upper diagram shows the order of his preferences among the three currencies; while the lower diagram indicates his currency choice. Two points are worth mentioning. First, paper currency users, as they get richer, will switch to digital currencies, i.e., either to the central bank digital currency or to the non-governmental digital currency. Second, as the digital payment cost of the central bank digital currency, x_c, declines relative to the storage cost of the paper currency, z, the area in which users prefer central bank digital currency expands. Once x_c falls below z, as shown in the diagrams for Case 2, where $x_c < z$, paper currency users disappear altogether, and non-governmental digital currency users become a minority.

If the shoe leather cost and tax-adjusted interest rate for paper currency are sufficiently large, similar results are obtained under the assumption of negative interest rates and positive tax rates on currency holdings. Otherwise, the results are strikingly different from those obtained above. In particular, paper currency use is more prevalent. See Appendix C for details.

3.5 The future of paper and digital currencies

Many are skeptical about the potential of digital currency, and it is generally considered too early to switch precipitately from the current paper-currency regime to a pure digital-currency regime. The skyrocketing rise in the bitcoin price in the recent years may or may not end miserably, as did the "tulip mania" and "South Sea" bubbles of yore. Nonetheless, the analysis above provides good reasons for us to believe that digital currency will play an important role in our economy, whether it happens in the near or remote future.

Figure 14 summarizes the heart of the matter. The share of paper currency is almost 100 percent at present. As discussed in Section 3.1, however, the share of digital currency is likely to increase over time, as economic growth proceeds. Progress in security technology will reduce the stress people experience making digital payments and cause the rate of adoption of digital currency to accelerate. A variety of altcoins will be created to facilitate increasingly diverse transactions, even if the timing of these developments cannot at present be precisely foreseen.

Younger generations are in general more accustomed to using information technology and thus less conservative than the older about currency choice. In fact, various payment media have been invented and popularized in their turn—coins, paper money, credit cards, and debit cards—and each has had its own important role to play. It is worth noting how credit cards are driving out paper money, as they are being used even for very small payments. This suggests that the notion of digital currency becoming a popular instrument of payment is far from merely the wishful thinking of an ardent technophile.

The successes of non-governmental digital currencies will attract central banks' attention, spurring them to issue their own digital currencies. Historically, we can observe the repetition of the same cycle, in which private money emerges first, only to be coopted either by the government or by the central bank. Our analysis shows that the central bank has access to a variety of policy levers with which to assert its hegemony over non-governmental digital currencies. In any situation, the government can enforce the use of central bank digital currency by bestowing it with the status of legal tender. Moreover, some central banks have built brand loyalty among individuals by managing the value of paper currency successfully.[27] This brand loyalty gives a great competitive advantage to central bank digital currency.

Nevertheless, some non-governmental digital currencies as well as paper currency will survive the competition with central bank digital currency. Although protected by cryptography, digital currency is always subject to attack by malicious

[27] The value of digital currencies, whether issued publicly or privately, should be measured in terms of a basket of consumer goods and services, and not in, say, dollars. For a central bank digital currency, the central bank will target some specific level of a consumer price index that summarizes the prices of such a basket of goods and services purchased using that currency. With no comparable stabilizer built into the scheme, the value of a non-governmental digital currency will inevitably be more volatile than that of the central bank digital currency, since nobody is responsible for keeping it stable.

hackers. This risk may be exacerbated by cyber terrorists. Paper currency will continue, therefore, to have a role at least as a failsafe. Furthermore, few people would welcome the idea of a big-brother government monitoring all of their transactions. Paper currency and non-governmental digital currencies will allow users to perform anonymous transactions that maintain their privacy.

The government does not need to monitor transactions as long as they are legal. But non-governmental digital currencies may also be used for illegal purposes in the underground economy. A balance is required between preserving privacy and banning illegal transactions. The government needs to construct a regulatory framework and to design efficient monitoring mechanisms in order to discourage illicit transactions effectively. At the same time, however, regulations must not be so strict that they drive out non-governmental digital currencies completely.

4. CONCLUSION

This paper extends the Baumol-Tobin model to investigate optimal currency choice for transactional purposes. We focus first on comparing paper and digital currencies, before examining the choice between central bank and non-governmental digital currencies. Which currency is preferred depends on an individual's preference parameters, and some of these vary widely across individuals. Nonetheless, the analysis suggests that digital currency, whether provided non-governmentally or issued by the central bank, has the potential to become a major medium of exchange in time. In fact, although not treated explicitly in this paper, some digital currencies already enable money to be transferred internationally at extremely low costs.

Some central bank officials have recently shown interest in the idea of issuing central bank digital currency. Central bank digital currency raises the efficacy of negative interest rate policy if introduced in concert with positive tax rates on cash holdings. Some argue that the government can constrain the underground economy by abolishing paper currency (see Rogoff, 2016) and providing central bank digital currency as a substitute. Our analysis shows that the central bank and government can design a digital currency with the potential to drive out paper and non-governmental currency, though not completely.

However, Hayek's (1976) argument about the various problems created as a result of currency monopolization by the government in the past demands attention. A government taking his argument seriously would not simply issue central bank digital currency, but would also ensure its provision of a legal and institutional infrastructure that achieves a good balance between fostering non-governmental digital currencies and regulating their activities so that *good money drives out bad* in a natural selection process. Further debate is needed on the advantages and disadvantages of the government or central bank issuing digital currency.

Lastly, the analysis detailed in this paper could be usefully extended in various ways in future research. First, the Baumol-Tobin model deals with a partial equilibrium. It would be interesting to extend the model to a general equilibrium setting, such as in Jovanovic (1982) and Romer (1986). Second, the paper investigates the effects of a tax on currency holdings, taking as given its feasibility. The effective tax rate on currency, however, is highly sensitive in practice to difficulties in tax collection. Thus, evaluating the feasibility of various taxation schemes is crucial for understanding optimal currency choice precisely. Third, this paper focuses exclusively on transactions demand for money. Some say, however, that most bitcoins are held for speculative purposes. So, another direction to pursue is to analyze the implications of considering digital currency holdings as financial assets for investment, like equities. Fourth, while the current paper has been predominantly theoretical, quantitative analysis would be particularly useful from the social planner's or regulator's point of view.[28]

APPENDIX A. PREFERENCE OVER CURRENCY MIXTURES

Consider an individual who prefers digital currency to paper currency. This means

[28] Barrdeara and Kumhof (2016) construct a dynamic stochastic general equilibrium model to study the macroeconomic impact of issuing central bank digital currency on GDP.

$$\sqrt{2ary} + z \geq \sqrt{2\hat{a}ry} + x. \tag{A1}$$

Suppose that he spends θ of his income using paper currency and $1-\theta$ using digital currency. The total cost is

$$\begin{aligned}
&\sqrt{2ar\theta y} + \theta z + \sqrt{2\hat{a}r(1-\theta)y} + (1-\theta)x \\
&= \sqrt{\theta}\sqrt{2ary} + \theta z + \sqrt{1-\theta}\sqrt{2\hat{a}ry} + (1-\theta)x \\
&\geq \theta\sqrt{2ary} + \theta z + (1-\theta)\sqrt{2\hat{a}ry} + (1-\theta)x \\
&= \theta(\sqrt{2ary} + z) + (1-\theta)(\sqrt{2\hat{a}ry} + x) \\
&\geq \theta(\sqrt{2\hat{a}ry} + x) + (1-\theta)(\sqrt{2\hat{a}ry} + x) \\
&= \sqrt{2\hat{a}ry} + x
\end{aligned} \tag{A2}$$

The third line inequality holds, because $\sqrt{\theta} \geq \theta$ for $\theta \leq 1$; the fifth line inequality comes from the assumption, i.e., inequality (A1). The result shows that the proposed currency mixture is more costly than pure digital currency. A similar result is obtained for an individual who prefers paper currency. Therefore, individuals use one of the currencies exclusively, not a mixture of them.

APPENDIX B. AN EXAMPLE SHOWING THE NETWORK EXTERNALITY FROM CURRENCY CHOICE

Assume that x and y follow joint distribution f, which is defined as follows.

$$f(x, y) \equiv \psi(y)\lambda e^{-\lambda x}, \tag{B1}$$

where

$$\psi(y) \equiv \begin{cases} 1 & for \quad \bar{y} - \frac{1}{2} \le y \le \bar{y} + \frac{1}{2}, \\ 0 & \qquad otherwise\,; \end{cases} \tag{B2}$$

and

$$\lambda \equiv \frac{1}{\bar{x}}. \tag{B3}$$

Figure B1 illustrates this joint distribution. Conditional on the value of y, the marginal distribution of x is an exponential distribution with mean $\bar{x} = 1/\lambda$. Conditional on the value of x, the marginal distribution of y is a uniform distribution with mean $\bar{y}$.

Define $\tilde{x} \equiv \sqrt{2r}(\sqrt{a} - \sqrt{\hat{a}})\sqrt{y} + z$. Using equations (12) and (B1), the share of digital currency users is given by

$$\begin{aligned} \theta &= \int_0^\infty \int_0^{\tilde{x}} \psi(y) \lambda e^{-\lambda x}\, dx\, dy \\ &= \int_{\bar{y}-1/2}^{\bar{y}+1/2} \int_0^{\tilde{x}} \lambda e^{-\lambda x}\, dx\, dy \\ &= 1 - \frac{\bar{x}^2 e^{-z/\bar{x}}}{r(a-\hat{a})^2} \{(1+v_2)e^{-v_2} - (1+v_1)e^{-v_1}\}, \end{aligned} \tag{B4}$$

where

$$v_1 \equiv \frac{1}{\bar{x}} \sqrt{2r}(\sqrt{a} - \sqrt{\hat{a}}) \sqrt{\bar{y} + \frac{1}{2}}, \tag{B5}$$

$$v_2 \equiv \frac{1}{\bar{x}} \sqrt{2r}(\sqrt{a} - \sqrt{\hat{a}}) \sqrt{\bar{y} - \frac{1}{2}}. \tag{B6}$$

Note that θ is a function of only $\bar{x}$ and $\bar{y}$, and that it is decreasing in $\bar{x}$, as assumed in Section 2.5. It can be easily shown that θ is increasing in $\bar{y}$.

