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International Recessions

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Abstract

The 2008-2009 crisis was characterized by an unprecedented degree of international synchronization as all major industrialized countries experienced large macroeconomic contractions. Countries also experienced large and synchronized contractions in the growth of financial flows. In this paper we present a two-country model with financial markets frictions where credit-driven recessions can explain these features of the recent crisis. A credit contraction can emerge as a self-fulfilling equilibrium caused by pessimistic but fully rational expectations. As a result of the credit contraction, in a financially integrated world, countries experience large and, endogenously synchronized, declines in asset prices and economic activity (international recessions).

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1 Introduction

During the 2007/2009 crisis all major industrialized countries experienced extraordinarily synchronized and extraordinarily large contractions in real and financial aggregates. The main objective of this paper is to provide an explanation for these two features of the crisis: its international dimension (both real and financial), and its depth.

Our hypothesis is that multiple self-fulfilling equilibria in credit markets can explain these features. We will make our point developing a stylized two-country incomplete-markets model where credit is used by firms both to pay dividends to owners and to finance hiring. For this reason credit markets are linked to labor markets and therefore disturbances in credit markets (i.e. tighter credit constraints) have adverse repercussions in the labor markets. We then go on to present our main theoretical contribution, that is to show that tighter/looser credit constraints can emerge endogenously as different self-fulling equilibria: in "bad" equilibria firms are financially constrained and economic activity, intermediation and asset prices are depressed; in "good" equilibria firms are un-constrained and economic activity, intermediation and asset prices are high. In a closed economy there is little difference between endogenous and exogenous credit shocks but in two financially integrated economies the two constructs have very different implications. Exogenous credit shocks imply strong co-movement of real activity but not necessarily co-movement of intermediation; endogenous shocks on the other hand always imply a strong co-movement of both real activity and intermediation. Since we document that empirically during the 2007-2009 crisis there has been strong co-movement along both dimensions, the theory of credit shocks emerging endogenously as different equilibria seems better suited to explain recent events. In other words we propose a theory of endogenous correlation of credit shocks which seems well suited to explain many features of "international recessions" like the one of 2007-2009. Modeling the shocks as an endogenous process has also important policy implications as it suggests that changes in structural features of the economy, such us financial integration or the public provision of liquidity, can change the volatility and the correlation of shocks (usually taken as exogenous variables) and hence the vulnerability of economies to credit shocks.

Our second contribution relates to the depth of the crisis. We argue that an "ordinary" credit shock can indeed generate a "extra-ordinary" recession like the current crisis. In order to show that this is the case we study a version of our model with occasionally binding constraints and show that there exist an equilibrium path in which credit constraints are not binding for a long time and, as a result, both economies undergo a long lasting expansion both in economic activity (gradual) and credit (rapid); if constraints become binding after this long expansionary phase, firms are forced to under-go a large de-leveraging which reduces the amount of credit they have available for hiring and thus causes a sharp recession, even if the possibility of the constraints becoming binding is fully anticipated by firms and households. We argue that this asymmetry between the expansion phase (with fast growing credit growth and mild real growth) and the recession (with sharp collapse in both economic activity and credit) captures well the macroeconomic developments of advanced economies during the recent cycle.

One important observation concerning the international dimension of the recent crisis is that although real GDP decline about the same amount in US and in the rest of the G7 countries, employment was hit particularly hard in the US but not in the remaining G7 countries (see Ohanian, 2010). As a consequence labor productivity soared in US but declined in the rest of the G7. Our baseline model with integrated credit markets and symmetric labor markets cannot explain this cross country difference. In the final part of the paper we argue that is not necessarily a problem of credit shocks but rather of how we model labor markets. We do so by introducing a very stylized asymmetry in labor markets (more flexibility in US and less flexiblity in the G6) in our baseline set-up: in this case credit shocks have the potential to explain both the symmetric behavior of GDP and the asymmetric behavior of employment (and labor productivity).

