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Abstract

This paper discusses the lean vs. clean policy debate in managing financial crises based on dynamic general equilibrium models with an occasionally binding collateral constraint. We show that a full state-contingent subsidy for debtors can restore the first-best allocations by forestalling disorderly deleveraging in a crisis. While this result appears to favor the clean policy against a lean policy that achieves the second-best allocation, further assessment points to various risks associated with the clean policy from a practical viewpoint. First, the optimal clean policy is likely to call for an unrealistically large amount of fiscal resources. Second, if the clean policy is activated with an empirically realistic intervention, the less-than-optimal clean policy incentivizes debtors to take on undue risks, exposing the economy to higher crisis probabilities. Finally, the less-than-optimal clean policy may give rise to huge welfare losses due to the policy maker's misrecognition of the state of the economy.

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1 Introduction

This paper aims to address an ongoing policy debate: should policy makers "lean against the upswing of a credit cycle" or "clean up afterwards once the boom has turned to bust?"[1] The fierce debate over these two opposing strategies in coping with financial crises has long continued among policy makers and academics. The first strategy (the lean policy) encourages policy makers to rely on an ex ante tightening of monetary conditions or a rise in the cost of credit to forestall financial crises. The lean view emphasizes the costly nature of postcrisis management as well as the possible moral hazard ignited by expectations for a precommitted bailout. The second strategy (the clean policy), on the other hand, favors an ex post easing policy to stabilize the economy. The clean view makes a strong case for bailouts on the grounds that, for example, economic booms may be driven by improving fundamentals and, in principle, should not be impeded by policy interventions.

In this paper, we reexamine the lean vs. clean debate using a dynamic general equilibrium model with an occasionally binding collateral constraint. The model provides an appropriate laboratory to discuss policy options in dealing with financial crises because it can endogenously create "rare but significant declines" in economic activity. In terms of policy measures, we solely focus on a state-contingent Pigouvian tax/subsidy on debt, and assume that the lean policy is implemented via the *tax* on debt *before* a crisis, whereas the clean policy is carried out via the *subsidy* on *debt* during a crisis.[2]

We begin by demonstrating that a full state-contingent clean policy per se can achieve the first-best (FB) allocation (i.e., the allocation without the collateral constraint). The economic intuition behind this result is as follows: with a massive subsidy on new debt anticipated in a crisis period, investors willingly continue to borrow. They would not engage in fire sales of their collateral assets, but would tend to keep the collateral assets at hand for rolling over their debts. On the other side of the investors' behavior, the policy maker would successfully inflate asset prices by discouraging the fire sale. Because of the high asset prices inflated by the intervention, investors can borrow freely as if there were no collateral constraint in the economy.

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[1]See White (2009) for the exact phrase expressing the central question of the debate.

[2]We do not translate the Pigouvian tax/subsidy into any particular policy tools in the real economy, but monetary policy is obviously one such tool in the sense that it changes the cost of debt via changing interest rates.

The foregoing result appears to favor those who advocate the clean policy. However, further inspections place doubt on this conclusion from the viewpoint of feasibility, and risk assessment, and welfare. First, the optimal clean intervention, which wipes out all the financial crises and achieves the FB allocation, is likely to require an unrealistically large amount of fiscal resources. Using the canonical model, we show that the optimal clean policy could require a subsidy on debt of more than 30 percent of GDP for a single year. In practice, however, such a large-scale wealth transfer from creditors to debtors is likely to give rise to a number of disputes and conflicts. In fact, empirically, government bailouts using taxpayers' money tend not to exceed five percent of GDP, which clearly falls short of the model's prescription of the optimal clean policy. Therefore, we perform a policy experiment in an attempt to assess the consequence of the less-than-optimal policy intervention. The policy experiment, assuming that policy makers can intervene with realistic "fire power," points to the possibility that a clean policy with a limited intervention size could fuel undue risk-taking by debtors while also increasing the probability of a crisis, which, in the end, would result in catastrophic consequences.

Second, if policy makers activate a clean policy with realistic "fire power", this less-than-optimal clean policy leaves the economy exposed to high crisis probability. We perform a policy experiment to assess the frequency of financial crises in the model with the less-than-optimal clean policy and find that the crisis probability amounts to 7.6 percent, which is much higher than the model without policy interventions. The policy experiment points to the possibility that such a clean policy could fuel undue risk-taking by debtors.

Third, noting that the optimal clean policy requires policy makers to recognize precisely when to embark on the intervention, we run another policy experiment to explore implications of mistimed policy intervention for welfare. We find that a poorly timed clean policy with a feasible size of intervention may give rise to huge welfare losses. It has been repeatedly argued by policy makers that timely interventions are, in general, not easy to undertake.[3] Our policy experiment points to the possibility that a preemptive activation of a precommitted clean policy would result in a catastrophic consequences. On the other hand, our policy experiments broadly suggest that the lean

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[3]For example, see Orphanides (2001), who argues that there are significant impacts on policy implementation due to the delay of data availability and inaccuracy of the initial data.

policy can be considered as the more practical and feasible option for two reasons. First, the optimal lean policy requires only a realistically small amount of fiscal resources, suggesting that it should be subject to less concern in terms of feasibility. Second, given the small size of the intervention, the lean policy does not tend to expose the economy to risks of disastrous outcomes even if the policy makers were not fully knowledgeable about when interventions could most appropriately be carried out.

This paper contributes to the growing literature analyzing financial crises using occasionally binding collateral constraint models. In terms of model structure, the three-period simplified model introduced in Section 2 of this paper shares a similar structure to that of Jeanne and Korinek (2010), among others, and the infinite horizon version in Section 3 employs a structure similar to that of Bianchi and Mendoza (2010) and Jeanne and Korinek (2012a).[4] In terms of motivation, we refer to Jeanne and Korinek (2012b), Bianchi (2012), and Benigno, Chen, Otrok, Rebucci, and Young (2009, 2011, 2012). These studies investigate the lean vs. clean debate using a model with a collateral constraint. Jeanne and Korinek (2012b) focus on underinvestment for capital-augmenting productivity rather than a fire sale externality and consider tax distortion as a cost of the clean policy. Bianchi (2012) focuses on an inefficiency arising from frictions between shareholders and firms, and discusses some iceberg costs for the clean policy. Benigno et al. (2009, 2011, 2012) develop models of the two-sector (tradable and nontradable good) small open production economy and argue that ex post interventions lead to larger welfare gains than ex ante interventions.[5]

As noted at the onset, this paper also contributes to the longstanding policy debate on the lean vs. clean options. Some policy makers have pointed out the possibility that a precommitted clean policy may result in undue risk-taking by investors and an increased probability of financial crises, and have referred to the experience of accommodative monetary conditions in the run-up to the global financial crisis of 2007–2008. (e.g., Taylor, 2009; Borio and Zhu, 2008; Shirakawa, 2012). On the other hand, for example, Kohn (2006) by and large favors the clean policy options, warning

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[4]A distinctive difference from their model is that our collateral constraint is formulated with respect to the market value of collateral assets at the end of a period rather than at the beginning of a period. This difference appears subtle at first glance, but the clean policy has no effect on mitigating the collateral constraint in their model setting.

[5]Using a model of banking, Green (2010) also argues that the government's bailouts can lead to a socially efficient outcome, rather than inefficient risk-taking by banks.

against "pricking the bubble." Our study could reconcile this debate in the sense that the optimal clean policy is theoretically possible, but if not correctly designed and implemented, it would result in a large welfare loss. If the clean option cannot be delicately performed, we could make a stronger case for lean policy options.

This paper proceeds as follows. Section 2 illustrates a three-period version of the model and demonstrates that a full state-contingent subsidy on debt per se can achieve the FB allocations. Section 3 introduces the infinite horizon model similarly to Bianchi and Mendoza (2010). In Section 4, we calibrate the infinite horizon model and summarize the quantitative implications. Section 5 reviews the real-world experiences of government policy interventions and discusses risks associated with the clean policy using a policy experiment. Section 6 concludes the paper.

2 A perfect foresight three-period model

2.1 Setup

We begin by analyzing a small open economy with three periods (i.e., $t = 0, 1, 2$). The three-period model analytically crystallizes comparisons between the optimal lean and clean policies considered in this paper. The model includes essentially the same ingredients in the models discussed in the literature.[6] For expository purposes, we introduce the following simplifying assumptions: (i) the discount factor and the real interest rate are equal to unity; (ii) households maximize their utility with perfect foresight; and (iii) a collateral constraint binds only in period 1. We define the "crisis" period as the period during which consumption is substantially reduced due to the binding collateral constraint. With this definition, we can interpret period 1 as the "crisis" period. While this model is simplified for expository purposes, the above-mentioned three assumptions will be relaxed later in Section 3.

The economy is inhabited by homogenous households with the utility function given by:

$$U = u(c_0) + u(c_1) + u(c_2), \tag{1}$$

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[6]Bianchi and Mendoza (2010) and Jeanne and Korinek (2012a) employ the infinite horizon framework and Jeanne and Korinek (2010) simplify their own model by assuming a three-period setting.

where c_t is the consumption in period t and u is the constant relative risk-aversion utility function. The households' budget constraints are:

$$c_0 + b_1 = 0 \tag{2}$$

$$c_1 + b_2 + q_1 k_2 = q_1 \bar{K} + b_1 + \varepsilon_1 \tag{3}$$

$$c_2 = b_2 + \varepsilon_2 k_2, \tag{4}$$

where b_t and k_t denote one-period bond holdings and collateral assets at the beginning of period t, respectively. In period 0, the consumption is fully financed by debt $(-b_1)$ in the same period because the initial bond holdings b_0 are zero. At the beginning of period 1, the households receive domestic income from two sources: endowments in the forms of consumption goods (ε_1) and of collateral assets $(\bar{K})$. The market value of the latter can be written as $q_1 \bar{K}$ in terms of the consumption goods, where q_1 is the price of the asset in period 1. In this period, the households spend their income on consumption goods (c_1), debt repayment $(-b_1)$, and collateral assets (k_2), while undertaking new borrowing $(-b_2)$. We assume that the collateral assets are not acquired by foreign investors because they cannot manage them to obtain any payoff from them. At the beginning of period 2, the collateral assets yield the dividend ε_2 per unit of the collateral assets. We assume that the supply of the collateral assets $\bar{K}$ is constant and normalized to one and, thus, the market-clearing condition for the collateral asset is $k_2 = \bar{K} (= 1)$.