APPENDIX C. INSUFFICIENT TAXATION ON PAPER AND/OR DIGITAL CURRENCIES

Taxation on currency holdings is not easy. Thus, the situation may emerge where effective tax rates on some currencies are not sufficient for the tax-adjusted interest rates to be positive. In that situation, individuals withdraw deposits from their bank accounts and exchange them for paper or digital currency. The criterion obtained in Section 3.2 is then no longer valid in its existing form. Nonetheless, as shown below, the main implications obtained in that section still hold. That is, the currency border rotates counterclockwise if a higher effective tax rate is imposed on paper currency, and clockwise otherwise.

Case 1: $\rho \geq 0 > \hat{\rho}$

In this case, only the tax-adjusted interest rate on digital currency is negative. Digital currency users withdraw deposits from their bank accounts all at once, i.e., $\hat{n}^* = 1$. The total cost is given by $\hat{c}^* = \hat{a} + \hat{\rho}y/2 + x$ for digital currency, and $c^* = \sqrt{2a\rho y} + z$ for paper currency. The currency border is obtained by equating these costs.

$$x = -\frac{\hat{\rho}}{2}\left(\sqrt{y} - \frac{\sqrt{2a\rho}}{\hat{\rho}}\right)^2 + \frac{a\rho}{\hat{\rho}} + z - \hat{a}\,. \tag{C1}$$

The currency border is not a linear function any longer, but a quadratic function of $\sqrt{y}$. In Figure C1 (1), the dotted curve indicates the currency border with $\rho = |\hat{\rho}|$, and the solid curve depicts the border with $\rho > |\hat{\rho}|$. The currency border rotates counterclockwise if a higher effective tax rate is imposed on paper currency.

Case 2: $\hat{\rho} \geq 0 > \rho$

In this case, only the tax-adjusted interest rate on paper currency is negative. Paper currency users withdraw deposits from their bank accounts all at once, i.e., $n^* = 1$. The total cost is given by $c^* = a + \rho y/2 + z$ for paper currency, and $\hat{c}^* = \sqrt{2\hat{a}\hat{\rho}y} + x$ for digital currency. The currency border is obtained by equating these costs.

$$x = \frac{\rho}{2}\left(\sqrt{y} - \frac{\sqrt{2\hat{a}\hat{\rho}}}{\rho}\right)^2 - \frac{\hat{a}\hat{\rho}}{\rho} + z + a\,. \tag{C2}$$

The currency border is not a linear function any longer, but a quadratic function of $\sqrt{y}$. In Figure C1 (2), the dotted curve indicates the currency border with $\hat{\rho} = |\rho|$, and the solid curve depicts the border with $\hat{\rho} > |\rho|$. The currency border turns clockwise if a higher effective tax rate is imposed on digital currency.

Case 3: $0 > \rho \geq \hat{\rho}$

In this case, the tax-adjusted interest rates on paper and digital currencies are both negative. All individuals withdraw deposits from their bank accounts all at once, i.e., $n^* = \hat{n}^* = 1$. The total cost is given by $c^* = a + \rho y/2 + z$ for paper currency, and $\hat{c}^* = \hat{a} + \hat{\rho} y/2 + x$ for digital currency. The currency border is obtained by equating these costs.

$$x = \frac{1}{2}(\rho - \hat{\rho})y + z + a - \hat{a}\,. \tag{C3}$$

The currency border is a linear function of y, but a quadratic function of $\sqrt{y}$. In Figure C1 (3), the dotted horizontal line indicates the currency border with $\rho = \hat{\rho}$, and the solid curve depicts the border with $\rho > \hat{\rho}$. The currency border rotates counterclockwise if a higher effective tax rate is imposed on paper currency.

Case 4: $0 > \hat{\rho} \geq \rho$

In this case, the currency border is the same as equation (C3). In Figure C1 (4), the solid curve depicts the border with $\hat{\rho} > \rho$. The currency border turns clockwise if a higher effective tax rate is imposed on digital currency.

APPENDIX D. AN OPTIMAL CHOICE AMONG THREE CURRENCIES WITH NEGATIVE INTEREST RATES AND POSITIVE CURRENCY TAXES

In this appendix, we address the case where there are negative interest rates on deposits but the government imposes sufficient taxes on currency holdings so that the adjusted interest rates are positive. The total costs are given by

$$\sqrt{2a\rho y} + z \qquad \textit{for paper currency;} \tag{D1a}$$

$$\sqrt{2\hat{a}\hat{\rho}_c y} + x_c \qquad \textit{for central bank digital currency;} \tag{D1b}$$

$$\sqrt{2\hat{a}\hat{\rho}_n y} + x_n \qquad \textit{for non-governmental digital currency,} \tag{D1c}$$

where the adjusted interest rate on central bank digital currency may differ from that on non-governmental digital currency, as in the main text. Then the three currency borders are given as

$$\sqrt{y} = \frac{x_c - z}{\sqrt{2}(\sqrt{a\rho} - \sqrt{\hat{a}\hat{\rho}_c})} \qquad \textit{between paper and central bank digital currencies;} \tag{D2a}$$

$$x_n = \sqrt{2}\left(\sqrt{\hat{a}\hat{\rho}_c} - \sqrt{\hat{a}\hat{\rho}_n}\right)\sqrt{y} + x_c \qquad \textit{between central bank and non-governmental digital currencies;} \tag{D2b}$$

$$x_n = \sqrt{2}\left(\sqrt{a\rho} - \sqrt{\hat{a}\hat{\rho}_n}\right)\sqrt{y} + z \qquad \textit{between paper and non-governmental digital currencies.} \tag{D2c}$$

Case 1: $a\rho \geq \hat{a}\hat{\rho}_c$ and $x_c \geq z$

In Figure D1 (1), the upper diagram shows an individual's order of preference among the three currencies; the lower diagram indicates the currency chosen by that individual. We obtain similar results to those in Section 3.4. First, as they get richer, paper currency users switch to one of the two digital currencies, either to the central bank digital currency or to the non-governmental digital currency. Second, as the digital payment cost of the central bank digital currency, x_c, declines, the area in which users prefer central bank digital currency expands.

Case 2: $a\rho \geq \hat{a}\hat{\rho}_c$ and $x_c < z$

Once x_c falls below z, as in Figure D1 (2), paper currency users disappear altogether, and non-governmental digital currency users become a minority

Case 3: $a\rho < \hat{a}\hat{\rho}_c$ and $x_c \geq z$

This is the case where the shoe leather cost and the effective tax rate of paper currency are smaller than those of central bank digital currency. The results are very different from those obtained in Cases 1 and 2 above. No one uses central bank digital currency in this case, as shown in Figure D1 (3). As they get richer, paper currency users switch to non-governmental digital currency, but never to central bank digital currency.

Case 4: $a\rho < \hat{a}\hat{\rho}_c$ and $x_c < z$

As the digital payment cost of central bank digital currency, x_c, declines, central bank digital currency users emerge, as in Figure D1 (4). Note, however, that it is poorer people who choose to use central bank digital currency in this case. This result contrasts with that obtained in Cases 1 and 2, where it is the rich who use central bank digital currency.

