IMES DISCUSSION PAPER SERIES

A Survey of Systemic Risk Measures: Methodology and Application to the Japanese Market

Akio Hattori, Kentaro Kikuchi, Fuminori Niwa, and Yoshihiko Uchida

Discussion Paper No. 2014-E-3

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

2-1-1 NIHONBASHI-HONGOKUCHO
CHUO-KU, TOKYO 103-8660
JAPAN

IMES Discussion Paper Series 2014-E-3
April 2014

A Survey of Systemic Risk Measures: Methodology and Application to the Japanese Market

Akio Hattori*, Kentaro Kikuchi, Fuminori Niwa***, and Yoshihiko Uchida******

Abstract

The recent financial crisis has prompted academia, country authorities, and international bodies to study quantitative tools to monitor the financial system, especially systemic risk measures. This paper aims to outline these measures and apply them to Japan's financial system. The paper demonstrates that they are effective tools for monitoring the robustness of financial system on a real-time basis, although there are some caveats.

Keywords: Systemic risk; Risk measure; Early warning indicators; Stress test; Scenario analysis; Macro-prudence; Financial crisis

JEL classification: C51, G01, G19

*****Economist, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: akio.hattori@boj.or.jp)
******Deputy Director and Economist, Institute for Monetary and Economic Studies, Bank of Japan (currently, Lecturer, Faculty of Economics, Shiga University, E-mail: kentaro-kikuchi@biwako.shiga-u.ac.jp)
*******Deputy Director and Economist, Institute for Monetary and Economic Studies (currently, Financial Markets Department), Bank of Japan (E-mail: fuminori.niwa@boj.or.jp)
********Director and Senior Economist, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: yoshihiko.uchida@boj.or.jp)

The authors would like to thank Toshiaki Watanabe (Hitotsubashi University) and the staff of the Bank of Japan for their useful comments. All the remaining errors are our own. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

1. Introduction

In the wake of the recent financial crisis, academia and policymakers have deepened their understanding of the mechanisms of financial crises. A financial crisis is triggered by a range of events (which we describe as a "trigger event"), and their negative effect on the financial system and real economy is amplified through an array of channels, as is demonstrated by the recent crisis episodes, which are summarized below:

- *The global financial crisis of 2007–08*: U.S. subprime mortgages triggered the global financial crisis through an increase in their default rates. Initially, because of the sudden decreases in price and liquidity in the subprime mortgage-backed securities market, the financial institutions that held such securities faced growing concerns over their creditworthiness, which caused funding problems for them in the interbank market. The problem severely impacted the soundness of the entire financial system through interconnectedness, and the deteriorated functioning of financial intermediation adversely affected the real economy. The negative impact was amplified by an adverse feedback loop between the financial system and the real economy.

- *The European debt crisis*: The fiscal solvency problem of some European peripheral countries caused concerns over the soundness of financial institutions that held government bonds issued by these countries. The market feared that the increasing cost of bailing out financial institutions incurred by the government would increase sovereign risk, which exacerbated the business position of financial institutions.

These episodes have prompted academia and policymakers to recognize the importance of appropriate macro-prudential policy and study effective methods to assess financial stability quantitatively. Based on the recognition from the recent episodes that a financial crisis rarely materializes without any trigger events or vulnerabilities in the financial system, two types of research on quantitative methods are being conducted.[1] The first strand of research, early warning indicators (EWIs), seeks to estimate the probability of trigger events that may cause systemic risk[2] to materialize. The second, systemic risk measures and macro stress tests, evaluates the vulnerability of the financial system. Specifically:

- EWIs predict trigger events such as drastic changes in asset prices, and are estimated using economic variables that predate trigger events by from one to three years.
- Systemic risk measures show the scale of the negative impact on the financial system (or real economy) brought about by a predetermined trigger event. They are obtained from stochastic models.
- Macro stress tests are types of scenario analysis used to evaluate the effects of predetermined stress scenarios on the financial system. They evaluate, for example, whether the accrual income of financial institutions can cover their incurred credit costs or their marketable security losses in the event of economic recession or a rise in interest rates, and also whether their capital reserves are sufficient to buffer against the losses that would arise under such scenarios.[3]

[1] A widely held idea is that the trigger event and the vulnerability of the financial system should be analyzed separately to anticipate the materialization of systemic risk in the future. For example, Bernanke (2013) cautioned that to monitor financial systemic risk it is necessary to discriminate between the probability of the trigger event's occurrence and the vulnerability of the financial system, because a trigger event may not solely cause systemic risk to materialize.

[2] Financial instability is often characterized as the materialization of systemic risk. However, the existing literature does not clearly define "systemic risk." We assume this is because systemic risk materializes through a variety of channels. According to most international organizations and central banks, systemic risk, broadly defined, is "the risk that an event will trigger a loss of economic value or confidence in, and attendant increases in uncertainty about, a substantial portion of the financial system that is serious enough to quite probably have significant adverse effects on the real economy" (Group of Ten [2001]). In most financial crises, trigger events in the financial sector have affected the real economy, and this shock then feeds back into the financial sector, so that financial instability and economic deterioration exacerbate each other. The above definition can be used to comprehend past crises.

[3] In macro stress tests, a macroeconomic model is often used to determine changes in individual economic variables based on a scenario. Because most current macroeconomic models use linear approximations around the equilibrium, they cannot capture nonlinear economic phenomena (Borio, Drehmann, and Tsatsaronis [2012]). In this paper, we explain EWIs and systemic risk measures, but not the use of scenarios in macroeconomic models.

This paper aims to outline systemic risk measures, which are the most intensively studied of the quantitative methods to assess financial stability, and apply them to Japan's financial system. In applying them to Japan, we analyze how they changed during Japan's crises of the late 1990s and early 2000s, and during the global financial crisis that began in 2007. The paper demonstrates that the systemic risk measures are effective tools for monitoring the robustness of the financial system on a real-time basis.

An important caveat applies to this conclusion, however. While systemic risk measures evaluate the loss to the financial system or real economy caused by particular trigger events, they are silent on the loss caused by other trigger events or the probability of such black swan events. Therefore, it is not sufficient to use only systemic risk measures to assess financial stability as a whole. Policymakers should pay due attention to the characteristics of quantitative tools including EWIs, systemic risk measures, and macro stress tests and use a combination of several tools to monitor and assess the financial system completely and comprehensively.

This paper is structured as follows. In Section 2, we review the quantitative methods used to evaluate financial stability. In Section 3, we classify the existing systemic risk measures based on channels of risk materialization, and then outline each measure. In Section 4, we obtain results by applying these systemic risk measures to Japanese data. Concluding remarks are presented in Section 5. In Appendix A, we briefly outline the EWIs. In Appendix B, we detail each systemic risk measure.

2. Overview of quantitative methods for evaluating the stability of the financial system[4]

2.1 EWIs

Typical features such as risk accumulation tend to appear before a systemic risk materializes. It may be possible, therefore, to capture the probability of its materialization

[4] As summarized in the previous section, quantitative methods relating to financial stability are generally classified into the three categories: EWIs, which estimate the probability of the trigger event; systemic risk measures, which evaluate the vulnerability of the financial system; and macro stress tests. In this section, we provide an overview of EWIs and systemic risk measures.

by choosing the appropriate data preceding the features and analyzing them statistically. EWIs are constructed on such an idea. Concretely, EWIs are estimated by modeling the relationship between the occurrence of the trigger event and the data relating to the event of systemic risk materialization and choosing economic variables that show specific regularity approximately from one to three years before the event.

Following the Mexican and Asian crises of the 1990s, EWIs on currency crises have been studied intensively; see, for example, Kaminsky, Lizondo, and Reinhart (1998), Demirgüç-Kunt and Detragiache (1998, 2000), Kaminsky and Reinhart (1999), and Goldstein, Kaminsky, and Reinhart (2000). Most of these studies use data on developing countries only or combined data on developed and developing countries. Recently, an increasing number of economists has studied EWIs for bubbles as the trigger events, paying attention to a significant increase in residential real estate prices both in the United States and Europe before the global financial crisis of 2007–08; see, for example, Alessi and Detken (2011), Barrell *et al*. (2010), Gerdesmeier, Reimers, and Roffia (2011), and Lo Duca and Peltonen (2013). These studies are based on data from developed countries only. Several of these studies identify credit outstanding as the most effective variable for predicting bubbles.

A detailed description of EWIs follows. For simplicity, we use a discrete-time representation; that is, we divide the time span between the present time (t) and the observation time of the trigger event ($t + N\Delta$) into N periods. Therefore, each period is written as $t, t + \Delta, \dots, t + N\Delta$. First, we introduce the random variable Y, which takes the value of unity when the supposed trigger event occurs at time $t + N\Delta$, and takes the value of zero otherwise. Next, we select economic variables ($X_{t+i\Delta}$, $0 \le i \le N$), including financial variables, that typically indicate the likely occurrence of the trigger event.[5] Suppose that $X_{t+i\Delta}$ for up to k periods before the trigger event (that is, $X_{t+i\Delta}$, where $0 \le i \le N - k$) has *ex ante* information about the probability of the trigger event at $t + N\Delta$, so that the value of $E(Y|X_{t+i\Delta}, 0 \le i \le N - k)$ approaches unity when the trigger event occurs,

[5] More than one variable can be selected. In that case, X_t is a vector.

and approaches zero otherwise.[6] Therefore, in the framework of EWIs, it is important to select the economic variables X such that the expectation of Y conditioned by X at $k = 0$ takes a value of unity in case of the outbreak of the trigger event or zero otherwise, and approaches unity or zero as k approaches zero. Then, one can choose the maximum value of k, which indicates statistically significant increase in the probability that Y is approaching unity.[7]

2.2 Systemic risk measures

Systemic risk measures, which are estimated by stochastic models, show the scale of the negative impact that a trigger event has on the financial system or the real economy. As an example, let us assume interconnectedness among the financial institutions is deeply related to the materialization of systemic risk.[8] In other words, strong interconnectedness among financial institutions amplifies the negative impact of a trigger event. To capture the extent of this negative impact, Adrian and Brunnermeier (2011) suggest quantifying the risk that a shock to an individual financial institution spills over to the entire financial sector. Acharya *et al*. (2010) and Huang, Zhou, and Zhu (2009, 2010, 2011) suggest quantifying the contribution of an individual financial institution to the risk of a financial sector-wide shock. Arguably, the negative impact of a trigger event in the financial sector not only affects the financial sector but also spills over to other sectors, and vice versa. For example, De Nicolò and Lucchetta (2010) and Giesecke and Kim (2011) study risk measures capturing the interdependence between the financial sector and the real economy. They include risk measures that quantify the risk to the financial sector induced by economic deterioration, and those that quantify the risk to the aggregate

---

[6] $E(\cdot)$ is the expectations operator relating to the historical probability measure. The historical measure is defined by the observation measure comprising the using historical data on economic variables, the observation period for which is set in advance. $E(Y|A)$ denotes the conditional expectation of Y given that A is realized.

[7] Representative methods and limitations of EWIs are discussed in Appendix A.

[8] Interconnectedness among financial institutions refers to the aggregated strength of relationships between different financial institutions based on correlations between their asset values, the number and volume of direct transactions, and so on. A shock to a financial institution that has strong interconnectedness with other financial institutions might have a severe impact on the financial sector as a whole.

economy induced by a worsening financial sector. Furthermore, to capture sovereign risk that attracts attention during the European debt crisis, Jobst and Gray (2013) suggest a measure related to interdependency risk between the financial and public sectors.

In addition, if volatile price fluctuations or a malfunctioning financial market generate price inconsistencies among financial products, most market participants, including financial institutions, cannot trade at fair values, which hampers funding and hedging. The global financial crisis that began in the summer of 2007 is often characterized as a materialization of "market-oriented systemic risk," indicating an extreme weakening of market functions. Such condition includes trading suspensions caused by loss of reasonable prices, funding difficulties caused by strict counterparty selection induced by increasing concern for counterparty risk, and plummeting prices boosted by asset fire-sales to meet funding demands. Severo (2012) and Diebold and Yilmaz (2009, 2014) propose systemic risk measures to capture such market malfunctions.

Most of the aforementioned systemic risk measures are based on market data observed daily, such as stock prices and the credit default swap (CDS) spreads of financial institutions. These measures enable monitoring at higher frequency, compared with measures based on the balance sheet data in financial statements. In addition, these systemic risk measures are designed to capture the tail risk that reflects the existing interconnectedness within and across sectors by using a range of data. The advantage of this is that they can convey complex information that may not be captured by simple time-series data. However, systemic risk measures do not convey system-wide vulnerability, because each systemic risk measure is based on a specific transmission channel for systemic risk, such as the interdependence between financial institutions and industry sectors, and the spillover to the public sector. Therefore, it is necessary to evaluate these measures comprehensively.