The key equation in this model is the collateral constraint that reins in the amount of debt to a certain fraction of the households' collateral:

$$-b_2 \leq \kappa q_1 k_2, \tag{5}$$

where κ denotes the ceiling on the leverage ratio satisfying $\kappa \in (0, 1)$. As in Kiyotaki and Moore (1997), among others, the collateral constraint builds on the limited enforceability of the debt contracts.

While the households are faced with "shocks" to income denoted by ε_1 and ε_2, they are assumed,

for the time being, to be deterministic. We further simplify the argument by making additional assumptions on ε_1, ε_2, and κ:

Condition 1 $\varepsilon_1 < 2\varepsilon_2$.

Condition 2 $\kappa < (2\varepsilon_2 - \varepsilon_1)/(3\varepsilon_1)$.

Condition 1 ensures that the households borrow, rather than save, in period 1 (i.e., $b_2 < 0$) and Condition 2 ensures that the collateral constraint binds in period 1.

2.2 The laissez-faire equilibrium

We briefly review the laissez-faire (LF) equilibrium in this economy. The households choose b_1, b_2 and k_2 to maximize the utility (1), subject to the budget constraint (2) - (4) and the collateral constraint (5), taking the asset price q_1 as given. Let $u'(c_t)$ and μ_t be the marginal utility of consumption in period t and the Lagrange multiplier on the collateral constraint, respectively. The first-order conditions are:

$$u'(c_0) = u'(c_1) \tag{6}$$

$$u'(c_1) = u'(c_2) + \mu_1, \tag{7}$$

$$q_1 = \frac{u'(c_2)}{u'(c_1) - \kappa\mu_1}\varepsilon_2. \tag{8}$$

Equations (6) and (7) mean that households attempt to smooth their consumption. However, the collateral constraint impedes their attempt to smooth out consumption between periods 1 and 2 because the binding collateral constraint increases the marginal cost of period 1 consumption. This impediment is reflected in the shadow price of the collateral constraint in (7), implying that $u'(c_1) > u'(c_2)$. Equation (8) represents the asset-pricing formula in this model. Notably, the asset price is influenced by the shadow value of the collateral constraint.

To understand the impact of the collateral constraint on the asset price, we eliminate μ_1, c_1 and c_2 from (8). From the budget constraints and (7), the asset-pricing formula is now rewritten

as:

$$q_1(b_1) = \frac{\varepsilon_2}{(1-\kappa)u'(b_1 + \varepsilon_1 - b_2)/u'(b_2 + \varepsilon_2) + \kappa} < \varepsilon_2, \tag{9}$$

where we substituted the market-clearing condition for the collateral assets: $k_2 = 1$. Equation (9) means that the collateral asset is undervalued compared to the case without the collateral constraint (i.e., $q_1 = \varepsilon_2$). Under the binding constraint, the households are fire-selling the collateral asset at hand to finance their period 1 consumption. With the downward pressure on the asset market underway, households (buyers as well as sellers) end up holding the asset at the fire-sale value.

2.3 The optimal lean policy as the macroprudential tax

We next characterize the optimal lean policy that reins in the households' borrowing. To characterize the optimal lean policy in this three-period model, we take three steps. First, we introduce the constrained social planner (CSP), subject to the same collateral constraint as that of the households. The key feature of the CSP is that he/she takes into account the changes in asset prices in response to the bond holdings, b_1. We then consider a decentralized economy with the new period 0 budget constraint of households, explicitly with a tax on debt τ_0. Finally, we compare the first-order conditions in the CSP's problem with those in the household's problem, including τ_0. From the comparison, we obtain the state-contingent Pigouvian tax that replicates the CSP's allocation.[7]

With these steps in mind, we first consider the CSP's maximization problem. The CSP's collateral constraint is:

$$-b_2 \le \kappa q(b_1)\bar{K}. \tag{10}$$

In the above equation, k_2 is replaced by $\bar{K}$, based on the market-clearing condition for the collateral assets. More importantly, q_1 is also replaced with the households' decision rule $q(b_1)$, given by (9). As a result, the CSP internalizes the fact that the individual decisions of households on the debt holdings at the beginning of period 1 affect the asset prices. This internalization of the CSP implies

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[7]Although there is only a single state in the economy due to the assumption of perfect foresight, we use the word "state-contingent" for this tax to emphasize that τ_t is a function of the state of the economy ε_t rather than a constant value. As noted, this assumption of the single state will be relaxed in the subsequent sections.

that the closed-form solution of $q(b_1)$ in this economy takes a different form from that in the LF economy, even though the CSP is faced with the same constraint as the households.

The CSP maximizes (1) by choosing b_1 and b_2, subject to the collateral constraint (10) and the resource constraints: $c_0 + b_1 = 0$, $c_1 + b_2 = b_1 + \varepsilon_1$, and $c_2 = b_2 + \varepsilon_2 \bar{K}$. The first-order condition for b_1 is:

$$u'(c_0) = u'(c_1) + \mu_1 \kappa q'(b_1), \tag{11}$$

whereas the first-order conditions for b_2 remains the same as (7). Intuitively, (11) suggests that, if $q'(b_1) > 0$, less consumption (i.e., less borrowing) in the precrisis period increases the asset price in the crisis period and gives the households an extra benefit, in that the collateral constraint will be relaxed by the increased asset prices. Thus, the period 0 consumption must be reduced by the CSP compared with the LF allocation. This extra effect arises from pecuniary externalities, as emphasized by Bianchi and Mendoza (2010) among others, and results in the households' overborrowing in the LF economy compared to the CSP's allocation because the decentralized households do not internalize this extra benefit of changing consumption in the precrisis period.

We next introduce the new period 0 budget constraint with a tax on debt:

$$c_0 + \frac{b_1}{1 + \tau_0} + T_0 = 0, \tag{12}$$

where τ_0 is a tax on debt and T_0 denotes the lump-sum transfers in period 0. Here, the government budget is balanced in period 0, so that $T_0 = \tau_0 b_1 / (1 + \tau_0)$. A higher τ_0 increases the cost of borrowing for the domestic households in this economy. In this regard, τ_0 is not only a tax on debt but can also be broadly understood as the policy instrument which affects the cost of borrowing for domestic debtors.

The following proposition summarizes the results of Jeanne and Korinek (2010, 2012a) who characterize the optimal lean policy (the macroprudential policy) via a tax on debt.

Proposition 1 *Suppose that the households' period 0 budget constraint is given by (12). Then, there exists an optimal lean policy τ_0^{**} that achieves the constrained social planner's allocation characterized by (7), (8), (10) and (11).*

Proof. See Jeanne and Korinek (2010, 2012a). In this problem with τ_0 and T_0, the first-order condition with respect to b_1 is $u'(c_0) = (1 + \tau_0) u'(c_1)$ and the other conditions remain the same as those in the CSP's problem. Comparing the first-order condition with (11), we obtain the optimal Pigouvian tax τ_0^{**} given by:

$$\tau_0^{**} = \frac{\kappa \mu_1 q'(b_1)}{u'(c_1)}.$$

∎

Note that τ_0^{**} can implement the optimal debt position from the CSP's point of view. It is clear that τ_0^{**} is strictly positive under $q'(b_1) > 0$, encouraging households to hold more b_1 (less debt in the precrisis period) than the case under the LF economy.[8]

2.4 The optimal clean policy as the bailout subsidy

The previous subsection reviewed the results of Jeanne and Korinek (2010, 2012a) and others, showing that the optimal lean policy replicates the second-best allocations that can be achieved by the CSP. In this subsection, by contrast, our analysis takes a different path. First, we consider the FB allocation, which simply corresponds to the unconstrained competitive equilibrium. Subsequently, we explore what kind of policy option, if any, can replicate the FB allocation even with the collateral constraint imposed in the economy.

We show that a well-designed, state-contingent "clean" policy that attempts to resolve the crisis taking place in period 1 can achieve the FB allocation. In our three-period model, the optimal clean policy aims to "mop up the mess" created at the time of the financial crisis by fueling the asset price with the subsidy on debt. Due to the inflated asset prices, the collateral constraint can be effectively removed and, hence, the optimal clean policy can achieve the FB allocation.

Suppose that the households are subject to the following period 1 budget constraints:

$$c_1 + \frac{b_2}{1 + \tau_1} + q_1 k_2 + T_1 = q_1 \bar{K} + b_1 + \varepsilon_1, \tag{13}$$

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[8]In our model, if the product of κ and the degree of relative risk aversion is sufficiently small, the condition $q'(b_1) > 0$ is satisfied.

where τ_1 represents the tax on debt in the crisis period. In line with the previous case of the lean policy, the lump-sum transfers T_1 are imposed such that $T_1 = \tau_1 b_2 / (1 + \tau_1)$ for the government budget to be balanced during the single period. Other constraints (2), (4), and (5) remain the same as in the LF economy. Although τ_1 and T_1 are referred to as taxes and transfers, similarly to the case of the lean policy, we note that, when the government intends to raise households' debt holdings during the crisis period, the government sets τ_1 to a negative value, meaning it creates a subsidy on debt. On the flip side, T_1 becomes a lump-sum tax rather than a transfer.

In this policy framework, the following proposition and corollaries hold:

Proposition 2 *Under Conditions 1 and 2, there exists a τ_1^* that, despite the presence of the collateral constraint (5), achieves the first-best allocation, i.e., $c_0 = c_1 = c_2 = (\varepsilon_1 + \varepsilon_2)/3$.*

Corollary 1 $\tau_1^* < 0$.