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Figure 1. Cost minimization

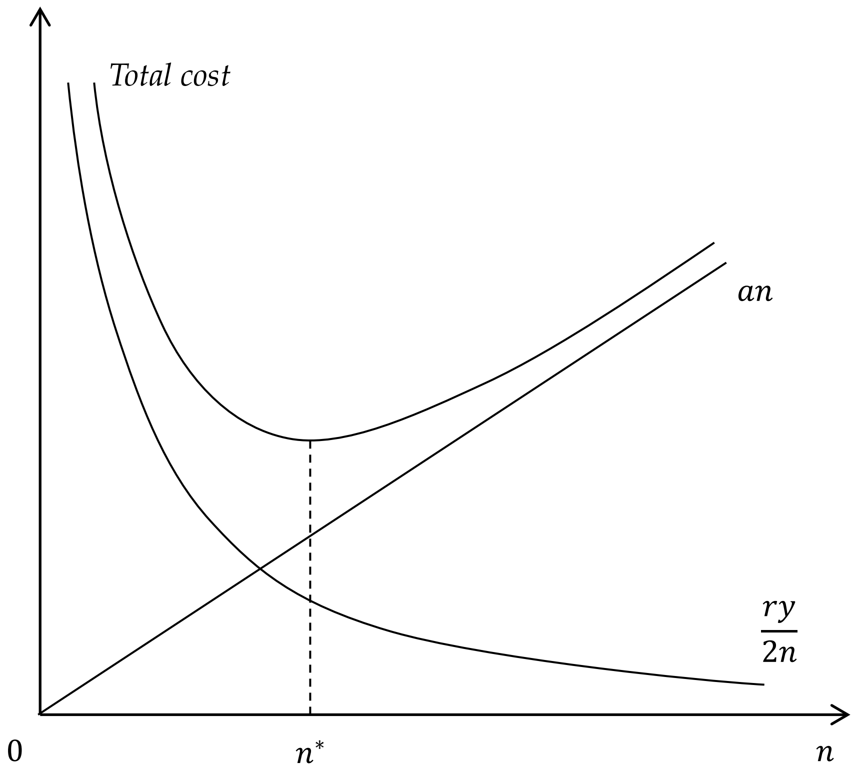


Figure 2. Cash holding

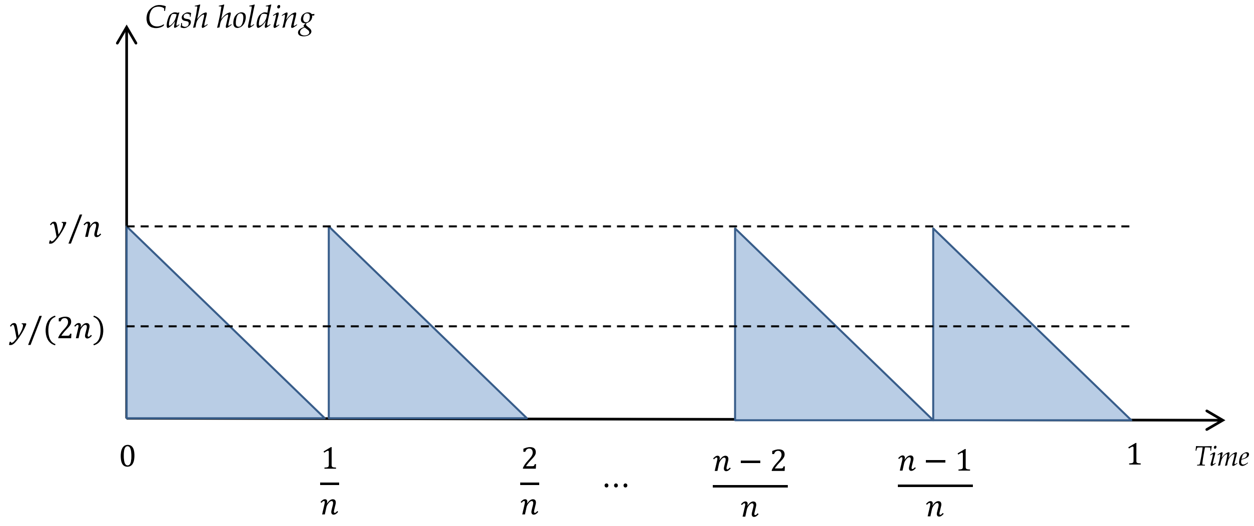


Figure 3. Currency border

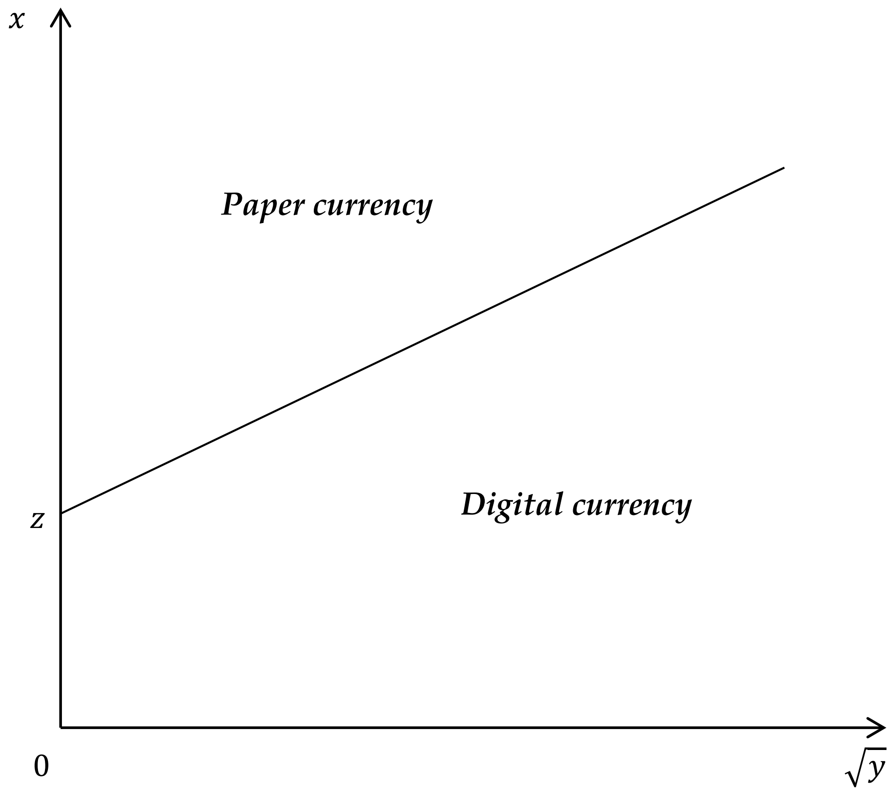
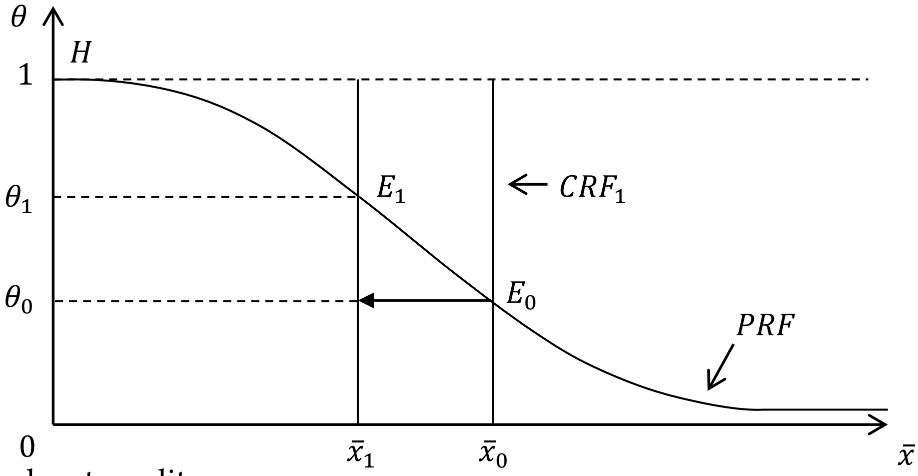
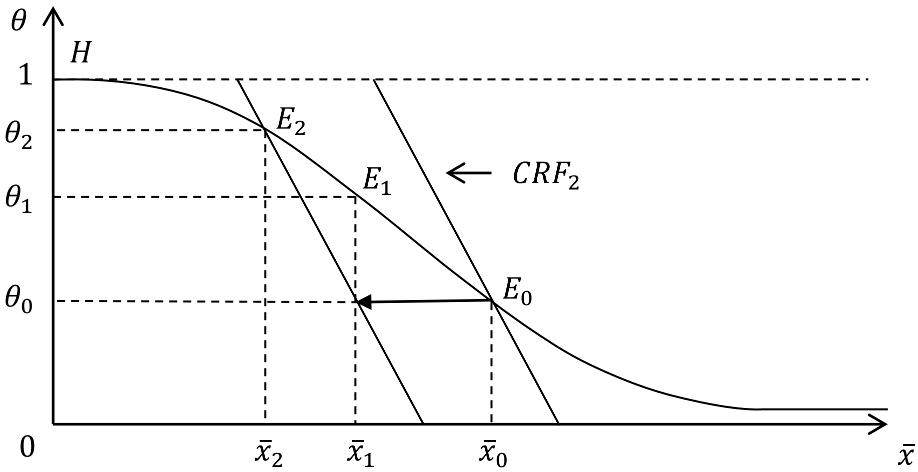


Figure 4. Reaction of the share of digital currency users to a shock affecting the digital payment cost

(1) No externality



(2) Weak externality



(3) Strong externality

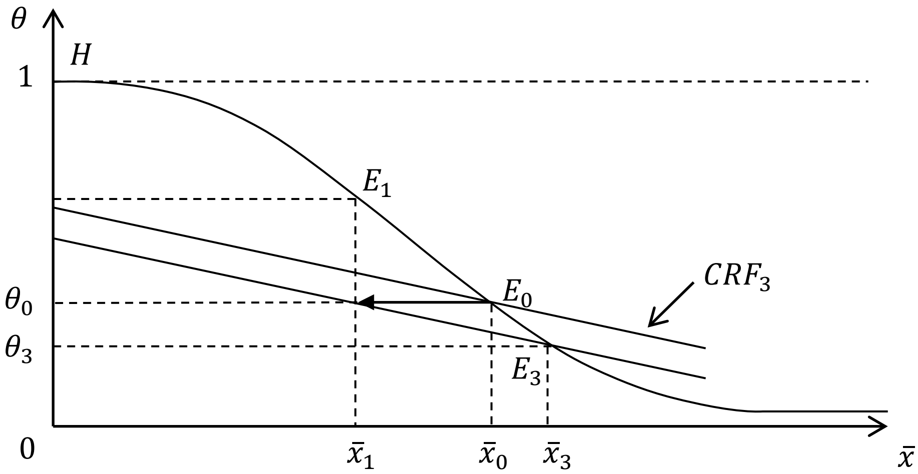


Figure 5. Critical mass for the diffusion of digital currency

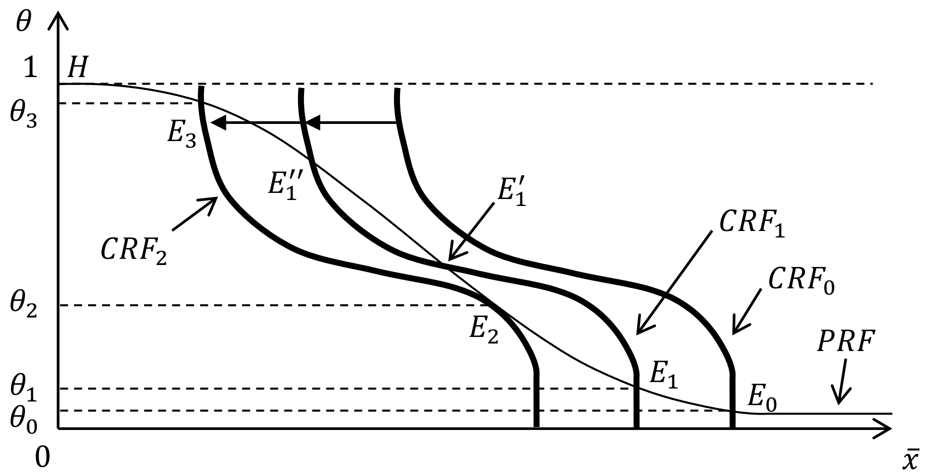


Figure 6. Reaction of the share of digital currency users to a shock affecting income growth