Let us detail the idea behind systemic risk measures. For simplicity, we use the discrete-time representation; that is, we divide the time span between the present time (t) and the observation time of a trigger event into N periods, with each period written as t, $t + \Delta$, … , $t + N\Delta$. First, we introduce the random variable V, which takes the value of unity when the trigger event occurs at $t + N\Delta$, and takes the value of zero otherwise. Let L denote the loss accruing to the financial system or the aggregate economy at $t + N\Delta$, and let $S(\cdot)$ denote the statistics obtained from the probability distribution of L.[9] Most researchers use quantiles or expectations for $S(\cdot)$. We assume that we can select economic variables ($X_{t+i\Delta}$, $0 \le i \le N - k$), including financial variables, that can indicate the occurrence of the trigger event and the size of loss when it occurs. If $X_{t+i\Delta}$, $0 \le i \le N - k$ (the data up to k periods before the trigger event occurs) has *ex ante* information about the size of the loss when the trigger event occurs at $t + N\Delta$, then the loss can be evaluated as $S(L|V = 1, X_{t+i\Delta}, 0 \le i \le N - k)$, which was obtained by processing the data up to $t + (N - k)\Delta$. Therefore, by monitoring this value we can evaluate the negative impact of the trigger event before its occurrence.[10]

When the trigger event occurs, the systemic risk measures change dramatically, which reflects the large fluctuations in the financial markets. Thus, one might argue that there exists no fundamental difference between the direct information from the financial markets, such as stock prices and CDS spreads, and the systemic risk measures estimated statistically from such data. According to this argument, it is better to monitor the market data directly than to monitor the systemic risk measures because these data are easier to use and interpret. However, what determines the robustness of a financial system is not only whether financial institutions have enough buffers to cover their market losses when a trigger event occurs, but also whether they have adequate buffers to withstand the biggest market losses suffered on the occurrence of the trigger event.[11] Therefore, systemic risk measures can be worth using to evaluate the robustness of the financial system.

[9] The functional $S(\cdot)$ denotes the statistics under the historical probability measure. However, a risk-neutral probability measure is sometimes used instead of a historical probability measure.
[10] In systemic risk measures, to evaluate the impact of a trigger event from observations on ($X_{t+i\Delta}$, $0 \le i \le N - k$), the period denoted by k typically ranges from one to six months.
[11] This relationship is the same that between the value at risk (VaR) and, for example, losses on bonds in the event of an increase in long-term interest rates.

3. Introduction of major systemic risk measures

In this section, we focus on the systemic risk measures mentioned in the previous section, and explain some representative methods suggested in previous studies.[12] We classify the systemic risk measures into four categories to reflect the channels through which these systemic risks materialize,: (1) measures of risk materializing from interconnectedness between financial institutions; (2) measures of risk materializing from interdependence between the financial sectors and real economy; (3) measures of risk materializing from interdependence between the financial and public sectors; and (4) measures of risk materializing from malfunctions in the financial market.

First, we define the systemic risk measures rigorously and mathematically. We introduce the probability space characterized by $(\Omega, \mathcal{F}, P)$. P is either a historical or risk-neutral probability measure, depending on the systemic risk measure. $\mathcal{F}_t$ represents filtration at time t. For simplicity, we use a discrete-time representation; that is, we divide the time span between the present time (t) and the observation time (when information on the occurrence of the trigger event is obtained) into N periods. Therefore, each period is written as $t, t + \Delta, \ldots, t + N\Delta$. We introduce the random variable V, which takes the value of unity when the trigger event arises at $t + N\Delta$, and zero otherwise. Let L denote the loss accruing to the financial system or the aggregate economy at $t + N\Delta$.[13] We use the functional $S(\cdot)$ to denote the statistics obtained from the probability distribution. By using the conditional distribution of L, namely $P(L \in dL|V = 1, \mathcal{F}_{t + (N - k)\Delta})$, we define the systemic risk measure as $S(P(L \in dL|V = 1, \mathcal{F}_{t + (N - k)\Delta}))$. The quantile or the expectation is often used for $S(\cdot)$ in a number of previous studies.[14] The risk evaluation period ($k\Delta$)[15] is

---

[12] For details of each measure, including details of data used in Section 4, see Appendix B. In this section, we also discuss systemic risk measures not used in Section 4 for reasons such as data unavailability.

[13] The term "loss" is not limited to monetary loss, but could also be the number of downgrades or the rate of change, for example. Here, "loss" refers to any adverse effect on the financial sector or economy.

[14] Some systemic risk measures used in this paper are defined directly as $P(L \in dL|\mathcal{F}_{t + (N - k)\Delta})$ without conditioning on $V = 1$. For example, let L denote a loss that may reduce financial stability, and define the risk measure as the VaR of $P(L \in dL|\mathcal{F}_{t + (N - k)\Delta})$ at the p% confidence level. Risk measures such as this represent "the degree of the adverse effect when an event with a probability of p% occurs" and can be thought of as measures based on information concerning tail events. In this paper, we consider this kind of risk measure to be a systemic risk measure.

[15] This is the same as the holding period of VaR. Although the term "holding period" is frequently used in the context of VaR, we use the term "risk evaluation period."

typically set to no longer than six months, and the observation period ($(N-k)\Delta$) is commonly set to range from six to 12 months. Table 1 summarizes a typical systemic risk measure.

Table 1: Example of a systemic risk measure

As an example, we define the systemic risk measure as the expected number of downgrades in the financial sector between $t+(N-k)\Delta$ and $t+N\Delta$, conditional on the number of downgrades in the nonfinancial sector during the same period exceeding its maximum value at the 99% confidence level. Each element of this risk measure is as follows:

Element	Example
Trigger event V	Between $t+(N-k)\Delta$ and $t+N\Delta$, the number of downgrades in the nonfinancial sector exceeds its maximum value at the 99% confidence level.
Loss L	The number of downgrades in the financial sector between $t+(N-k)\Delta$ and $t+N\Delta$
Observation period	$(N-k)\Delta$ (between t and $t+(N-k)\Delta$)
Risk evaluation period	$k\Delta$ (between $t+(N-k)\Delta$ and $t+N\Delta$)
Function S	Expectation
Probability measure P used to calculate risk	Historical probability measure

The value of the systemic risk measure depends mostly on the magnitude of interdependence between the trigger event V and the loss L. The value of the systemic risk measure tends to be large when the trigger event V and the loss L always occur simultaneously. For example, let V denote some sort of trigger event occurring in the financial sector, and let L denote the loss incurred in the aggregate economy. Then, $P(L \in dL|V=1, \mathcal{F}_{t+(N-k)\Delta})$ is the probability distribution of the loss incurred in the aggregate economy conditional on the occurrence of the trigger event in the financial sector, and $S(P(L \in dL|V=1, \mathcal{F}_{t+(N-k)\Delta}))$ is the risk measure constructed from the distribution. This risk measure takes a large value because of the high degree of interdependence between the financial and economic sectors. By defining the trigger event and the loss in this way, the measurement of the systemic risk can be close to the definition of the systemic risk itself, as described in Footnote 1: both economic conditions and the financial sector deteriorate at the same time.

An advantage of defining $S(P(L \in dL|V = 1, \mathcal{F}_{t+(N-k)\Delta}))$ as the systemic risk measure is that we can compare the seriousness between using different trigger events or losses. For example, let L_D denote the loss given defaults in the entire financial sector, and let V_A and V_B denote the default events of financial institutions A and B at $t + N\Delta$,[16] respectively. By comparing $S(P(L_D \in dL|V_A = 1, \mathcal{F}_{t+(N-k)\Delta}))$ with $S(P(L_D \in dL|\, V_B = 1, \mathcal{F}_{t+(N-k)\Delta}))$, we can compare the supposed contribution of financial institutions A and B to the financial system as a whole when they default. For another example, let L_G denote the growth of the aggregate economy and let V_{AB} denote the event that two or more financial institutions simultaneously default at $t + N\Delta$. By comparing $S(P(L_D \in dL|V_{AB} = 1, \mathcal{F}_{t+(N-k)\Delta}))$ with $S(P(L_G \in dL|V_{AB} = 1, \mathcal{F}_{t+(N-k)\Delta}))$, we can determine whether the event in the financial sector remains within that sector or spills over to the entire economy, and thereby determine the impact of these simultaneous defaults on the real economy.

Furthermore, by calculating and comparing multiple systemic risk measures defined by the combination of various trigger events and losses, we can determine the type of events to which the financial and economic systems are most vulnerable.

3.1 Measures of risk materializing from interconnectedness between financial institutions

As an example of risk materialization in the financial system as a whole, we can imagine that a management crisis in a specific financial institution damages the soundness of the other financial institutions through transactions between these institutions. We could also consider that a single shock causes the significant deterioration in the asset values of financial institutions and systemic concerns.

Since 2007, researchers have been actively quantifying the risk posed by the interconnectedness of financial institutions such as direct business relationships or the commonality of exposures. For example, Adrian and Brunnermeier (2011) suggest the "CoVaR," Acharya *et al*. (2010) suggest the "marginal expected shortfall" (MES), Huang,

[16] That is, a period of $(t + (N-1)\Delta, t + N\Delta]$.

Zhou, and Zhu (2009, 2010, 2011) suggest the "distress insurance premium" (DIP), and Segoviano and Goodhart (2009) suggest the "joint probability of distress" (JPoD).

CoVaR

The CoVaR is a measure indicating how the banking sector stock index falls when an individual bank stock price declines. It is based on the rate of return of the individual stock and the banking sector stock index. Plotting the CoVaR over time conditioned by a plunge in bank *A*'s stock as a trigger event, we can analyze several situations such as the soundness of the entire financial sector in the case of trouble involving bank *A* and the current conditions compared with those from the past. In addition, by comparing the level of the CoVaR based on the fall in bank *A*'s stock price relative to bank *B*'s, we can ascertain which of the two banks affects the vulnerability of the financial sector more (Table 2).

Table 2: CoVaR

	Trigger event *V*	Loss *L*	Functional *S*	Probability measure *P*
CoVaR	The rate of return on an individual bank stock decreases to its lower fifth percentile (95% VaR).	The rate of return on the banking sector stock index during the risk evaluation period	Lower fifth percentile (95% VaR)	Historical probability measure

MES

Like the CoVaR, the MES is a risk measure based on banks' stock prices. The MES quantifies the risk that an individual stock price decreases when the banking sector stock index decreases. This measure indicates which banks contribute to financial sector-wide risks when risk materializes, and the magnitude of these contributions (Table 3).

Table 3: MES

	Event *V*	Loss *L*	Functional *S*	Probability measure *P*
MES	The rate of return on the banking sector stock index decreases to its lower fifth percentile (95% VaR).	The rate of return on an individual bank stock during the risk evaluation period	Expectation	Historical probability measure

Based on the MES, Acharya *et al*. (2010) propose "systemic expected shortfall" (SES) and Brownlees and Engle (2012) propose "SRISK" to capture capital shortage risk in the financial sector. Details are given in Appendix B.

DIP

The DIP defines a trigger event as the situation when the gross loss of the defaulted financial institutions exceeds some fixed percentage of the total liabilities of all sampled financial institutions. It measures risk in the financial system as the expected total loss conditional on the occurrence of the trigger event. As a risk measure, the DIP uses the contributions of individual financial institutions to the overall loss, in addition to the overall loss itself. Unlike the CoVaR and the MES, the DIP uses the incidence of default instead of the plunge of stock prices. The variable that this measure utilizes is the default probability of the financial institutions based on their CDS spreads. The possibility that multiple financial institutions default simultaneously affects the behavior of the DIP. Such simultaneous defaults are assumed to arise from high correlations between the assets of these financial institutions. Huang, Zhou, and Zhu (2009, 2010, 2011) substitute correlations between equities for those between assets due to the difficulty of the measurement of asset correlation. The DIP uses a risk-neutral measure in its calculation, unlike the CoVaR and the MES. Consequently, the DIP reflects both a financial institution's substantial creditworthiness and its risk premium. The risk premium comprises two components: the default risk premium, which reflects uncertainty about a financial institution's creditworthiness; and the liquidity risk premium, which reflects market liquidity. These risk premiums are affected by the risk aversion of market participants and price volatility. Concerns about the financial system can arise from not only deteriorating creditworthiness among financial institutions but also increased risk aversion or price volatility. The DIP is designed to capture the risk emanating from both types of concerns (Table 4).

Table 4: DIP

	Trigger event V	Loss L	Functional S	Probability measure P
DIP_I	The gross loss of the defaulted financial institutions exceeds a fixed level (for example, 15% of the gross liability of all the sampled financial institutions).	The total loss incurred by all financial institutions	Expectation	Risk-neutral measure
DIP_{II}		The loss incurred by an individual financial institution		

JPoD

Like the DIP, the JPoD is a systemic risk measure using the information concerning defaults of financial institutions. In Segoviano and Goodhart (2009), who suggest JPoD, three risk measures are introduced. The first is the JPoD, the probability that all sampled financial institutions default at the same time. The second is the "Banking Stability Index," the expected number of defaulting financial institutions conditional on the default of at least one financial institution in the sample. The third is the probability of a "cascade" of defaults, the probability that a specific financial institution defaults conditional on the default of another particular financial institution in the sample (Table 5).

Table 5: JPoD

	Trigger event V	Loss L	Functional S	Probability measure P
$JPoD_I$ (joint probability of distress)	None (empty set)	The event that all financial institutions in the sample default	Expectation	Risk-neutral measure
$JPoD_{II}$ (Banking Stability Index)	Default of at least one financial institution in the sample	The number of defaulting financial institutions in the sample		
$JPoD_{III}$ ("cascade" probability)	Default of a specific financial institution	Another particular financial institution defaults		

3.2 Measures of risk materializing from interdependence between the financial sector and the real economy

According to the previous systemic risk materialization, it is reasonable to develop a systemic risk measure based on interdependence between the financial sector and the real economy. In this context, Giesecke and Kim (2011) propose the "default intensity model" (DIM),[17] and De Nicolò and Lucchetta (2010) propose "GDP at risk."