Corollary 2 *Under τ_1^*, $q_2 > \varepsilon_1$ in equilibrium.*

Proof. See Appendix A.1. ■

Proposition 2 indicates that the optimal lean policy may no longer be the welfare-maximizing option if we expand the scope of the policy options by including ex post bailout measures.[9] If the government is restricted to using τ_0 as its instrument, τ_0^{**} is the best policy option among all ex ante interventions. When the government has an option of using τ_1 as the instrument, however, the government can achieve the FB allocation via the subsidy on debt τ_1^*, despite the presence of the collateral constraint. Proposition 2 also suggests that policy makers may not need to rely on additional instruments, but that the subsidy on debt τ_t, if optimally designed, should be sufficient in achieving the FB allocation. This starkly contrasts with the previous related studies which

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[9]Proposition 2 depends crucially on the assumption that the policymakers have the capacity to finance the funds for bailing-out via lamp-sum tax. While we take it as a purely theoretical result rather than a realistic policy recommendation, the proposition holds under a more realistic tax system other than the lamp-sum tax. For example, we can consider a tax system where a consumption tax is levied universally in the economy and its revenue is distributed as a lump-sum transfer. A brief inspection would confirm that the consumption tax does not distort inter-temporal decision makings of the households and, hence, if the bailout funds are financed by such a consumption tax, Proposition 2 still holds. The key element behind the result is that the labor supply is not considered explicitly in the economy. In a more general case, Proposition 2 may not hold under the consumption tax, which affects incentives of the households, distorting their consumption-leisure decisions via labor supply channel.

consider extra bailout measures other than τ_t to achieve the FB allocation.[10]

The intuition behind the proposition and corollaries is as follows. The subsidy ($\tau_1^* < 0$) incentivizes households to have a larger amount of debt. They would not engage in a fire sale of their collateral assets, but would tend to keep the collateral asset at hand for rolling over their debt. The demand stimulus for asset purchases inflates asset prices, enhancing the households' ability to borrow because the booming asset market relaxes the collateral constraint. Corollary 2 shows that the government should aim to inflate the asset price beyond the level that prevails in the unconstrained economy. Put differently, as long as the government can choose the size of subsidy freely, it can make the collateral constraint effectively nonbinding as a result of the price-keeping operation in the asset market.[11]

Technically, our results rely on the assumption that the household borrowing in period 1 is constrained by the asset value $q_1 k_2$, on which households can make decisions in period 1, rather than on $q_1 \bar{K}$, which has already been chosen by that point in time. If the households borrow in period 1 against $q_1 \bar{K}$ as their collateral, the households cannot increase their period 1 borrowing capacity by changing the amount of the collateral assets k_2. Because k_2 is not collateralizable for period 1 borrowing, cutting the cost of period 1 borrowing ($-b_2$) through the government subsidy provides no incentives for households to increase their demand for k_2. As a result, the asset price in period 1 remains unchanged.[12]

In evaluating the optimal clean policy, quantitative prerequisites may be called into question. Hence, the next step we take is to quantitatively reassess the lean vs. clean debate by extending

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[10]For example, Benigno et al. (2009) developed a two-sector small open-economy model and suggest that there is a parameter set where the subsidy on nontradable goods consumption can achieve the FB allocation. Jeanne and Korinek (2010) suggest that the CSP could completely eliminate the collateral constraint from the household's problem by intertemporal transfers from the CSP to the domestic debtors. In particular, when the CSP is not subject to the collateral constraint, he/she can borrow the bailout funds from outside lenders and then repay the fund after his/her bailout. If this is the case, the intertemporal transfers effectively remove the collateral constraint.

[11]Under some parameter sets, this FB allocation achieved by the optimal clean policy could be a locally unstable equilibrium, in the sense that a small perturbation in borrowing around the equilibrium could lead the economy into another inefficient equilibrium. This has been pointed out by Jeanne and Korinek (2012a). While this local instability might be theoretically interesting, the FB allocation remains *an equilibrium* in this model. Therefore, we limit our attention to the FB allocation achieved by the optimal clean policy and do not investigate the other inefficient equilibrium. In fact, the log-utility guarantees the globally stable unique equilibrium under the optimal clean policy.

[12]While we recognize that this assumption may be debatable, it could be the case that, in practice, borrowers normally collateralize their new asset (e.g., houses) that they are planning to purchase at the time of drawing new loans.

the deterministic three-period model to a stochastic infinite horizon model with an *occasionally binding* collateral constraint.

3   The infinite horizon model

3.1   Setup

We consider the stochastic infinite horizon model in an attempt to provide a more realistic assessment of state-contingent policies by introducing shocks to the dividend. The model here, as we have mentioned, relaxes the simplifying assumptions that we made in the previous section. The household preference is:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where β is the discount factor satisfying $\beta \in (0,1)$ and $\mathbb{E}_0 [\cdot]$ denotes the expectations operator conditional on the information available at period 0. Each household is faced with the period-by-period budget constraint:

$$c_t + \frac{b_{t+1}}{R} + q_t k_{t+1} = q_t \bar{K} + b_t + \varepsilon_t k_t, \tag{14}$$

and the occasionally binding collateral constraint:

$$-\frac{b_{t+1}}{R} \leq \kappa q_t k_{t+1}. \tag{15}$$

The period-by-period budget constraint in the infinite horizon model slightly differs from the period 1 budget constraint (3) in the three-period model. The one-period bond b_{t+1} is now interpreted in terms of non-state-contingent one-period bonds. The real interest rate on the non-state-contingent bonds is $R > 1$ rather than unity.[13] We further assume that, as in the period 2 budget constraint

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[13]We intentionally slightly abuse the notation for R in (14) and (15). In the computation, R in (14) is replaced with $R_{t+1} = \bar{R} + \psi[\exp(-b_{t+1}) - 1]$ where $\bar{R}$ is an exogenous and constant risk-free interest rate. Here, $\psi[\exp(-b_{t+1}) - 1]$ represents a household-specific risk premium. Throughout this paper, this risk premium is always included in the period-by-period budget constraint. As discussed in Schmitt-Grohe and Uribe (2003), the lack of a risk premium leads to nonstationary bond holdings in the class of the models without the collateral constraint. Because we are interested in the comparison between models with and without the collateral constraint, we introduce the risk premium into the budget constraint. By contrast, however, R in (15) is replaced with $\bar{R}$, implying that the non-state-contingent one-period bond holdings are discounted by the risk-free rate. This assumption is reasonable because the assets that

(4) in the three-period model, the households receive a dividend by holding collateral assets. The collateral asset k_t is useless after yielding the dividend at the beginning of period $t + 1$, but the households receive the endowment of new collateral assets $\bar{K}$ as in the period 1 budget constraint (3).

3.2 The laissez-faire equilibrium

The LF equilibrium can be characterized in a recursive form. The household's state variables in period t consist of bond holdings b, collateral asset holdings k, the aggregate bond holdings B, and the dividend ε. In what follows, a subscript attached to functions refers to the function under the corresponding equilibrium. For example, $B'_{LF}(B, \varepsilon)$ and $q_{LF}(B, \varepsilon)$ are the aggregate law of motion for the aggregate bond holdings and the asset price function under the equilibrium in the LF economy, respectively. We can also define the value function for each household in the LF economy as $V_{LF}(b, k; B, \varepsilon)$. With this notation in mind, the household's recursive optimization problem is:

$$V_{LF}(b, k; B, \varepsilon) = \max_{b', k'} \left\{ u(c) + \beta \mathbb{E} \left[V_{LF}(b', k'; B', \varepsilon') \right] \right\} \tag{16}$$

$$\text{s.t.} c + \frac{b'}{R} + q_{LF}(B, \varepsilon)k' = b + q_{LF}(B, \varepsilon)\bar{K} + \varepsilon k \tag{17}$$

$$B' = B'_{LF}(B, \varepsilon) \tag{18}$$

$$-\frac{b'}{R} \leq \kappa q_{LF}(B, \varepsilon)k', \tag{19}$$

where $\mathbb{E}[\cdot]$ denotes the expectations across the states of ε', conditional on ε, and variables with a prime are the corresponding variables in period $t + 1$ (e.g., $b' \equiv b_{t+1}$).

We formally define a recursive competitive equilibrium in the LF economy as follows.

Definition 1 (*Recursive competitive equilibrium in the laissez-faire economy*) *A recursive competitive equilibrium in the laissez-faire economy consists of a price function for collateral assets $q_{LF}(B, \varepsilon)$, an aggregate law of motion, $B'_{LF}(B, \varepsilon)$, a value function $V_{LF}(b, k; B, \varepsilon)$, and policy*

outside creditors can take as the collateral are risk free and the gross return of risk-free assets is $\bar{R}$, rather than R_{t+1}.

functions $b'_{LF}(b, k; B, \varepsilon)$, $k'_{LF}(b, k; B, \varepsilon)$, *and* $c_{LF}(b, k; B, \varepsilon)$ *such that:*

1. *Given the price function* $q_{LF}(B, \varepsilon)$ *and the law of motion for the aggregate bond holdings* $B'_{LF}(B, \varepsilon)$, *the value function* $V_{LF}(b, k; B, \varepsilon)$ *solves the Bellman equation (16) and* $b'_{LF}(b, k; B, \varepsilon)$, $k'_{LF}(b, k; B, \varepsilon)$ *and* $c_{LF}(b, k; B, \varepsilon)$ *are the associated policy functions.*

2. *The law of motion for the aggregate bond holdings is consistent with that for the individual bond holdings:* $B'_{LF}(B, \varepsilon) = b'_{LF}(B, \bar{K}; B, \varepsilon)$.

