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Abstract

Negative correlations between inflation and demographic aging have been observed across developed nations recently. To understand the phenomenon from a political economy perspective, we embed the fiscal theory of the price level into an overlapping-generations model. We suppose that short-lived governments successively choose income tax rates and bond issues, considering political influence from existing generations and the expected policy responses of future governments. Our analysis reveals that the effects of aging depend on its causes; aging is deflationary when caused by an unexpected increase in longevity, but is inflationary when caused by a decline in the birth rate. Our analysis also sheds new light on the traditional debate about the burden of national debt. Because of price adjustment, the accumulation of government debt imposes no burden on future generations.

Keywords: Deflation; Fiscal theory of the price level; Population aging; Redistribution across generations
JEL classification: D72, E30, E62, E63, H60

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1 Introduction

In this paper, we analyze the political economy of the concurrent progress of aging and deflation by embedding the fiscal theory of the price level (FTPL) within the framework of an overlapping-generations (OLG) model and by considering the consequences of policy choices by successive short-lived governments.

Negative correlations between inflation and demographic aging have been observed among developed nations recently. Masaaki Shirakawa, the former Governor of the Bank of Japan, acknowledged this in his opening remarks at a conference focusing on demographic changes and macroeconomic performance:

Seemingly, there would be no linkage between demography and deflation. But it may not be the case. A cross-country comparison among advanced economies reveals intriguing evidence: Over the decade of the 2000s, the population growth rate and inflation correlate positively across 24 advanced economies. That finding shows a sharp contrast with the recently waning correlation between money growth and inflation. How could we square those facts with each other? [Shirakawa (2012)]

In Japan, deflation and rapid demographic aging have proceeded simultaneously in the last two decades, as shown in Figure 1. In conjunction with the fact that the Bank of Japan has been taking a passive stance on monetary policy by fixing nominal interest rates around zero, the conventional fiscal viewpoint, termed the FTPL, seems an appropriate unified framework within which to understand the concurrent progress of deflation and aging. Other economies may soon follow Japan. Aging is starting to pose a challenge to many developed countries as well as emerging economies such as China. Disinflation is underway in developed countries. Because central banks in these countries have been setting low nominal interest rates similarly to Japan, particularly following the recent financial crisis, we believe that the FTPL can shed light on disinflation in aging societies.

However, when attempting to understand the concurrent progress of deflation and aging by applying the FTPL, one encounters a potential puzzle. According to the FTPL, the price level is determined to balance the government's consolidated budget in present value terms. The current price level adjusts to equate the real value of the government's outstanding debt to the discounted sum of current and future fiscal surpluses in real terms. In this context, population

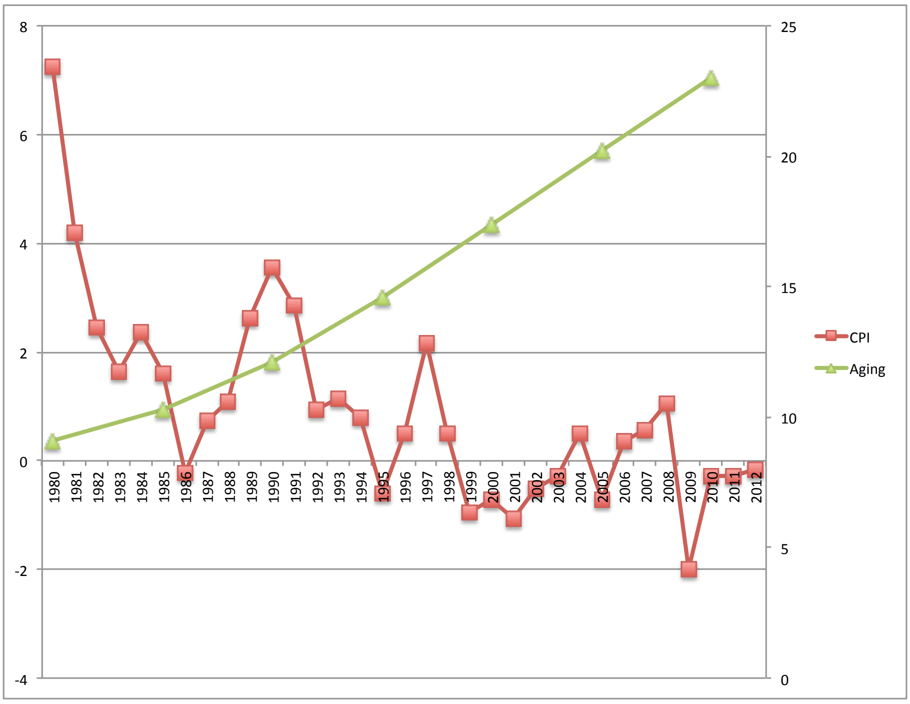

Figure 1: Recent Aging and Deflation in Japan

Note: The red line with squares represents the annual rate of change in the consumer price index (measured in % on the left vertical axis), and the green line with triangles records the change in the share of the population aged 65 or above (measured in % on the right vertical axis).

Source: IMF World Economic Outlook Databases, 2012.

aging is expected to reduce future fiscal surpluses by increasing social security expenditures and by reducing the working-age population and hence income tax revenues. One would expect this to generate *inflationary* pressure (that is, upward pressure on current prices), rather than the deflation observed recently in Japan and other developed countries.

We argue that the standard FTPL fails to explain the simultaneous progress of aging and deflation because its fiscal-policy parameters, such as income tax rates and per-capita government expenditure, are exogenously fixed. One should consider how fiscal policies respond to changes in the demographic structure. In other words, we need to consider not only the economic effects, but also the political effects of aging on the current price level.

In this paper, we undertake two main extensions. First, we extend the standard FTPL, which is based on a model of an infinitely lived representative consumer, by incorporating it into an OLG model with two generations. By explicitly modeling different generations, we can decompose the determinants of demographic change into factors affecting life expectancy and those affecting the birth rate. This enables us to analyze the different effects of these factors on prices and policy responses.

Second, we consider endogenous policy making by a succession of short-lived governments. We analyze the Markov perfect equilibrium, in which governments choose income tax rates and issue public bonds to incorporate the political effects of existing generations and the expected strategic responses of future governments.[1] This represents an extension of the standard FTPL because it ignores changes of government. When fiscal policies are chosen by a succession of short-lived governments, because the tax smoothing argument pioneered by Barro (1979) is unlikely to apply, any reduction in future fiscal surpluses may be absorbed by future tax increases, thereby leaving the current price level unaffected.

Our main findings are summarized below.

First, although our model shows that the current price level is determined to balance the intertemporal budget of the public sector, as does the standard FTPL, the underlying logic is

[1]The usefulness of these extensions is not limited to understanding recent deflation in Japan. Indeed, our modeling contributes to the following three (of four) important research topics proposed by Leeper and Walker (2011) regarding the FTPL: integrating heterogeneity and policy uncertainty; identifying policy behavior; and quantifying fiscal limits. The remaining issue concerns the appropriate anchoring of fiscal expectations so that monetary policy can be used to control inflation. We do not incorporate strategic interaction between a government and a central bank.

different. In the standard FTPL, future budget surpluses of the public sector mirror losses in the lifetime income of an infinitely lived representative consumer. Hence, future budget surpluses produce a wealth effect in the form of a reduced demand for goods and a fall in the current price level [see, for example, Woodford (1995)]. In our model, however, because consumers have a finite lifetime, changes in budget surpluses that occur after their death cannot affect their consumption behavior. This means there is no wealth effect. Instead, future budget surpluses affect the current price level through the response of savings to changes in real interest rates. Whether the predictions of the standard FTPL survive in our OLG model depends on the specification of the utility functions. For example, with log utility functions, changes in fiscal surpluses beyond the lifetime of a consumer have no effect on the current price level. By contrast, with linear utility functions, the corresponding effects are as predicted by the standard FTPL.

Second, the accumulation of government debt produces temporary inflation that only affects the well-being of existing generations, but has no impact on future generations. The future price level continues to rise to restore the future government's real debt repayment burden to its initial level. [2] This finding is closely related to two important controversies about national debt. One concerns the burden of national debt. Our analysis demonstrates that the burden of national debt does not shift from current to future generations, even in the absence of altruistic bequest motives between generations. This finding contrasts sharply with those from the literature on the long-running controversy over the burden of national debt, including those of Bowen *et al.* (1960) and Barro (1974). The other controversy concerns the strategic use of debt accumulation. Our analysis shows that debt accumulation has no commitment effect on the budgetary decisions of a succeeding government. A government cannot "tie the hands" of a successor of which it disapproves by using strategic debt creation. This contradicts the arguments pioneered by Tabellini and Alesina (1990) and Persson and Svensson (1989).[3]

Third, although population aging tends to increase the current price level *via* its economic impact (as suggested by the standard FTPL), it tends to decrease the current price level *via* its

[2]Shedding light on defaults on both domestic local currency bonds and foreign currency bonds, Reinhart and Rogoff (2011) point out that governments have used inflation to wipe out part of the debt in the case of domestic local currency bonds. According to recent estimates obtained by Hall and Sargent (2011), the US federal government's debt-to-GDP ratio declined by 80.3% from 1945 to 1974, 50.2% of which was attributed to inflation.

[3]See also Aghion and Bolton (1990) for the analysis of strategic debt creation in the presence of future government's defaults.

political impact. This is because, in the context of financing government expenditure, inflation can be regarded as a substitute for increasing income taxes. Used in this way, inflation redistributes income from the old to the young by lowering the current real value of the government bonds held by the old generation. Thus, as population aging increases the political influence of the old generation, the government becomes more reluctant to raise the price level and more willing to increase income taxes.

Fourth, population aging affects the price level differently depending on whether it is caused by an increase in life expectancy or a decline in the birth rate. Its effects also depend on whether households anticipate the changes in population aging. In a version of our model that incorporates log utility functions, we show that deflation is more likely to occur when an increase in life expectancy that causes population aging is unexpected than when it is expected. By contrast, population aging caused by a decline in the birth rate is unambiguously inflationary. The difference arises because of how the government responds to distributional concerns. If life expectancy increases unexpectedly, the government is motivated to support the well-being of the old by inducing deflation through the imposition of higher income taxes on the young. By contrast, when the birth rate declines, the government is inclined to accept inflation and make the old generation share the increased fiscal burden generated by the decline in the birth rate.