DIM

In previous studies of the DIM,[18] either the number of defaults or downgrades is used to define the trigger event or the loss. Here, we explain the DIM based on the number of downgrades, because we focus on the increase in the number of downgrades in the empirical analysis using Japanese data in the following section.

In the DIM, the number of downgrades is described by the hazard model, both for the aggregate economy including the financial sector and for the financial sector.[19] The model parameters are estimated from the number of downgrades observed during the sample period. Then, the numbers of downgrades during the risk evaluation period are calculated for the entire economy excluding the financial sector, and separately for the financial sector. By interchanging the sectors to which the trigger event V and the loss L relate, one can define two risk measures. The one is the number of downgrades in the aggregate economy conditional on extreme downgrading in the financial sector ($\mathrm{DIM_I}$). The other is the number of downgrades in the financial sector conditional on extreme downgrading in the economy ($\mathrm{DIM_{II}}$). In addition, we use $\mathrm{DIM_{III}}$ as a measure of risk materializing from interconnectedness between financial institutions. For $\mathrm{DIM_{III}}$, we have the following: the trigger event V is empty, the loss L is the number of downgrades

---

[17] Yamanaka, Sugihara, and Nakagawa (2012) use the same concept, but they deal with the risk on a loan portfolio.

[18] This includes the research mentioned in Footnote 20.

[19] In the DIM, the hazard intensity is modeled for each group of corporations. In the field of credit risk modeling, this modeling method is sometimes called the "top-down approach." In another method, the hazard intensities are modeled separately for each corporation and the number of downgrades among corporations is described using these intensities. This is sometimes called the "bottom-up approach."

in the financial sector during the risk evaluation period, and functional *S* is the 99th percentile (Table 6).

Table 6: DIM

	Trigger event *V*	Loss *L*	Functional *S*	Probability measure *P*
DIM_{I}	The number of downgrades in the financial sector during the risk evaluation period exceeds the upper first percentile (1% VaR) of its probability distribution based on the sample data.	The number of downgrades in the aggregate economy during the risk evaluation period	Expectation	Historical probability measure
DIM_{II}	The number of downgrades in the aggregate economy during the risk evaluation period exceeds the upper first percentile (1% VaR) of its probability distribution based on the sample data.	The number of downgrades in the financial sector during the risk evaluation period	Expectation	Historical probability measure
DIM_{III}	None (empty set)	The number of downgrades in the financial sector during the risk evaluation period	99th percentile	Historical probability measure

GDP at Risk

GDP at Risk models the interdependence between real GDP growth and the rate of the banking sector stock index return. By interchanging the sectors for the trigger event *V* and the loss *L*, we can define two types of risk measures (Table 7).

Table 7: GDP at risk

	Trigger event *V*	Loss *L*	Functional *S*	Probability measure *P*
GDP at risk_{I}	The rate of the banking sector stock index return during the risk evaluation period decreases below the lower fifth percentile (95% VaR) of its probability distribution based on the sample data.	Real GDP growth during the risk evaluation period	Lower fifth percentile (5% VaR)	Historical probability measure
GDP at risk_{II}	Real GDP growth decreases below the lower fifth percentile (95% VaR) of its probability distribution based on the sample data.	The rate of return on the banking sector stock index during the risk evaluation period	Lower fifth percentile (5% VaR)	Historical probability measure

3.3 Measures of risk materializing from interdependence between the financial and public sectors

Consider an example of how systemic risk materializes through interdependence between the financial and public sectors. First, suppose that the government is likely to bail out financial institutions in financial distress. In that case, the financial institution's risk of default is transferred to the government. If several financial institutions default at the same time, then the government's fiscal position may deteriorate because of the soaring cost of the bailouts. This may generate a lack of confidence in the government's solvency and a rise in government bond yields. Then, financial institutions may face further problems, including a decline in the value of their assets (which include government bonds) and funding difficulties arising from collateral devaluation. Hence, as already mentioned, there may be negative feedback effects between the financial system and the government's fiscal position.

SCCA

Assuming that the government bails out defaulted financial institutions, Jobst and Gray (2013) develop a measure for the risk induced by an increase in the bailout cost incurred by the government, which they term "systemic contingent claims analysis" (SCCA).

Based on the contingent claims analysis (CCA) concept,[20] we assume that the equity value of a defaulted financial institution is equal to zero. We also define the loss given default (LGD) incurred by unsecured creditors as the net face value of the financial institution's liabilities minus its asset value and financial support provided by the government. The stock price of the financial institution does not reflect how creditors are repaid, because the equity falls to zero on default. That is, the LGD of the equity is simply the difference between the asset value and the face liability value. However, the CDS

[20] CCA, suggested by Merton (1974), is a method of evaluating corporations by using stock prices. It considers the asset value as the underlying asset and defining default as an event in which the asset value falls below the face value of the liability at maturity, with the stock price the value of the European call option whose strike price is the face value of the liability. Based on this concept, a corporation that has an asset deficit at maturity is liquidated instantly, and the stockholder receives nothing. In this case, the LGD of unsecured creditors is equal to the asset deficit based on the face value of the liability.

spreads on the bonds issued by the financial institution reflect the loss that unsecured creditors expect to suffer in the event that the financial institution defaults. If participants in the CDS market expect the defaulting financial institution to be bailed out by the government, this will narrow CDS spreads.[21] Jobst and Gray (2013) assume that there is a difference between the loss suffered by unsecured creditors based on the stock price and that based on the CDS spread, and that this difference represents the expected bailout cost incurred by the government.[22] Based on this, they calculate the cost to the government of individual bailouts and then sum them up, taking into account the interdependence between financial institutions.[23] This sum represents the risk of an increased bailout cost across the financial system. Details are given in Table 8.

Table 8: SCCA

	Trigger event *V*	Loss *L*	Function *S*	Probability measure *P*
SCCA	None (empty set)	The amount of financial support provided to defaulting financial institutions by the government	Upper fifth percentile (5% VaR)	Historical probability measure

3.4 Measures of the risk of a malfunctioning financial market

During the global financial crisis in 2007–08, the consistency of the relative prices between related financial products was often violated. Consequently, the price discovery function of financial markets was severely damaged. This arose from increasing concerns about counterparty risk against U.S. and European financial institutions as well as the unwinding of positions taken by market participants facing tightened funding constraints. This caused stock prices and other asset prices to decline dramatically, which further increased risk aversion among market participants. Consequently, market liquidity declined further, and financial markets became turbulent. To measure the risk of

[21] Although the CDS spread is generally determined by the probability of default given that the LGD is constant, the change in the LGD is also taken into account when there is doubt about whether the corporation remains a going concern. If the corporation is a financial institution that is in trouble and if the market participants can predict the bailout amount, then one can use CDS spreads to extract the relevant information about the expected cost incurred by the government.
[22] We assume that there is no financial support other than that from the government, although some unsecured creditors may comply with debt reductions, for example.
[23] See Appendix B for details.

such market malfunctions, the systemic liquidity risk index (SLRI) and the volatility spillover index have been proposed.

SLRI

Severo (2012) suggests the SLRI. This index is based on the event that the intermarket no-arbitrage relationship, which is generally satisfied under normal market conditions, is violated because of substantial position unwinding by market participants.

The SLRI is calculated by integrating the deviations of various basis spreads, which are normally close to zero. Severo (2012) selects the following basis spreads: covered interest parity; the on-the-run versus the off-the-run interest rate spread on government bonds; the interest rate spread between the overnight index swap (OIS) and short-term government bonds; and the basis spread between corporate bonds and the CDS. When the no-arbitrage relationship between these markets breaks down, these basis spreads significantly deviate from zero. To represent the degree of the co-movement of these basis spreads, Severo (2012) used the first component score from principal components analysis based on historical time-series data on these spreads.

We consider the SLRI differently to examine the performance of forecasting the materialization of systemic risk. Suppose that the systemic risk materialization is that the SLRI hits the two standard deviation point (at the upper 2.3 percentile) at the end of the risk evaluation period, assuming that the first component score follows the same probability distribution in the period. However, this value does not necessarily reflect the risk that basis spreads deviate from zero during the risk evaluation period, because the distribution of the first principal component score is estimated from the times series of the first principal component of multiple basis spreads in each past time.

Therefore, we simulate the individual basis spreads at the end of the risk evaluation period assuming that individual basis spreads follow their historical distributions, and define the upper first percentile of the first principal component of the simulated spreads as a risk measure (Table 2).

Table 9: SLRI

	Trigger event V	Loss L	Function S	Probability measure P
SLRI	None (empty set)	The first principal component score based on six-months-ahead multiple time-series data, representing deviation from the prevailing no-arbitrage relationship in the market	Upper first percentile (1% VaR)	Historical probability measure

Volatility spillover index

Based on the suggestion that volatility in certain markets affects volatility in other markets, Diebold and Yilmaz (2009, 2014) propose the volatility spillover index. See Appendix B for details.

4. Results from applying systemic risk measures to Japanese data

In this section, we report results of the application of the systemic risk measures to Japanese data from the 1990s. We select several of the measures explained in the previous section, subject to data availability. In addition, we explain how these measures behaved from 1997 to 2004 and from 2007 to 2012. In the former period, most Japanese financial institutions suffered losses on write-downs of nonperforming loans (NPLs), and in the latter period the global financial crisis and the European sovereign debt crisis occurred. Moreover, we use the behavior of the measures to examine differences in the robustness of the financial system between these periods.

4.1 Systemic risk from 1997 to 2004

First, we focus on measures regarding the risk on interconnectedness between financial institutions. Both the CoVaR and MES increased from early 1997, particularly toward the end of the year. They remained fairly stable at that level until the end of 2000. They increased in 2001 and remained high until the middle of 2004 (Figure 1, top and middle panels). $\mathrm{DIM}_{\mathrm{III}}$, the 99th percentile of the number of downgrades in the banking sector, reached a high level in 1998.[24] The measure was at a low level before increasing again in early 2000, and remained high until early 2003 (Figure 1, bottom panel). These

[24] Because of data availability, DIM was calculated from April 1998.

movements in the risk measures reflected the instability of the Japanese financial system at the time. In fact, there were concerns about the solvency of a major commercial bank, Hokkaido Takushoku Bank, in early 1997. In November, three financial institutions, Sanyo Securities, Hokkaido Takushoku Bank, and Yamaichi Securities, went bankrupt. Furthermore, the real economy deteriorated partly because of the collapse of the information technology bubble at the beginning of the 2000s. This effect spilled over to the financial system, as described later. Consequently, although the Japanese financial system stabilized temporarily, it experienced instability again until 2004.

Next, we examine measures regarding the risk on the interconnectedness between the financial sector and the real economy, namely, DIM_{I} (the number of downgrades in the aggregate economy conditional on extreme downgrades occurring in the banking sector) and DIM_{II} (the number of downgrades in the banking sector conditional on extreme downgrades occurring in the aggregate economy). From a high level in late 1998, these risk measures fell around the spring of 2000 before increasing again to reach a historical high during the spring of 2001, where they remained until the middle of 2003 (Figure 2). These movements suggest that risk materializing from interdependence between the financial sector and the real economy increased as the Japanese financial system became unstable in the late 1990s and 2000s. That is, after the bubble collapsed at the start of the 1990s, the financial soundness of Japanese financial institutions deteriorated in the late 1990s because of increasing bankruptcies and deteriorating performance of Japanese corporations. As a result, several Japanese financial institutions entered bankruptcy in 1997. During this period, other Japanese financial institutions tightened their lending attitudes due to capital constraints. This depressed the real economy because of falling business confidence. In the early 2000s, the Japanese economy, particularly exports, began a downturn after the collapse of the information technology bubble. Consequently, the amount of NPLs increased once again reflecting increasing bankruptcies and deteriorating company performance. These conditions forced Japanese financial institutions to strengthen their capital positions through mergers with other Japanese banks or public funds to compensate for losses from write-downs of NPLs. Such conditions continued until 2003.

4.2 Systemic risk from 2007 to 2012

First, we examine measures regarding the risk on interconnectedness between financial institutions. The CoVaR, MES, and DIM_{III} stabilized at low levels from 2005 to the middle of 2007. These increased in August 2007, when an announcement by BNP Paribas, a major French bank, shocked global markets. (Hereafter, we refer to this event as the BNP Paribas shock.) The measures rose sharply following the collapse of U.S. investment bank Lehman Brothers in September 2008. (Hereafter, we refer to this event as the Lehman shock.) This upward trend was reversed in 2009–10, and the measures remained low until 2012 (Figure 1).

Next, we comment on measures regarding the risk on interdependence between the financial sector and the real economy. DIM_{I} and DIM_{II} rose slightly from August 2007, and soared from the winter of 2008 to the spring of 2009. Having then begun a downward trend, these risk measures stabilized at quite low levels in 2010–12 (Figure 2). These movements can be interpreted as follows. Although risk arising from interdependence between the financial sector and the real economy was not much of a concern around the time of the BNP Paribas shock, it materialized because of the sudden decline in the production activity following the Lehman shock. In other words, while the BNP Paribas shock had a limited impact on the Japanese economy, corporate performance worsened drastically following the Lehman shock and the accompanying sudden decline in global demand, which was described as "instantaneous evaporation." The measures indicate that interdependency risk between the financial sector and the real economy materialized, albeit temporarily. This set off a negative chain of events in which weakened financial intermediation caused the real economy to deteriorate, which in turn further damaged financial intermediation.