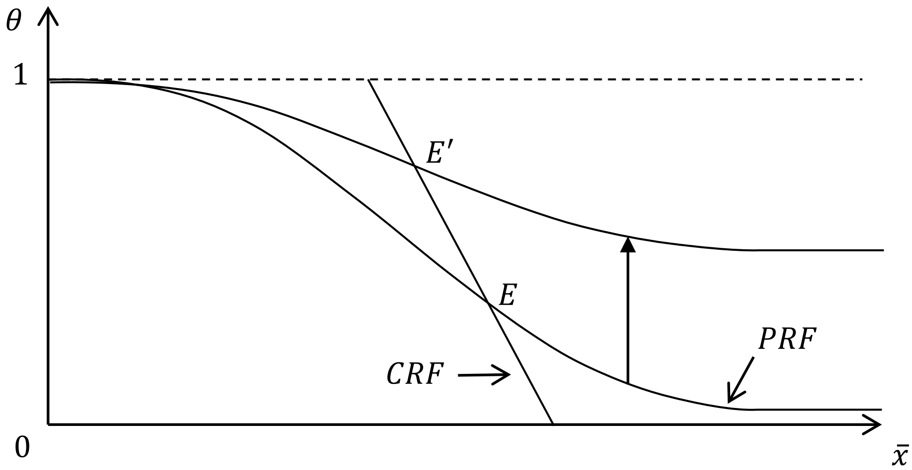


Figure 7. Cross-country preference variation

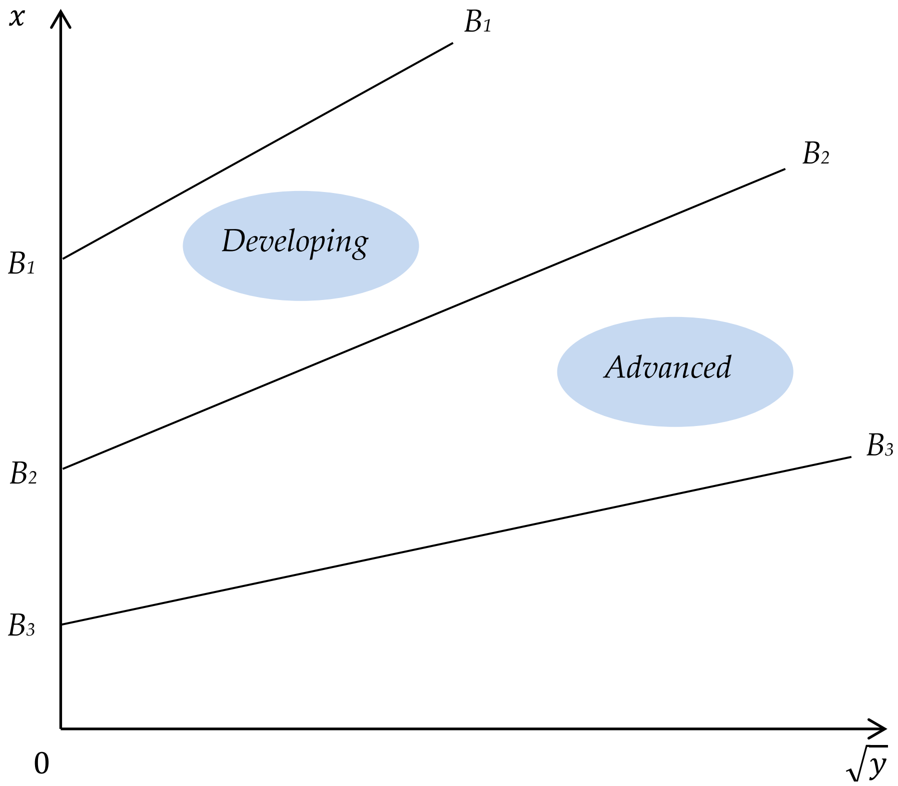


Figure 8. Cross-country variation in storage costs

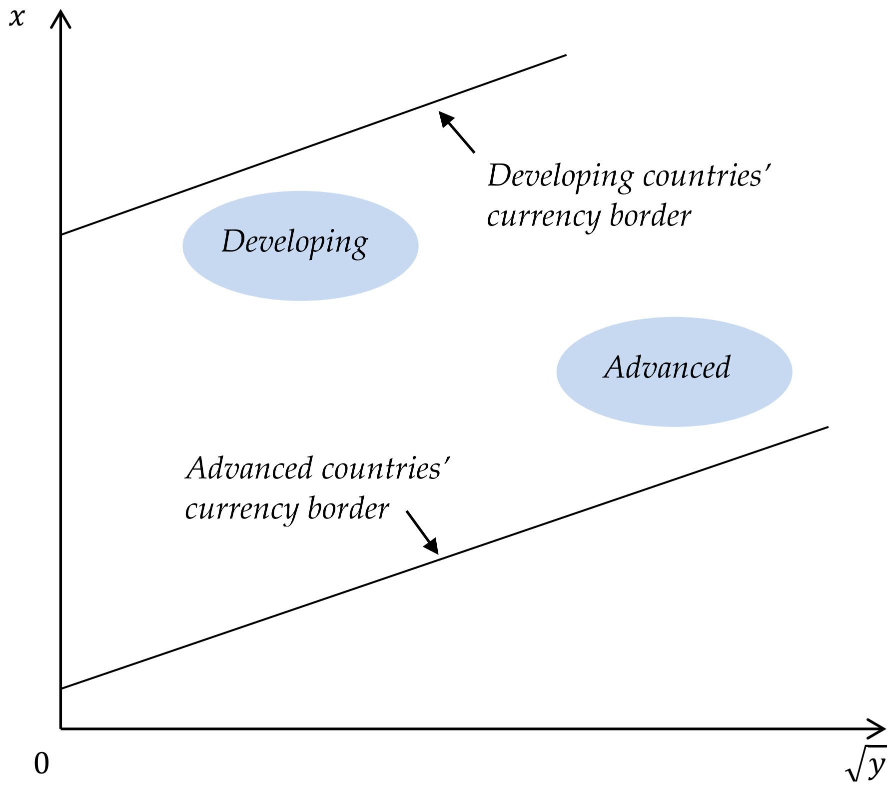


Figure 9. Cross-country variation in shoe leather costs and interest rates

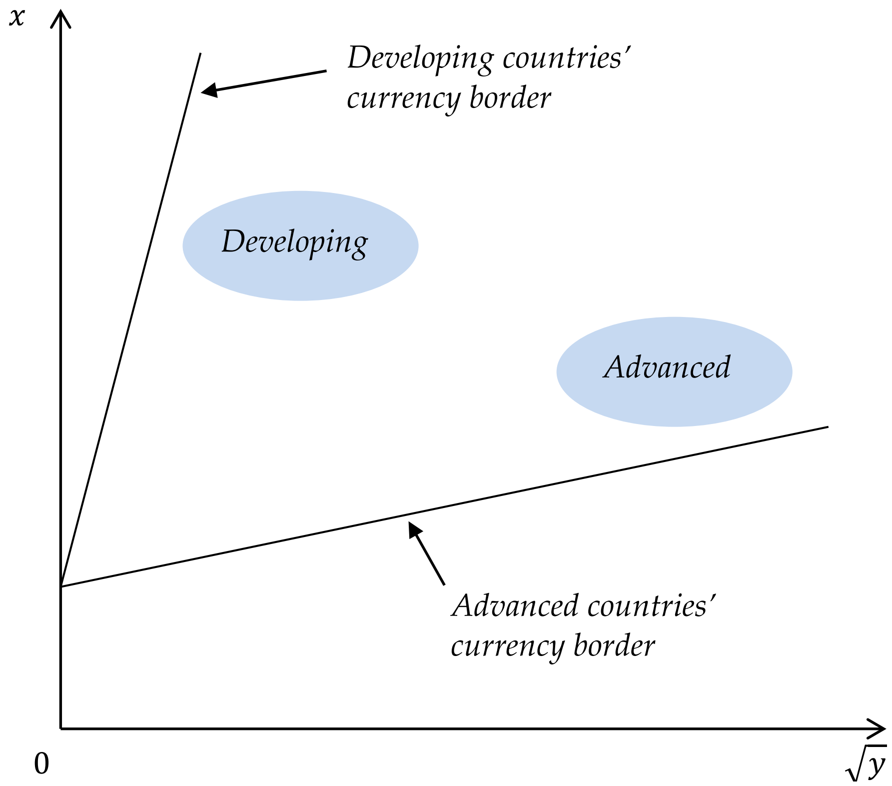


Figure 10. Different effective tax rates on paper and digital currencies

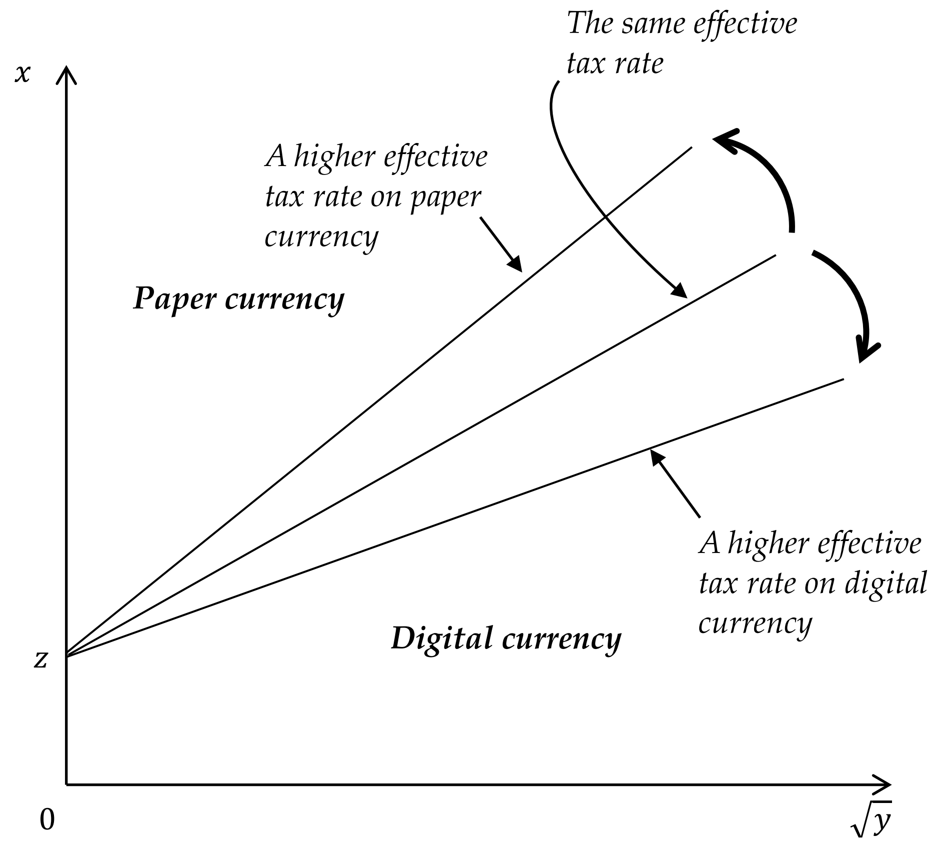