Our paper is related to the vast literature (both empirical and theoretical) studying the sources of macroeconomic co-movement and international transmission of shocks. Usually co-movement is explained as the result of synchronized disturbances (global or common shocks, see for example Crucini, Kose and Otrok, 2011) and/or as the result of country-specific shocks that spill to other countries (international transmission of country specific shocks). In this paper we show that credit shocks generate co-movement for both reasons: exogenous credit shocks spill-over from one country to the other, and endogenous credit shocks will appear to the econometrician like a common-shock or a global factor. This finding is consistent with the empirical results of Helbling, Huidrom, Kose & Otrok (2010) according to which credit market shocks matter in explaining global business cycles, especially during the 2009 global recession. Recent contributions that analyze directly the strong international co-movement during the 2007-2009 crisis include Dedola & Lombardo (2010), Devereux & Yetman (2010) and Enders, Kollmann & Muller (2010). All these studies focus on the international transmission of shocks in models with financial market frictions and they do not consider the possibility of an endogenously generated common credit shock.

The role of credit shocks and in particular of tightening credit constraints for macroeconomic fluctuations has recently, not surprisingly, been extensively studied (See, for example Gertler and Kiyotaki 2009, Jermann and Quadrini 2009, Goldberg 2010, Khan and Thomas 2010, Lorenzoni and Guerrieri, 2010) but in a closed economy. Furthermore, while in those contexts credit shocks are purely exogenous, in our paper we provide a micro foundation for these shocks which is based on self-fulling expectations. In this respect there are some similarities with the multiple equilibria property of the model studied in Kocherlakota (2009) and with the idea of a liquidity crisis as a multiple equilibrium discussed in Lucas and Stokey (2011). In our model the multiplicity of equilibria derive from 'occasionally binding' enforcement constraints. This is another important difference between our paper and other studies that investigate the macroeconomic impact of financial shocks (for example, Christiano, Motto and Rostagno (2009) and Jermann and Quadrini 2009). Most of these contributions limit the analysis to equilibria with always binding constraints and the quantitative properties are studied using linear approximation techniques. In our model, instead, borrowing constraints are only occasionally binding and this is important to generate the asymmetry between long and gradual credit driven booms and sharp credit driven contractions. Mendoza (2010) also studies an economy with occasionally binding constraints but does not investigate the importance of financial shocks. Furthermore, by focusing on a small open economy, this study does not address the issue of international co-movement which is one of the central issues studied in our paper. Occasionally binding constraints are also central to Brunnermeier and Sannikov (2010) but the analysis is limited to productivity shocks in a closed economy.

The paper is organized as follows. Section 2 discusses macroeconomic evidence regarding the recent crisis. We then present our theoretical framework in steps. First, to develop intuition, section 3 presents the model with fixed capital and with exogenous credit shocks. Section 4 makes credit shocks endogenous and section 5 introduces capital accumulation with occasionally binding constraints. Section 6 presents the results of the model and section 7 shows the effect of modifying the assumption of symmetric labor markets. Section 8 concludes.

2 Macroeconomic evidence

In this section we first present some facts about international co-movement in different variables during the 2007-2009 crisis Figure 1 plots the GDP dynamics for the G7 countries in six of the most recent US recessionary episodes. In each panel we plot percent deviations for GDP of each country from GDP in the quarter preceding the start of the US recession (as dated by the NBER). Comparison of the bottom right panel of the figure with the other panels suggests how the 2007-2009 recession stands out both in terms of depth and in terms of macroeconomic synchronization among all G7 countries.