3. *The price of collateral assets satisfies the following asset price function:*

$$q_{LF}(B, \varepsilon) = \frac{\beta \mathbb{E}\left(u'\left\{c_{LF}\left[B'_{LF}(B, \varepsilon), \bar{K}; B'_{LF}(B, \varepsilon), \varepsilon'\right]\right\} \varepsilon'\right)}{u'\left[c_{LF}(B, \bar{K}; B, \varepsilon)\right] - \kappa \max\left[0, u'\left[c_{LF}\left(B, \bar{K}; B, \varepsilon\right)\right] - \beta R \mathbb{E}\left(u'\left\{c_{LF}\left[B'_{LF}(B, \varepsilon), \bar{K}; B'_{LF}(B, \varepsilon), \varepsilon'\right]\right\}\right)\right]}.$$

4. *All markets clear:*

$$c_{LF}(B, \bar{K}; B, \varepsilon) + \frac{B'_{LF}(B, \varepsilon)}{R} = B + \varepsilon \bar{K}$$

$$k'_{LF}(B, \bar{K}; B, \varepsilon) = \bar{K}.$$

The following first-order conditions, which generalize those in the three-period problem, characterize the equilibrium allocation in the LF economy:

$$u'[c_{LF}(B, \bar{K}; B, \varepsilon)] = \beta R \mathbb{E}\left(u'\left\{c_{LF}\left[B'_{LF}(B, \varepsilon), \bar{K}; B'_{LF}(B, \varepsilon), \varepsilon'\right]\right\}\right) + \mu_{LF}(B, \varepsilon) \qquad (20)$$

$$q_{LF}(B, \varepsilon) = \frac{\beta \mathbb{E}\left(u'\left\{c_{LF}\left[B'_{LF}(B, \varepsilon), \bar{K}; B'_{LF}(B, \varepsilon), \varepsilon'\right]\right\} \varepsilon'\right)}{u'\left[c_{LF}(B, \bar{K}; B, \varepsilon)\right] - \kappa \mu_{LF}(B, \varepsilon)} \qquad (21)$$

$$0 = \left[\frac{B'_{LF}(B, \bar{K}; B, \varepsilon)}{R} + \kappa q_{LF}(B, \varepsilon) \bar{K}\right] \mu_{LF}(B, \varepsilon) \qquad (22)$$

where $\mu_{LF}(B, \varepsilon) \geq 0$ and $-B'_{LF}(B, \bar{K}; B, \varepsilon)/R \leq \kappa q_{LF}(B, \varepsilon) \bar{K}$.

3.3 The lean and clean policies in the infinite horizon model

Before characterizing the equilibria under the optimal lean and clean policies, we should clarify how these policies can be distinguished in the infinite horizon model. Recall that, in the three-period model, the distinction between the two policies is clear because the lean option uses τ_0, whereas the clean option relies on τ_1. In the infinite horizon model, however, the recursive structure of the maximization problem does not allow us to distinguish between the two policies based on the time subscript of τ_t. As we will confirm in the subsequent subsections, the two policies face the common household's period-by-period budget constraint: $c_t + b_{t+1}/\left[(1 + \tau_t) R\right] + q_t k_t + T_t = b_t + q_t \bar{K} + \varepsilon_t k_t$, where $T_t = \tau_t b_{t+1}/(1 + \tau_t)$.

A simple litmus test to distinguish the two policies is the sign of τ_t. The lean policy in the infinite horizon model reduces borrowing by imposing a tax on period t debt, aiming to support the asset price which may be falling in period $t + 1$ in the case of a crisis. Thus, the lean policy means that $\tau_t \geq 0$. In this context, the optimal lean policy can be interpreted as the macroprudential Pigouvian tax, as discussed in Bianchi and Mendoza (2010) and Jeanne and Korinek (2010, 2012a). In contrast, the clean policy in the infinite horizon model subsidizes borrowing in period t, when the collateral constraint is about to bind in period t, aiming to inflate the period t asset price beyond the level that prevails in the unconstrained economy. Hence, the clean policy implies that $\tau_t \leq 0$. Later, we will confirm that the optimal clean policy achieves the FB allocation in line with the three-period model in Section 2. Based on the above-mentioned distinction, we can quantitatively assess the welfare gains from the two policies in the infinite horizon model.

3.4 The optimal lean policy

We now consider the optimal lean policy in the infinite horizon model. As in the three-period model, we take three steps to characterize the optimal lean policy.

First, we consider the CSP, who internalizes the effect of bond holdings on the asset price. Let $V_{CSP}(B, \varepsilon)$ be the value function of the CSP. Taking the asset price function in this economy $q_{CSP}(B, \varepsilon)$ as given, the CSP chooses the aggregate bond holdings B' in the following recursive optimization problem:

$$V_{CSP}(B,\varepsilon) = \max_{B'} \left[u(c) + \beta \mathbb{E} V_{CSP}(B',\varepsilon') \right] \tag{23}$$

$$\text{s.t.} \quad c + \frac{B'}{R} = B + \varepsilon \bar{K} \tag{24}$$

$$-\frac{B'}{R} \leq \kappa q_{CSP}(B,\varepsilon) \bar{K}. \tag{25}$$

Here, the CSP faces the same collateral constraint as the households in the LF economy. As we discussed in Section 2, however, the asset price function $q_{CSP}(B,\varepsilon)$ is not the same as $q_{LF}(B,\varepsilon)$ in the equilibrium. Following Jeanne and Korinek (2012a), the decision rule for the collateral assets in the LF economy, namely (21), is used to implicitly define the asset price in the CSP's problem. In particular, $q_{CSP}(B,\varepsilon)$ is given by:

$$q_{CSP}(B,\varepsilon) = \beta \frac{\mathbb{E}\left(u'\left\{c_{CSP}\left[B'_{CSP}(B,\varepsilon),\varepsilon'\right]\right\}\varepsilon'\right)}{u'\left[c_{CSP}(B,\varepsilon)\right] - \kappa \mu_{CSP}(B,\varepsilon)}. \tag{26}$$

The first-order condition for B' is:

$$u'\left[c_{CSP}(B,\varepsilon)\right] = \beta R \mathbb{E}\left[u'\left\{c'_{CSP}\left[B'_{CSP}(B,\varepsilon),\varepsilon'\right]\right\} \right. \tag{27}$$

$$\left. + \kappa \mu'_{CSP}\left[B'_{CSP}(B,\varepsilon),\varepsilon'\right] \left.\frac{\partial q_{CSP}(B',\varepsilon')}{\partial B'}\right|_{B'=B'_{CSP}(B,\varepsilon)} \right] + \mu_{CSP}(B,\varepsilon).$$

As in the three-period model, $\partial q_{CSP}(B',\varepsilon')/\partial B'$ appears in the Euler equation.

Next, we consider the decentralized economy with the tax on debt. The period-by-period budget constraint with tax on debt is:

$$c + \frac{b'}{(1+\tau)R} + q^{**}(B,\varepsilon)k + T = b + q^{**}(B,\varepsilon)\bar{K} + \varepsilon k, \tag{28}$$

where T is the lump-sum transfers satisfying $T = \tau b'/\left[(1+\tau)R\right]$ and $q^{**}(B,\varepsilon)$ is the asset price function in the decentralized economy with the tax on debt. This debt tax introduces a wedge into

the household's first-order condition for b':

$$u'(c) = (1 + \tau) \left\{ \beta R \mathbb{E} \left[u'(c') \right] + \mu^{**} (B, \varepsilon) \right\}, \tag{29}$$

where $\mu^{**} (B, \varepsilon)$ denotes the Lagrange multiplier under the decentralized economy with the debt tax τ.

Finally, we compare (29) with (27) to obtain the state-contingent Pigouvian tax that achieves the CSP's allocation. By equating $\mu^{**} (B, \varepsilon) = \mu_{CSP} (B, \varepsilon)$ for all B and ε, the government can choose the optimal tax rate $\tau^{**} (B, \varepsilon)$ for all states. The tax rate $\tau^{**} (B, \varepsilon)$ is:

$$\tau^{**} (B, \varepsilon) = \frac{\beta R \kappa \mathbb{E} \left\{ \mu'_{CSP} \left[B'_{SP} (B, \varepsilon), \varepsilon' \right] \partial q_{CSP} (B', \varepsilon') / \partial B' \right\}}{\beta R \mathbb{E} \left(u' \left\{ c_{CSP} \left[B'_{CSP} (B, \varepsilon), \bar{K}; B'_{CSP} (B, \varepsilon), \varepsilon' \right] \right\} \right) + \mu_{CSP} (B, \varepsilon)}, \tag{30}$$

where the partial derivative $\partial q_{SP} (B', \varepsilon') / \partial B'$ is obtained from the asset price function (26). It can be easily shown that $\tau^{**} (B, \varepsilon)$ ensures that $q^{**} (B, \varepsilon) = q_{CSP} (B, \varepsilon)$ for all states because the decision rule for the collateral assets is the same across the CSP's problem and the decentralized economy's problem. Therefore, the set of functions, $\tau^{**} (B, \varepsilon)$, $\mu^{**} (B, \varepsilon)$, and $q^{**} (B, \varepsilon)$ replicates the CSP's allocation, $c_{CSP} (B, \varepsilon)$ and $B'_{CSP} (B, \varepsilon)$, as the decentralized equilibrium.

3.5 The optimal clean policy

The other policy option for the government is to implement the optimal clean policy. As in the previous section, we first characterize the FB allocation. The FB allocation is obtained from the unconstrained competitive equilibrium, where the optimization problem consists of (16) - (18). The first-order conditions for the individual bond holdings and the collateral assets are:

$$u' \left[c_{FB} (B, \bar{K}; B, \varepsilon) \right] = \beta R \mathbb{E} \left(u' \left\{ c_{FB} \left[B'_{FB} (B, \varepsilon), \bar{K}; B'_{FB} (B, \varepsilon), \varepsilon' \right] \right\} \right) \tag{31}$$

$$q_{FB} (B, \varepsilon) = \beta \frac{\mathbb{E} \left(u' \left\{ c_{FB} \left[B'_{FB} (B, \varepsilon), \bar{K}; B'_{FB} (B, \varepsilon), \varepsilon' \right] \right\} \varepsilon' \right)}{u' \left[c_{FB} (B, \bar{K}; B, \varepsilon) \right]}, \tag{32}$$

respectively.