Following Sargent and Wallace (1981), the foundations of the FTPL literature were built by Leeper (1991), Sims (1994), and Woodford (1995), among others. Despite extensive research on the FTPL, to the best of our knowledge, there is no study in which fiscal policy variables such as tax rates and bond issues are treated as endogenous. Researchers have ignored potential interactions between exogenous shocks in the economy and the government's choice of policy variables. Although we treat expenditure variables as exogenous, revenue variables, such as income tax rates and government bond issues, are endogenously determined based on the relative degrees of prevailing political power held by different constituents. Moreover, our paper represents one of few in the literature to incorporate an OLG model within the FTPL framework; most papers in the literature are based on models incorporating an infinite horizon and a representative household. Among the former is the seminal paper by Sargent and Wallace (1981), but their concern is real debt rather than the FTPL. Among the latter are papers by Cushing (1999) and Braun and Nakajima (2012). Although it does not deal with the FTPL, the paper by Bullard *et al.* (2012) is similar to ours. They construct an OLG model that incorporates capital and obtain a result similar to ours; that is, as society ages, the capital stock and the rate of inflation decrease. Our model differs from theirs by explicitly incorporating decisions about fiscal policy in terms of government bond issues and taxes.

This paper is organized as follows. In Section 2, we document several recent features of Japan's political economy. In Section 3, we explain our model and characterize its Markov perfect equilibrium. In Section 4, we obtain an explicit solution for the equilibrium of the model by specifying a utility function that is logarithmic in consumption. We then discuss the policy implications of the effects of an increase in government expenditure, an increase in the political influence held by the old generation, a fall in the birth rate, and an increase in life expectancy (greater longevity). Section 5 concludes the paper.

2 Features of Japan's Political Economy in Recent Times

Before explaining the model, we identify and analyze four recent features of Japan's political economy:

1. It is well known that around 90% of Japanese government bonds (JGBs) are held by domestic investors. Many Japanese people use JGBs as an important savings instrument (directly and indirectly through commercial banks).

2. Japan's nominal interest rates have been set close to zero for at least a decade. Because of *de facto* fixed nominal interest rates, fluctuations in the real interest rates have been correlated negatively with inflation.

3. To some extent, demographic aging in Japan has been unexpected. Figures 2 and 3 show that official forecasts of Japan's birth rate (formally, the total fertility rate) have been repeatedly revised downward, while those of life expectancy have been continually revised upward.[4]

4. The political influence of old people in Japan has overtaken that of young people recently because of changes in political participation. Figure 4 not only shows that the voter turnout rate (the fraction of eligible voters who cast a ballot) in elections for the Japanese lower

[4]These figures are from a speech by Nishimura (2012), the former Deputy Governor of the Bank of Japan.

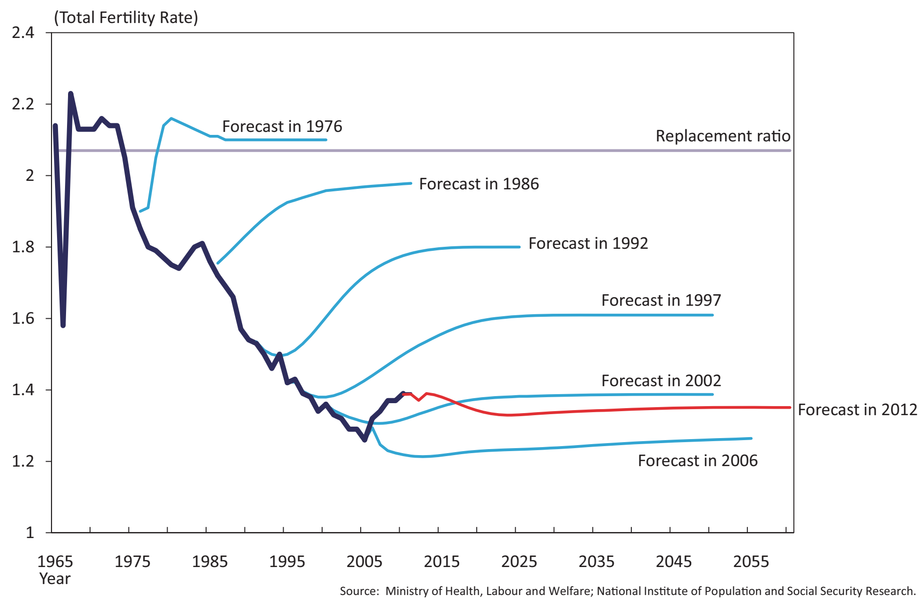

Figure 2: Revisions to Japan's Total Fertility Rate Forecasts

Source: http://www.boj.or.jp/en/announcements/press/koen_2012/ko121114a.htm/

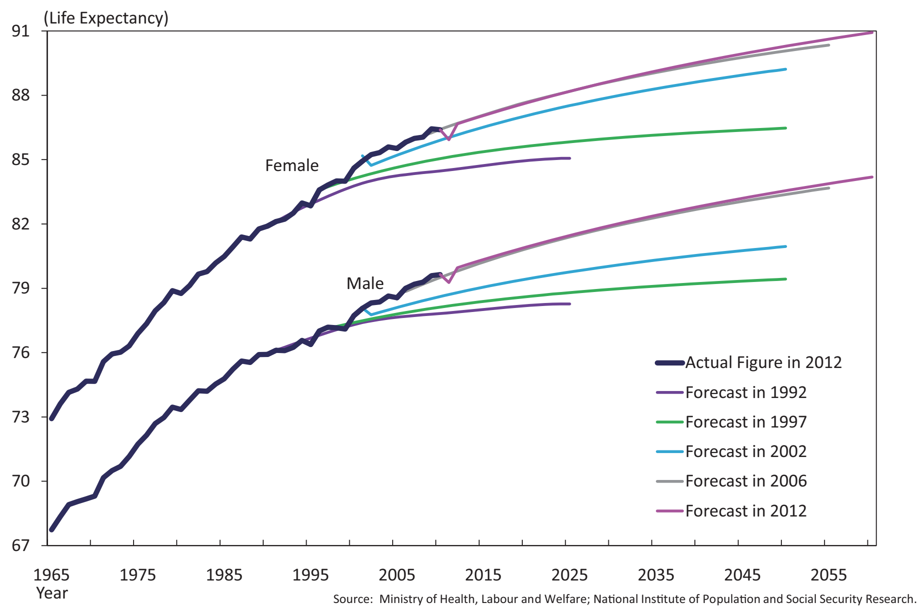

Figure 3: Revisions to Japan's Life Expectancy Forecasts

Source: http://www.boj.or.jp/en/announcements/press/koen_2012/ko121114a.htm/

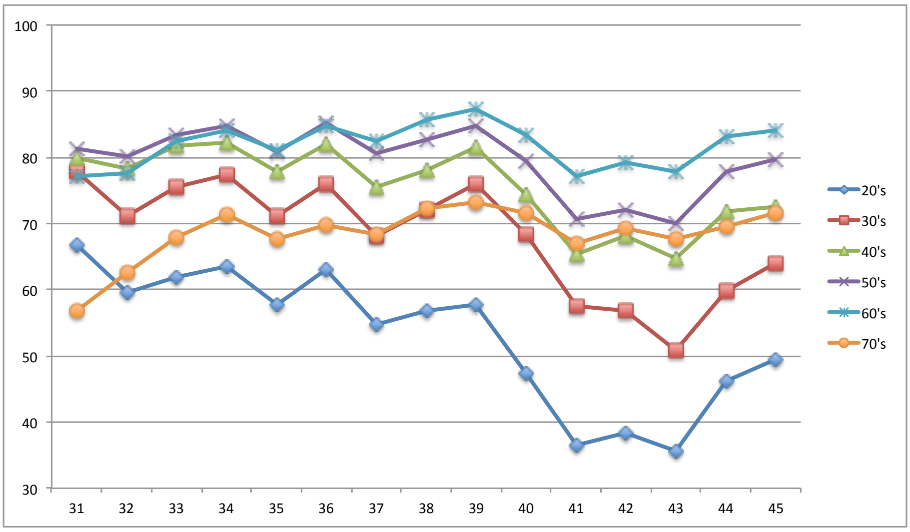

Figure 4: Voter Turnout Rates by Age in Japan

Note: The voter turnout rate is the fraction of eligible voters who cast a ballot in an election. The figure illustrates the voter turnout by age in Japanese lower house elections Nos. 31–45 (from 1967 to 2009). Specifically, the lines with diamonds, squares, triangles, Xs, asterisks, and circles, respectively, represent the turnout rates of people in their 20s, 30s, 40s, 50s, 60s, and 70s.

Source: The Association For Promoting Fair Elections.

house has been increasing with age, but also that differences in turnout rates between age groups have widened.

We incorporate these four features into the standard FTPL, and then examine the predicted effects of population aging on fiscal balances and general prices.

3 The Model

The economy's population comprises two generations in each period, the young and the old. The number of young households in period t is denoted by N_t. They face a longevity risk: young household j in period t becomes old in period $t+1$ with probability $0 < \theta^j_{t+1} \leq 1$ or dies young with probability $1 - \theta^j_{t+1}$. The survival probability θ^j_{t+1} is subject to both idiosyncratic and aggregate risks; that is, $\theta^j_{t+1} = \theta_{t+1} + \varepsilon^j_{t+1}$, with support Θ_{t+1}, where $\theta_{t+1} = \mathbb{E}_t[\theta^j_{t+1}]$ and $\mathbb{E}_t[\varepsilon^j_{t+1} \,|\, \theta_{t+1}] = 0$. $\mathbb{E}_t$ is the expectations operator conditional on information available in period t. By the law of large numbers, the population of old people in period $t+1$ is $\theta_{t+1} N_t$, where θ_{t+1}

is unknown at the beginning of period $t+1$. Idiosyncratic longevity risk is insured by a state contingent claim, which is provided by insurance companies.