To make this point in a more general fashion in figure 2 we plot the average (with 2 standard

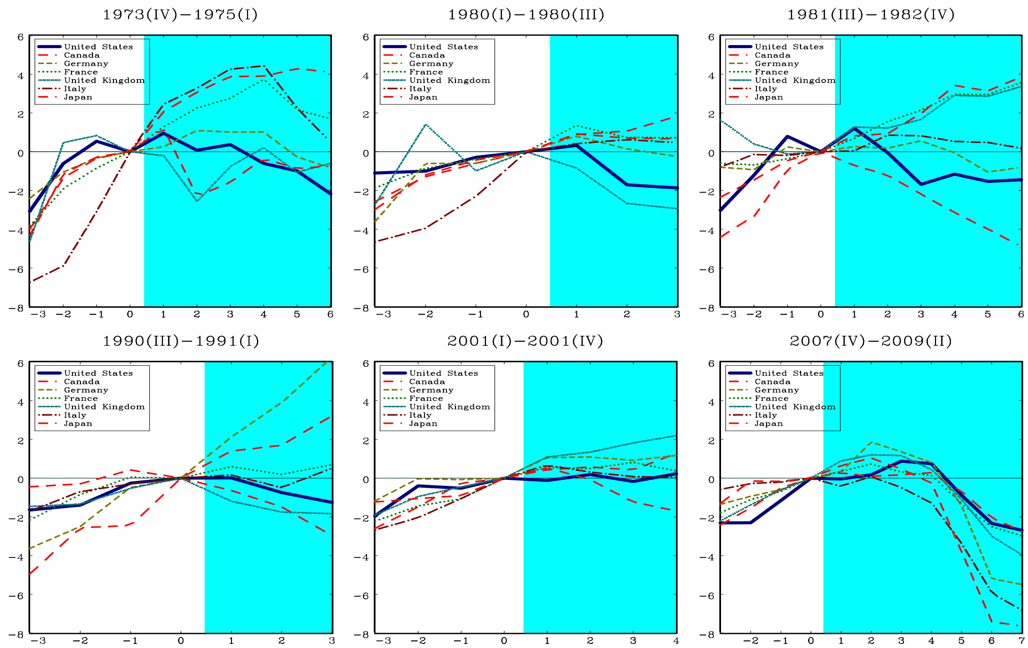


Figure 1: The dynamics of GDP during the six most recent recessions in the G7 countries.

deviations bands) of 10 years rolling windows pairwise correlations of quarterly GDP growth between all G7 countries. The dates in the graph correspond to the end points of the window used to compute the correlation. The figure shows that in 2007 the average correlation jumps from 0.3 to 0.7 and at the same time the sample standard deviation of the correlations falls from 0.19 to 0.09, confirming that the 2007-2009 stands out in the post-war as a period of extraordinarily high co-movement for all developed countries. For a similar point see also Imbs (2010).

The high degree of international co-movement between US and other major countries is also observed in other real and financial variables. In figure 3 we analyze the pattern of GDP, consumption, investment and employment in the period 2005-2010 for the US and an aggregate of the other countries in the G7 group (from now on G6). The pictures highlights how GDP consumption and investment are hit almost equally hard in US and in the G6. For employment, as noted for example by Ohanian, 2010, the picture is different. In US employment declines about 6% which is more than the 4% decline in GDP. This implies that in US labor productivity during this recession is counter-cyclical. On the contrary in the G6 employment falls but much less than GDP, hance labor productivity is counter-cyclical. In a sense employment/and labor