The SCCA, a measure regarding the risk on interdependence between the financial and public sectors, rose slightly in the autumn of 2007 and more strongly in the winter of 2008. It fell to a low level by the middle of 2009 and remained there until 2012 (Figure 3).

This implies that at the time of the Lehman shock, the costs of potential bailouts of financial institutions increased in Japan, which gave rise to concerns about risk in the public sector.

Next, we examine measures regarding the risk of a malfunctioning financial market. The SLRI began an upward trend after 2007. It rose steeply in the middle of March 2008 and again in October 2008 before trending downward and stabilizing at a low level after 2010 (Figure 4). These movements reflected the events of 2007–09, including turmoil in the short-term money market and greatly reduced market liquidity. In fact, in late 2007 heavy declines in the prices of securitized products and the subsequent illiquidity in these markets gave rise to solvency concerns about financial institutions that had structured such products aggressively. This uncertainty brought market illiquidity to the market generally. Furthermore, in the middle of March 2008, prices in the Japanese fixed income and domestic CDS markets swung up and down significantly as overseas investors adjusted their positions to deal with funding constraints.[25] In the wake of the Lehman shock, domestic investors also started unwinding their positions to reduce risk. This caused further reduction in market liquidity and deepened the malfunction of the financial market.

4.3 Using the risk measures to compare the two periods

As mentioned above, the behavior of the systemic risk measures describes Japan's financial crises accurately. However, there are differences in the risks represented by these measures between 1997–2004 and 2007–12. Below, we evaluate differences in the vulnerability of the financial system between these periods through investigation of the differences in the behavior of the measures and their implication.

First, we consider differences in the persistence and peak levels of the measures. In 1997–2004, the systemic measures remained high for a long time, once they rose. In 2007–12, they quickly returned to their normal levels after they rose. To evaluate both periods, we examine the measures regarding the risk on interconnectedness between

[25] This point is referred to by the Bank of Japan (2008).

financial institutions and those regarding the risk on interdependence between the financial sector and the real economy. The peaks of these measures are lower in 1997–2004 than in 2007–12. This suggests that Japan's financial crisis of the 1990s was prolonged, whereas the financial crisis of 2008–09 was severe.

Hence, we can interpret the vulnerability of Japan's financial system during each period differently. In 1997–2004, there was slow adjustment of three excesses (excess debt, excess business capacity, and excess employment), and the financial system was vulnerable for a period. In 2007–12, although the financial system was temporarily vulnerable because of a huge shock (the Lehman shock and the subsequent instantaneous evaporation of global demand), it was considered relatively sound as a result of the relatively healthy positions of financial institutions and industrial firms.

Figure 1: Measures of risk materializing from interconnectedness between financial institutions

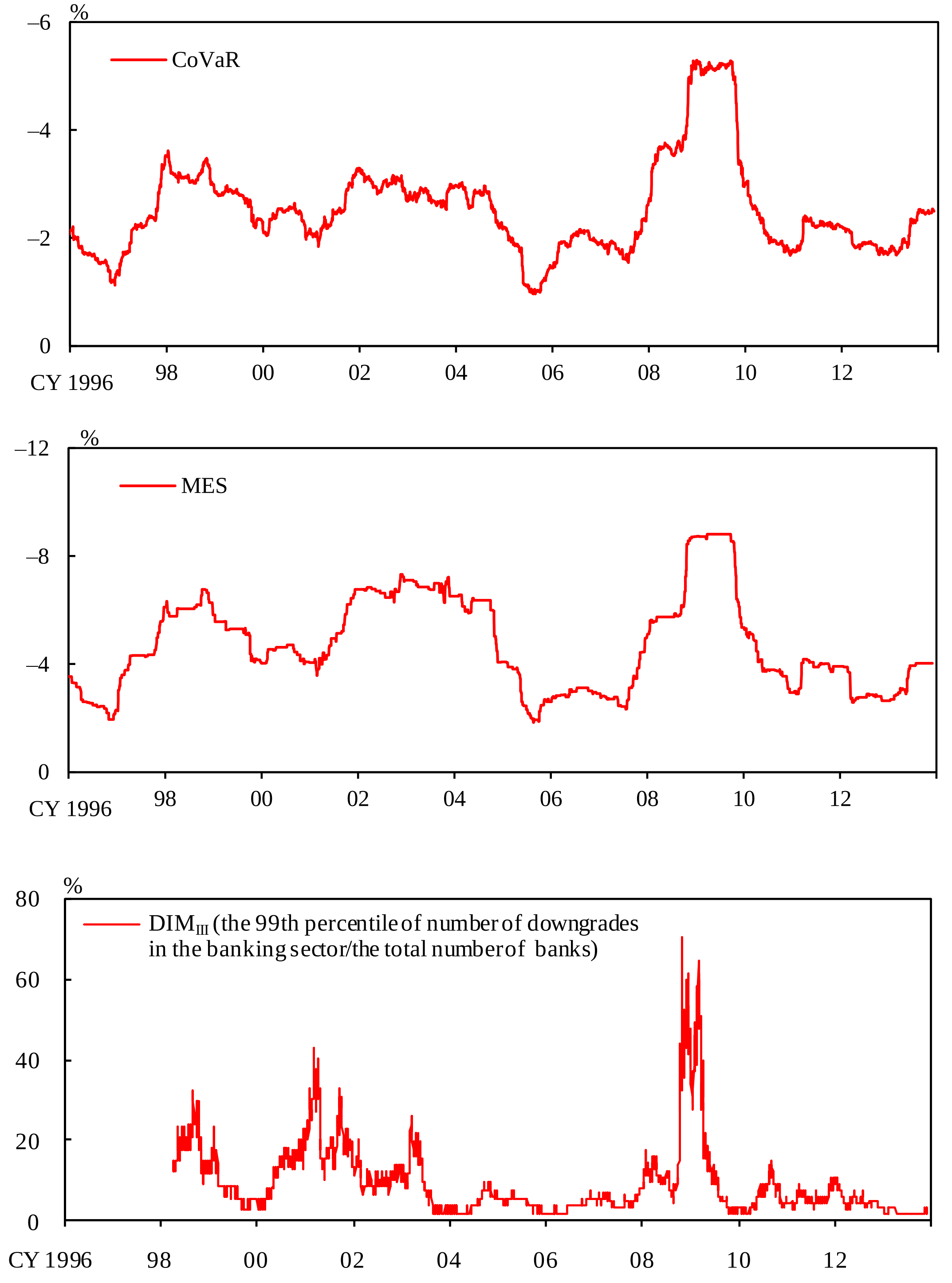


Figure 2: Measures of risk materializing from interdependence between the financial sector and the real economy

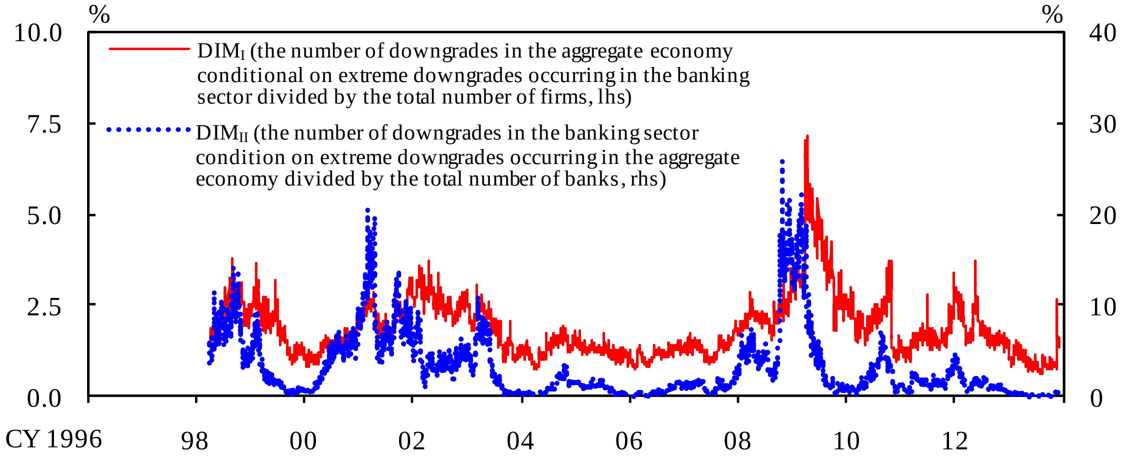


Figure 3: Measures of risk materializing from interdependence between the financial and public sectors

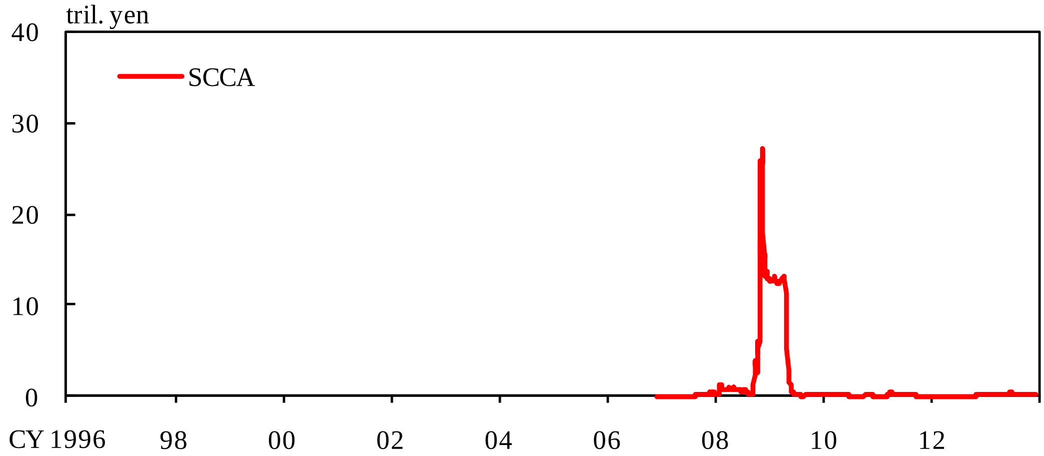
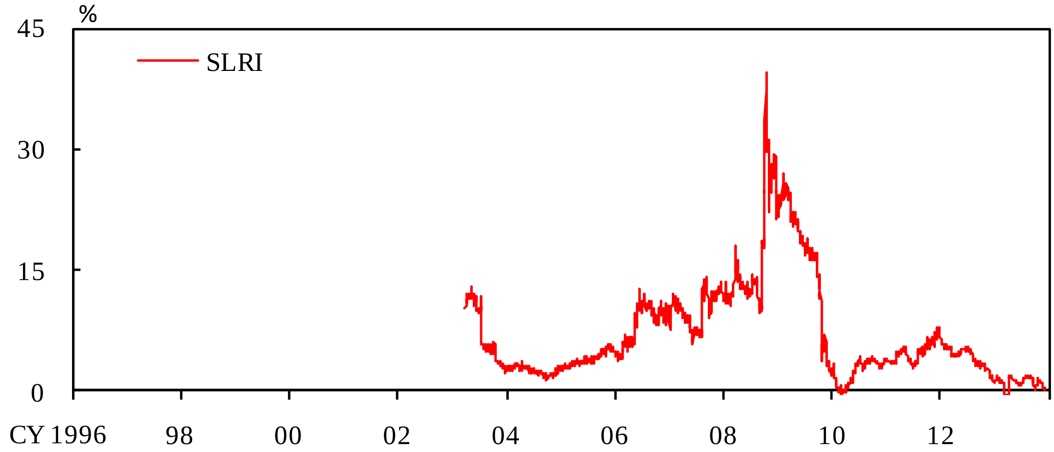


Figure 4: Measures of the risk of a malfunctioning financial market



5. Concluding remarks

In this paper, we considered quantitative methods used to analyze financial stability, focusing on systemic risk measures. We selected and outlined several risk measures, and presented the results of our empirical analysis based on applying these measures to Japanese data. The results demonstrate that using systemic risk measures is an effective way of analyzing the robustness of the financial system. This suggests that systemic risk measures can be powerful tools for monitoring the financial system. In fact, some international organizations and central banks, which attach importance to macro-prudence, actively use tools such as these.

However, systemic risk measures have a few limitations. The first concerns scenario setting. Using systemic risk measures can be considered a form of scenario analysis because, under certain assumptions about a trigger event, the measures are used to quantify the impact on the financial system of that trigger event when it occurs. Since using systemic risk measures can be considered a kind of scenario analysis, the corresponding scenario is defined by statistical procedures that are applied to past data. Therefore, if the looming crisis occurs in an unprecedented way, it is difficult to appropriately evaluate the robustness of the financial system using systemic risk measures. The second limitation relates to the characteristics of the data used. Most recent research involving systemic risk measures is based on market data (so that updates can be made frequently). However, the importance of short-term changes to market data might be overestimated,[26] because such research does not necessarily analyze the mechanism of systemic risk materialization directly. In particular, when markets function poorly, market data are a poor indicator of financial environments. So far, it is not possible to extract only useful information from market data when systemic risk is about to materialize. To use systemic risk measures proficiently, we must overcome these restrictions. To conclude this paper, we suggest ways of addressing these difficulties.

[26] This short-term change can be considered as noise in *ex post* analysis. In addition to white noise, which does not compromise the results, it includes fluctuations with a structure that makes it impossible to analyze.