Governments remain in power for only one period, after which they are replaced by another. Each government maximizes a weighted average of the utilities of the young and the old who are alive when it is in power. It does so by choosing government bond issues and the income tax rate, taking as given the outstanding government debt inherited from its predecessor. The weights on the utilities of respective generations are determined in accordance with a probabilistic voting model. As in the standard FTPL, the price level is determined to maintain government's intertemporal fiscal balance. Next, we explain each agent's consumption–savings behavior, and describe the general equilibrium conditions that characterize the model's Markov perfect equilibrium.

3.1 Households

Households live for two periods at most. Young household j supplies labor ℓ_t and consumes c_t^y.[5] This household survives in period $t+1$ with probability θ_{t+1}^j, which is unknown in period t. Faced with a longevity risk, the household saves by buying A_t units of annuities. An annuity is a contingent nominal contract that repays R_{t+1}^A dollars in period $t+1$ for a payment of one dollar in period t if the holder is alive in period $t+1$. The nominal rate of return on annuities, R_{t+1}^A, is stochastic, depending on the aggregate survival rate in period $t+1$, θ_{t+1}, as shown below. In old age, the household does not supply labor, but consumes c_{t+1}^o out of its annuity income. This annuity income is also stochastic, dependent on the realized survival rate.

Specifically, young households born in period t face the budget constraint:

$$c_t^y + \frac{A_t}{P_t} = (1-\tau_t)\ell_t, \tag{1}$$

where P_t is the price level realized in period t and τ_t is an income tax rate (strictly, a wage tax rate) determined by the government before households choose their consumption. The real wage rate is normalized to unity. When old, surviving households consume:

$$c_{t+1}^o = \frac{R_{t+1}^A A_t}{P_{t+1}} + g_{t+1}^T, \tag{2}$$

where $g_{t+1}^T > 0$ is an exogenously fixed government transfer paid to each old household. P_{t+1} is

[5]Because information is symmetric, every young household born in the same period makes the same choices. Hence, in what follows, we omit the index j, except for θ_{t+1}^j.

the price level realized in period $t+1$, contingent on the realization of the survival rate in period $t+1$.

Insurance companies invest their revenues obtained from selling annuities in purchasing government bonds. In the next period, they pay back these returns to the living annuity holders. (For simplicity, we assume that there are no administration costs associated with supplying annuities.) If we use R_t to denote the one-period gross nominal interest rate on government bonds issued in period t, then competition among insurance companies ensures that annuities take the form of the *Tontine* pension. [6] That is, investment returns are shared equally among the surviving contractors and, thus, the nominal return on annuities depends on the realization of the aggregate survival rate such that:

$$R^A_{t+1} = \frac{R_t}{\theta_{t+1}}. \tag{3}$$

In what follows, unless stated otherwise, we assume that the nominal interest rate on government bonds is constant over time; that is, $R_t = R$ for all t. This assumption reflects the fact that nominal interest rates in Japan have been set close to zero for at least a decade.[7]

Combining (1), (2) and (3) yields each young household's intertemporal budget constraint:

$$c^o_{t+1} = \frac{r_{t+1}}{\theta_{t+1}}\left[(1-\tau_t)\ell_t - c^y_t\right] + g^T_{t+1}, \tag{4}$$

where $r_{t+1} \equiv RP_t/P_{t+1}$ is the real interest rate realized in period $t+1$. Then, young household j in period t chooses c^y_t and ℓ_t to maximize its expected utility:

$$u^y(c^y_t, \ell_t) + \beta \mathbb{E}_t\left[\theta^j_{t+1} u^o(c^o_{t+1})\right], \tag{5}$$

subject to (4), where $\beta \in (0,1)$ is a discount factor. Optimal consumption and labor supply are then characterized by the following first-order conditions:

$$u^y_c(c^y_t, \ell_t) = \beta \mathbb{E}_t\left[r_{t+1} u^o_c\left(c^o_{t+1}\right)\right] = -\frac{u^y_l(c^y_t, \ell_t)}{1-\tau_t}, \tag{6}$$

combined with (4), where $u^y_c = \partial u^y / \partial c^y_t$, $u^y_\ell = \partial u^y / \partial l_t$, and $u^o_c = du^o / dc^o_{t+1}$.

[6] Tontine pensions are named after their founder, Lorenzo de Tonti. In the 17th and 18th centuries, many European governments used Tontine pension plans to raise revenues. See Lange *et al.* (2007) for a brief introduction.

[7] Contrary to our assumption, the excessive accumulation of government bonds may cause defaults and lead to a surge in nominal interest rates. However, Reinhart and Rogoff (2011) argue that some restructuring of debt has involved the imposition of financial restrictions in the form of explicit or implicit caps on interest rates.

From these conditions, for ease of exposition, we formulate c_t^y and ℓ_t as functions of the current income tax rate, τ_t, and the *decision rule* for the real interest rate in period $t+1$, $\mathbf{r}_{t+1}$, as follows:

$$c_t^y = c_t^y(\tau_t\,;\,\mathbf{r}_{t+1}) \tag{7}$$

and

$$\ell_t = \ell_t(\tau_t\,;\,\mathbf{r}_{t+1}). \tag{8}$$

As shown subsequently, the decision rule, $\mathbf{r}_{t+1}$, expresses the real interest rate dictated by the government's fiscal policy in period $t+1$ as a function of the realization of θ_{t+1}.

The real amount of savings per young household in period t, $a_t \equiv A_t/P_t$, depends on τ_t and $\mathbf{r}_{t+1}$ such that:

$$a_t = a_t(\tau_t\,;\,\mathbf{r}_{t+1}) \equiv (1-\tau_t)\ell_t(\tau_t\,;\,\mathbf{r}_{t+1}) - c_t^y(\tau_t\,;\,\mathbf{r}_{t+1}). \tag{9}$$

Consumption per old household in period $t+1$ is stochastic such that:

$$c_{t+1}^o = \frac{r_{t+1}a_t(\tau_t\,;\,\mathbf{r}_{t+1})}{\theta_{t+1}} + g_{t+1}^T. \tag{10}$$

From equations (7)–(10), we define the indirect utility function for young household j in period t as:

$$\begin{aligned} v_t^y(\tau_t\,;\,\mathbf{r}_{t+1}) \equiv\, & u^y\big(c_t^y(\tau_t\,;\,\mathbf{r}_{t+1}), \ell_t(\tau_t\,;\,\mathbf{r}_{t+1})\big) \\ & +\beta\mathbb{E}_t\left[\theta_{t+1}^j u^o\left(\frac{r_{t+1}a_t(\tau_t\,;\,\mathbf{r}_{t+1})}{\theta_{t+1}} + g_{t+1}^T\right)\right]. \end{aligned} \tag{11}$$

The consumption of the old in period t, c_t^o, is not a choice variable in period t, but is determined automatically by the realizations of θ_t and r_t. This is because the old generation's real savings, $a_{t-1}(\tau_{t-1}\,|\,\mathbf{r}_t)$, have been already chosen. Thus, the indirect utility function for the old in period t is:

$$v_t^o(r_t, \theta_t) \equiv u^o\left(\frac{r_t a_{t-1}(\tau_{t-1}\,;\,\mathbf{r}_t)}{\theta_t} + g_t^T\right). \tag{12}$$

The indirect utility functions for the young and old are used to define the government's objective function, which is explained in the next subsection.

3.2 The Government

3.2.1 Budget Balance

In each period, a short-lived government, which remains in power just for one period, makes policy choices.[8] The government in period t (hereafter, government t), having taken over the nominal government debt, B_{t-1}, from government $t-1$, chooses an income tax rate, τ_t, and the outstanding government debt, B_t, based on the known realization of θ_t. It is important to note that government bonds are issued in nominal terms. As we explain below, in equilibrium, the government can effectively choose the price level, P_t, to balance its budget.

The government's budget balance is:

$$B_t + P_t N_t \tau_t \ell_t = R B_{t-1} + P_t G_t, \tag{13}$$

where $G_t \equiv (N_t + \theta_t N_{t-1}) g_t^C + \theta_t N_{t-1} g_t^T$ is government spending, with $g_t^C > 0$ being per-capita government consumption in period t.[9] The left- and right-hand sides of (13) represent government revenue and expenditure, respectively. The budget balance can be expressed in real terms per young household by dividing the previous equation through by $P_t N_t$:

$$n_t(b_t + \tau_t \ell_t) = r_t b_{t-1} + (n_t + \theta_t) g_t^C + \theta_t g_t^T, \tag{14}$$

where $b_t \equiv B_t/(P_t N_t)$ is the real value of outstanding government debt per young household at the end of period t, and $n_t \equiv N_t/N_{t-1}$ is the ratio of young populations in consecutive periods, which we refer to as the birth rate.

Note that although our model is intrinsically based on nominal government debt, variables are expressed in real terms for analytical convenience. Hence, in equation (14), b_{t-1} is predetermined at the beginning of period t, but r_t is *not* because it depends on the current price level P_t. In fact, P_t is chosen to satisfy (14) given governments' choices of income tax rates and public bond issues; this suggests the mechanism behind the FTPL, which is explained in the next subsection.

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[8] Forni (2005) and Song (2011) used a model of successive short-lived governments to analyze the political economy of social security. However, their models do not incorporate government bonds. Song *et al.* (2012) considered debt accumulation within a political–economic model similar to ours under the assumption that no government defaults occur.

[9] In our analysis, we assume that g_t^C has no effect on private consumption, except for its income effect, which arises because of the implicit assumption of preference separability.

3.3 The Mechanism for the FTPL in an OLG Model

The mechanism for the FTPL in our OLG model differs from the one in the standard model based on an infinitely lived representative consumer. From the fiscal perspective, even in an OLG model, the equilibrium price level in period t, P_t, balances the government's budget (14). By using equilibrium prices from periods t to infinity, then repeatedly eliminating b_t and recalling that $r_1 b_0 = RB_0/(n_1 P_1)$, we have:

$$\frac{RB_0}{P_1} = n_1 \mathbb{E}_1 \left[s_1 + \sum_{t=2}^{\infty} \left(\prod_{k=2}^{t} \frac{r_k}{n_k} \right)^{-1} s_t \right], \tag{15}$$

where $s_t \equiv (n_t \tau_t \ell_t - (n_t + \theta_t) g_t^C - \theta_t g_t^T)/n_t$ is the fiscal surplus per young household in period t. In the context of this expression, it is assumed that the non-Ponzi game condition is satisfied.[10] This indicates that the real value of outstanding government debt at the beginning of period 1 equals the expected discounted present value of the future fiscal surplus from period 1 onwards.