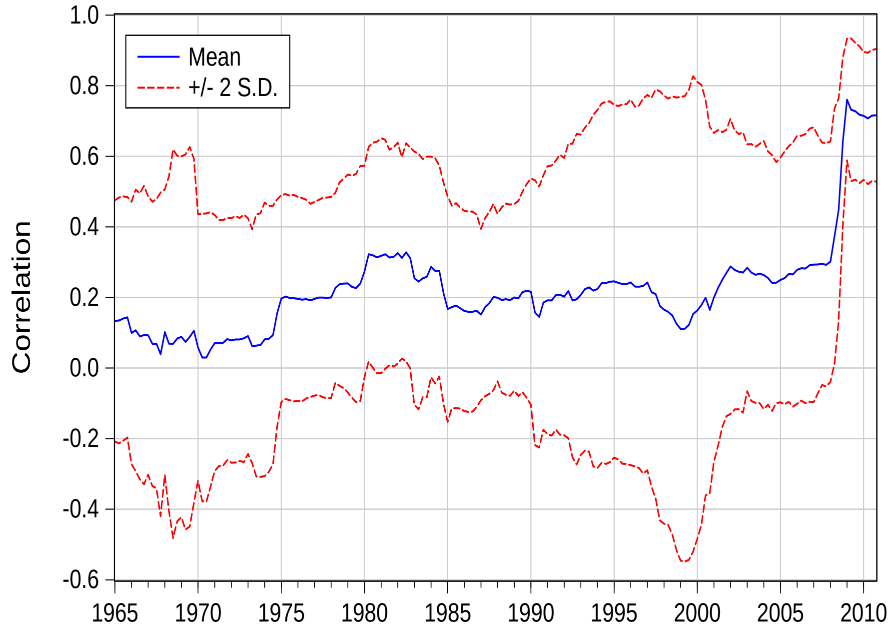


Figure 2: Rolling correlations of quarterly GDP growth among G7 countries.

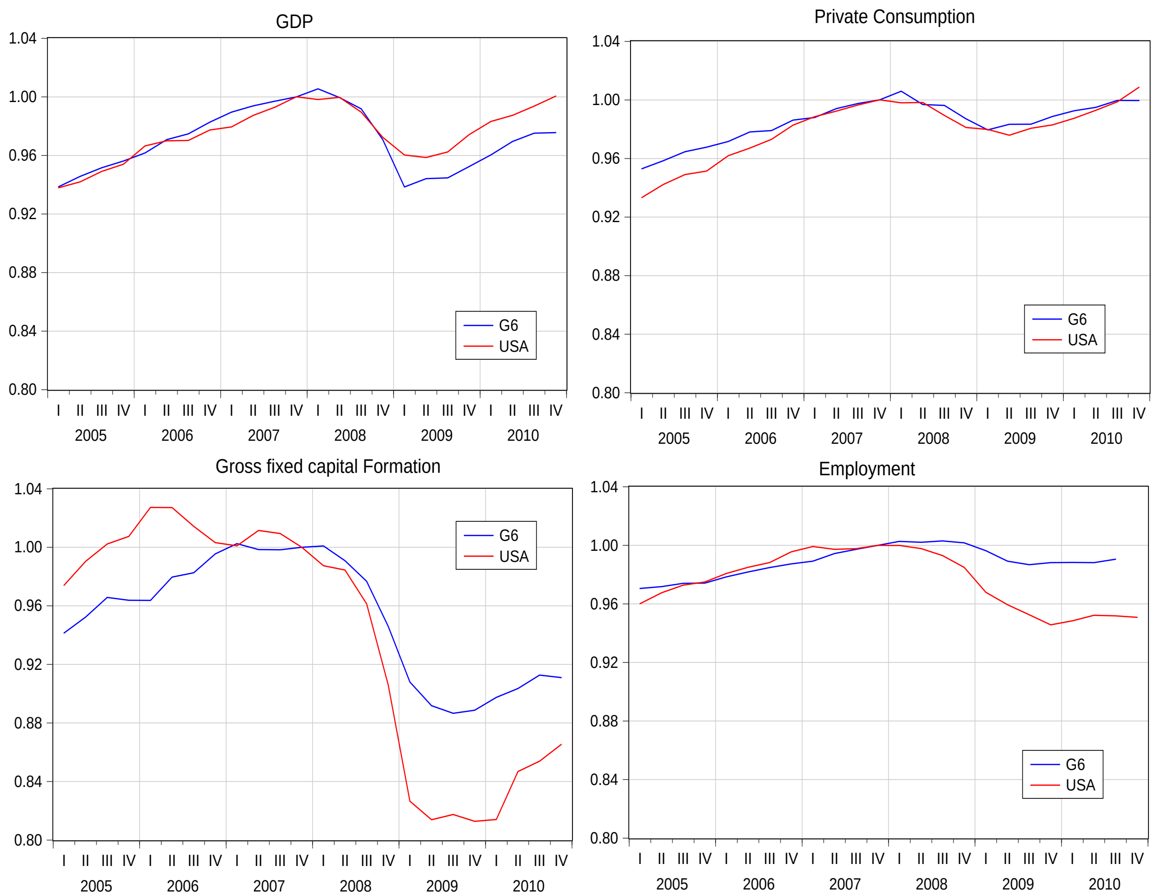
productivity are dimensions in which the current crisis is "less" international. Later in the theory section we'll consider a potential explanation for this asymmetry.