We then consider the government's policy instrument τ to achieve the FB allocation. The

period-by-period budget constraint is:

$$c + \frac{b'}{(1+\tau)R} + q^*\left(B,\varepsilon\right)k' + T = b + q^*\left(B,\varepsilon\right)\bar{K} + \varepsilon k, \tag{33}$$

where $q^*\left(B,\varepsilon\right)$ is the asset price function associated with the optimal clean policy. Together with (33), the equilibrium in the decentralized economy is characterized by the following first-order conditions:

$$u'(c) \quad = \quad (1+\tau)\left\{\beta R\mathbb{E}\left[u'(c')\right] + \mu^*(B,\varepsilon)\right\} \tag{34}$$

$$q^*(B,\varepsilon) \quad = \quad \frac{\beta\mathbb{E}\left[u'(c')\varepsilon'\right]}{u'(c) - \kappa\mu^*(B,\varepsilon)}$$

$$= \quad \frac{\beta\mathbb{E}\left[u'(c')\varepsilon'\right]}{\{1 - \kappa/(1+\tau)\}u'(c) + \kappa\beta R\mathbb{E}[u'(c')]} \tag{35}$$

$$0 \quad = \quad \left[\frac{B'}{R} + \kappa q^*\left(B,\varepsilon\right)\bar{K}\right]\mu^*(B,\varepsilon) \tag{36}$$

$$\mu^*(B,\varepsilon) \geq 0 \quad \text{and} \quad -B'/R \leq q^*\left(B,\varepsilon\right)\bar{K}. \tag{37}$$

The following proposition indicates that there exists the state-contingent optimal clean policy $\tau^*(B,\varepsilon)$ that achieves the FB allocation in the infinite horizon model.

Proposition 3 *Suppose the government adopts the clean policy $\tau^*(B,\varepsilon)$ such that*

1. *if $-B'_{FB}(B,\varepsilon)/\left(\kappa R\bar{K}\right) < q_{FB}\left(B,\varepsilon\right)$, then $\tau^*(B,\varepsilon) = 0$;*

2. *otherwise, $\tau^*(B,\varepsilon) = \bar{\tau}(B,\varepsilon)$, satisfying:*

$$\frac{-B'_{FB}(B,\varepsilon)}{\kappa R\bar{K}} = \frac{\beta\mathbb{E}\left(u'\left\{c_{FB}\left[B'_{FB}\left(B,\varepsilon\right),\bar{K};B'_{FB}\left(B,\varepsilon\right),\varepsilon'\right]\right\}\varepsilon'\right)}{\{1 - \kappa/\left(1+\bar{\tau}(B,\varepsilon)\right)\}u'\left[\left(c_{FB}\left(B,\bar{K};B,\varepsilon\right)\right] + \kappa\beta R\mathbb{E}\left(u'\left\{c_{FB}\left[B'_{FB}\left(B,\varepsilon\right),\bar{K};B'_{FB}\left(B,\varepsilon\right),\varepsilon'\right]\right\}\right)}.$$

Then, there exists an asset price function $q^*(B,\varepsilon)$ and a Lagrange multiplier $\mu^*(B,\varepsilon)$ that

18

achieve the first-best allocation, $c_{FB}(B, \varepsilon)$ and $B'_{FB}(B, \varepsilon)$, as the decentralized equilibrium charac-

terized by (33) - (37).

Proof. See Appendix A.2. ∎

This proposition implies that, under full knowledge of the policy and the price functions under the FB allocation, the policy maker who sets τ to $\tau^*(B, \varepsilon)$ can create the asset and shadow price functions $q^*(B, \varepsilon)$ and $\mu^*(B, \varepsilon)$ that are consistent with the FB allocation. We emphasize that the asset price function $q^*(B, \varepsilon)$ is not the same as $q_{FB}(B, \varepsilon)$. The basic idea is the same as Corollary 2 in Proposition 2: the asset price is inflated by the subsidy during the period of the binding collateral constraint. In the next section, we will numerically confirm that this is the case and discuss the quantitative implications.

4 Simulation results

4.1 Calibration

Our simulation exercise aims to understand the quantitative implications of the lean and clean policies in the infinite horizon model. To this end, we calibrated the parameters of our model mainly following Bianchi and Mendoza (2010), where a similar framework is employed. For the household's preference, $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$, with $\sigma = 2.0$. The discount factor β is set to 0.96, calibrating the model to the annual frequency. The constant and exogenous world gross interest rate, which effectively discounts the household's bond holdings in (15), is 1.028.[14] The ceiling of the household borrowing per collateral asset, which is denoted as κ, is set at 0.36 and the total supply of the collateral asset $\bar{K}$ is normalized to one. For parameterization, we assume that the total factor productivity in Bianchi and Mendoza (2010) can be translated into the stochastic process of the dividend ε_t in our model. Bianchi and Mendoza (2010) assume that the stochastic process of the total factor productivity follows a log-normal AR(1) process. We assume the same stochastic

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[14] As discussed in the footnote 13, our computation replaces the interest rate in the budget constraint with $R_{t+1} = \bar{R} + \psi \left[\exp \left(\bar{b} - b_{t+1} \right) - 1 \right]$ and the interest rate in the collateral constraint with $\bar{R}$. In our calibration, we set $\bar{R} = 1.028$, $\bar{b} = 0$, and $\psi = 0.01$.

process for ε_t as theirs: $\log(\varepsilon_{t+1}) = \rho \log(\varepsilon_t) + \eta_{t+1}$, where $\eta_t \sim N(0, \sigma_\varepsilon)$ for all t and ρ and σ_ε are set to 0.53 and 0.014, respectively.[15]

We confirm that our model-based moments are broadly in line with the previous studies which calibrate their models to the US or to emerging countries. Although we do not directly target the debt-to-GDP ratio and the crisis probability in parameterizing β and κ, the model performs remarkably well in replicating these moments compared with the data. For example, bearing in mind that GDP in our model can be interpreted as $\varepsilon_t \bar{K}$, we find that the mean of the debt-to-GDP ratio in the LF economy is 33.5 percent, which is in close proximity to the calibration target in the literature.[16] We also compute the probability that the collateral constraint binds and a decrease in the credit exceeds one standard deviation of the credit growth (the first-difference of credit). This probability is 2.3 percent, only slightly lower than the 3.0 percent targeted in Bianchi and Mendoza (2010).[17]

4.2 Numerical results on the optimal lean and clean policies

Figure 1 plots the optimal tax and subsidy on debt against B_t. In the figure, the solid red and blue lines represent the optimal lean policy $\tau^{**}(B, \varepsilon)$ and the optimal clean policy $\tau^*(B, \varepsilon)$, respectively. Each line shows how the policy responds to the aggregate debt when the dividend ε_t is kept at the mean. The dotted lines point to the cases of the low dividend, where the log-dividend takes a value of minus two standard deviations from the mean. As the figure indicates, the optimal lean policy corresponds to the debt tax $(\tau^{**}(B, \varepsilon) \geq 0)$, whereas the optimal clean policy has the opposite sign, indicating the debt subsidy $(\tau^*(B, \varepsilon) \leq 0)$.

Apart from the sign of τ_t, Figure 1 clearly points to the notable difference in the size of the two types of interventions. Taking the example of the low-dividend state in the figure, we note that τ_t^{**} takes a value of 1.9 percent at the maximum for $B_t = -0.345$. In contrast, the blue dashed line shows that τ_t^* amounts to 19.5 percent at the same state of B_t and ε_t. The unrealistically large size

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[15]The solution method is the policy function iterations employed by Bianchi and Mendoza (2010). In solving the models, we discretize the state variables B_t and ε_t to 100 and 13 grids, respectively.

[16]Bianchi and Mendoza (2010) target the mean of the debt-GDP ratio at 38 percent in calibrating the model to the US economy. Benigno et al. (2012), who calibrated the model to the Mexican economy, target it at 35 percent.

[17]This probability is defined as the crisis probability in Bianchi and Mendoza (2010). Later, we will provide our own definition of the crisis probability, using the previous empirical studies.

suggested by the optimal clean policy is highlighted even more if we examine the average size of the tax and the subsidy in stochastic simulations. Table 1 summarizes the computational results, including the averages and standard deviations of the variables of interest, as well as the average rate of the debt tax/subsidy across the optimal lean and clean policies. The table clearly indicates that the optimal clean policy requires a large-scale bailout that reduces the cost of borrowing for the domestic debtors by, on average, 66.5 percent.

Table 1 indicates that the average debt position, as a result of the massive interventions, amounts to 0.47, which is more than 40 percent higher than that in the LF economy.[18] Despite the collateral constraint, such high debt positions are made attainable and sustainable under the optimal clean policy because the government's bailout effectively removes the collateral constraint by inflating the asset prices. In fact, the third row of the table indicates that the average asset price is 1.27, which is significantly higher than the 0.96 prevailing in the LF economy. Table 1 also reports that, under the optimal clean policy, while consumption remains largely at the same level as that in the LF economy, the volatility is significantly reduced. This is because the optimal clean policy promotes consumption smoothing by more flexibly changing the debt position to absorb income shocks.

In contrast, in the economy with the optimal lean policy, allocations, including the low tax rate itself, remain close to those in the LF economy. The average tax rate is only 0.26 percent. The other variables remain essentially the same as in the LF economy, while the households are slightly less indebted, which reduces the probability of crises. Reflecting the similar allocations across the two economies, the welfare improves only moderately under the optimal lean policy.

Figure 2 shows policy functions for the bond holdings and the equilibrium asset price functions, given ε_t at a low level. We note the following two points. First, across the two panels, we can confirm that, as widely discussed in the literature, the policy functions and the asset pricing functions in the LF economy and in the economy with the optimal lean policy exhibit sharp bends. The kinks in the functions indicate that, if the households' debt exceeds a certain level, a crisis takes place and the households need to accept a large contraction of their borrowing and consumption. Second, in

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[18]We compute the moments from the numerically approximated ergodic distribution. In particular, we take the time-series average and standard deviations of variables across 300 periods and then take the cross-sectional average across 500 simulations.

the upper panel, the policy function for B_{t+1} under the economy with the optimal clean policy (the blue solid line in the upper panel) fully replicates that under the unconstrained FB economy (the black dashed line), which confirms Proposition 3. In the lower panel, the asset-pricing function under the optimal clean policy (the blue solid line in the lower panel) reconfirms the proposition as well: as long as the households' debt falls short of a certain level of B_t, the asset price function $q^*(B, \varepsilon)$ replicates $q_{FB}(B, \varepsilon)$. If the households' debt exceeds a certain level, however, the clean policy is activated and the asset price is inflated beyond the level suggested by $q_{FB}(B, \varepsilon)$.