To illustrate the FTPL, suppose that $B_0 > 0$, and consider an expected decrease in future fiscal surpluses brought about by a reduction in τ_t for some $t \geq 2$. Then, other things being equal, it must induce an increase in P_1 in the new equilibrium; this is a well-known implication of the FTPL.

A standard explanation for the mechanism behind this result, based on models with an infinitely lived representative household, involves the wealth effect [see, for example, Woodford (1995)]. A future tax reduction increases the value of the representative household's net real assets and increases consumption, which generates excess demand in the current goods market. To clear the goods market, the current price level must increase so that the value of household net assets remains unchanged.

However, this mechanism does not operate in our OLG model because of households' limited lifespan. An income tax cut in period $t \geq 2$ does not affect household consumption in period 1 because it has no direct effect on the wealth of households living in period 1.

[10] In an infinite-horizon model, such as the standard FTPL, the non-Ponzi game condition, which is $\lim_{T\to\infty} \mathbb{E}_t \left[\left(\prod_{k=t}^{T} n_k/r_k \right) b_T \right] = 0$, is satisfied because of the transversality condition. However, because the transversality condition does not generally hold in an OLG model, satisfaction of the non-Ponzi game condition must be assumed to exclude the possibility of an explosive equilibrium. Another related issue that arises in an OLG model is dynamic inefficiency: on the balanced- growth path, the level of the capital stock may exceed the golden rule level. Because our model does not incorporate the capital stock, dynamic inefficiency is beyond the scope of this paper.

In our OLG model, the mechanism through which the FTPL operates is not a wealth effect. Rather, it is households' consumption–savings behavior responding to changes in real interest rates. Consider a reduction in τ_2. In response to this, young households in period 2 consume more, which generates excess demand in the goods market, and this leads to an increase in P_2. P_2 continues to rise until the associated reduction in the old generation's consumption compensates for the increase in the young's consumption, which is induced by the lower r_2 realized in period 2. At the new equilibrium price level, the government's budget is balanced in period $t = 2$ (that is, equation (14) holds). Now, consider period 1. In that period, young households that anticipate a fall in the real interest rate may change their savings behavior. If they save less (and consume more) in anticipation of a lower real interest rate, the ensuing excess demand in the goods market causes P_1 to increase until the market clears. The new equilibrium price balances the budget of government 1 by reducing the real value of its outstanding debt. At the same time, government 1 can reduce its bond issue in response to reduced savings by the young generation.

Clearly, how savings by the young respond to an expected decrease in the real interest rate will depend on the specification of their utility functions. In the case of log utilities, as shown below, a reduction in τ_t, $t \geq 2$, does not change P_1 because savings are invariant with respect to real interest rates.

3.3.1 Economic Equilibrium

We now consider the general equilibrium of the model. We assume that all government bonds are held by domestic investors through insurance companies. This is reasonable considering that around 90% of JGBs are held by domestic investors.[11] The market clearing condition for government bonds in period t is:

$$a_t(\tau_t\,;\,\mathbf{r}_{t+1}) = b_t. \tag{16}$$

Given the satisfaction of equation (14), equations (7)–(10) show that (16) is equivalent to the goods market clearing condition in period $t \geq 1$ based on the realized values of θ_t and r_t:

$$n_t c_t^y(\tau_t\,;\,\mathbf{r}_{t+1}) + \theta_t c_t^o(\tau_{t-1}\,;\,\mathbf{r}_t) + (n_t + \theta_t) g_t^C = n_t \ell_t(\tau_t\,;\,\mathbf{r}_{t+1}). \tag{17}$$

This is because $c_t^o(\tau_{t-1}, \mathbf{r}_t)$ depends on r_t and θ_t in period t. This equivalence derives from Walras' law, which implies that (16) and (17) are not independent of each other given that (14)

[11] Allowing the government to sell its bonds to foreign investors would be an interesting extension of our model.

is identically satisfied. Accordingly, if fiscal policy is so *passive* as to satisfy the intertemporal budget constraint even at disequilibrium prices, then the sequence of $\mathbf{r}_{t+1}$ is determined through (17), with r_t, θ_t, and the sequence of τ_t being taken as given. (This mirrors Diamond 's (1965) textbook treatment of the OLG model.) This implies that even with a fixed nominal interest rate, price levels are indeterminate, and only expected inflation rates are endogenously determined in equilibrium. Specifically, for period 1, it is not P_1 but the stochastic distribution of P_2/P_1 that is determined in equilibrium for given values of r_1 and θ_1.

The regime on which the FTPL is based involves a quite different interaction between monetary and fiscal policy: governments set such an *active* fiscal policy that although the intertemporal budget may fail to balance at disequilibrium prices, it balances in equilibrium because of a passive monetary policy.[12] Therefore, in our model, equation (14) is subsumed into the equilibrium condition:

$$r_t = \frac{n_t\left(b_t + \tau_t \ell_t(\tau_t\,;\,\mathbf{r}_{t+1})\right) - (n_t + \theta_t)g_t^C - \theta_t g_t^T}{b_{t-1}}. \tag{18}$$

This determines the price level, P_t, taking R and P_{t-1} as given (because $r_t \equiv RP_{t-1}/P_t$).

Note that (16) and (17) are no longer equivalent in this regime because (18) only holds in equilibrium. However, we can say that either (16), (17), or (18) is redundant as an equilibrium condition. Unlike under the passive fiscal-policy regime, P_t is determined through (18) depending on the realization of θ_t and the policy decisions of governments. Specifically for period 1, by combining (18) for $t \geq 1$ with repeated elimination of b_t, we obtain the discounted present-value budget-balancing condition for the public sector. This is similar to (15), which determines P_1.

Thus, in our framework, it is not only the expected future inflation rate, but also the realization of the current inflation rate that can be analyzed by examining changes in the equilibrium price level. In what follows, we treat (16) and (18) as the relevant conditions for the achievement of *economic* equilibrium in our model. Under these conditions, the sequences of r_t and b_t are endogenously determined, with τ_t and future policy decisions being taken as given.

3.3.2 The Optimization Problem Facing Short-lived Governments

In our model, fiscal policies are endogenous. This contrasts with the standard FTPL, in which fiscal policy is assumed to be exogenous. We suppose that, when setting their fiscal policies,

[12] Although there is controversy over the validity of the FTPL, we ignore this debate. See Bassetto (2002) and Buiter (2002) for a critical view of its assumptions.

governments consider their effects on the equilibrium price level. As Woodford (2001, p. 693) states: "For the government is a large agent, whose actions can certainly change equilibrium prices, and an optimizing government surely should take account of this in choosing its actions."

In the spirit of the probabilistic voting model, we define the objective function of each short-lived government as a weighted average of the utility levels of those households in existence when it is in power:[13]

$$W_t = \gamma_t v_t^o(r_t, \theta_t) + v_t^y(\tau_t\,;\,\mathbf{r}_{t+1}), \tag{19}$$

where the weight assigned to the utility of the old, γ_t, represents the old generation's influence over government policy. According to probabilistic voting theory, γ_t increases with population aging because of the old generation's high turnout rate, their strong preference for practical policies over ideological ones, and their active campaign contributions, relative to the young. In particular, if $\gamma_t = \theta_t/n_t$, the government is a *myopic* utilitarian one that maximizes the sum of utilities across those households living when it is in power.

Government t, being in power for period t only, maximizes the above objective function by choosing the income tax rate τ_t and bond issues b_t, subject to the following conditions: (i) private agents' first-order conditions; (ii) the bond market clearing condition (16); (iii) the government's balanced budget condition (18); and (iv) the policy decisions of government $t+1$, as represented by $\mathbf{r}_{t+1}$. These constraints bind government t's policy choices in specific ways. For example, if a_t decreases with τ_t in (16), then government t cannot simultaneously increase both τ_t and b_t, other things being equal. Note also from (16) that, when choosing policy, government t must consider the policy response of government $t+1$, which is conveyed by the decision rule $\mathbf{r}_{t+1}$. Furthermore, through equation (18), government t must take into account the change in P_t (and the associated change in r_t) induced by its own choice of τ_t and b_t.

What must the government consider when choosing the income tax rate and bond issues? In equilibrium, because the price level adjusts to balance the budget, the government need not bother about the revenues raised from these financial instruments. However, it does have to care about their distributive effects on existing constituents.

Redistribution occurs through three channels. The first is a direct channel, through which

[13] Song *et al.* (2012) used a similar formulation. In Appendix A, we provide a brief explanation of the probabilistic voting model and an interpretation of the government's objective function. For details of the probabilistic voting model, see, for example, Persson and Tabellini (2000) and Grossman and Helpman (2000).

an increase in the income tax rate reduces the utility of the young by decreasing their after-tax income. The current price level provides a second channel for redistribution. For example, if government t imposes a higher income tax, other things being equal, the current price level falls (through (18)), which provides unexpected windfalls to the old. The agent for the third channel is the subsequent government. For example, if an increase in bond issues in period t increases the price in period $t+1$ (which transpires in theory, as shown below), the young in period t are worse off because of a reduction in the expected real return on their investments in government bonds.

3.3.3 The Markov Perfect Equilibrium and its Implications

In this subsection, we consider the *political–economic* equilibrium of the model. In this formulation, ignoring the realization of θ_t, the only state variable for government t is b_{t-1}, the per-capita amount of outstanding debt inherited from government $t-1$. Hence, one can expect the government's optimal choice of r_t to depend on b_{t-1} as well as θ_t. Given that government $t+1$ acts the same way, the decision rule, $\mathbf{r}_{t+1}$, has the following form:

$$r_{t+1} = r_{t+1}(\theta_{t+1}, b_t). \tag{20}$$

This equation expresses the real interest rate, and hence the price level, in period $t+1$ for each realization of $\theta_{t+1} \in \Theta_{t+1}$. As shown below, the decision rules for taxes, τ_t, and bond issues, b_t, depend only on θ_t, not on b_{t-1}.