Figure 11. Different effective tax rates on central bank and non-governmental digital currencies

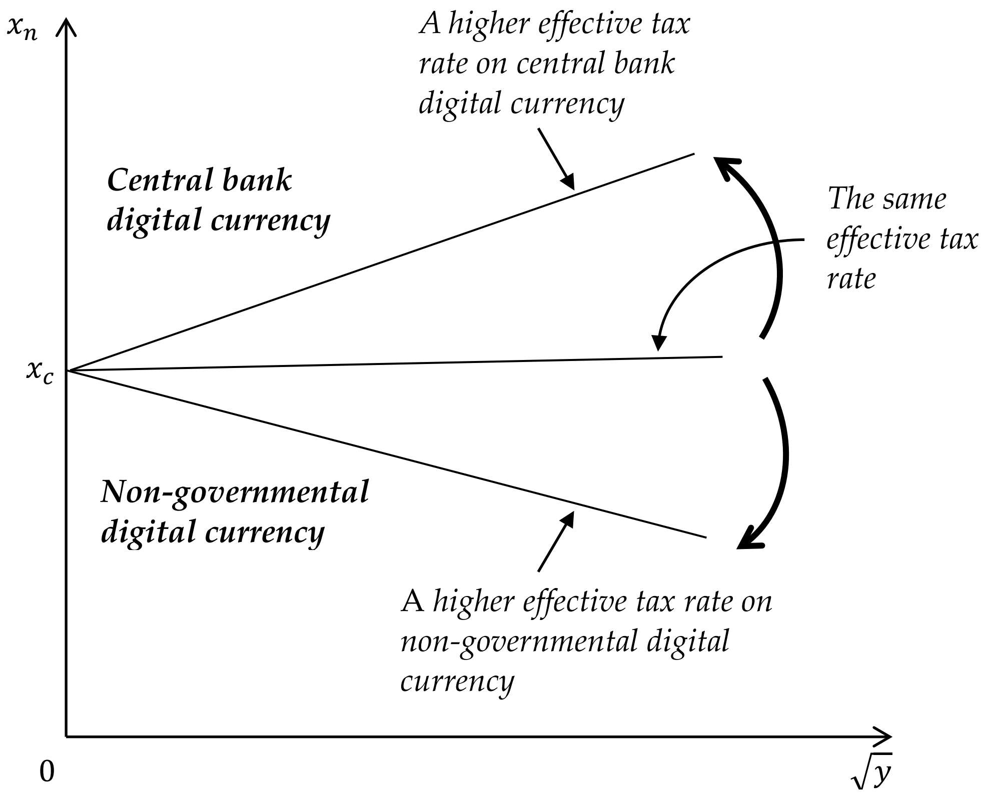


Figure 12. Monopolization of currency issue by the central bank

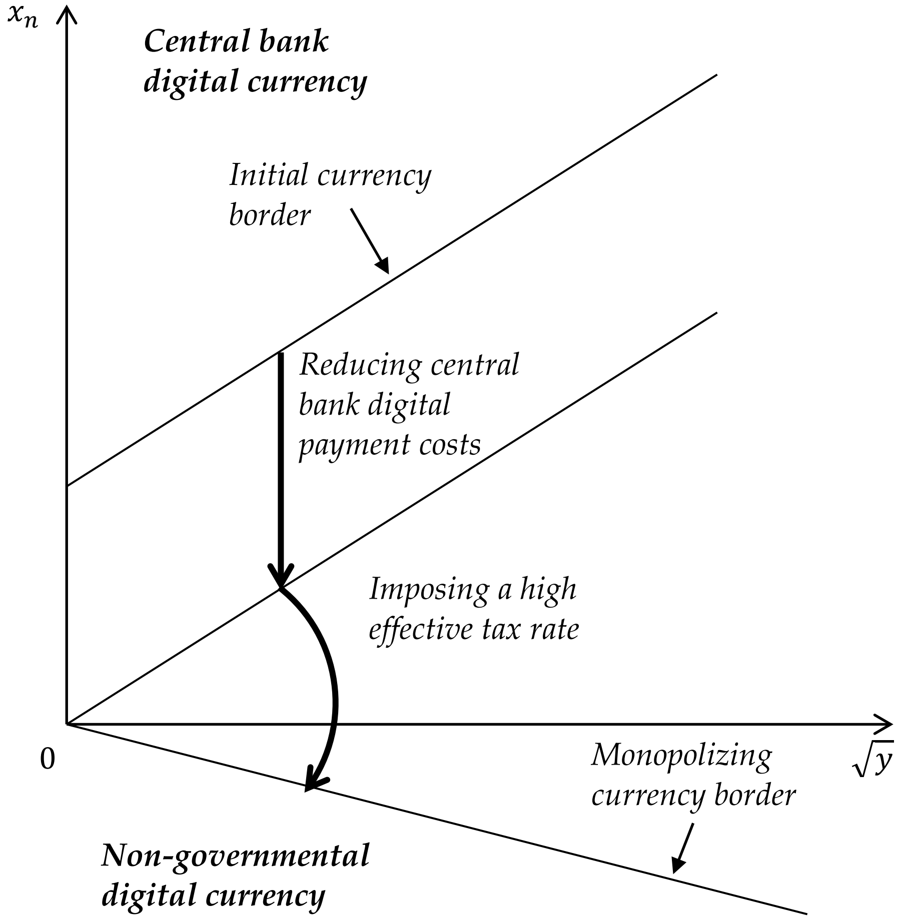
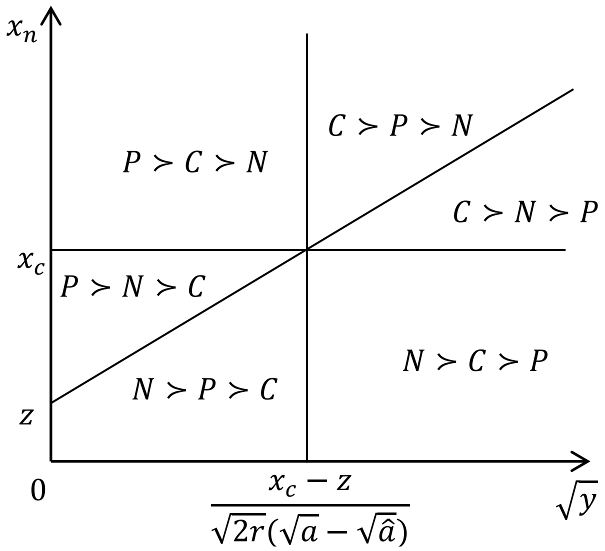


Figure 13. Comparison among three currencies with positive interest rates and no taxes

Case 1: $x_c \geq z$

(a) Currency preferences



Note. P: paper currency; C: central bank digital currency; N: non-governmental digital currency