Comparisons of the policy functions for bond holdings and the equilibrium asset price functions with the optimal lean and clean policies to those in the LF economy give us a clue to understand why the size of the interventions varies markedly across the two policies. The upper panel of Figure 2 indicates that the policy function under the optimal lean policy (the solid red line in the upper panel of Figure 2) is, overall, fairly close to that of the LF economy, while small deviations are observed around the kink.[19] The distance between the two policy functions is, on average, 0.5 percent in terms of B_{t+1}.[20] In other words, the policy maker, if he/she is about to carry out the optimal lean policy, is required to adjust the households' debt holdings only by a small amount. In a similar vein, the comparison of the policy function under the optimal clean policy (the solid blue line in the upper panel of Figure 2) with that of the LF economy provides insight into the size of the interventions. The distance between the policy functions, evaluated by the same above-mentioned measure, is 30.5 percent, on condition that the collateral constraint is binding (and 4.1 percent otherwise). The measured distance here suggests that the policy maker should embark on large-scale interventions to achieve the FB allocation in the form of the optimal clean policy.[21]

The final issue in relation to the optimal clean policy is that it requires frequent bailouts of

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[19]As pointed out by Bianchi and Mendoza (2010), this region of bond holdings is a "high-externality region" where the CSP should strongly rein in the households' risk taking on debt to correct overborrowing in the LF economy.

[20]To obtain the distance reported, we set ε to the value corresponding to the low dividend state and take the simple average of $|\ln[B'_{LF}(B, \varepsilon)/B'_{CSP}(B, \varepsilon)]|$ across various bond holdings ranging from -0.2 to -0.4.

[21]Our result regarding the massive size of the intervention indicated in Figures 1 and 2 may be somewhat understated, because we set the risk-premium parameter ψ addressed in the footnote 13 to 0.01, somewhat larger than the standard value in the literature. The use of the standard value would increase the households' debt position in the unconstrained economy and, thus, the debt subsidy under the optimal clean policy would be further increased in its magnitude, compared to the baseline parameterization. In this sense, the use of the standard value does not undermine, but supports, our findings that the size of the intervention is unrealistically large in the optimal clean policy.

debtors to allow them to enjoy consumption at the FB level. In our stochastic simulations, the collateral constraint is almost always binding in the economy under the optimal clean policy, due to high debt positions. The reason is that, as long as the government aims to achieve the FB allocation through the clean policy, activated correctly in terms of its size and timing, households do not worry about making reductions in consumption due to the binding collateral constraint. Even if the households take high risks in relation to the binding collateral constraint, they are fully insured by the government's state-contingent clean policy. Considering that a number of crises have unfolded throughout history, however, the idea that the FB allocation is always achieved by such a finely tuned clean policy would not be acceptable in reality. In the next section, we discuss what would be the consequence of the failed attempts to carry out the optimal clean policy, in comparison with the optimal lean policy.

5 Discussion: Empirical perspectives on crisis management

This section discusses how the full state-contingent interventions, including the two options to use lean or clean policies, could be compared with real-world experiences. From an empirical viewpoint, we ask, (i) whether the optimal clean policy has been carried out in a real economy; if not, we further discuss (ii) why the optimal clean option is hard to implement for the policy makers, and we consider what impediments would prevent them from carrying out the optimal intervention; more importantly, based on the consideration of impediments and practical constraints, we use our model to answer the following two questions: (iii) what would be the consequence in terms of the probability of a financial crisis if the size of the clean policy is too small in comparison to the optimal clean policy? And (iv) what would be the welfare implications if the state-contingent policy intervention is poorly timed in terms of the response to the state of the economy? To wrap up this section, we evaluate the practical advantage of the lean option compared to the clean option.

Regarding the first question, perhaps it would be quite acceptable without relying on formal empirical analysis to believe that the optimal policy has rarely been carried out in a real economy. In fact, a number of financial crises, which cannot take place in the presence of the optimal clean

policy have hit economies around the world over the past decades and centuries. In response to these crises, government bailouts using taxpayers' money to rescue troubled debtors during crisis periods are an equally common observation in history, implying that governments attempt to clean up such messes after or during the financial crises, but repeatedly fail to fully ward off the crises.

Given the fact that full-fledged financial crises, typically followed by sharp contractions in overall economic activity, have repeatedly taken place in the real world, Question (ii) is a natural point of departure. Unquestionably, there are a number of disturbances that could forestall the best practice of bailout interventions in particular, such as political conflict, uncertainties regarding the hard-to-estimate size of the rescue packages required, and, similarly, the hard-to-recognize timing of the activation of such bailout plans. Putting all these issues together, in this section, we discuss to what extent the model that we introduced in the previous section can explain the observations of financial crises and the remedial measures. Based on the discussion, we aim to derive insights into the lean vs. clean debate to better prevent crises or bail out troubled debtors.

5.1 Why is the optimal clean policy hard to implement?

The quantitative results articulated in the previous section provide important insights into why the optimal clean policy is difficult to implement in real economies. In particular, we compare the size of the optimal clean interventions simulated in Section 4 with those of the actual bailout interventions carried out in real economies. For the comparisons, we take empirical estimates reported in a comprehensive study by Frydl (1999). An empirically realistic size for bailouts, as estimated by Frydl (1999), would normally amount to somewhere between 1 to 4 percent of GDP for a single year in the aftermath of a crisis.[22] By contrast, the optimal clean-type intervention requires an unrealistically massive fiscal budget equal to 31.8 percent of GDP ($\varepsilon \bar{K}$) on average. Clearly, there is a huge gap between the optimal prescription suggested by the model and the policies that have been carried out in real economies. In this context, an answer to the question

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[22]Frydl (1999) picks up five recent empirical works estimating (i) fiscal costs and (ii) the length of financial crises, and he chooses two of these works for his baseline estimations. The two empirical papers he chooses argue, respectively, that the fiscal cost is 13.6 percent (7.2 percent) of GDP and that crises last 4.5 years (6.2 years) on average. We compute the average fiscal cost for a single year by dividing average total fiscal costs by the average length of financial crises, suggesting that the average fiscal cost for a single year ranges from 1.16 (=7.2/6.2) to 3.02 (=13.6/4.5) percent of GDP.

why the optimal clean policy is hard to implement is simply the government's inability to raise such a massive amount of money to finance the bailouts. This view simply suggests that there could be some "too-big-to-save" crises in the real world because of some costs or impediments such as fiscal issues and/or political pressures that have been left out of the model.

5.2 What if the size of the clean policy is too small?

If the government had inadequate fiscal resources to bail out troubled debtors, as a natural consequence, the FB allocations could not be achieved. We note that this is not the end of the bad news, but rather a beginning. A government lacking in resources would not only be inadequate from the perspective of its own needs, but would also reduce welfare substantially. In particular, the expectations of such inadequate bailouts raise the probability of a crisis compared to the LF economy. As a point of departure, we reassess the LF economy. We then ask how frequently a crisis would take place if a bailout that is "too small" (i.e., less than optimal) is expected in the same economy. To quantify a less-than-optimal clean policy in the experiment, we assume that the government employs the policy instrument as follows:

$$\tilde{\tau}^*(B, \varepsilon) = \Delta_1 \tau^*(B, \varepsilon).$$

Here, the government commits to a debt subsidy and $\tilde{\tau}^*(B, \varepsilon)$ is commonly known to all agents in the economy. The parameter Δ_1 is calibrated to the actual size of the bailout interventions during financial crises. Specifically, we set Δ_1 to 0.2 and, as a result, the average size of the intervention in the experiment is kept at 2.2 percent of GDP, which lies in the range of the estimated fiscal cost shown by Frydl (1999).

With the less-than-optimal bailout in place, the individual households take on higher risks because they all know that each of them cannot affect the probability of being bailed out. For each household, borrowing more is a rational choice because, otherwise, households give up returns from more consumption yet remain exposed to a higher crisis risk. In this regard, expectations of an insufficiently large bailout would exacerbate the coordination failure among the debtors/households

and, as a result, would amplify the credit externality.

To compute the crisis probability in our policy experiment, we simply identify a "crisis period" as one in which the aggregate consumption declines more than 4 percent in that period. We carefully pick this number considering the following factors. From the viewpoint of the calibration and simulations, this specific definition involves a small supplement to the definition that we introduced in Section 2: the binding collateral constraint and a substantial reduction in consumption. We note that, if the aggregate consumption decreases more than 4 percent in the model, the collateral constraint is always binding. Hence, simply looking at the periods when consumption decreases more than 4 percent sensibly identifies the crisis periods in our model. Further, this quantification, the 4 percent decrease in consumption, is broadly in line with the estimates shown in a variety of empirical studies.[23]

With this specific definition, the crisis probability can be computed easily in simulations. First, we note that, under the baseline calibration, the crisis probability is zero in the FB economy. This result reconfirms that consumption does not decrease by more than 4 percent as long as the borrowing constraint is not binding. In the LF economy, a financial crisis takes place with a probability of 1.7 percent. Our focus is on how much this probability would be raised if an insufficiently large bailout is expected. The simulation result of the policy experiment shows that financial crises that erode consumption by more than 4 percent take place 7.6 times in every 100 periods. The predicted frequency of a crisis in this case with a "too-small bailout " is more than double that in the LF economy. As we have noted, the results indicate that households are incentivized to be more leveraged and take undue risks in a booming period, but they are not provided with bailouts large enough to fully ward off financial crises.

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[23]For example, Figure 7a in Reinhart and Rogoff (2008) effectively suggests that the detrended output growth declines, on average, by 4 to 5 percent in the aftermath of a banking crisis. In a more recent context, the International Monetary Fund (2011) indicated that outputs in advanced economies decreased by 3 to 5 percent in 2009 in the wake of the global financial crisis. We also note that Barro and Jin (2011) suggest that output and consumption tend to decrease proportionally in crisis periods.

5.3 The case of a poorly timed policy intervention

As well as the issue of insufficient resources for government bailouts, policy makers need to recognize under what conditions such optimal clean policy could be enacted. That is, to carry out the optimal policy, policy makers need to recognize the state-variable vector, which is represented by (B, ε) in the model. In practice, however, such perfect recognition of the state variable vector on a real-time basis may be questionable, owing to, for example, lagged publication of statistics or noise in the data.