In figure 4 we look at financial variables, in particular stock prices[1] and the stock of net debt of non financial businesses.[2]

The stock market panel documents well the massive (one order of magnitude larger than the decline in GDP) and extraordinarily synchronous (correlation of stock price growth during the crisis approaches 1) decline in stock prices that took place during the crisis. The right panel shows that also corporate debt, albeit with a delay, declined substantially both in the US and in the rest of the G6 during the crisis: this evidence will be particularly important as it will allow us to identify more precisely the source of credit disturbances. A final observation

[1]The stock prices in US are the MSCI BARRA US stock market index, while stock prices in the G6 are computed using the MSCI BARRA EAFE+Canada index which is an average of stock prices in advanced economies except the US.

[2]For the US data is from the Flows of Funds Accounts and for the whole nonfinancial business sector. For the other countries it only includes the corporate non-financial sector. Net Debt is defined as credit markets instruments minus liquid assets i.e. the sum of foreign deposits, checkable deposits and currency, time and savings deposits, money market funds, securities RPs, commercial paper, treasury securities, agency and GSE backed securities, municipal securities and mutual fund shares. For other countries data is for nonfinancial corporations and the definition of variables is similar to the US.



Note: Data for GDP, private consumtion, gross fixed capital formation are from OECD Quarterly National Accounts in PPP constant dollars.
Data for employment are from OECD Main Economic Indicators. All series are normalized to 1 in the fourth quarterof 2007.

Figure 3: GDP, Consumption, Investment and Employment in US and G6: 2005-2010

regards an asymmetry between the credit expansion period before the crisis and the credit collapse period after the crisis. In the years before the crisis although debt experiences a very rapid growth the other variables display standard or sub-standard growth. In the crisis periods debt contracts but all real variables contracts very strongly. This feature is not unique of the 2007-2009 recessions and other authors that analyze empirically historical episodes of credit booms have noticed that these booms are not necessarily associated with rapid real growth, but when they collapse they are often associated with sharp real contractions.[3]

The facts presented here, in particular the high international correlation in real and financial

[3]See for example Reinhart and Rogoff, 2009, Classens, Kose and Terrones, 2011 or Jord, Schularick, and Taylor, 2011

Figure 4: GDP, Stock market and corporate debt in US and G6: 2005-2010

variables, the counter-cyclical (US) productivity, the high employment and stock markets volatility and the asymmetry between expansion and recessions are hard to explain with a standard work-horse international model so in the next section start developing a framework with credit disturbances that we believe is useful to understand the evidence presented in this section and eventually the causes of such a large and internationally diffused crisis.

3 The model with fixed capital and exogenous credit shocks

We start with a simple model without capital accumulation and with exogenous credit shocks. This allows us to provide the intuitions for some of the key results of the paper analytically. After the presentation of the simple model it will be easy to extend it with capital accumulation.

There are two types of atomistic agents, investors and workers. A key difference between these two types of agents is the availability of different investment opportunities. Due to the assumption of markets segmentation only investors have access to the ownership of firms while workers can only save in the form of bonds. Investors discount the future at rate β while the discount factor of workers is $\delta > \beta$. The different discounting between the owners of firms (investors) and workers implies that firms borrow from workers subject to the enforcement constraints as we will describe below.