5.1 Combining risk measures with the macro stress test

When the use of systemic risk measures is regarded as a kind of scenario analysis, although systemic risk measures and macro stress tests both function to evaluate the vulnerability of the financial system, they do so differently because of differences in scenario settings. With systemic risk measures, as mentioned above, the stress events deduced from the historical data are used to assume scenarios. Stress events in macro stress tests are not deduced from the historical data; they are defined as extreme but plausible possibilities.

Financial crises take different forms, and some arise through channels unforeseen by market participants. For instance, during the European sovereign debt crisis, sovereign default was the key to risk materialization, which was expected neither by policymakers nor by market participants. Unlike macro stress tests, systemic risk measures are ill equipped to capture risks based on unfamiliar risk scenarios. This suggests that systemic risk measures might be more effective when used in combination with macro stress tests.

5.2 Combining different systemic risk measures[27]

As shown in this paper, there are various systemic risk measures. While many of them are based on market data such as those on stock prices and CDS spreads, some measures are based on data on credit downgrades and GDP growth. Combining multiple measures may minimize the limitations of using market data without modifying the concept of systemic risk measures that the mechanism behind the risk is not explicitly modeled.

In any case, research into systemic risk measures is ongoing, and further study is needed.

[27] When combining multiple measures, it is necessary to decide which measures to use and how they should be weighted. However, the problem is that there is no theoretical guidance for doing this in practice. The current approach is to simply list the various risk measures in a chart (termed the "dashboard" by Blancher *et al*. [2013]), which provides an overview, and then use expert judgment to choose the risk measures and their weights.

Appendix A: EWIs

To predict the currency crises seen in developing countries, research on EWIs has been conducted actively since the 1990s. In recent studies, following the global financial crises, the methods adapted to EWIs in the past have been used to predict trigger events such as dramatic changes in asset prices. In this section, we detail analysis based on EWIs and make several comments on them.

A.1 The framework of EWI-based analysis: the signaling approach

EWIs are determined in four steps. First, the period in which the trigger event occurs is specified. Second, indicators that predict the trigger event are found. Third, the prediction period is set for the trigger event (normally from one to three years). Fourth, the methods signaling the occurrence of the trigger event are determined.

In an example of the first step, one could define the occurrence period as the period during which the deviation of residential mortgage prices from trend is larger than a specific threshold. For the second step, candidates used in the literature for EWIs include indicators of external balances such as the current and capital accounts, economic indicators such as residential and equipment investment, financial variables such as credit and money, and market prices such as interest rates. The third step is to set the prediction period, the length of time before the crisis the warning is received. In many previous studies, a period of around one to three years is used, which is considered adequate for policymakers to plan and implement the actions, and to verify predictions. The fourth step is to choose effective predictors of trigger events. Most researchers use the signaling approach.[28]

Under the signaling approach, certain thresholds for candidate variables are determined in advance, and the trigger event is signaled when these variables exceed their thresholds. EWIs are chosen to minimize the weighted average of type I errors, which

[28] Another approach is to use multivariate regression analysis. In this approach, the probability that a trigger event occurs is estimated by using a probit or logit model which includes independent variables useful for explaining the trigger event. The predicted probability of the event exceeding a predetermined threshold constitutes a warning signal.

arise if a trigger event occurs but no signal is sent, and type II errors, which arise if a trigger event does not occur but a signal is sent.[29]

A.2 Limitations of EWIs

The signal of EWIs selected in this way inevitably involves prediction errors, which can be substantial. For example, the International Monetary Fund (2009) reports a probability of about 35% that a bubble (the trigger event) is signaled but does not occur, and a probability of about 45% that a bubble occurs when there is no signal. Such errors arise because choosing EWIs for bubbles is akin to predicting the future paths of asset prices. Therefore, the more efficient the market, the more difficult it is to identify effective EWIs.[30]

In addition, if economic agents other than the government perceive that a policymaker is using a specific EWI, the economy or financial sector may overheat without making the EWI respond. In this case, the EWI's usefulness is limited.

Therefore, when using EWIs, one must be aware of their limitations, and it is necessary to review the risk of a trigger event occurring by combining EWIs with soft information of the behavior of market participants.

[29] Depending on whether or not the trigger event occurs with or without a preceding signal, there are four possibilities: (A) the event occurs with a preceding signal; (B) the event does not occur with a preceding signal; (C) the event occurs without a preceding signal; and (D) the event does not occur without a preceding signal. To be precise, the EWIs are selected to minimize the noise-to-signal ratio, which is generally defined as the probability of a type II error divided by (1 – the probability of a type I error). In this case, this is (B/(B + D))/(A/(A + C)).

[30] Under the strong form of the efficient markets hypothesis, future prices cannot be predicted because any available information, including that held by insiders, is instantly factored into the market price. In this case, bubbles cannot be predicted from available information, and EWIs cannot be constructed. Under the semistrong form of the hypothesis, it is impossible to construct EWIs from public information because such information is instantly factored into the market price. Under the weak form of the hypothesis, all past information on the price of an asset is factored into its market price. In this case, it is possible to construct EWIs that can predict the bursting of an asset price's bubble by using past public information other than that on the price of the asset. If the efficient markets hypothesis does not hold, EWIs can be constructed. In short, the construction of EWIs assumes that the efficient markets hypothesis does not hold, or assumes that only the weak form of the hypothesis holds.

Appendix B: Details of previous research and the model used in this paper

B.1 Measures of risk materializing from interconnectedness between financial institutions

a. CoVaR

The CoVaR is a risk measure based on the idea of systemic risk materializing on which a management crisis in a financial institution undermines the soundness of another financial institution.

The CoVaR, denoted by $CoVaR^i_{q,p}$, is defined as the $(100 - q)$th percentile of the stock market rate of return r for the financial sector as a whole, conditional on the stock market rate of return r^i for a financial institution i, being equal to its $(100 - p)$th percentile, denoted as VaR^i_p. The formal definition is as follows:[31]

$$\Pr(r \le CoVaR^i_{q,p} \mid r^i = VaR^i_p) = q/100.$$

From this definition, $CoVaR^j_{q,p}$ can be calculated by replacing financial institution i with financial institution j, and it is possible to compare the impacts of individual financial institutions on the materialization of systemic risk by comparing these values. That is, $\Delta CoVaR^i_{q,p}$ can be defined as the difference between $CoVaR^i_{q,p}$ and $CoVaR^i_{q,50}$:[32]

$$\Delta CoVaR^i_{q,p} = CoVaR^i_{q,p} - CoVaR^i_{q,50}.$$

From this definition, a larger value of $\Delta CoVaR^i_{q,p}$ implies a greater impact of financial institution i on the other financial institutions when it faces a management crisis.

The CoVaR is estimated by using quantile regression.[33] Adrian and Brunnermeier (2011) run a linear regression of the $(100 - q)$th percentile of the rate r_t (the financial

[31] Pr($A|B$) is the conditional probability of event A conditional on the occurrence of event B.
[32] The $(100 - q)$th percentile of the stock market rate of return for the financial sector as a whole, conditional on the stock market rate of return of financial institution i, is at its average level (its 50th percentile).
[33] In quantile regression, the quantile of the dependent variable is regressed on the explanatory variables.

sector-wide return at time t, namely, $VaR_{t,q}$) on the vector X_{t-1}, which represents financial indices at $t-1$ (such as the volatility index [VIX], the repo-treasury bills [TB] interest rate spread, and the TB rate) and r_t^i (the return for financial institution i at time t):

$$VaR_{t,q} = \alpha_q^i + \beta_q^i r_t^i + \gamma_q^i X_{t-1} + u_t^i, \tag{A-1}$$

where α^j_q, β^i_q, and γ^i_q are parameters and u^i_t is the error term. Because equation (A-1) relates a financial institution's stock market rate of return to the $(100-p)$th percentile of the whole financial sector's stock market rate of return, $CoVaR^i_{t,q,p}$ and $\Delta CoVaR^i_{t,q,p}$ can be obtained by substituting the $(100-p)$th percentile of the stock market rate of return of the financial institution into the equation

$$\begin{aligned} CoVaR^i_{t,q,p} &= \alpha_q^i + \beta_q^i VaR^i_{t,p} + \gamma_q^i X_{t-1}, \\ \Delta CoVaR^i_{t,q,p} &= \beta_q^i (VaR^i_{t,p} - VaR^i_{t,50}). \end{aligned} \tag{A-2}$$

Equation (A-2) implies that a larger value of $VaR^i_{t,p}$ or a larger β^j_q means that financial institution i has greater impact on any systemic risk that materializes.

In this paper, we calculate $\Delta CoVaR^i_{q,p}$ for three major Japanese banks by using the Banks Index of the Tokyo Stock Exchange as the stock price of the financial sector as a whole.[34] We use daily data from January 1984 to November 2013. Because of bank mergers, we reconstruct the stock price retrospectively by using the market-value-weighted average of the rates of return on the stocks of the participating bank before the merger.[35] The observation period is one year, the risk evaluation period is one day, and the confidence level is 95% ($p = q = 95\%$). We use the historical method to calculate the VaR on the rate of return of an individual stock. For the financial indices vector X_t, we use the daily rate of return on the Nikkei 225 index, the spread between the

---

Linear quantile regression was developed by Koenker and Bassett (1978) and applied to VaR estimation by Chernozhukov and Umantsev (2001). We use linear quantile regression to estimate the CoVaR. VaR estimation based on nonlinear quantile regression was developed by Engle and Manganelli (2004). Under this method, robust estimation of the regression coefficients requires a substantial quantity of data.

[34] The three major banks are Mizuho Bank, Bank of Tokyo–Mitsubishi UFJ, and Sumitomo Mitsui Banking Corporation.

[35] Hereafter, the same treatment is applied to individual bank stock prices.

three-month Libor and three-month government bills, and the term spread between three-month government bills and 10-year government bonds.

b. MES, SES, and SRISK

The MES and SES scale the connectedness between an individual financial institution and the financial sector as a whole by measuring the deteriorating soundness of a specific financial institution given that the financial sector as a whole is becoming less sound (Acharya *et al*. [2010]).

The MES of financial institution i is defined as the expected stock market rate of return r^i of financial institution i, conditional on the stock market rate of return of the financial sector as a whole r being less than its (100 – p)th percentile (that is, VaR_p):

$$MES^i = E[r^i \mid r \leq VaR_p]. \tag{A-3}$$

From this definition, the stock-market-value-weighted average of the MES over all financial institutions equals the expected shortfall of the rate of return of the financial sector as a whole.

The SES is a risk measure based on the individual contributions of financial institutions to overall financial sector capital shortages, which arise when the financial sector falls into capital deficit (Acharya *et al*. [2010]).[36]

The SES of financial institution i is defined as the expected amount of the capital shortage in financial institution i, conditional on the stock value across the financial sector as a whole (W) being less than the value of all assets across the financial sector (A) multiplied by the required capital adequacy ratio k. With the value of total assets and the stock value of financial institution i being denoted by A^i and W^i, respectively, the SES of financial institution i is as follows:[37]

[36] Capital shortages for the overall financial sector are calculated by aggregating information from the balance sheets of the individual financial institutions included in this analysis.
[37] The required capital ratio for depositary financial institutions is calculated on a risk-asset basis according to the regulations, but this measure assumes that the requisite capital can be calculated from financial statements.

$$SES^i = E[kA^i - W^i \mid W - kA < 0].$$

From this definition, the aggregate SES over all financial institutions is the expected amount of capital shortage in the financial sector as a whole.[38] Acharya *et al*. (2010) assume a regression in which the SES is explained by the MES and the leverage ratio in the last period:

$$SES_t^i = \alpha + \beta MES_{t-1}^i + \gamma LVG_{t-1}^i + u_t^i,$$

where LVG_t^i denotes the leverage ratio of financial institution i, u_t^i is the error term, and α, β, and γ are parameters to be estimated from data on aspects of financial crises, such as capital deficiency of the financial sector. Because of the lagged regressors, the period-ahead SES can be predicted from the current MES and leverage ratio.[39]

However, the SES is only valid for previous financial crises. When crises change the parameters, the SES using the past parameters is invalid for future crises. The SRISK is designed to address this problem. Like the SES, it is a systemic risk measure that is based on the capital shortages of financial institutions, but unlike the SES it is based on the trigger event of a downturn in the rate of return on financial sector stocks. The advantage of the SRISK is that it measures risk independently of specific parameters (Brownlees and Engle [2012] and Acharya, Engle, and Richardson [2012]). The SRISK of financial institution i is defined as follows:

$$SRISK^i = \max(E[kA^i - W^i \mid r^s \le r^*], 0), \tag{A-4}$$

[38] Because we are using the expected capital shortage rather than the expected amount of capital, this statistic is similar to the expected shortfall, and is based on the required amount of capital instead of the quantile.

[39] Acharya *et al*. (2010) estimate these parameters for U.S. financial institutions based on the following: (1) the amount of additional capital required according to the stress tests conducted in February 2009; (2) the rate of decrease in financial institutions' stock prices during the financial crisis of July 2007–September 2008; and (3) the increase in financial institutions' CDS spreads during the global financial crisis.

where r^* denotes the threshold rate of return indicating financial sector weakness. By denoting the face value of the liability held by financial institution i as B^i, equation (A-4) can be rewritten as

$$SRISK_t^i = \max(kB_t^i - (1-k)W_t^i(1+MES(r^*)),0). \tag{A-5}$$

Therefore, the SRISK can be calculated from the face value of liability, the stock value, and the MES when the stock market rate of return index falls below the threshold.[40]

In this paper, we use equation (A-3) to calculate the MES for three major Japanese banks using the Banks Index of the Tokyo Stock Exchange as the stock price of the financial sector as a whole. We use daily stock return data from January 1984 to November 2013.

c. DIP

The DIP is a risk measure that is based on overall financial sector losses conditional on the default of a particular financial institution. It can be interpreted as the insurance premium paid against loss (Huang, Zhou, and Zhu [2009, 2010, 2011]).