Given the limited ability to recognize the state of the economy, a practical question for policy makers is what would happen if the clean and lean policies are preemptively activated with the wrong timing in terms of both the indebtedness (B_t) and the productivity (ε_t) of the economy. For example, policy makers may wonder what would happen if a clean-type intervention is activated in the state where (i) the household is not heavily indebted, and/or (ii) a sharp contraction in output is yet to materialize. In order to understand the consequence of the poorly timed policy interventions, including a clean policy that is implemented "too early,' we run another policy experiment using our calibrated dynamic model. In this policy experiment, we redefine the policy instrument $\tilde{\tau}^*(B, \varepsilon)$ as:

$$\tilde{\tau}^*(B, \varepsilon) = \Delta_1 \tau^*(B - \Delta_2, \varepsilon),$$

to investigate the poorly timed clean policy. Here, in this experiment, we are considering possible misrecognition of the timing represented by Δ_2 as well as the size of the policy intervention denoted by Δ_1. If $\Delta_2 < 0$, then the policy is activated "too early," whereas if $\Delta_2 > 0$, then the policy is carried out "too late."[24] The size of the policy intervention is also adjusted by $\Delta_1 = 0.2$ because, otherwise, the unrealistically large fiscal resources are required. For comparison, we similarly define the policy instrument for the lean policy as $\tilde{\tau}^{**}(B, \varepsilon)$ given by $\tilde{\tau}^{**}(B, \varepsilon) = \Delta_1 \tau^{**}(B - \Delta_2, \varepsilon)$. In simulations for the lean option, however, we set $\Delta_1 = 1$ because the optimal lean policy is not infeasible in the terms of the size of the intervention.

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[24] Note that because $\tau^*(B, \varepsilon)$ is increasing with respect to both B and ε, $\tilde{\tau}^*(B, \varepsilon)$ can be interpreted as the policy that is activated for an inappropriate timing with respect to ε as well.

Figure 3 shows the results of the experiments in terms of the economic welfare gain/loss relative to the LF economy for a variety of cases where poorly timed clean and lean policies are employed. The horizontal axis is Δ_2, which represents the magnitude of the misrecognition by the policy makers. The blue and red lines show the results for the poorly timed clean and lean policies, respectively. The results can be summarized as follows. First, clean interventions that are "too early" give rise to large welfare losses compared to the LF economy because of the same reason discussed in Section 5.2. We can reconfirm that, even if the clean intervention is implemented in a timely manner (i.e., $\Delta_2 = 0$), it would result in a large welfare loss mainly due to the increased crisis probability. Because of the excess risk taking of households, ensuing financial crises are noticeably exacerbated compared to the LF case in the absence of such bailouts. Second, if the clean policy is activated "too late," it still improves welfare relative to the LF economy, but the welfare gain diminishes to zero if activation is considerably delayed. Moreover, if the government is not able to collect money for the clean policy through a non-distorting tax such as a lump sum tax, the clean policy would cause a tax distortion, possibly reducing the welfare gain under the clean policy to negative. Third, the lean policies broadly tend to improve welfare unless they are activated "too late." The "too late" lean interventions reduce welfare relative to the LF economy simply because they penalize debtors by increasing the burden of the debt exactly when the debtors require reductions in their debt burden. Quantitatively, the welfare loss in the case of the too late lean interventions is rather small compared to that for the clean policies that are "too early," implying that the lean option tends to be more robust against wide-ranging mistimed policy activations.

5.4   Lean vs. clean debate: Recap

To what extent are the experiments relevant in the light of the recent experience of the global financial crisis of 2007–2008? While there are many potentially comparable experiences of poorly timed interventions, the case involving the federal funds rate between 2002 and 2006 may be a good example of a clean option that is "too early" as argued by Taylor (2009). Figure 1 in Taylor (2009) shows that the actual federal funds rate during this period was slightly lower than that predicted by the standard Taylor rule. Given that the standard Taylor rule indicates the desirable levels of

28

borrowing costs, this means that, during the period in question, debtors were effectively subsidized by a low interest rate. With hindsight, this case can be considered an example of "too- early" policy activation because such a low cost of debt in the booming period could work as subsidy to debtors, who were only moderately, not heavily, indebted at that point in time, and hence it could increase the probability of financial crises.[25] These policy episodes provide a prime example of the pursuit of an optimal clean policy producing catastrophic outcomes.

We note the general point from Figure 3: broadly speaking, the computational results in the presence of policy errors indicate that the lean policy tends to be resilient against the problems involving a variety of misrecognitions of the state of the economy and a lack of fiscal resources. As already discussed in Section 4.2, the size of the optimal lean policy appears to be reasonable compared with the real world episodes and practically realistic budgetary resources. On the other hand, attempts to carry out the optimal clean policy run a higher risk because, if it is enacted too early, such a poorly timed intervention could easily result in catastrophic outcomes. The clean policy, if pursued, requires intricate fine tuning and large fiscal resources to deal with the crisis. Only if such fine tuning is possible and a huge fiscal measure is acceptable, would the optimal clean policy achieve the FB outcome, as demonstrated in Section 3.

6 Conclusion

In this paper, we discuss the lean vs. clean policy debate in dealing with financial crises based on dynamic general equilibrium models with an occasionally binding constraint. First, using a three-period perfect foresight model, we show that it is possible that a full state-contingent clean policy per se, without relying on any policy mix, achieves the FB allocation by forestalling deleveraging in a crisis period. While this result appears to favor the clean policy against the lean policy, further assessment reveals that the optimal clean policy requires an unrealistically large intervention using taxpayer's money. Our calibration exercise suggests that the FB allocation may not be achievable by the clean policy, from a practical viewpoint. Moreover, our policy experiments, assuming that

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[25]Taylor (2009) argues that "...this extra easy policy was responsible for accelerating the housing boom and, thereby, ultimately leading to the housing bust."

policy makers can only intervene with insufficient resources, show that the clean policy is prone to incentivize debtors to take on more risks in a noncrisis period and, as a result, may give rise to a large welfare loss because of the elevated probability of crises. We also find that, if policy makers cannot precisely recognize the state of the economy and, consequently, they do not activate the clean policy in a timely manner, then the clean policy is subject to risks of massively destabilizing the economy. Given such an undesirable consequence of the pursuit of the clean policy, it is worth positively evaluating the lean policy from a practical viewpoint.

Finally, a few caveats should be noted. First, the overall result does not necessarily invalidate the effectiveness of the clean policy intervention during a crisis period. Even though the welfare gain provided by the (suboptimal) clean option would be small relative to its risks in our model, the clean policy could have other favorable effects, which are not modeled here. For example, the clean option may serve as liquidity provision to financial and nonfinancial firms during financial crises because lowering the cost of debt makes it easy for firms to hold ample liquidity at a low cost. Accordingly, such a policy could mitigate the liquidity shortage and prevent the exacerbation of financial crises. Second, to facilitate policy in practice, we need to investigate which specific policy measure(s) should be chosen to adjust the cost of debt in practice. In particular, whether policy makers should rely on monetary policy or on macroprudential policies, including financial regulations, remains an open question. We leave these issues to future work.

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A Appendix

A.1 Proof of Proposition 2

The proof is straightforward. Under the intervention being included, the first-order conditions are:

$$u'(c_0) = u'(c_1) \tag{38}$$

$$\frac{u'(c_1)}{1 + \tau_1} = u'(c_2) + \mu_1 \tag{39}$$

$$q_1 = \frac{\varepsilon_2}{[1 - \kappa/(1 + \tau_1)] \, u'(c_1)/u'(c_2) + \kappa}. \tag{40}$$

It immediately follows from (38) that $c_0 = c_1$. Then, (2), and (13) together with $T_1 = \tau_1 b_2/(1 + \tau_1)$ and $k_t = 1$ in equilibrium imply that $c_0 = c_1 = (\varepsilon_1 - b_2)/2$. Under the full consumption smoothing, consumption in period 2 is $c_2 = b_2 + \varepsilon_2 = (\varepsilon_1 - b_2)/2$. The collateral constraint, however, implies that the asset price must be inflated to:

$$q_1 = \frac{2\varepsilon_2 - \varepsilon_1}{3\kappa} > \varepsilon_2 \tag{41}$$

for a sufficiently small $\kappa \ (< (2\varepsilon_2 - \varepsilon_1)/(3\varepsilon_2))$. On the other hand, the asset price must satisfy the asset-pricing formula (40) under the full consumption smoothing, $u'(c_1) = u'(c_2)$:

$$q_1 = \frac{\varepsilon_2}{1 - \kappa/(1 + \tau_1) + \kappa}. \tag{42}$$

Eliminating q_1 from (41) and (42) and solving for τ_1 yields the optimal subsidy on debt that achieves the FB allocation:

$$1 + \tau_1^* = \frac{1}{1 + 1/\kappa - 3\varepsilon_2/(2\varepsilon_2 - \varepsilon_1)},$$

where τ_1^* is strictly negative because $\kappa < (2\varepsilon_2 - \varepsilon_1)/(3\varepsilon_2)$ implies that the denominator on the right-hand side of the equation is strictly positive. ∎

A.2 Proof of Proposition 3

First, we examine the state (B, ε) satisfying:

$$\frac{-b'_{FB}(B, \bar{K}; B, \varepsilon)}{\kappa R \bar{K}} < q_{FB}(B, \varepsilon).$$

In this case, if we set $\mu^*(B, \varepsilon) = 0$ and $q^*(B, \varepsilon) = q_{FB}(B, \varepsilon)$, then (36) - (37) are obviously satisfied under the FB allocation. By setting $\tau^*(B, \varepsilon) = 0$, (34) and (35) are identical to the first-order conditions in the unconstrained economy, (31) and (32).