To facilitate the presentation we first describe the closed-economy version of the model. Once we have characterized the autarkic equilibrium, it will be easy to extend it to the environment with international mobility of capital.

3.1 Investors and firms

Investors have lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. They are the owners of firms and derive income only from dividends. Denoting by d_t the dividends paid by firms, the effective discount factor for investors is $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$. This is also the discount factor used by firms since they maximize shareholders' wealth. As we will see, fluctuations in the effective discount rate play a central role in the analysis of this paper.

Firms operate the production function $F(h_t) = \bar{k} h_t^{\nu}$, where $\bar{k}$ is a fixed input of capital and h_t is the variable input of labor. The parameter ν is smaller than 1 implying decreasing returns to scale in the variable input. In this version of the model without capital accumulation we can think of $\bar{k}$ as a normalizing constant.

Firms start the period with intertemporal debt b_t. Before producing they choose the labor input h_t, the dividends d_t, and the next period debt b_{t+1}. The budget constraint is

$$b_t + w_t h_t + d_t = F(h_t) + \frac{b_{t+1}}{R_t},$$

where R_t is the gross interest rate.

The payments of wages, $w_t h_t$, dividends, d_t, and current debt net of the new issue, $b_t - b_{t+1}/R_t$, are made before the realization of revenues. This implies that the firm faces a cash flow mismatch during the period. The cash needed at the beginning of the period is $w_t h_t + d_t + b_t - b_{t+1}/R_t$. To cover the cash flow mismatch, the firm contracts the intra-period loan $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t$ which is repaid at the end of the period, after the realization of revenues. From the budget constraint we can also see that the intra-period loan is equal to the revenue $F(h_t)$.

Debt contracts are not perfectly enforceable as the firm can default. Default takes place at the end of the period before repaying the intra-period loan. At this stage the firm holds the revenues $F(h_t)$ which are equal to the intra-period loan l_t. These are liquid funds that can be easily diverted in the event of default. Default gives the lender the right to liquidate the firm's assets. But after the diversion of $l_t = F(h_t)$, the only remaining asset is the physical capital $\bar{k}$. Suppose that the liquidation value of capital is $\xi_t \bar{k}$, where ξ_t is stochastic. Since default arises at the end of the period, the total liabilities of the firm are $l_t + b_{t+1}/R_t$. To ensure that the firm does not default, the total liabilities are subject to the enforcement constraint[4]

$$\xi_t \bar{k} \geq l_t + \frac{b_{t+1}}{R_t}.$$

Fluctuations in ξ_t affect the ability to borrow and, as we will see, they generate pro-cyclical movements in real and financial variables.[5] Our goal is to derive the variable ξ_t endogenously from liquidity considerations. As we will describe below, fluctuations in this variable are induced by self-fulfilling expectations leading to multiple equilibria. For the moment, however, we treat ξ_t as an exogenous stochastic variable. Once we have characterized the equilibrium with an exogenous ξ_t, we will make ξ_t endogenous.

To illustrate the role played by fluctuations in ξ_t, consider a pre-shock equilibrium in which the enforcement constraint is binding. Starting from this equilibrium, suppose that ξ_t decreases. In response to the decline in ξ_t the firm is forced to reduce either the dividends and/or the input of labor. To see this, let's start with the case in which the firm is unwilling to change the input of labor. This implies that the intra-period loan $l_t = F(h_t)$ also does not change. Thus, the only way to satisfy the enforcement constraint is by reducing the intertemporal debt b_{t+1}. We can then see from the budget constraint, $w_t h_t + d_t + b_t = b_{t+1}/R_t + F(h_t)$, that the reduction in b_{t+1} requires a reduction in dividends. Thus, the firm is forced to substitute debt

[4] Here we adopt a similar approach as in Hart and Moore (1994). After defaulting the firm bargains the repayment with the lender. Under the assumption that the firm has all the bargaining power, the lender would recover only the threat value $\xi_t \bar{k}$. In anticipation of this, the lender will never lend more than $\xi_t \bar{k}$.