The political–economic equilibrium that we focus on is a Markov perfect equilibrium, defined as a sequence of policy decision rules, $\mathbf{q}_t = \{(\tau_t(\theta_t), b_t(\theta_t), r_t(\theta_t, b_{t-1})) \mid \theta_t \in \Theta_t\}$ for $t = 1, 2, 3, \cdots$, such that for each given θ_t, $\mathbf{q}_t$ maximizes (19) subject to (14), (16), and $\mathbf{q}_{t+1}$.

To characterize the equilibrium, it is useful to eliminate r_t from the expression for W_t by using (12) and (18). This reduces the optimization problem to:

$$\max_{\tau_t, b_t} W_t = \gamma_t u^o \left(\frac{n_t(b_t + \tau_t \ell_t) - (n_t + \theta_t) g_t^C}{\theta_t} + g_t^T \right) + v_t^y(\tau_t\,; \mathbf{r}_{t+1}) \quad \text{s.t. (16).}$$

Note that b_{t-1} does not appear in the above formulation. Hence, government t's optimal choices of τ_t and b_t should be independent of b_{t-1} (because they appear in the specification of $\mathbf{q}_t$), and only the optimal choice of r_t depends on b_{t-1}, which satisfies (18). This property also applies to the policy choices of government $t+1$. Accordingly, government t rationally anticipates the next government's best response, at the realization of $\theta_{t+1} \in \Theta_{t+1}$, to be:

$$r_{t+1}(\theta_{t+1}, b_t) = \frac{1}{b_t}\Big\{ n_{t+1}[b_{t+1}(\theta_{t+1}) + \tau_{t+1}(\theta_{t+1})\ell_{t+1}(\tau_{t+1}(\theta_{t+1})\,;\, \mathbf{r}_{t+2})] \\ -(n_{t+1} + \theta_{t+1})g^C_{t+1} - \theta_{t+1}g^T_{t+1}\Big\}. \tag{21}$$

These observations provide important implications for understanding the equilibrium (provided one exists).

First, (21) implies $\partial r_t/\partial b_{t-1} < 0$; other things being equal, an increase in the amount of outstanding debt inherited by government t from government $t-1$ leads the former to increase the price level. As the government's outstanding debt increases, the old, who hold it, become wealthier relative to the young under a fixed price level. Hence, the government balances intergenerational utilities by allowing unexpected inflation to redistribute real income from the old to the young.

Second, (21) implies $\partial r_{t+1}/\partial b_t < 0$; other things being equal, for any realization of θ_{t+1}, an increase in bond issues in period t makes the young worse off by reducing the expected real return on their savings. To be precise, the fall in the gross real interest rate is proportional to the increase in the amount of outstanding debt. Hence, the young, having borne the cost of increased government debt by reducing consumption, gain nothing in the future.[14]

Third, combining $\partial b_{t+1}/\partial b_t = \partial \tau_{t+1}/\partial b_t = 0$ with $\partial r_{t+1}/\partial b_t < 0$ has an interesting implication for the long-running debate about whether the costs of national debt are passed on to future generations. Our analysis reveals that these burdens cannot be shifted to future generations even in the absence of altruistic bequests.

Suppose that government t were to issue additional bonds, simultaneously reducing income taxes to clear the bond market. Such a debt financing policy would not affect the utilities of future generations, but would affect those alive today through price changes. Because, in our model, the next government would not increase taxes or issue additional bonds, accumulating national debt today would raise future prices and perhaps the current one. These price increases would reduce the expected real interest rate on the bonds held by today's young generation, and might

[14]This argument relies on the *ceteris paribus* effects of accumulating more debt. Later, we use an example based on log utility functions to show that the government must lower income taxes to induce more savings by the young so that they accept greater debt. These two effects combine so that an increase in the amount of government bonds issued makes the young better off overall, while the resulting increase in the current price level makes the old worse off.

also make today's old generation worse off were a higher price to combine with a deterioration in the current government's fiscal position.

This implication contrasts sharply with the findings obtained from the same OLG framework by, among others, Bowen *et al.* (1960) and Barro (1974). As is well known, Bowen *et al.* (1960) argue that the young can avoid these burdens by selling their bonds to the next generation. Barro (1974) argues that the young generation would neutralize the burden of increased debt by increasing bequests by the same amount to keep the next generation's welfare unchanged. Our findings conflict with these traditional views because our model incorporates a succession of short-lived governments within the framework of the FTPL; interaction between these governments affects the price level so as to eliminate the effects of current debt on future debts and taxes.[15]

Fourth, in contrast to the strategic debt creation arguments pioneered by Persson and Svensson (1989) and Tabellini and Alesina (1990), according to our model, each government cannot affect its successor's policy decisions by accumulating public debt. This is confirmed by $\partial b_{t+1}/\partial b_t = \partial \tau_{t+1}/\partial b_t = 0$. In equilibrium, debt financing is absorbed by the acceleration of expected inflation and has no commitment effects on future governments' policy decisions.

Let us now return to the characterization of the Markov perfect equilibrium. By incorporating (21) into the optimization problem facing government t, and by making use of the envelope condition obtained from the households' maximization problem, we have the following first-order conditions for τ_t and b_t, respectively:[16]

$$\frac{\gamma_t}{\chi_t} u_c^o \left[\ell_t + \tau_t \frac{\partial \ell_t}{\partial \tau_t} \right] - u_c^y \ell_t + \lambda_t \frac{\partial a_t}{\partial \tau_t} = 0 \tag{22}$$

and

$$\frac{\gamma_t}{\chi_t} u_c^o - u_c^y + \lambda_t \left[\lim_{b_t' \to b_t} \frac{a_t(\tau_t\,;\,\mathbf{r}_{t+1}') - a_t(\tau_t\,;\,\mathbf{r}_{t+1})}{b_t' - b_t} - 1 \right] = 0, \tag{23}$$

where λ_t is the Lagrangian multiplier for the constraint (16) and $\mathbf{r}_{t+1}'$ is the decision rule of government $t+1$ on having inherited outstanding debt of b_t'. We define the population structure as the ratio of the old and young:

$$\chi_t \equiv \theta_t / n_t. \tag{24}$$

[15] Our argument suggests that the burden of national debt may depend on whether government bonds are price indexed. To our knowledge, this issue has not been discussed in the academic literature.

[16] When (21) is incorporated, given (6), (16), and $\partial r_{t+1}/\partial b_t = -r_{t+1}/b_t$, the derivatives of the young's indirect utility function satisfy $\partial v_t^y/\partial \tau_t = -u_c^y \ell_t$ and $\partial v_t^y/\partial b_t = -u_c^y$.

The Markov perfect equilibrium is fully characterized by conditions, (16), (21), (22), and (23), used in a recursive fashion.

In particular, if $\partial \ell_t / \partial \tau_t = 0$, (22), and (23) yield $\lambda_t = 0$ so that, in equilibrium, income is redistributed between the old and the young to equalize their per-capita *politically weighted* marginal utilities, then we have:

$$\frac{\gamma_t}{\chi_t} u_c^o = u_c^y, \tag{25}$$

Theoretically, this arises because both income taxes and price changes redistribute income between young and old in a lump-sum fashion. Equation (25) implies that the government favors the old in terms of income redistribution when their political weight, γ_t, is biased upward relative to the population ratio, χ_t.

4 Policy Implications

To analyze these equilibrium conditions further, we parameterize the household utility functions and compute explicitly the Markov perfect equilibrium in our OLG model. Specifically, we let utility depend on the log of consumption and squared labor supply. This specification ensures that $\partial \ell_t / \partial \tau_t = 0$. For simplicity, we assume $g_t^T = 0$. In Appendix B, we describe the case in which utility is linear in consumption; in this case, $\partial \ell_t / \partial \tau_t = 0$ does not hold. In Appendix C, we examine the case in which labor supply is fixed, the utility function exhibits constant relative risk aversion, and governments provide public benefits to the old generations. Having obtained the equilibrium solution based on this parametrization, we discuss policy implications, focusing on the effects of population aging on fiscal balances and the price level.

4.1 Parameterization

The utility functions are specified as $u^y(c_t^y, \ell_t) = \log c_t^y - \ell_t^2/2$ and $u^o(c_t^o) = \log c_t^o$. We assume $g_t^T = 0$. These specifications yield the following optimization problem for young households:

$$c_t^y(\tau_t\, ; \mathbf{r}_{t+1}) = \frac{1-\tau_t}{\bar{\ell}_t}, \tag{26}$$

$$\ell_t(\tau_t\, ; \mathbf{r}_{t+1}) = \bar{\ell}_t, \tag{27}$$

and

$$a_t(\tau_t\, ; \mathbf{r}_{t+1}) = \frac{(1-\tau_t)(\bar{\ell}_t^2 - 1)}{\bar{\ell}_t}, \tag{28}$$

where $\bar{\ell}_t \equiv \sqrt{1+\beta\mathbb{E}_t[\theta_{t+1}]} > 1$. Labor supply is independent of the income tax rate, and the consumption and savings of each young household are independent of the interest rate. From (16) and (17), in the economic equilibrium with a given income tax rate, the consumption of the old and the real interest rate are determined to satisfy:

$$c_t^o = \frac{n_t\ell_t - (n_t+\theta_t)g_t^C - n_t c_t^y}{\theta_t} = \frac{\bar{\ell}_t\left\{\bar{\ell}_t - (1+\chi_t)g_t^C\right\} - (1-\tau_t)}{\bar{\ell}_t\chi_t}. \tag{29}$$

Now consider the political–economic equilibrium. By substituting these equations into (25), we obtain the policies chosen by government t in the Markov perfect equilibrium:

$$b_t(\theta_t) = \frac{(\bar{\ell}_t^2 - 1)\left\{\bar{\ell}_t - (1+\chi_t)g_t^C\right\}}{\gamma_t + 1}, \tag{30}$$

$$\tau_t(\theta_t) = 1 - \frac{\bar{\ell}_t\left\{\bar{\ell}_t - (1+\chi_t)g_t^C\right\}}{\gamma_t + 1}, \tag{31}$$

and

$$r_t(\theta_t, b_{t-1}) = \frac{\gamma_t}{1+\gamma_t}\frac{n_t(\bar{\ell}_t - (1+\chi_t)g_t^C)}{b_{t-1}}. \tag{32}$$

In the context of these equations, we assume that $\bar{\ell}_t - (1+\chi_t)g_t^C > 0$, which must hold to satisfy the goods market equilibrium condition, (17).