Next, we examine the state (B, ε) satisfying:

$$\frac{-b'_{FB}(B, \bar{K}; B, \varepsilon)}{\kappa R \bar{K}} \geq q_{FB}(B, \varepsilon).$$

If we set:

$$q^*(B, \varepsilon) = \frac{\beta \mathbb{E}\left(u'\left\{c_{FB}\left[B'_{FB}(B, \varepsilon), \bar{K}; B'_{FB}(B, \varepsilon), \varepsilon'\right]\right\} \varepsilon'\right)}{\{1 - \kappa/[1 + \tau^*(B, \varepsilon)]\}\, u'\left[(c_{FB}(B, \bar{K}; B, \varepsilon)\right] + \kappa \beta R \mathbb{E}\left(u'\left\{c_{FB}\left[B'_{FB}(B, \varepsilon), \bar{K}; B'_{FB}(B, \varepsilon), \varepsilon'\right]\right\}\right)},$$

then (35) is satisfied by definition. When we set $\tau^*(B, \varepsilon) = \bar{\tau}$, as in the statement of the proposition, the collateral constraint binds, meaning that (36) is satisfied. Finally, if we select a $\mu^*(B, \varepsilon)$ so that it satisfies (34), then we have:

$$
\begin{aligned}
\frac{\beta \mathbb{E}\left(u'\left\{c_{FB}\left[B'_{FB}(B, \varepsilon), \bar{K}; B'_{FB}(B, \varepsilon), \varepsilon'\right]\right\} \varepsilon'\right)}{u'\left[c_{FB}(B, \bar{K}; B, \varepsilon)\right] - \kappa \mu^*(B, \varepsilon)} &= q^*(B, \varepsilon) \\
&= \frac{-b'_{FB}(B, \bar{K}; B, \varepsilon)}{\kappa R \bar{K}} \\
&\geq q_{FB}(B, \varepsilon) \\
&= \frac{\beta \mathbb{E}\left(u'\left\{c_{FB}\left[B'_{FB}(B, \varepsilon), \bar{K}; B'_{FB}(B, \varepsilon), \varepsilon'\right]\right\} \varepsilon'\right)}{u'\left[c_{FB}(B, \bar{K}; B, \varepsilon)\right]}.
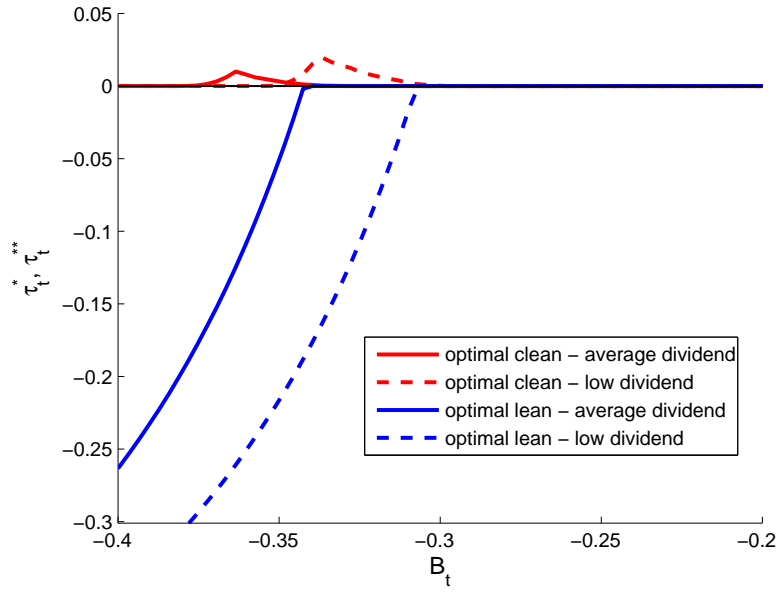\end{aligned}
$$

implying that $\mu^*(B, \varepsilon) \geq 0$. ∎

Table 1: The average and standard deviations of variables

	Laissez-faire economy	Optimal lean policy	Optimal clean policy
Debt position	0.335	0.331	0.470
	(0.016)	(0.016)	(0.046)
Asset price	0.962	0.962	1.270
	(0.018)	(0.016)	(0.122)
Consumption	0.990	0.990	0.985
	(0.015)	(0.014)	(0.009)
tax/subsidy (%)	-	0.26	-66.51

Note: The numbers in each entry report the average and the standard deviations of the debt position, the asset price, and consumption in the LF economy, the economy with the optimal lean policy, and the economy with the optimal clean policy. The numbers without the parentheses refer to the average of variables and the numbers in parentheses below the average are the standard deviations of the corresponding variables. Together with the moments, the second and third columns report the optimal debt tax τ_t^{**} or the optimal debt subsidy τ_t^*.
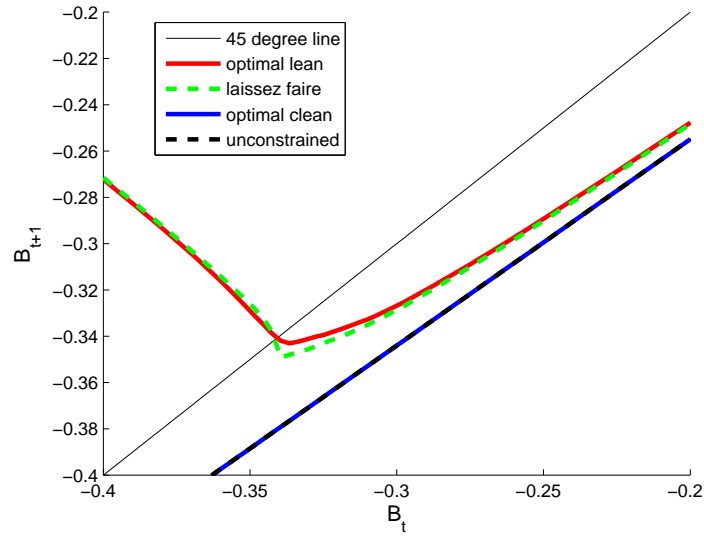
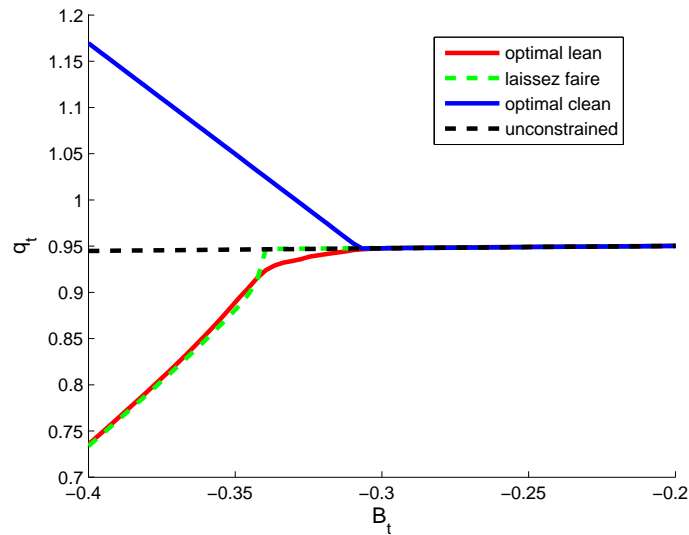Figure 1: Tax/subsidy under the optimal lean policy and the optimal clean policy

Note: The solid lines plot the tax/subsidy against B_t when the log-dividend takes the mean value. The dashed lines represent the tax/subsidy when the log-dividend takes a value of minus two standard deviations from the mean.

Figure 2: Policy functions for bond holdings and asset price functions
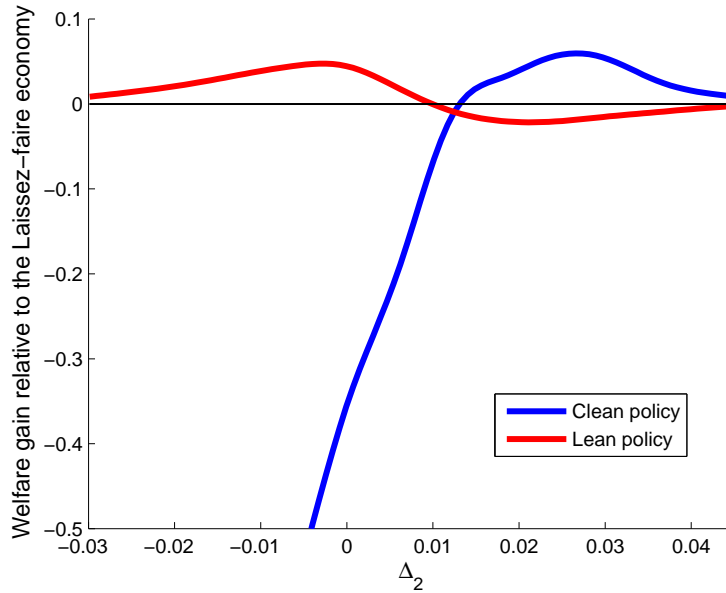
(a) Bond holdings

(b) Asset price

Note: The upper panel plots the policy functions for the next period's bond holdings B_{t+1} while the lower panel plots the asset prices q_t. The solid lines represent the functions under the economy with the optimal lean (red) and clean (blue) policies. The dashed green line is the function for the laissez-faire economy while the dashed black line is for the unconstrained economy.

Figure 3: Average welfare gains/losses for mistimed policies

Note: Lines represent the average welfare gains/losses relative to the laissez-faire economy when the government's policy deviates from the optimal policy. On the blue line, the clean policy is activated based on $\tilde{\tau}^*(B,\varepsilon) = \Delta_1 \tau^*(B - \Delta_2, \varepsilon)$. On the red line, the lean policy is activated based on $\tilde{\tau}^{**}(B,\varepsilon) = \Delta_1 \tau^{**}(B - \Delta_2, \varepsilon)$. For each line, the welfare gains/losses are plotted against Δ_2, which measures how much the policy misrecognizes the aggregate bond holdings. To replicate the realistically sized intervention, $\Delta_1 = 0.2$ for the clean policy and $\Delta_1 = 1.0$ for the lean policy. To compute the welfare gains/losses, we simulate 500 consumption paths over 300 periods and compute the relative welfare gain from $(V_{Policy} - V_{LF}) \times 100$, where V_{Policy} is the welfare under the $\tilde{\tau}^*(B,\varepsilon)$ or $\tilde{\tau}^{**}(B,\varepsilon)$. The average welfare gains/losses are computed by taking the cross-sectional average over 500 simulations. In each simulation, the steady state values are used for the initial value.