[5] Eisfeldt and Rampini (2006) provide some evidence that the liquidity of capital ξ_t must be procyclical to match the amount of capital reallocation observed in the data.

with equities.

Alternatively the firm could keep the dividend payments unchanged and reduce the intra-period loan $l_t = F(h_t)$. This would also ensure that the enforcement constraint is satisfied but it requires the reduction in the input of labor. Therefore, after a reduction in ξ_t, the firm faces a trade-off: paying lower dividends or cutting employment. The optimal choice depends on the relative cost of changing these two margins which, as we will see, depends on the stochastic discount factor for investors $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$.

Firm's problem: The optimization problem of the firm can be written recursively as

$$V(\mathbf{s}; b) = \max_{d,h,b'} \left\{ d + Em'V(\mathbf{s}'; b') \right\} \tag{1}$$

subject to:

$$b + d = F(h) - wh + \frac{b'}{R} \tag{2}$$

$$\xi\bar{k} \geq F(h) + \frac{b_{t+1}}{R_t}, \tag{3}$$

where $\mathbf{s}$ are the aggregate states, including the shock ξ, and the prime denotes the next period variable. The enforcement constraint takes into account that the intra-period loan is equal to the firm's output, that is, $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(h_t)$.

In solving this problem the firm takes as given all prices and the first order conditions are

$$F_h(h) = \frac{w}{1-\mu}, \tag{4}$$

$$REm' = 1 - \mu, \tag{5}$$

where μ is the Lagrange multiplier for the enforcement constraint. These conditions are derived under the assumption that dividends are always positive, which will be the case if the investors' utility satisfies $u_c(0) = \infty$. The detailed derivation is in Appendix A.

We can see from condition (4) that there is a wedge in the demand for labor if the enforcement constraint is binding ($\mu > 0$). This derives from the fact that the input of labor needs to be financed and part of the financing has to come from equity (through lower payment of dividends). As long as the cost of equity ($1/Em'$) is greater than the cost of debt (R), expanding the input of labor is costly in the margin because the firm needs to substitute debt with equity. It is then the equity premium $1/Em' - R$ that determines the labor wedge as can be seen from condition (5).[6] The wedge is strictly increasing in μ and disappears when $\mu = 0$, that is, when the enforcement constraint is not binding.

Some partial equilibrium properties: The characterization of the firm's problem in partial equilibrium provides helpful insights about the property of the model once extended to a general equilibrium set-up. For partial equilibrium we mean the allocation achieved when the interest rate and the wage rate are both exogenously given and constant.

Under these conditions, equation (5) shows that μ decreases with the expected discount factor Em'. A decrease in ξ makes the enforcement constraint tighter. Because firms reduce the payment of dividends, the investors's consumption has to decrease. This induces a decline in the discount factor $m' = \beta u_c(d')/u_c(d)$ and an increase in the multiplier μ (condition (5)). Condition (4) then shows that the demand for labor declines.

Intuitively, when the credit conditions become tighter, firms need to rely more on equity financing and less on debt. This requires investors to cut consumption (dividends) which is costly since they have concave utility. Because of this, in the short-term firms do not raise enough equity needed to keep the pre-shock production scale and cut employment. If investors' utility were linear (risk-neutrality), the discount factor would be equal to $Em' = \beta$ and the credit shock would not affect employment. This also requires that the interest rate does not change, which is the case in the partial equilibrium considered here. In the general equilibrium, of course, prices do change. In particular, movements in the demand of credit and labor affect the interest rate R and the wage rate w. To derive the aggregate effects we need to close the model and characterize the general equilibrium.