The DIP in the financial sector as a whole, DIP_I, is defined as the expected loss incurred by the financial sector as a whole conditional on the loss exceeding a certain proportion[41] of the financial sector's total liabilities:

$$DIP_I = E[L \mid L \geq L^*].$$

The DIP of an individual financial institution, DIP_{II}, is defined as the expected loss incurred by an individual financial institution on the same basis:

---

[40] If the observation period is reasonably long, the historical distribution of the rate of return on a stock might change substantially. In this case, an additional process or simplification is needed to calculate the MES. Acharya, Engle, and Richardson (2012) circumvent this problem by using extreme value theory. They first consider the long-term MES, whose conditioning event is a fall in financial sector stock prices of more than 40% in six months. They also consider the short-term MES, whose conditioning event is a fall in financial sector stock prices of more than 2% in one day. Then they derive theoretically the following:

$$(\text{long-term } MES^i) \approx 1 - \exp(-18 \times (\text{short-term } MES^i)),$$

which is used to calculate the MES. The result is substituted into equation (A-5) to calculate SRISK.

[41] Huang, Zhou, and Zhu (2009) use a ratio of 15%.

$$DIP_{\mathrm{II}}^{i} = E[L^{i} \mid L \geq L^{*}].$$

In this expression, L^i denotes the LGDs of financial institution i, L denotes the loss incurred by the financial sector as a whole (which equals the sum of all L^i), and L^* denotes the threshold of loss corresponding to the certain proportion of the financial sector's total liabilities (which equals the sum of all financial institutions' total liabilities). Because the sum of $\mathrm{DIP}_{\mathrm{II}}{}^i$ equals $\mathrm{DIP}_{\mathrm{I}}$ by definition, $\mathrm{DIP}_{\mathrm{II}}{}^i$ can be interpreted as the contribution of each financial institution to the financial sector's overall loss incurred because of the materialization of systemic risk.

The DIP is calculated by using simulation. In this simulation, under the assumption that the rate of change in the value of assets held by the financial institution follows a multivariate normal distribution,[42] multidimensional normal random numbers are generated, and the event of default by an institution is determined based on a default boundary consistent with the default probability implied by the CDS spreads. The LGD is calculated by using simulation based on a different distribution.

d. JPoD

The joint probability of distress, the banking stability index, and the cascade probability are risk measures based on chain defaults of financial institutions (Segoviano and Goodhart [2009]).

The joint probability of distress is defined as the probability of simultaneous defaults by all financial institutions across the financial sector:

$$\text{joint probability of distress} = \Pr(Y_t^1 < y^{1*}, Y_t^2 < y^{2*}, \ldots, Y_t^n < y^{n*}).$$

In this expression, Y_t^i ($i = 1, 2, \ldots, n$) is a random variable representing the soundness of individual financial institutions at time t, and $y^{i}*$ ($i = 1, 2, \ldots, n$) is the default boundary of each institution; that is, the institution defaults when Y_t^i falls below the

[42] Because the normal distribution cannot account for the fat tails and tail dependency of the distribution, assuming the normal distribution may cause risk to be underestimated.

boundary. The banking stability index is defined as the expected number of defaulting financial institutions conditional on the default of at least one financial institution:

$$\text{banking stability index} = \frac{\sum_{i=1}^{n} \Pr(Y_t^i < y^{i*})}{1 - \Pr(Y_t^1 \geq y^{1*}, Y_t^2 \geq y^{2*}, \ldots, Y_t^n \geq y^{n*})}.$$

The cascade probability is defined as the probability that at least one financial institution defaults conditional on the default of a certain financial institution *i*:

$$\text{cascade probability}^i = 1 - \frac{\Pr(\{Y_t^i < y^{i*}\} \cap \{Y_t^j \geq y^{j*}\}_{j \neq i})}{\Pr(Y_t^i < y^{i*})}.$$

Calculating these risk measures requires the joint probability density function of Y_t^i ($i = 1, 2, \ldots, n$) and the default boundaries y^i* ($i = 1, 2, \ldots, n$). To determine y^i*, Segoviano and Goodhart (2009) obtain the joint probability density function $p_t(y^1, y^2, \ldots, y^n)$ for each period t by minimizing its difference from the multivariate standard normal distribution, whose correlation coefficient is the default correlation coefficient for financial institutions. To be precise, they first determine the default boundary y^i* ($i = 1, 2, \ldots, n$) such that the default probability of financial institution *i* that is implied by the CDS spread is equal to $\Pr(Y_t^i < y^i*)$ for each period *t*, where the marginal distribution of Y_t^i ($i = 1, 2, \ldots, n$) for each *i* is assumed to be standard normal. Next, they define the "distance," as the Kullback–Leibler divergence,[43] which is the difference between the objective joint probability density function and the standard normal joint probability function $\phi(y^1, y^2, \ldots, y^n)$, whose correlation matrix represents the default correlation for all financial institutions, and then they choose $p_t(y^1, y^2, \ldots, y^n)$ to minimize this distance. In this

[43] The Kullback–Leibler divergence between the objective joint probability density function and the standard normal density function is used to measure the distance between two distributions. This divergence can be written as $H(P,\Phi) - H(P)$, where $H(P)$ is referred to as the entropy of $p_t(y^1, y^2, \ldots, y^n)$ and $H(P,\Phi)$ is referred to as the cross entropy between $p_t(y^1, y^2, \ldots, y^n)$ and $\phi(y^1, y^2, \ldots, y^n)$, defined respectively as

$$H(P) := -\int \cdots \iint p_t(y^1, y^2, \cdots, y^n) \ln p_t(y^1, y^2, \cdots, y^n) dy^1 dy^2 \cdots dy^n,$$
$$H(P,\Phi) := -\int \cdots \iint p_t(y^1, y^2, \cdots, y^n) \ln \phi(y^1, y^2, \cdots, y^n) dy^1 dy^2 \cdots dy^n.$$

The smaller the Kullback–Leibler divergence, the closer the two distributions.

calculation, the default probabilities based on the objective density function and the default boundary should be consistent with the observed default probability. Then, $p_t(y^1, y^2, \ldots, y^n)$ is

$$p_t(y^1, y^2, \ldots, y^n) = \phi(y^1, y^2, \ldots, y^n)\exp(-(1+\lambda_0+\sum_{i=1}^{n}\lambda_i 1_{\{y^i<y^{i*}\}})). \quad \text{(A-6)}$$

The joint probability density function given by equation (A-6) is obtained by minimizing the Kullback–Leibler divergence using the Lagrange multiplier method under $n + 1$ constraints.[44,45] Calculating the joint probability density function $p_t(y^1, y^2, \ldots, y^n)$ makes it possible to represent the tail dependency of time-dependent state variables, with the default probability of each financial institution being matched to market data.[46]

B.2 Measures of risk materializing from interdependence between the financial sector and the real economy

a. DIM

The DIM is a measure based on the change in the default intensities[47] of financial institutions and corporations through spillover channels such as the default chains that are caused by negative synergy within and between sectors (Giesecke and Kim [2011]).

The DIM is defined as statistics from the distribution (such as quantiles or expectations) on, for example, the proportion of corporations in a sector that default. The model used in measuring DIM has two features. First, the event that a corporation defaults in a sector increases default intensity of that sector (the self-exciting property).[48] Second, a sector's default intensity increases with default intensity of all economic

[44] There are n constraints on the default probability of an individual financial institution; that is, $\Pr(Y_t^i < y^i*)$. In addition, the joint probability density function is constrained to integrate to unity ($\int\cdots\int\int p_t(y^1, y^2, \cdots, y^n)dy^1dy^2\cdots dy^n = 1$).

[45] See Segoviano and Goodhart (2009) for a derivation.

[46] These risk measures are calculated by integrating the joint probability density function for each period t according to the definition.

[47] This is defined as the expected number of defaults that occurs per period, which can be interpreted as the inverse of the length of time before default occurs.

[48] The financial sector is usually considered.

sectors. These two properties of the model enable DIM to capture default spillover within and between sectors.

In Japan, because legal liquidation tends to be avoided when a financial institution faces management crisis, changes in default intensity might not reflect the materialization of systemic risk effectively. Therefore, following Yamanaka, Sugihara, and Nakagawa (2012),[49] we model the downgrade intensity of corporations, instead of the default intensity. In this context, we calculate the three types of statistics. The first is the proportion of expected downgrades in the nonfinancial sector (all economic sectors excluding the financial sector), conditional on the number of downgrades exceeding its 99th percentile in the financial sector. (Hereafter, the risk evaluation period is set to six months.) The second is the proportion of expected downgrades in the financial sector, conditional on the number of downgrades exceeding its 99th percentile in the nonfinancial sector. The third is the proportion of the 99th percentile of downgrades in the financial sector.

Giesecke and Kim (2011) only used the third statistic, and we measure anew the first and second statistics. Because these measures reflect the propagation of a shock from the originating group to other groups, they can be regarded as risk measures based on the interdependence between the financial sector and the real economy.

In the DIM framework, the downgrade intensity λ_t^* of all economic sectors at time t is defined as follows:

$$\begin{aligned} &\lambda_t^* = \exp(\beta^* X_t) + \int_0^t \exp(-\kappa(t-s))dJ_s, \\ &\text{where, } J_s = \nu_1 + \cdots + \nu_{N_s^*},\ \nu_n = \max(\gamma, \delta\lambda_{d(n)-}^*), \end{aligned} \qquad \text{(A-7)}$$

where N_s^* denotes the number of downgrades from the initial period to period s, $d(n)-$ denotes the period immediately before the nth downgrade occurs, X_t is a vector of financial variables, and β^*, γ, δ, and κ are parameters. In equation (A-7), the downgrade

[49] In their model, Yamanaka, Sugihara, and Nakagawa (2012) consider the self-exciting property in a way similar to Giesecke and Kim (2011). The model is used to evaluate the credit risk on the corporate bond portfolio from data on changes in corporate ratings.

intensity λ_t^* is explained by the two terms on the right-hand side. The first of these represents the normal level of the intensity, which is expected to depend on financial variables at time t. The second represents the effect of the self-exciting property; that is, previous defaults in the sector increase the downgrade intensity of that sector. In particular, the effect of the nth downgrade on downgrade intensity is governed by a magnitude ν_n with a decay effect $\exp(-\kappa(t - T(n)))$. The ν_n is determined by downgrade intensity immediately before the nth downgrade $\lambda^*_{d(n)-}$, and depends on two parameters, γ and δ, which denote the impact floor and the scale of the effect, respectively. The decay effect depends on the parameter κ, which represents the speed of decay. In this paper, the following financial variables are used for X_t: the year-on-year rate of return on the Nikkei 225 index; the one-year lagged term spread of Japanese government bonds between 10 years and three months; and the credit spread between corporate bonds rated BBB and those rated AA.[50]

The downgrade intensity of the financial sector λ_t is defined by the downgrade intensity of all economic sectors λ_t^*; that is, $\lambda_t = \lambda_t^* Z_t$, where $Z_t = \Phi(\beta X_t)$.[51] In this expression, X_t is again a vector of financial variables, Φ denotes the standard normal distribution function, and β is a parameter. The variable Z_t denotes the ratio of downgrades in the financial sector to downgrades in all economic sectors.[52] The DIM reflects interdependence between sectors, because downgrades in the financial or nonfinancial sectors increase the downgrade intensity of all economic sectors through equation (A-7), which then affects the downgrade intensity of other sectors.

The parameter $\theta_1 = (\beta^*, \kappa, \gamma, \delta)$, which determines λ_t^*, and the parameter $\theta_2 = \beta$, which determines Z_t, are estimated by using the maximum likelihood method. Because each variable has an explicit likelihood function, estimation is straightforward.[53]

---

[50] This is calculated from the yield of each Rating and Investment Information Inc. (R&I) rating published by the Japan Securities Dealers Association.

[51] The downgrade intensity in the nonfinancial sector is $\lambda_t^*(1 - Z_t)$.

[52] Hence, Z_t is called the thinning process.

[53] The log likelihood function $l_1(\cdot, \theta)$ for λ_t^* is

$$l_1(\cdot, \theta_1) = \int_0^T \log(\lambda^*_{s-}(\theta)) dN^*_s - \int_0^T \lambda^*_s(\theta) ds,$$

where [0,T] is the period covered by the data. This likelihood can be obtained by simulating a sample

By using the estimated model, the distribution of downgrade numbers in the financial and nonfinancial sectors for each period can be calculated from Monte Carlo simulation. To be precise, simulating downgrade numbers in all economic sectors proceeds in four steps. The first step is to generate a sample pass of future values of the financial variables X_t from the historical distribution, which is based on daily data covering the observation period (the previous six months). The second step is to determine the subsequent downgrade time τ by using random samples of the exponential distribution with intensity λ^*, under the assumption that the interval of two successive downgrades follows this distribution. The third step is to update the value of λ^* at time τ by using the financial variables X_τ and the simulated downgrade data up to time τ. The fourth step is to iterate the second and third steps until time τ exceeds the risk evaluation period. In this simulation, the sector in which downgrades occur is determined by random samples based on $Z_t = \Phi(\beta X)$, which represents the downgrade ratio in each period. The distribution of the number of downgrades in individual sectors can be obtained by iterating these steps. The DIM is the conditional expectation or 99th percentile of this distribution.