(b) Currency choice

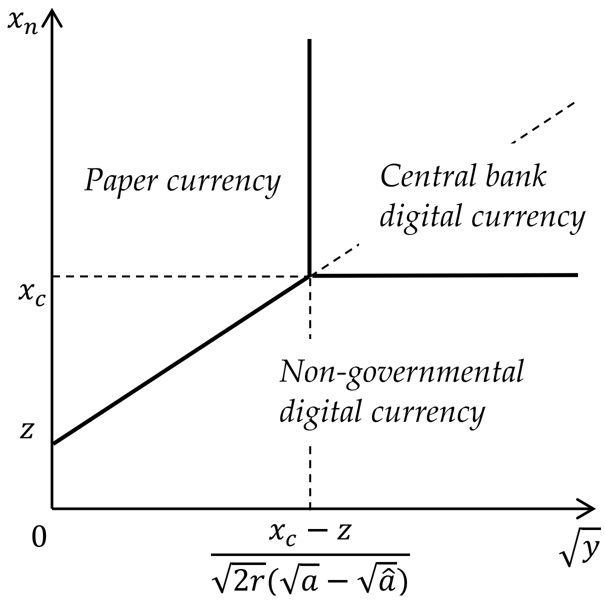
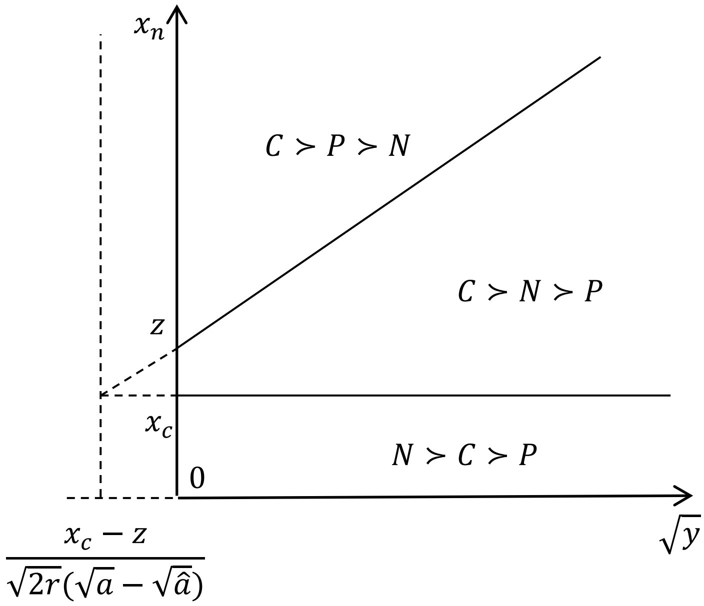


Figure 13. (continued)

Case 2: $x_c < z$

(a) Currency preferences



Note. P: paper currency; C: central bank digital currency; N: non-governmental digital currency

(b) Currency choice

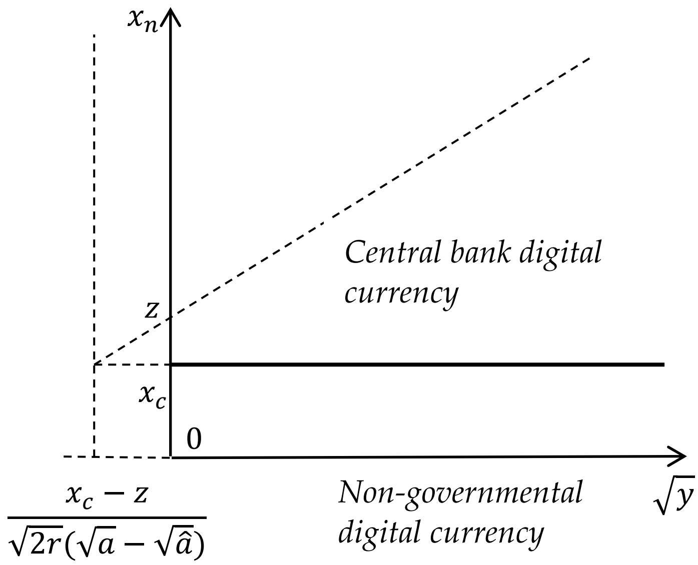


Figure 14. Rise and fall of currencies

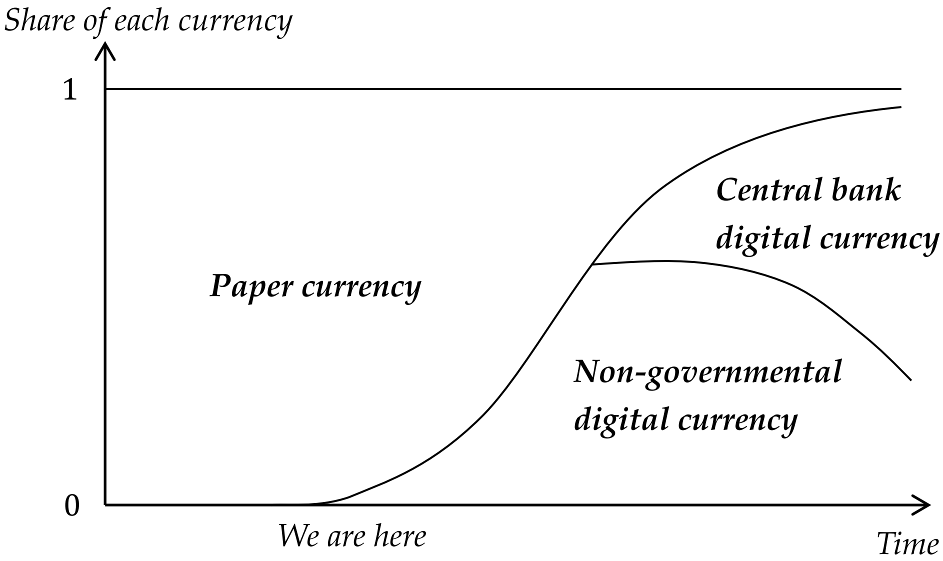


Figure B1. Joint distribution of income and digital payment cost: an example

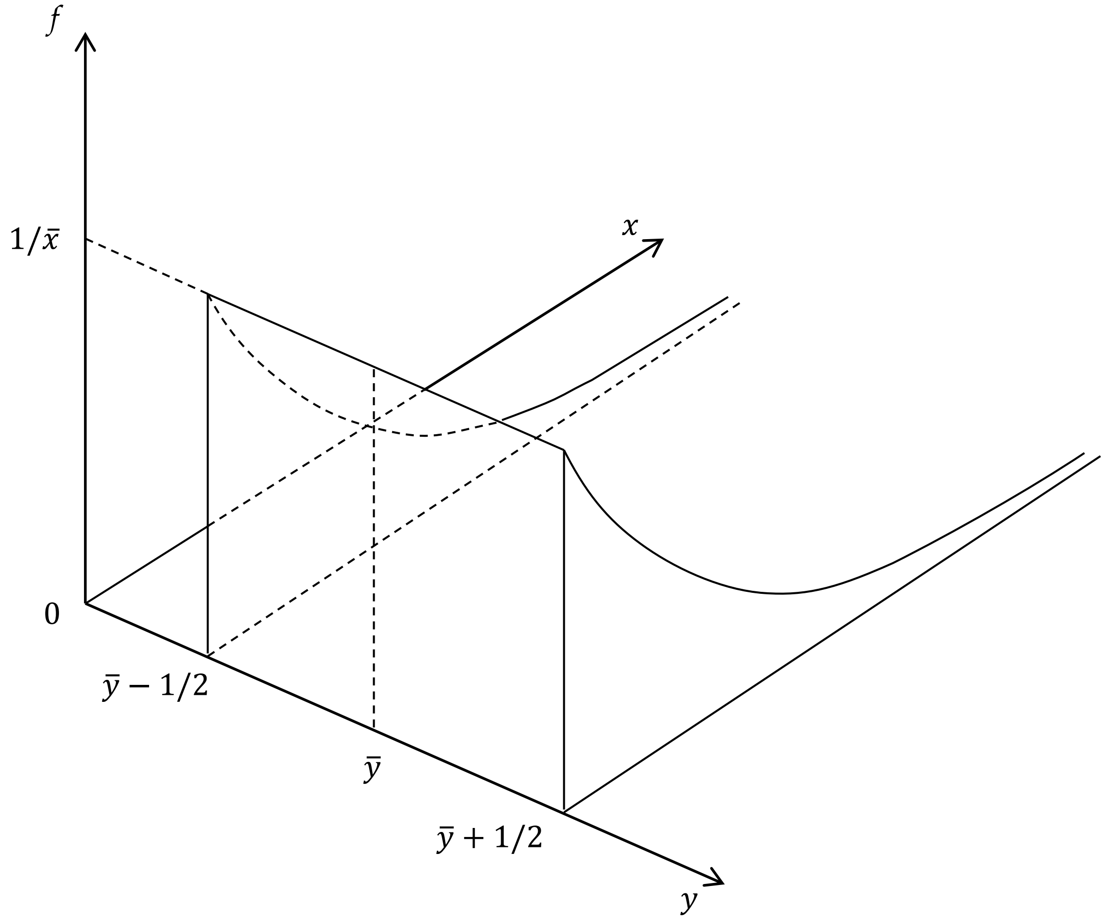
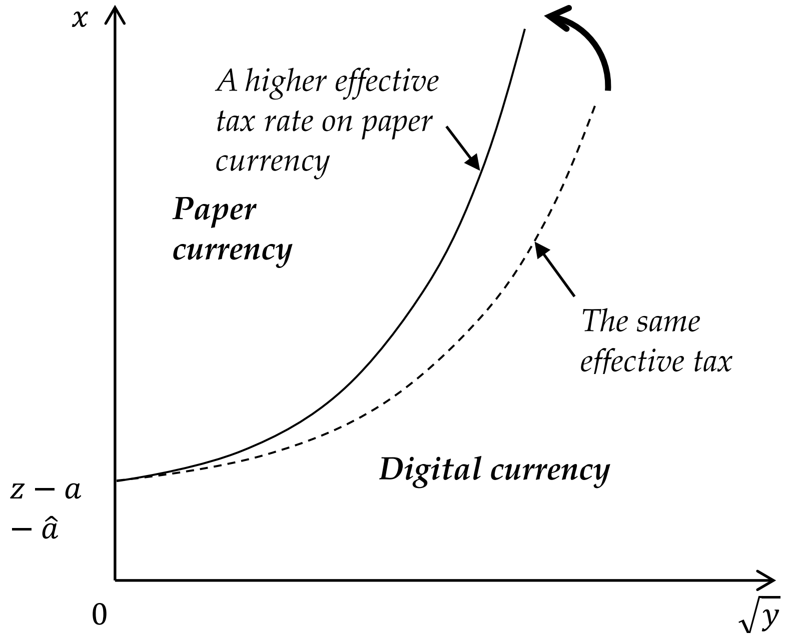