4.2 Discussion

The above results are interpreted as variations of those based on the FTPL. As in the standard FTPL, the price level, P_t, is determined through (32); this is because $r_t = RP_{t-1}/P_t$, with R fixed and P_{t-1} predetermined.

Unlike in the standard FTPL, in our model, it is only the fiscal policy parameters set in period t that affect P_t; future ones have no effect. From the fundamental budget balancing equation in the FTPL (15), this is because changes in fiscal surpluses in period $k > t$ lead government k to adjust the price level P_k so that proportional changes in the gross real interest rate leaves discounted present values unchanged. Details are provided below.

To understand the changes in the price level, it is useful to distinguish between political and economic factors. On the right-hand side of (32), the first term, $\gamma_t/(1+\gamma_t)$, is interpreted as the political factor, while the second term is interpreted as the economic factor. The former represents the relative political influence of the old in period t, and the latter represents the maximum fiscal surplus available to government t relative to the outstanding debt inherited from government $t-1$.

In this subsection, we use the above results to examine the effects of the following: (i) an exogenous increase in government consumption; (ii) an exogenous increase in the political influence of the old; (iii) population aging brought about by a decline in the birth rate; and (iv) population aging brought about by increased longevity. Given that (iii) and (iv) have opposing effects on government expenditure, the cause of population aging matters.

4.2.1 An Increase in Government Consumption

Suppose that per-capita government consumption, g_t^C, increases unexpectedly, other thing being equal. From (30)–(32) and $r_t \equiv RP_{t-1}/P_t$, we then have:

$$\frac{\partial b_t}{\partial g_t^C} < 0, \quad \frac{\partial \tau_t}{\partial g_t^C} > 0, \quad \text{and} \quad \frac{\partial P_t}{\partial g_t^C} > 0. \tag{33}$$

In response to an exogenous increase in per-capita government consumption, the government chooses to reduce deficits in real terms, increase income tax revenues, and induce a higher price level.

To understand these responses, first note the surprising result that increasing the amount of bonds reduces the fiscal surplus because of the fall in income taxes necessary to make the additional bonds affordable to young households.[17] If the fiscal surplus decreases, the current price level rises; accordingly, the young are better off and the old are worse off. Because such a redistribution causes the politically weighted marginal utilities to diverge, the government cannot issue a large amount of bonds in equilibrium following an exogenous expenditure increase.[18] Government t's optimal response is to increase both income taxes and bond issues, but this does not yield enough additional revenue to cover the increased government expenditure. As a result, the price level increases and both the young and old are worse off.[19]

[17]Formally, by using (16) and (28), we can demonstrate that total revenues satisfy $b_t + \tau_t \bar{\ell} = \bar{\ell} - b_t/(\bar{\ell}^2 - 1)$.

[18]Because the labor supply is fixed, we can make use of (25).

[19]In Appendix C, we show that, under the assumption of a fixed labor supply, the decline in current government transfers to the old, g_t^T, lowers the price level. Governments in developed countries, including Japan, are under increasing pressure to cut these transfers. According to our model, this would reduce the price level, which is consistent with Japan's deflation.

4.2.2 An Increase in the Old Generation's Degree of Political Influence

We next consider the effects of aging, or a change in the demographic structure, which is potentially generated by two causes: one is a decrease in the birth rate, regarded as a decrease in n_t in our model, and the other is an increase in longevity, identified as an increase in θ_t. Although aging might affect the government budget, it might also influence policy by tipping the balance of political influence toward the old. To capture this, we assume that γ_t is linear in the population ratio, $\chi_t \equiv \theta_t/n_t$, such that:

$$\gamma_t = \omega_t \chi_t, \tag{34}$$

in which ω_t is interpreted as a measure of the old generation's political influence, which is based on such factors as their high turnout rate in elections relative to the young. Then, as does an increase in θ_t, a decrease in n_t increases both γ_t and χ_t, which appear in equations (30)–(32).

Let us examine the effects of the old generation's political influence and aging separately. Following an unexpected increase in ω_t, with χ_t unchanged, from (30)–(32) and $r_t \equiv RP_{t-1}/P_t$, we have:

$$\frac{\partial b_t}{\partial \omega_t} < 0, \quad \frac{\partial \tau_t}{\partial \omega_t} > 0, \quad \text{and} \quad \frac{\partial P_t}{\partial \omega_t} < 0. \tag{35}$$

As the old generation's political influence increases under a fixed demographic structure, the government lowers deficits, increases income taxes, and induces a lower price level. The adjustments in deficits and income taxes follow from the equalization of the per-capita politically weighted marginal utilities between the young and the old. In response to the old generation's increased political influence, the government favors them by increasing the fiscal surplus, which reduces the equilibrium price level. In this case, the price level is used as a means of redistribution. Given (32), the political factor, represented by the first term on the right-hand side, increases following an increase in γ_t. This generates unexpected deflation.

4.2.3 A Decline in the Birth Rate

Consider next the case of an unexpected decrease in the birth rate in period t. By incorporating the associated changes in γ_t and χ_t under the assumption implied by (34), from (30)–(32) and $r_t \equiv RP_{t-1}/P_t$, we obtain:

$$\frac{\partial b_t}{\partial n_t} > 0, \quad \frac{\partial \tau_t}{\partial n_t} < 0 \quad \text{and} \quad \frac{\partial P_t}{\partial n_t} < 0. \tag{36}$$

The qualitative effects are the same as those of an increase in government consumption: an unexpected decline in the birth rate reduces deficits, increases income taxes, and increases the price level.

Political and economic factors are behind these effects. The economic effects of a decline in the birth rate are captured by the effects of changes in χ_t and n_t on the right-hand sides of (30)–(32). Like the effects of an increase in government consumption, changes in χ_t and n_t trigger a fall in the fiscal surplus (induced by the contraction in the labor force), and this leads the government to reduce deficits, increase income taxes, and induce a higher price level. The political effects are represented by the changes brought about by a change in γ_t. Because a decline in the birth rate weakens the relative political power of the young, the government prefers increasing income taxes to increasing the price level.

Hence, the political effects *exacerbate* the economic effects on taxes and deficits, but *mitigate* the economic effects on prices.

4.2.4 An Increase in Life Expectancy

Under the assumption implied by (34), the effects on equilibrium deficits and income taxes of population aging caused by an unexpected increase in life expectancy, which is captured by an increase in θ_t, obtained from (30) and (31), respectively, are:

$$\frac{\partial b_t}{\partial \theta_t} < 0, \text{ and } \frac{\partial \tau_t}{\partial \theta_t} > 0. \tag{37}$$

The intuition behind these effects is the same as in the case of a decline in the birth rate. By inducing increases in χ_t and γ_t, an increase in longevity raises government expenditure, which reduces the fiscal surplus. It also leads the government to favor the old as they gain political influence at the expense of the young.

Unlike in the case of a decline in the birth rate, the effect on the price level (given by $\partial P_t/\partial \theta_t$) is ambiguous. From (32) and $r_t \equiv RP_{t-1}/P_t$, we have:

$$\frac{\partial P_t}{\partial \theta_t} < 0 \tag{38}$$

if and only if:

$$\bar{\ell}_t - (1 + 2\chi_t + \omega_t \chi_t^2) g_t^C > 0. \tag{39}$$

The condition in (39) demonstrates that an unexpected increase in longevity *lowers* the price level unless the population structure is dominated by the old (χ_t or ω_t is high) or unless government expenditure (g_t^C) is sufficiently high. For example, given $\chi_t = 0.5$, $\omega_t = 2$, and $g_t^C = \bar{g}$, then $\partial P_t/\partial\theta_t < 0$ if and only if $\bar{g}/\bar{\ell}_t < 0.4$. The intuition behind the price change is as follows. When an unexpectedly high θ_t is realized, the old generation's per-capita income, $r_t b_{t-1}/\theta_t$, falls. The government responds by using deflation to restore the incomes of the old to meet their political demands. However, to do this, the government must increase the fiscal surplus. This may be politically impossible if the increase in longevity induces an excessive increase in government expenditure. This is because such an increase would require an excessive increase in income taxes. In this case, an unexpected increase in longevity causes inflation rather than deflation.

As mentioned in the introduction, Japan experienced mild deflation and population aging during its so-called lost decades. Our results suggest that this puzzling phenomenon might be the combined result of the political and economic effects of unexpected population aging stemming from an increase in longevity. Given repeated upward revision of Japan's forecast on population aging, unexpected shocks to longevity are apparently common in Japan.

This result has implications for Japan's inflation outlook too. As equation (39) suggests, excessive population aging might lead to $\partial P_t/\partial\theta_t > 0$. Then, further increases in longevity may cause inflation, as does a decline in the birth rate.

4.2.5 Future Changes in Fiscal Policy, Voter Turnout Rates, Birth Rates, and Longevity

Our predicted effects on the price level differs from those based on the standard FTPL. According to our model, the current price level is independent of future government policies and population aging. For example, an increase in g_{t+1}^C, lowers the expected real interest rate, r_{t+1}, but has no effect on the young's savings in period t. This is because the intertemporal substitution effect offsets the income effect in the log utility function. As a result, future government spending becomes irrelevant for the current economy. This means that the succession of short-lived governments does not engage in tax smoothing. That is, tax rates are modified only when government spending changes or when there are demographic changes.

This result depends crucially on the assumption of log utility functions. In Appendix B, we examine the case of linear utility functions, and show that future changes in fiscal policy, birth rates, and longevity can affect the current price level, as in the standard FTPL, even when there is a succession of short-lived governments.