In this paper, we use R&I rating data from April 1998 to November 2013. The data cover all corporations in Japan rated by R&I, except for public organizations such as governments. We regard a rating cut of one notch or more as a downgrade, and create time-series data on the number of downgrades accordingly. The financial sector includes only banks.

---

pass of λ_t^* using data on corporation downgrade based on equation (A-7). The log likelihood function $l_2(\cdot,\theta)$ for Z_t is

$$l_2(\cdot,\theta_2) = \sum_{N=1}^{N_t^*} Y_{d(N)} \log(Z_{d(N)}(\theta_2)) + (1 - Y_{d(N)}) \log(1 - Z_{d(N)}(\theta_2)).$$

If default (downgrading) occurs throughout the entire economy sector at time t, Y_t is unity if the corporation belongs to the financial sector, and is zero otherwise. $d(N)$ is the time at which the Nth downgrade occurs.

b. GDP at risk

GDP at risk is a risk measure focusing on shocks to the real economy and on interdependence between the financial sector and the real economy. It is obtained from a factor-augmented vector autoregression (VAR) model with two variables representing the real economy and the financial sector (De Nicolò and Lucchetta [2010]).

GDP at risk is defined as the (100 – p)th percentile of quarterly real GDP growth ($GDPG_t$) or that of an indicator representing the risk to which the financial system as a whole is exposed (FS_t). [54] These two variables are formulated by using a factor-augmented VAR.

The factor-augmented VAR in the two variables is as follows:[55]

$$\begin{aligned} GDPG_t &= \alpha_1 + \beta_1 F_t + \gamma_{11}(L) GDPG_t + \gamma_{12}(L) FS_t + u_t^1, \\ FS_t &= \alpha_2 + \beta_2 F_t + \gamma_{21}(L) GDPG_t + \gamma_{22}(L) FS_t + u_t^2, \\ F_t &= H(L) F_{t-1} + K v_t. \end{aligned} \tag{A-8}$$

This model has the multidimensional common factor F_t, in addition to standard VAR terms. The common factor affects the original two variables. The parameters and the common factor are jointly estimated.[56] In (A-8), α_i, β_i, γ_{ij} (i,j = 1, 2), $H = (\eta_{ij})$, and $K = (\kappa_{ij})$ are parameters, L denotes the lag operator (so that $\gamma_{ij}(L)$ and $\eta_{ij}(L)$ are lag polynomials with parameters), and u_t^1, u_t^2, and v_t are error terms. By using quantile

[54] De Nicolò and Lucchetta (2010) suggest examples of FS_t, such as excess returns on the stock portfolios of financial institutions and a measure of risk within the financial system (distance to default and DIP, etc.).

[55] In matrix form, the third equation of (A-8) is

$$\begin{pmatrix} f_t^1 \\ f_t^2 \\ \vdots \\ f_t^l \end{pmatrix} = \begin{pmatrix} \eta_{11}(L) & \eta_{12}(L) & \dots & \eta_{1l}(L) \\ \eta_{21}(L) & \eta_{22}(L) & \cdots & \eta_{2l}(L) \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{l1}(L) & \eta_{l2}(L) & \cdots & \eta_{ll}(L) \end{pmatrix} \begin{pmatrix} f_{t-1}^1 \\ f_{t-1}^2 \\ \vdots \\ f_{t-1}^l \end{pmatrix} + \begin{pmatrix} \kappa_{11} & \kappa_{12} & \dots & \kappa_{1m} \\ \kappa_{21} & \kappa_{22} & \cdots & \kappa_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{l1} & \kappa_{l2} & \cdots & \kappa_{lm} \end{pmatrix} \begin{pmatrix} v_t^1 \\ v_t^2 \\ \vdots \\ v_t^m \end{pmatrix},$$

where $F_t = [f_t^1, f_t^2, ..., f_t^l]'$, $v_t = [v_t^1, v_t^2, ..., v_t^m]'$, $H = (\eta_{ij})$, $K = (\kappa_{ij})$.

[56] De Nicolò and Lucchetta (2010) use an asymptotic method to estimate the factor-augmented VAR. This involves alternating iterative estimation of the common factor F_t and the parameters. The common factor is estimated by using principal components analysis, and the parameters are estimated as normal from a VAR model given the common factor. The details are explained by Stock and Watson (2002, 2005). To determine the lag order and the number of factors, the sum of final prediction errors and Akaike's information criterion is minimized.

regression, the fifth percentiles of $GDPG_t$ and FS_t (denoted by $GDPaR_t$, and $FSaR_t$, respectively) are estimated from the following:

$$\begin{aligned} GDPaR_t &= \hat{\alpha}_1 + \hat{\beta}_1 F_t + \hat{\gamma}_{11}(L) GDPG_t + \hat{\gamma}_{12}(L) FS_t + \hat{u}_t^1, \\ FSaR_t &= \hat{\alpha}_2 + \hat{\beta}_2 F_t + \hat{\gamma}_{21}(L) GDPG_t + \hat{\gamma}_{22}(L) FS_t + \hat{u}_t^2, \end{aligned} \qquad \text{(A-9)}$$

where $\hat{\alpha}_i$, $\hat{\beta}_i$, and $\hat{\gamma}_{ij}$ denote the estimated parameters of equation (A-8) and $\hat{F}_t$ denotes the estimated common factor of equation (A-8).

The risk measures are obtained by using equation (A-9). In the context of the CoVaR, described in section B.1a of this Appendix, De Nicolò and Lucchetta (2010) suggest using $Co(GDPaR_t)$, which is the fifth percentile of the distribution of current real GDP growth conditional on the decline of both real GDP growth and the indicator of risk to the financial system to their fifth percentiles in the previous period. Analogously, they suggest using $Co(FS_t)$, which is the fifth percentile of the distribution of the current indicator of risk to the financial system, conditional on the decline of both real GDP growth and the indicator to their fifth percentiles in the previous period. These risk measures are expressed as follows:

$$\begin{aligned} Co(GDPaR_t) &= \hat{\alpha}_1 + \hat{\beta}_1 F_t + \hat{\gamma}_{11}(L) GDPaR_t + \hat{\gamma}_{12}(L) FSaR_t, \\ Co(FSaR_t) &= \hat{\alpha}_2 + \hat{\beta}_2 F_t + \hat{\gamma}_{21}(L) GDPaR_t + \hat{\gamma}_{22}(L) FSaR_t. \end{aligned}$$

The risk measures are sometimes defined in terms of difference from their fifth percentiles not conditional on them in the previous period:

$$\begin{aligned} \Delta Co(GDPaR_t) &:= Co(GDPaR_t) - GDPaR_t, \\ \Delta Co(FSaR_t) &:= Co(FSaR_t) - FSaR_t. \end{aligned}$$

To emphasize the synergic interaction between the financial sector and the real economy, we can consider two types of risk measure: (1) the $Co(GDPaR_t)'$, the fifth percentile of current real GDP growth, conditional on the decline of the indicator representing the risk to the financial system as a whole to its fifth percentile in the previous period; and (2) the $Co(FSaR_t)'$, the fifth percentile of the current indicator representing the risk to the financial system as a whole, conditional on the decline of real

GDP growth to its fifth percentile in the previous period. Then, we define each risk measure in terms of the difference from its normal level. We denote these measures by $\Delta Co(GDPaR_t)'$ and $\Delta Co(FSaR_t)'$, which are expressed as follows:

$$\begin{aligned} Co(GDPaR_t)' &= \hat{\alpha}_1 + \hat{\beta}_1 F_t + \hat{\gamma}_{11}(L)GDPG_t + \hat{\gamma}_{12}(L)FSaR_t, \\ Co(FSaR_t)' &= \hat{\alpha}_2 + \hat{\beta}_2 F_t + \hat{\gamma}_{21}(L)GDPaR_t + \hat{\gamma}_{22}(L)FS_t, \\ \Delta Co(GDPaR_t)' &:= Co(GDPaR_t)' - GDPaR_t, \\ \Delta Co(FSaR_t)' &:= Co(FSaR_t)' - FSaR_t. \end{aligned}$$

GDP at risk is updated quarterly given the use of real GDP growth, and might be renewed monthly using another variable as alternative for real GDP growth. However, this measure cannot be used to monitor a risk in a timely manner, because its updating is less frequent than that of measures based on mainly market data. This measure requires a long sample period, because quantile regression needs a large number of data points for accuracy.

B.3 Measures of risk materializing from interdependence between the financial and public sectors

a. SCCA

The SCCA is an index of the risk that the government incurs an increased cost of bailing out financial institutions, based on the assumption that the government will continue to bail out financial institutions in difficulty (Gray and Jobst [2011]). In the SCCA framework, the bailout cost implied by the financial markets is estimated from the stock prices and the CDS price based on option pricing theory.

The SCCA is based on statistics[57] of the present value of the cost incurred by the government to bail out financial institutions in the financial sector as a whole. Normally, the present value of the LGD implied by the stock prices of financial institutions is similar to that implied by their CDS spreads. However, when systemic risk materializes, the stock-price-implied loss is much greater than the CDS-spread-implied loss. This

57 Either the quantile or the expected shortfall is typically used.

difference reflects a perception in the market that because of the government bailout, stockholders suffer the loss while bondholders avoid some of the loss. Therefore, we calculate the present values of individual financial institutions' LGDs from stock prices and CDS spreads, respectively. Then, by summing up their differences, for the financial sector as a whole, we measure the present value of the cost to the government of bailing out financial institutions.

First, we explain how to estimate future LGDs from stock market data. Based on the CCA suggested by Merton (1974),[58] the present value of a financial institution's liability is equivalent to the present face value of its liability minus the present value of the put option, the underlying value of which is its asset value and the strike price of which is the face value of its liability. In other words, the present value of the liability of the financial institution $D_E(t)$ satisfies the equation $P_E(t) = Be^{-r(T-t)} - D_E(t)$, where $P_E(t)$ is the present value of the put option whose underlying asset is its asset value, B is the face value of the liability, r is the risk-free interest rate, and T is the risk evaluation period. Under the assumption that movements in the asset value are governed by geometric Brownian motion,[59] the present value of the put option $P_E(t)$ is as follows:

$$\begin{aligned} P_E(t) &= Be^{-r(T-t)}\Phi(-d_2) - A(t)\Phi(-d_1), \\ \text{where}\quad d_1 &= (\log(A(t)/B) + (r + \sigma_A^2/2)(T-t))(\sigma_A\sqrt{T-t})^{-1}, \\ d_2 &= d_1 - \sigma_A\sqrt{T-t}, \end{aligned} \tag{A-10}$$

where $A(t)$ denotes the asset value of the financial institution at time t, σ_A denotes the volatility of the asset value, and Φ denotes the standard normal distribution function. Given equation (A-10), we need $A(t)$ and σ_A to estimate the LGD. The market capitalization of the financial institution $E(t)$ is

$$E(t) = A(t)\Phi(-d_1) - Be^{-r(T-t)}\Phi(-d_2).$$

58 See Footnote 23.

59 Making this assumption may cause the present values of financial institutions' LGDs to be underestimated, because it ignores fatness in the tails of the distribution. Nevertheless, following Gray and Jobst (2011), we make this assumption.

By using Ito's Lemma to differentiate this equation, and then comparing volatility terms, we can express σ_A as

$$\sigma_A = \frac{E(t)}{A(t)\Phi(d_1)}\sigma_E,$$

where σ_E denotes stock volatility. From these two equations, $A(t)$ and σ_A are obtained. Therefore, a financial institution's LGD implied by the stock market can be estimated from information on market capitalization, stock volatility, and total liabilities.

Next, from the viewpoint of the CDS market, the present value of a financial institution's LGD is

$$P_{CDS}(t) = (1 - \exp(-S_{CDS}(t)(1 - D_B(t)/B)L_{CDS}^{-1}(T-t)))Be^{-r(T-t)}, \qquad \text{(A-11)}$$

where $D_B(t)$ denotes the market value of bonds and L_{CDS} denotes the CDS-based LGD ($L_{CDS} = 0.65$ in this paper).[60] Equation (A-11) is based on several assumptions. First, let h denote the hazard rate of default on a bond issued by the financial institution, and let L_{Bond} denote the LGD. The credit spread of the corporate bond S_{Bond} is $S_{Bond} = hL_{Bond}$, and the CDS is $S_{CDS} = hL_{CDS}$. By assuming that the LGD of this bond is determined by $B - D(t)$, or equivalently, that $L_{Bond} = 1 - D(t)/B$, the bond's credit spread is $S_{Bond} = h((B - D(t))/B)$. Given that the CDS spread is $S_{CDS} = hL_{CDS}$, it follows that[61]

$$\begin{aligned} S_{Bond} &= h(1 - D(t)/B) \\ &= S_{CDS}(1 - D(t)/B)L_{CDS}^{-1}. \end{aligned}$$

Equation (A-11) can be derived from this equation and the following identity relating to the market value of the bond:

---

[60] Gray and Jobst (2011) use corporate bond prices calculated from stock market information for $D_E(t)$ instead of the market prices of corporate bonds. To calculate $P_{CDS}(t)$ without using stock market information, we avoid using $D_E(t)$. Then, based on different corporate bonds in the secondary market, we derive the theoretical price of corporate bonds whose maturity matches that of CDS, and define the theoretical price as the price of the corporate bond.