Figure C1 Insufficient taxation on paper and/or digital currencies

(1) Case 1: $\rho \geq 0 > \hat{\rho}$



(2) Case 2: $\hat{\rho} \geq 0 > \rho$

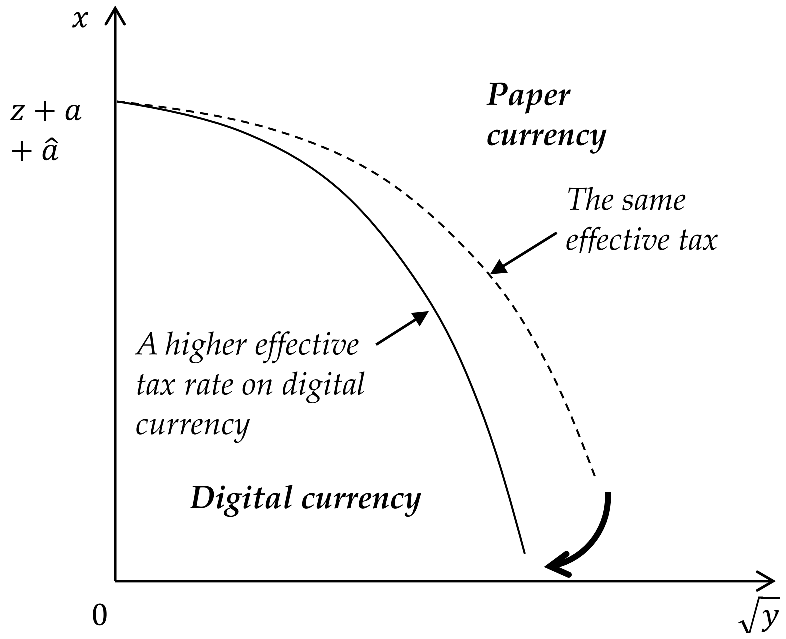
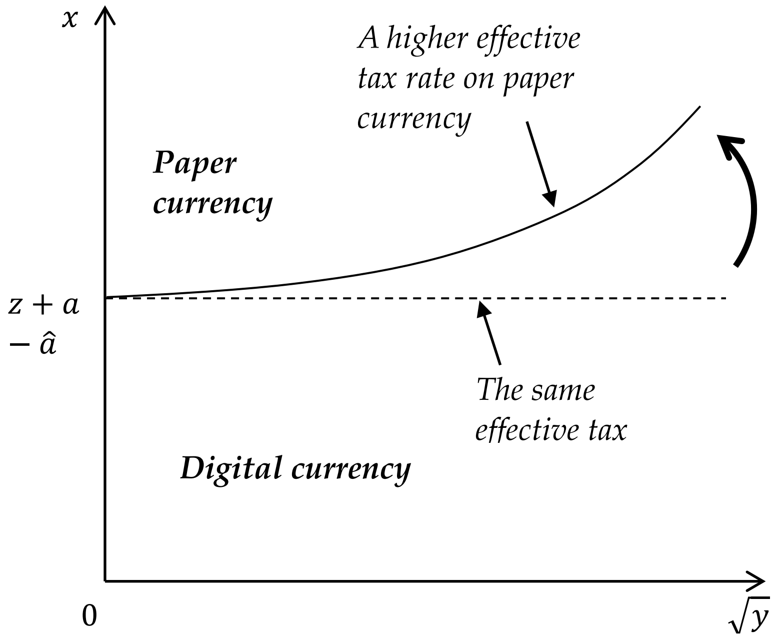


Figure C1. (continued)

(3) Case 3: $0 > \rho \geq \hat{\rho}$



(4) Case 4: $0 > \hat{\rho} \geq \rho$

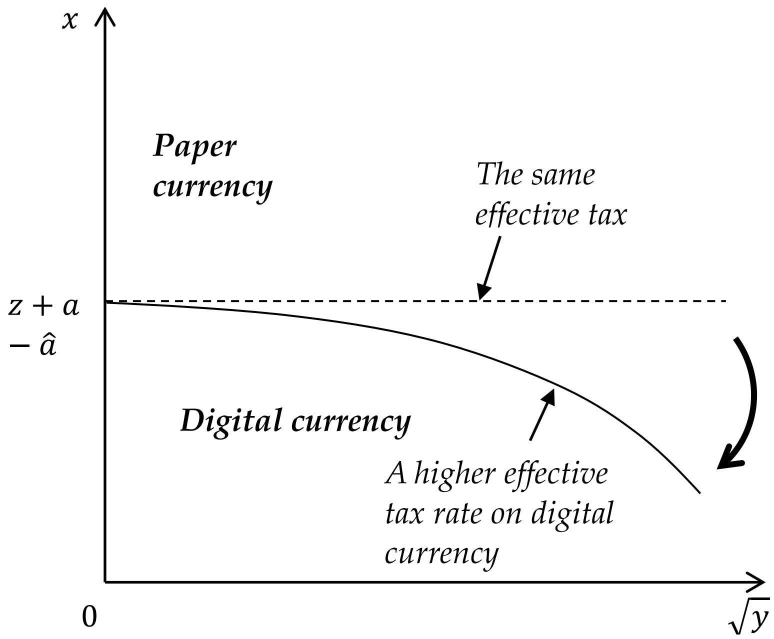
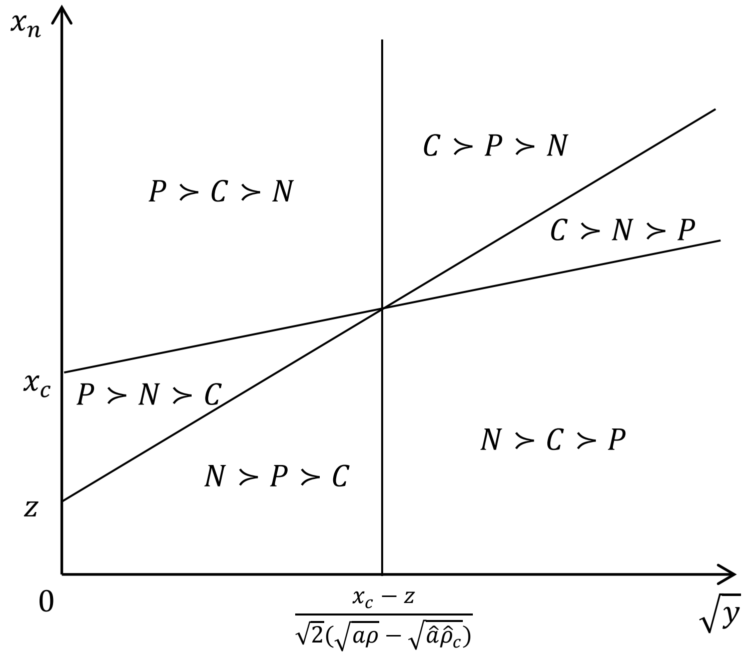


Figure D1. Comparison among three currencies with negative interest rates and positive taxes

(1) Case 1: $a\rho \geq \hat{a}\hat{\rho}_c$ and $x_c \geq z$

(a) Currency preferences



Note. P: paper currency; C: central bank digital currency; N: non-governmental digital currency

(b) Currency choice

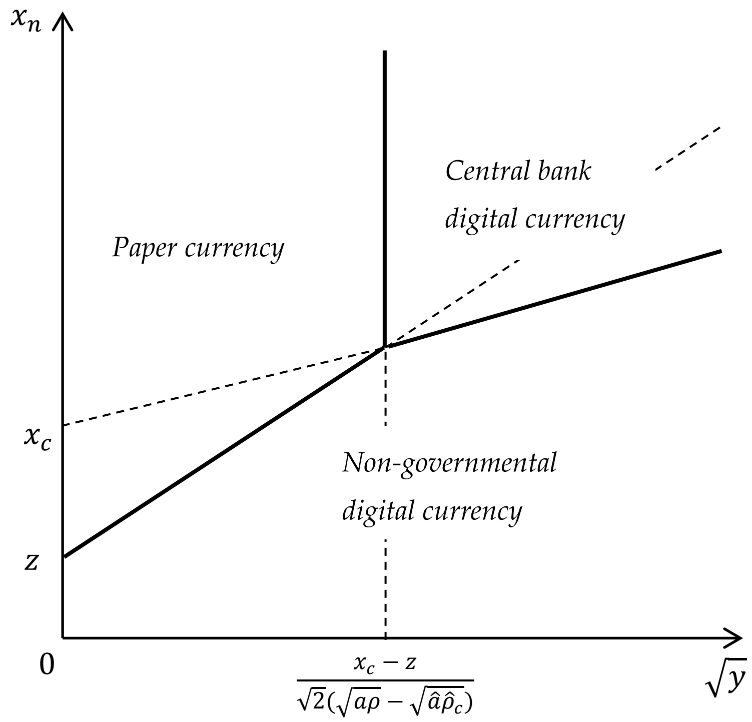
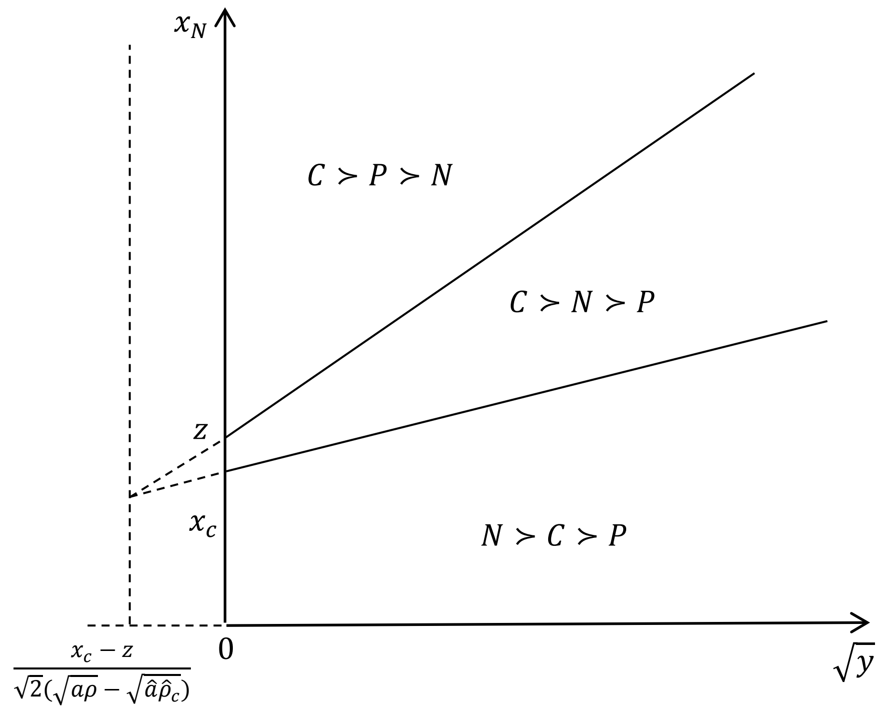