4.2.6 Expected Longevity and the Inflation Rate

In this subsection, we examine the relationship between expected longevity and the inflation rate. The inflation rate in period $t+1$ is $\pi_{t+1} \equiv R/r_{t+1}$. By using (30) and (32), this can be reduced to:

$$\pi_{t+1} = \frac{R\left(\bar{\ell}_t^2 - 1\right)\left\{\bar{\ell}_t - (1 + \frac{\theta_t}{n_t})g_t^C\right\}}{1 + \frac{\omega_t \theta_t}{n_t}} \left(\frac{n_{t+1}}{1 + \frac{n_{t+1}}{\omega_{t+1}\theta_{t+1}}}\bar{\ell}_{t+1} - \frac{n_{t+1} + \theta_{t+1}}{1 + \frac{n_{t+1}}{\omega_{t+1}\theta_{t+1}}} g_{t+1}^C\right)^{-1}. \quad (40)$$

This expression shows that the inflation rate in period $t+1$ depends on expected longevity, $\mathbb{E}_t[\theta_{t+1}]$. This is because $\bar{\ell}_t \equiv \sqrt{1 + \beta \mathbb{E}_t[\theta_{t+1}]}$. Longevity affects inflation differently depending on whether it is expected or not. Although the effect of (un)expected longevity on the future inflation rate is generally ambiguous, an *expected* increase in longevity is considered to be less likely to generate future deflation than an *unexpected* increase in longevity.

The reason is as follows. When the young in period t expect to live longer (that is, when θ_{t+1} is higher), they tend to save more. Thus, as they age, government in $t+1$ does not have to use deflation to increase the real value of their assets. When increased longevity is *unexpected*, the old in period $t+1$ do not have sufficient savings, to which, for political reasons, the government responds by boosting their resources through the creation of deflation.[20]

It is worth highlighting the different effects on prices of expected and unexpected increases in longevity by conducting a numerical experiment. Assuming a myopic utilitarian government, we consider two scenarios. In the perfect-foresight scenario, longevity follows a deterministic process, and $\mathbb{E}_{t+k}[\theta_{t+s}] = \theta_{t+s}$ for all k and s. In the other scenario, longevity follows a stochastic process, with $\mathbb{E}_{t+s}[\theta_{t+s+1}] = \theta_{t+s}$ for all s. Our simulations are based on the assumption that the longevity increases by 1% in each period from period 5 to period 10, with parameter settings of $\beta = 0.99^{20}$, $g_t^C = 0.1$, $\omega_t = 2$, $n_t = 1$, and $R = 1/r_1$.

The blue and green lines in Figure 5 represent the responses of P_t to the increase in longevity in the perfect-foresight and stochastic cases, respectively. In accordance with our intuitive explanation, there is *inflation* under perfect foresight and there is *deflation* in the stochastic case. This indicates that public expectations about aging determine how it affects prices.

[20] The log-linearized version of (40) indicates that the inflation rate in period $t+1$ is increasing in the log-linearized deviation of expected longevity ($\mathbb{E}_t[\theta_{t+1}]$) and decreasing in the log-linearized deviation of actual longevity (θ_{t+1}).

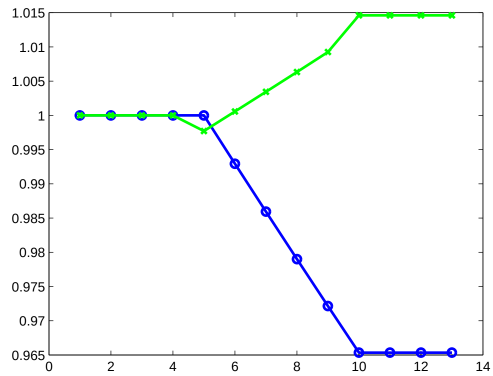

Figure 5: The Response of Prices to an Increase in Longevity

Note: The green line with the dots and the blue line with the circles represent the responses of P_t to an increase in longevity when aging is perfectly foreseen ($\mathbb{E}[\theta_{t+1}] = \theta_{t+1}$) and when it is completely unforeseen ($\mathbb{E}_t[\theta_{t+1}] = \theta_t$), respectively. We assume that longevity increases by 1% in each period from period 5 to period 10.

5 Concluding Remarks

In this paper, we considered the effects on fiscal balances and general prices of population aging by applying the FTPL within a simple OLG model in which fiscal policy decisions were endogenized in the spirit of the probabilistic voting model. A key feature of our model is that the government uses the fiscal impact on general prices as a means of redistributing income between the current young and old generations, and because of this, short-lived governments cannot pass fiscal burdens on to future generations by issuing government debt.

Our main findings are as follows. First, population aging stemming from an increase in longevity causes deflation by increasing the political influence of the older generation, as does an increase in the election turnout rate among older voters. This happens because, to appease older voters, the government increases income tax rates (which harms the young) to avoid increasing prices (which would harm the old). Second, population aging stemming from a decline in the birth rate generates inflation by shrinking the tax base and raising fiscal expenditure. These findings reveal that the effects of population aging on general prices depend on the cause of the aging.

Our study represents a first step toward embedding the FTPL in a political–economic framework. This presents four main challenges. The first is to address Japan's accumulation of government bonds over the last 40 years. Our finding that aging improves fiscal balances by increasing the tax rate on the young seem questionable in reality. One way to solve this problem might be to introduce into the model a long-lived government. Such a government may be motivated to postpone a rise in the tax rate. Second, it would be worth extending our model by incorporating endogenous monetary policy. As shown by Leeper (1991), the implications of the FTPL depend on how the monetary authority responds to inflation changes. Third, it would be interesting to introduce foreign investors buying government bonds into the model. If the government cares less about foreign investors than domestic ones, it might devalue bonds if the proportion of bonds held by foreign investors were to increase. Fourth, we should make our stylized OLG model more quantitative.

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Appendix A: The Probabilistic Voting Model

In this appendix, following Persson and Tabellini (2000), we first explain the basic idea behind the probabilistic voting model. Then we use the probabilistic voting model to interpret the government objective function used in our model.

Consider an election in which two parties, A and B, offer policy vectors, x_A and x_B, respectively, as their campaign promises. For simplicity, we assume that voters are categorized into two groups, 1 and 2, and their populations are denoted by N_1 and N_2, respectively. They may have different political preferences, even within the same group. We assume that voter j in group i votes for party A if and only if:

$$\delta_i u_i(x_A) + \varepsilon_{Aij} + \sigma_A \geq \delta_i u_i(x_B) + \varepsilon_{Bij} + \sigma_B,$$

where $u_i(x_A)$ represents policy preferences for party A common across voters in group i, and $\varepsilon_{Aij} + \sigma_A$ represents preferences unrelated to policy issues such as ideological preferences for A. (The subscript B is used to denote corresponding items for party B). In the context of the latter, ε_{Aij} is specific to voter j in group i, and σ_A is common among all voters. $\delta_i > 0$ measures the relative importance of policy preferences in the formation of the political preferences of each voter in group i. The above condition simplifies to:

$$\delta_i \Delta u_i + \varepsilon_{ij} + \sigma \geq 0,$$

where $\Delta u_i \equiv u_i(x_A) - u_i(x_B), \varepsilon_{ij} \equiv \varepsilon_{Aij} - \varepsilon_{Bij}$, $\sigma \equiv \sigma_A - \sigma_B$. We assume that ε_{ij} and σ are independently and uniformly distributed over $[-\overline{\varepsilon}, \overline{\varepsilon}]$ and $[-\overline{\sigma}, \overline{\sigma}]$, respectively. Then, the conditional probability, given σ, that voter j in group i votes for party A is:

$$s_{Ai}(\sigma) = \text{Prob}(\text{voter } j \text{ in group } i \text{ votes for } A \mid \sigma) = \frac{1}{2\overline{\varepsilon}}\left[\overline{\varepsilon} + \delta_i \Delta u_i + \sigma\right]. \tag{A.1}$$

This conditional probability is equal to the share of votes for party A in group i given σ. Suppose that a fraction ξ_i of voters in group i go to the polls. Given (A.1), the number of votes for party A given σ is:

$$V_A(\sigma) = \sum_{i=1,\ 2} \xi_i N_i s_{Ai}(\sigma).$$

This expression yields the following condition for party A to win the election:

$$V_A \geq V_B \Longleftrightarrow V_A \geq \frac{\sum_{i=1,\ 2} \xi_i N_i}{2} \Longleftrightarrow \sigma \geq \frac{\sum_{i=1,\ 2} \xi_i N_i \delta_i \Delta u_i}{\sum_{i=1,\ 2} \xi_i N_i \delta_i}.$$

Hence, party A's probability of winning the election, p_A, is:

$$\begin{aligned} p_A = \mathrm{Prob}(V_A \geq V_B) &= \frac{1}{2\overline{\sigma}} \left[\overline{\sigma} + \frac{\sum_{i=1,\ 2} \xi_i N_i \delta_i \Delta u_i}{\sum_{i=1,\ 2} \xi_i N_i \delta_i} \right] \\ &= \frac{1}{2} + \frac{1}{\overline{\sigma} \sum_{i=1,\ 2} \xi_i N_i \delta_i} \left[\sum_{i=1,\ 2} \xi_i N_i \delta_i u_i(x_A) - \sum_{i=1,\ 2} \xi_i N_i \delta_i u_i(x_B) \right]. \end{aligned}$$

If each party is interested only in winning the election, it would choose policy to maximize the probability of winning, taking the other party's policy as given. Because of symmetry, we have $x_A = x_B = x^*$ in the Nash equilibrium, where:

$$x^* = \arg\max \sum_{i=1,\ 2} \xi_i N_i \delta_i u_i(x). \tag{A.2}$$

We can use the probabilistic voting model to interpret the government's objective function in our model. In the text, government t's objective function is:

$$\gamma_t u^o(c_t^o) + u^y(c_t^y).$$

Given (A.2), γ_t can be interpreted as the ratio of $\xi_i N_i \delta_i$ between the young and the old. Thus, γ_t increases following an increase in the turnout rate, an increase in the aged population, or an increase in the importance attached to policy issues by the old when deciding how to vote. For example, if we set $\gamma_t \equiv \omega_t \theta_t N_{t-1}/N_t$, as in (34), then ω_t is the ratio of turnout rates between the old and the young.