[61] Jobst and Gray (2013) make different assumptions about the recovery rates of CDS and bonds; a recovery rate of 100% is assumed for CDS and one based on the market value is assumed for bonds. We assume $(1 - D_B(t)/B)L_{CDS}^{-1}$ for the adjustment term of the CDS spread. This adjustment term differs from that used in Jobst and Gray (2013).

$$Be^{-r(T-t)} - P_{CDS}(t) = Be^{-(r+S_{Bond})(T-t)}.$$

As explained in Section 3, given several assumptions, the present value of the cost to the government of bailing out a financial institution is

$$P_E(t) - P_{CDS}(t).$$

Next, we use time-series data obtained from applying the method described above to estimate the joint distribution for the cost of bailing out multiple financial institutions. Gray and Jobst (2011) estimated the multidimensional generalized extreme value distribution, which they assumed for the joint distribution.[62] They constructed the distribution for the future total cost to the government of bailing out financial institutions across the financial sector as a whole, and defined its quantile or expected shortfall as the risk measure.

We obtained the distribution of the potential total cost to the government of bailing out the three major Japanese banks mentioned earlier. We used daily market data from January 2006 to November 2013 on market capitalization, the CDS spread, and the price of corporate bonds. The value of total liabilities was taken from the latest financial statements at the time. The cost to the government of bailing out individual financial institutions was measured by applying the method described above. To specify the joint distribution of the total future cost to the government of bailing out financial institutions across the financial sector, we used a simpler estimation method than that suggested by Gray and Jobst (2011). For each individual financial institution, we obtained the marginal distribution for the government's bailout cost by using its historical distribution from the previous year. We then used the 99th percentile of the sum of these distributions as the risk measure.[63]

[62] According to Gray and Jobst (2011), the multidimensional generalized extreme value distribution provides more detailed tail information. For details about this distribution, including its form and approaches to its estimation, see Jobst and Gray (2013).

[63] So that the simple sum of the quantiles is not smaller than the total cost of multiple bailouts, the condition that financial institutions' costs are not highly correlated must be imposed. Hence, given strong nonlinear correlations between financial institutions' loss distributions, the simple sum is sometimes inappropriate. Thus, before using the simple sum, we checked that it was appropriate.

Because bond prices did not fall dramatically during the financial crisis, applying these methods to data on Japanese financial institutions reveals that, from 2006, financial institutions' LGDs implied by CDS spreads were low relative to those implied by the stock market. In other words, data on Japanese financial institutions imply that the market strongly expects the government to bail out financial institutions. Therefore, the SCCA can be roughly explained by the LGD implied by the stock market.

B.4 Measures of the risk of a malfunctioning financial market

a. SLRI

The SLRI is based on market liquidity in the financial markets in the event that systemic risk materializes. It is measured by synthesizing data validating the no-arbitrage relationship (Severo [2012]).

The SLRI is defined as the first component score from principal components analysis of time-series data validating the no-arbitrage relationship in the market. Severo (2012) uses 36 such series, classifying them into four groups: covered interest parity; on-the-run and off-the-run government bond spreads; the spread between short-term government bills and the OIS; and the spread between the CDS and corporate bonds.

We predict the distribution of the first principal components score from historical data, and define its quantile as the risk measure. We use data for the entire period to obtain the coefficients from the principal components analysis. These coefficients are used to compute the first principal component score. Then, we use daily data for the observation period (the last six months) to determine the historical distributions of the variables that validate the no-arbitrage relationship, and use simulation to predict these distributions at the end of the risk evaluation period (six months later). Then, by using the principal components coefficients and the distributions of the variables that validate the no-arbitrage relationship, we construct the distribution of the first principal component score, and define its (100 – p)th percentile as the risk measure. To do this, we use time-series data on the following six variables: the covered interest parities between the Japanese yen and the U.S. dollar and those between the yen and the euro; the on- and off-the-run spreads of government bonds maturing in five years; the spreads between short-term government bonds and the OIS at maturities of three and six months; and the spread between the CDS and corporate bonds (formed by aggregating the spreads of five highly liquid companies). We use daily data from March 2003 to November 2013.

b. Volatility spillover index

The volatility spillover index is a risk measure based on the degree of interconnectedness between the rates of return or levels of volatility of financial assets. It is calculated from the variance decomposition of a VAR model (Diebold and Yilmaz [2009, 2014]).

First, we estimate a VAR model in which the endogenous variables are rates of return or volatilities of financial assets. Then, to define the volatility spillover index, we use the variance decomposition d_{ij}^{H} following the risk evaluation period (H periods ahead) for the response of asset i to a shock to asset j. d_{ij}^{H} represents the extent to which a shock to asset j affects asset i. Diebold and Yilmaz (2009, 2014) estimate the following second-order VAR:[64]

$$X_t = \Psi_1 Y_{t-1} + \Psi_2 Y_{t-2} + u_t.$$

To measure the degree of interconnected between the variables, they use

$$C^H = \frac{1}{N} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} d_{ij}^{H}.$$

C^H is the average prediction error for the variables excluding those emanating from their own shocks. A large value of C^H indicates that a shock to a particular asset spreads easily to other assets.[65,66]

---

64 X_t denotes the vector of variables under consideration, Ψ_i is the parameter matrix, and u_t is the error vector. For X_t, Diebold and Yilmaz (2009) use two ranges of data: that is, weekly stock return ratios for 19 countries and their volatility. To compute stock volatility, they use the simple method suggested by Garman and Klass (1980) and Alizadeh, Brandt, and Diebold (2002). This is described by the high (H), low (L), opening (O), and closing (C) prices during the observation period as follows:

$$\sigma^2 = 0.511(H_t - L_t)^2 - 0.019((C_t - O_t)(H_t + L_t - 2O_t) - 2(H_t - O_t)(L_t - O_t)) - 0.383(C_t - O_t)^2.$$

65 Diebold and Yilmaz (2009, 2011) obtain time-series data on C^H by lagging the observation period of the VAR estimation day by day. In their case, the observation period is 100 days and H is set to 12 days.

References

Acharya, Viral, Robert Engle, and Matthew Richardson, "Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks," *The American Economic Review*, 102 (3), 2012, pp. 59–64.

——, Lasse H. Pedersen, Thomas Philoppon, and Matthew Richardson, "Measuring Systemic Risk," Federal Reserve Bank of Cleveland Working Paper No. 10-02, 2010.

Adrian, Tobias, and Markus K. Brunnermeier, "CoVaR," NBER Working Paper No. 17454, National Bureau of Economic Research, 2011.

Alessi, Lucia, and Carsten Detken, "Quasi Real Time Early Warning Indicators for Costly Asset Price Boom/Bust Cycles: A Role for Global Liquidity," *European Journal of Political Economy*, 27 (3), 2011, pp. 520–533.

Alizadeh, Sassan, Michael W. Brandt, and Francis X. Diebold, "Range-Based Estimation of Stochastic Volatility Models," *The Journal of Finance*, 57 (3), 2002, pp. 1047–1091.

Bank of Japan, Financial Markets Department, "Financial Markets Report, September 2008," Bank of Japan, 2008.

Barrell, Ray, E. Philip Davis, Dilruba Karim, and Iana Liadze, "Bank Regulation, Property Prices and Early Warning Systems for Banking Crises in OECD Countries," *Journal of Banking and Finance*, 34 (9), 2010, pp. 2255–2264.

Bernanke, Ben S., "Monitoring the Financial System," speech at the 49th Annual Conference on Bank Structure and Competition, Board of Governors of the Federal Reserve System, May 10, 2013.

Blancher, Nicolas, Srobona Mitra, Hanan Morsy, Akira Otani, Tiago Severo, and Laura Valderrama, "Systemic Risk Monitoring ("SysMo") Toolkit—A User Guide" IMF Working Paper No. 13/168, International Monetary Fund, 2013.

---

[66] Stock volatility is nonstationary and has long-term memory (see, for example, Watanabe [2007]), but this is ignored by Diebold and Yilmaz (2009).

Borio, Claudio, Mathias Drehmann, and Kostas Tsatsaronis, "Stress-Testing Macro Stress Testing: Does It Live Up to Expectations?" BIS Working Paper No. 369, Bank for International Settlements, 2012.

Brownlees, Christian T., and Robert Engle, "Volatility, Correlation and Tails for Systemic Risk Measurement," working paper, 2012 (available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1611229).

Chernozhukov, Victor, and Len Umantsev, "Conditional Value-at-Risk: Aspects of Modeling and Estimation," *Empirical Economics*, 26, 2001, pp. 271–292.

Demirgüç-Kunt, Asli, and Enrica Detragiache, "The Determinants of Banking Crises in Developing and Developed Countries," *IMF Staff Papers*, 45 (1), 1998, pp. 81–109.

——, and ——, "Monitoring Banking Sector Fragility: A Multivariate Logit Approach," *The World Bank Economic Review*, 14 (2), 2000, pp. 287–307.

De Nicolò, Gianni, and Marcella Lucchetta, "Systemic Risks and the Macroeconomy," IMF Working Paper No. 10/29, International Monetary Fund, 2010.

Diebold, Francis X., and Kamil Yilmaz, "Measuring Financial Asset Return and Volatility Spillovers, with Application to Global Equity Markets," *The Economic Journal*, 119 (534), 2009, pp. 158–171.

——, and ——, "On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms," *Journal of Econometrics*, 2014 (forthcoming).

Engle, Robert, and Simone Manganelli, "CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles," *Journal of Business & Economic Statistics*, 22 (4), 2004, pp. 367–381.

Garman, Mark B., and Michael J. Klass, "On the Estimation of Security Price Volatilities from Historical Data," *The Journal of Business*, 53 (1), 1980, pp. 67–78.

Gerdesmeier, Dieter, Hans-Eggert Reimers, and Barbara Roffia, "Early Warning Indicators for Asset Price Booms," *Review of Economics and Finance*, 1, 2011, pp. 1–19.

Giesecke, Kay, and Baeho Kim, "Systemic Risk: What Defaults Are Telling Us," *Management Science*, 57 (8), 2011, pp. 1387–1405.

Goldstein, Morris, Graciela Kaminsky, and Carmen M. Reinhart, "Assessing Financial Vulnerability: An Early Warning System for Emerging Markets," Institute for International Economics, 2000.

Gray, Dale F., and Andreas A. Jobst, "Modelling Systemic Financial Sector and Sovereign Risk," *Sveriges Riksbank Economic Review*, 2, 2011, pp. 68–106.

Group of Ten, "Report on Consolidation in the Financial Sector," Bank for International Settlements, January, 2001.

Huang, Xin, Hao Zhou, and Haibin Zhu, "A Framework for Assessing the Systemic Risk of Major Financial Institutions," *Journal of Banking and Finance*, 33 (11), 2009, pp. 2036–2049.

——, ——, and ——, "Assessing the Systemic Risk of a Heterogeneous Portfolio of Banks during the Recent Financial Crisis," BIS Working Paper No. 296, Bank for International Settlements, 2010.

——, ——, and ——, "Systemic Risk Contributions," Finance and Economics Discussion Series, 2011-8, Federal Reserve Board, 2011.

International Monetary Fund, "World Economic Outlook: Sustaining the Recovery," International Monetary Fund, October 2009.

Jobst, Andreas A., and Dale F. Gray, "Systemic Contingent Claims Analysis—Estimating Market-Implied Systemic Risk," IMF Working Paper No. 13/54, International Monetary Fund, 2013.

Kaminsky, Graciela, Saul Lizondo, and Carmen M. Reinhart, "Leading Indicators of Currency Crises," *IMF Staff Papers*, 45 (1), 1998, pp. 1–48.

——, and Carmen M. Reinhart, "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems," *The American Economic Review*, 89 (3), 1999, pp. 473–500.

Koenker, Roger, and Gilbert Bassett, "Regression Quantiles," *Econometrica*, 46 (1), 1978, pp. 33–50.

Lo Duca, Marco, and Tuomas A. Peltonen, “Assessing Systemic Risks and Predicting Systemic Events,” *Journal of Banking and Finance,* 37 (7), 2013, pp. 2183–2195.

Merton, Robert C., “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *The Journal of Finance,* 29 (2), 1974, pp. 449–470.

Segoviano, Miguel A., and Charles Goodhart, “Banking Stability Measures,” IMF Working Paper No. 09/4, International Monetary Fund, 2009.

Severo, Tiago, “Measuring Systemic Liquidity Risk and the Cost of Liquidity Insurance,” IMF Working Paper No. 12/194, International Monetary Fund, 2012.

Stock, James H., and Mark W. Watson, “Forecasting Using Principal Components From a Large Number of Predictors,” *Journal of the American Statistical Association*, 97 (460), 2002, pp. 1167–1179.

——, and ——, “Implications of Dynamic Factor Models for VAR Analysis,” NBER Working Paper No. 11467, National Bureau of Economic Research, 2005.

Watanabe, Toshiaki, “Realized Volatility: A Survey with Application to the Japanese Stock Market,” *Economic Review,* 58 (4), 2007, pp. 352–373.

Yamanaka, Suguru, Masaaki Sugihara, and Hidetoshi Nakagawa, “Modeling of Contagious Credit Events and Risk Analysis of Credit Portfolios,” *Asia-Pacific Financial Markets*, 19, 2012, pp. 43–62.