Figure D1. (continued)

(2) Case 2: $a\rho \geq \hat{a}\hat{\rho}_c$ and $x_c < z$

(a) Currency preferences



Note. P: paper currency; C: central bank digital currency; N: non-governmental digital currency

(b) Currency choice

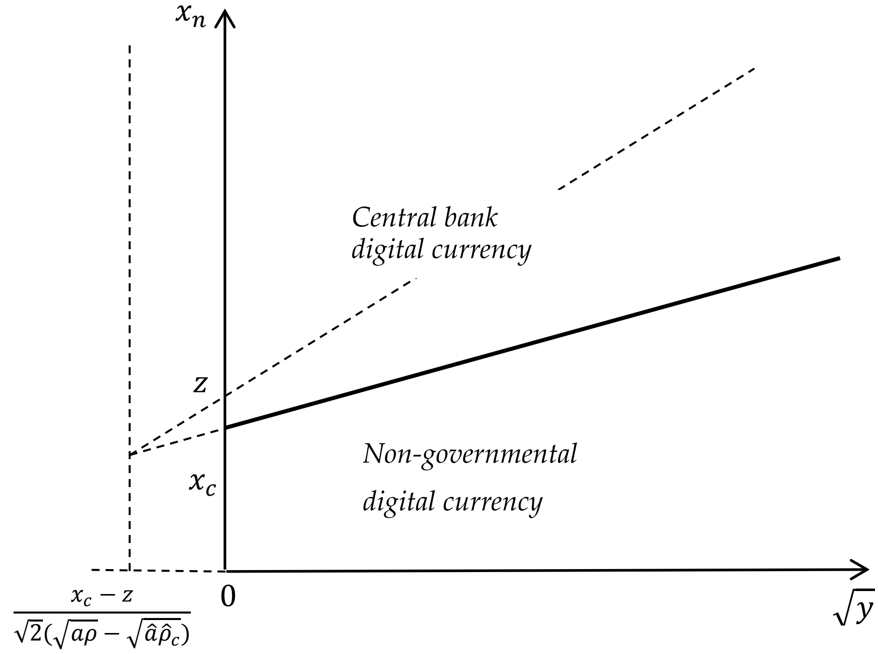
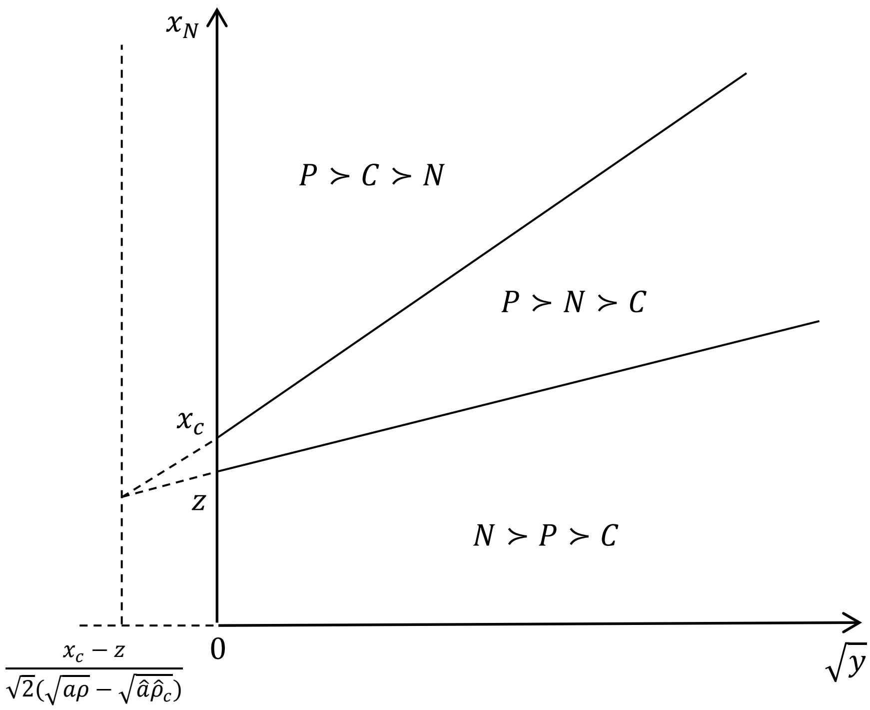


Figure D1. (continued)

(3) Case 3: $a\rho < \hat{a}\hat{\rho}_c$ and $x_c \geq z$

(a) Currency preferences



Note. P: paper currency; C: central bank digital currency; N: non-governmental digital currency

(b) Currency choice

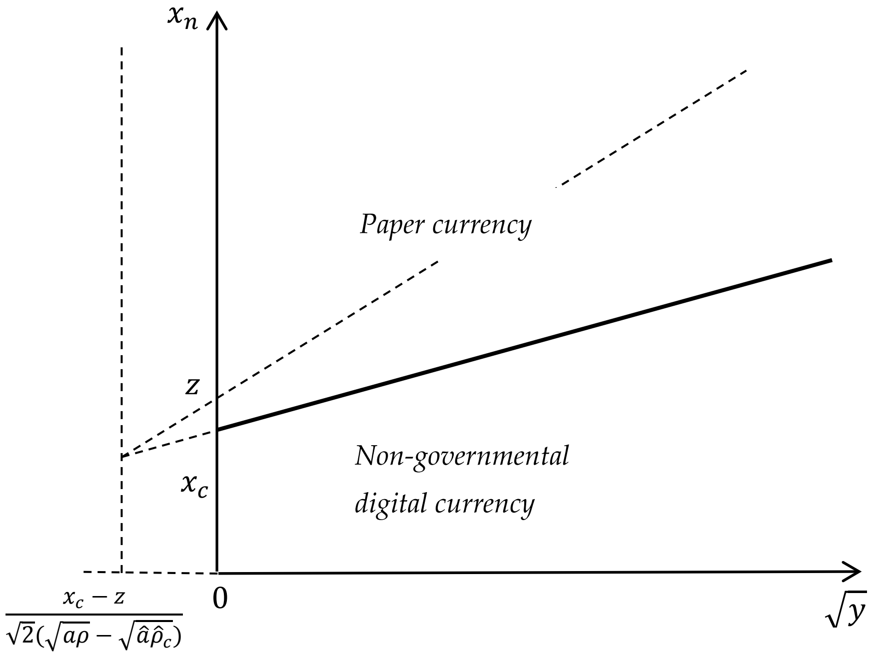
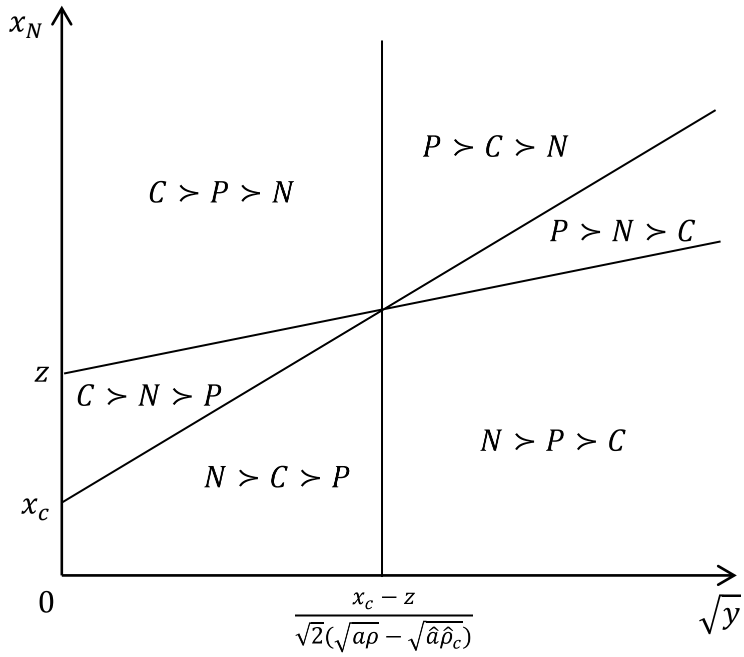


Figure D1. (continued)

(4) Case 4: $a\rho < \hat{a}\hat{\rho}_c$ and $x_c < z$

(a) Currency preferences



Note. P: paper currency; C: central bank digital currency; N: non-governmental digital currency

(b) Currency choice