Appendix B: The Case of Linear Utility

In this appendix, we show that if the household's utility function exhibits constant marginal utilities for consumption, then in contrast to the case of log utilities analyzed in the text, we obtain the results aligned with the standard FTPL, in which future fiscal surpluses affect the current price level.

Consider the following utility functions:

$$u^y(c_t^y, \ell_t) = c_t^y - \frac{1}{2}\ell_t^2 \tag{A.3}$$

and

$$u^o(c_t^o) = c_t^o. \tag{A.4}$$

For simplicity, we assume that θ_t and n_t are deterministic and both equal to one. We also assume the existence of an internal solution, at which the household's consumption is strictly positive.

Given the above utility function, the first-order conditions for a young household are:

$$r_{t+1} = \frac{1}{\beta}, \tag{A.5}$$

$$\ell_t = 1 - \tau_t, \tag{A.6}$$

and

$$s_t = (1-\tau_t)^2 - c_t^y, \tag{A.7}$$

where $c_t^y \in [0, (1-\tau_t)^2]$ is arbitrary, and the indirect utility functions are reduced to:

$$v_t^y(r_{t+1}, \tau_t) = \frac{1}{2}(1-\tau_t)^2 + \beta g_{t+1}^T \tag{A.8}$$

and

$$v_t^o(r_t) = r_t s_{t-1} + g_t^T. \tag{A.9}$$

The optimization problem for government t is then expressed as follows:

$$\max_{b_t, \tau_t, r_t} \gamma(r_t b_{t-1} + g_t^T) + \frac{1}{2}(1-\tau_t)^2 + \beta g_{t+1}^T, \tag{A.10}$$

subject to

$$b_t + \tau_t(1-\tau_t) = r_t b_{t-1} + \bar{g}_t, \tag{A.11}$$

$$0 \leq a_t = b_t \leq (1-\tau_t)^2, \tag{A.12}$$

and

$$r_{t+1}(b_t) = 1/\beta, \tag{A.13}$$

where we assume that the weight on the old generation's utility, γ, is constant over time and $\bar{g}_t = 2g_t^C + g_t^T$. The second constraint implies that outstanding bonds should not exceed the maximum level of private savings, $(1-\tau_t)^2$. The third constraint implies that because purchases of government bonds in period t require government $t+1$ to achieve a real return of $1/\beta$, government t must issue an amount of bonds that will guarantee it in the next period. Let us define $\bar{b}_t = r_{t+1}^{-1}(1/\beta)$. If government t were to issue more bonds than $\bar{b}_t$, then government $t+1$ would set the real interest rate below $1/\beta$. Anticipating this, the young in period t would not purchase government bonds. Government t would eventually be forced to issue bonds in the amount of $b_t = \bar{b}_t$. Thus, we can treat b_t as given in the above optimization problem.

Given that τ_t and r_t are choice variables, we can obtain the optimal tax rate as:

$$\tau_t = \tau^* = \frac{\gamma - 1}{2\gamma - 1}. \tag{A.14}$$

If $\gamma > 1$, $\tau^* > 0$ is guaranteed: when the well-being of the old is prioritized over that of the young, every government imposes a positive tax on the young's income. The tax rate τ^* increases with γ, and converges to 50%. Note that the path of government spending, g_k, does not influence τ^*. This suggests that the optimal tax rate is smoothed over time despite each government being short-lived.

The governments maintain their fiscal balances by issuing and denominating bonds, without changing the tax rate. Substituting τ^* in (A.11) reveals that b_t and r_t satisfy:

$$r_t b_{t-1} = \bar{b}_t + \tau^*(1 - \tau^*) - \bar{g}_t, \tag{A.15}$$

where $b_t = \bar{b}_t$. Therefore, in a Markov perfect equilibrium, b_{t+1} and r_{t+1} also satisfy:

$$r_{t+1} b_t = b_{t+1} + \tau^*(1 - \tau^*) - \bar{g}_{t+1}. \tag{A.16}$$

Because $r_{t+1} = 1/\beta > 1$, the equilibrium is determinate. Iterating the above equations yields:

$$b_t = \frac{\beta}{1 - \beta}\tau^*(1 - \tau^*) - \sum_{j=1}^{\infty} \beta^j \bar{g}_{t+j}.$$

Then, from (A.15), we obtain:

$$r_t b_{t-1} = \frac{1}{1 - \beta}\tau^*(1 - \tau^*) - \sum_{j=0}^{\infty} \beta^j \bar{g}_{t+j}. \tag{A.17}$$

Recall that $r_t b_{t-1} \equiv R B_{t-1}/P_t$, in which only P_t is endogenous in period t. Equation (A.17) coincides with the fundamental equation in the standard FTPL: the real value of a government bond is determined by the discounted present value of future fiscal surpluses. This equation implies that a future increase in public spending, $\bar{g}_{t+j}$, lowers the current real interest rate, r_t, and hence raises the current price level, P_t. This result differs from that based on the log utility case analyzed in the text. The source of the difference is the response of savings to the real interest rate: when $\bar{g}_{t+j}$ increases, P_{t+j} increases and r_{t+j} decreases. However, to keep the government bonds attractive to the young, the real interest rate must be at least $1/\beta$. Thus, P_{t+j-1} increases by the same amount to keep r_{t+j} constant. This requires an increase in P_{t+j-2}, and eventually, these effects are translated into an increase in P_t. Compared with the log utility case, where savings do not respond to the real interest rate at all, the linear utility case gives a similar result to the standard FTPL.

Appendix C: The Case of Fixed Labor Supply

In this appendix we use a utility function that exhibits constant relative risk aversion to generalize our results in the text. Suppose that the utility function satisfies $u^y(c_t^y, \ell_t) = \frac{(c_t^y)^{1-\sigma}}{1-\sigma}$ and $u^o(c_t^o) = \frac{(c_t^o)^{1-\sigma}}{1-\sigma}$, where $\sigma > 0$ measures risk aversion, and labor supply is fixed at $\ell_t = 1$. Except for the restriction of a fixed labor supply, these specifications are more general than those used in the text, particularly in terms of the form of the utility function with respect to consumption and the inclusion of nonzero government transfers to the old.

Substituting c_t^y and c_t^o into (10) and substituting (17) into (25) yields:

$$r_t = \frac{1}{b_{t-1}} \left(\frac{\omega_t^{-\frac{1}{\sigma}}}{\theta_t} + \frac{1}{n_t} \right)^{-1} \left\{ 1 - \left(1 + \frac{\theta_t}{n_t}\right) g_t^C - \left(\omega_t^{-\frac{1}{\sigma}} + \frac{\theta_t}{n_t} \right) g_t^T \right\}, \tag{A.18}$$

$$c_t^y = 1 - \frac{r_t b_{t-1}}{n_t} - \left(1 + \frac{\theta_t}{n_t}\right) g_t^C - \frac{\theta_t}{n_t} g_t^T, \tag{A.19}$$

and

$$c_t^o = \frac{r_t b_{t-1}}{\theta_t} + g_t^T. \tag{A.20}$$

Equation (6) implies:

$$b_t = \beta \left[\frac{r_{t+1} b_t \left(\frac{r_{t+1} b_t}{\theta_{t+1}} + g_{t+1}^T \right)^{-\sigma}}{\left(1 - \frac{r_t b_{t-1}}{n_t} - \left(1 + \frac{\theta_t}{n_t}\right) g_t^C - \frac{\theta_t}{n_t} g_t^T \right)^{-\sigma}} \right], \tag{A.21}$$

where $r_t b_{t-1}$ and $r_{t+1} b_t$ are given by (A.18). The tax rate is then chosen to satisfy:

$$\tau_t = 1 - b_t - c_t^y. \tag{A.22}$$

We now use (A.18) to examine the effect of various shocks on the real interest rate r_t, and in turn, the price level P_t. Recall that, because $r_t \equiv RP_{t-1}/P_t$, an increase in r_t implies a decrease in P_t.

Suppose first that current per-capita government consumption, g_t^C, increases unexpectedly, other thing being equal. Because:

$$\frac{\partial r_t}{\partial g_t^C} < 0, \tag{A.23}$$

an exogenous increase in per-capita government consumption leads to an increase in the price level.

Consider next an unexpected increase in current government transfers to the old, g_t^T. We have:

$$\frac{\partial r_t}{\partial g_t^T} < 0. \tag{A.24}$$

The government opts to raise the price level. An increase in government transfers to the old improves the old generation's welfare, other things being equal. However, this does not equalize the politically weighted marginal utilities. The price level must rise to reduce the old generation's welfare. Governments in developed countries, including Japan, have tended to reduce government transfers to the old. Our model implies that this will reduce prices, which is consistent with Japan's recent deflation.

If the relative political influence of the old, $\gamma_t = \omega_t \theta_t / n_t$, increases unexpectedly, without the old or young populations changing, we have:

$$\frac{\partial r_t}{\partial \gamma_t} > 0. \tag{A.25}$$

Because the old generation's political influence increases given the population structure, the government opts to induce a lower price level to improve the well-being of the old.

Next, consider the effect of population aging. As in the text, we distinguish between two causes of aging: a decline in the birth rate and an increase in longevity. Consider first the impact of an unexpected decline in the birth rate in period t. We have:

$$\frac{\partial r_t}{\partial n_t} > 0. \tag{A.26}$$

Hence, an unexpected decline in the birth rate increases the price level. Consider next the impact of an unexpected increase in longevity. An increase in θ_t affects the real interest rate such that:

$$\frac{\partial r_t}{\partial \theta_t} > 0 \tag{A.27}$$

if and only if:

$$1 - \left(1 + \frac{2\theta_t}{n_t} + \frac{\theta_t^2}{n_t^2}\omega_t^{\frac{1}{\sigma}}\right) g_t^C - \left(\omega_t^{-\frac{1}{\sigma}} + \frac{2\theta_t}{n_t} + \frac{\theta_t^2}{n_t^2}\omega_t^{\frac{1}{\sigma}}\right) g_t^T > 0. \tag{A.28}$$

This condition demonstrates that an unexpected increase in longevity *reduces* the price level unless government expenditure is sufficiently high (that is, if either g_t^C, g_t^T, or both are high).