3.2 Closing the model and general equilibrium

There is a representative household/worker with lifetime utility $E_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t)$, where c_t is consumption, h_t is labor and δ is the intertemporal discount factor. It will be convenient to

[6] Notice that we are using the term 'equity premium' to denote the differential between the expected shareholders' return and the interest rate on bonds. Since shareholders and bondholders are different agents, the equity premium is not only determined by the cost of risk (risk premium).

assume that the period-utility takes the form

$$U(c_t, h_t) = \log(c_t) - \alpha \frac{h_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}.$$

The worker's budget constraint is

$$w_t h_t + b_t = c_t + \frac{b_{t+1}}{R_t},$$

and the first order conditions for labor, h_t, and next period bonds, b_{t+1}, are

$$U_h(c_t, h_t) + w_t U_c(c_t, h_t) = 0, \tag{6}$$

$$\delta R_t E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \right\} = 1. \tag{7}$$

We can now define a competitive general equilibrium. The aggregate states, denoted by $\mathbf{s}$, are given by the credit conditions ξ and the aggregate stock of bonds B.

Definition 3.1 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for (i) workers' policies $h_w(\mathbf{s})$, $c_w(\mathbf{s})$, $b_w(\mathbf{s})$; (ii) firms' policies $h(\mathbf{s}; b)$, $d(\mathbf{s}; b)$, $b(\mathbf{s}; b)$; (iii) firms' value $V(\mathbf{s}; b)$; (iv) aggregate prices $w(\mathbf{s})$, $R(\mathbf{s})$, $m(\mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s})$. Such that: (i) household's policies satisfy the optimality conditions (6)-(7); (ii) firms' policies are optimal and $V(\mathbf{s}; b)$ satisfies the Bellman's equation (1); (iii) the wage and interest rates are the clearing prices in the markets for labor and bonds, and the discount factor for firms is $m(\mathbf{s}') = \beta u_c(d_{t+1})/u_c(d_t)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic processes for z and ξ.*

To illustrate the main properties of the model we look at some special cases. Consider first the economy without uncertainty, that is, ξ is constant. In this economy the enforcement constraint binds in a steady state equilibrium. To see this, consider the first order condition for the bond, equation (7), which in a steady state becomes $\delta R = 1$. Using this condition to eliminate R in (5) and taking into account that in a steady state $Em' = \beta$, we get $\mu = 1 - \beta/\delta > 0$ (since $\delta > \beta$). Firms want to borrow as much as possible because the cost of borrowing—the interest rate—is smaller than their discount rate.

With uncertainty, however, the enforcement constraint may be binding only occasionally.

In particular, after a large and unexpected decline in ξ. In this case firms will be forced to cut dividends inducing a change in the discount factor Em'. Furthermore, the change in the demand for credit impacts on the equilibrium interest rate. Using condition (5) we can see that these changes affect the multiplier μ, which in turn impacts on the demand for labor (see equation (4)). On the other hand, an increase in ξ may leave the enforcement constraint non-binding without direct effects on the demand of labor. Therefore, the responses to credit shocks could be highly asymmetric: negative shocks induce large falls in employment and output while the impacts of positive shocks is moderate.

3.3 Capital mobility

Let's consider now two countries, domestic and foreign, with the same size, preferences and technology as described in the previous section. Although we consider the case with only two symmetric countries, the model can be easily extended to any number of countries and with different degrees of heterogeneity. For the moment we continue to assume that ξ_t is an exogenous stochastic variable, specific to each country.

Once we allow for cross-country capital mobility, we have to specify what agents can do in an integrated financial market. We continue to assume that there is market segmentation in the ownership of firms, that is, workers are unable to purchase shares of firms. However, in addition to domestic bonds they can purchase foreign financial assets as specified below. Furthermore, investors are now able to purchase shares of foreign firms.

Investors/firms: Because firms are subject to country specific shocks, investors would gain from diversifying the cross-country ownership of shares. Therefore, in an economy that is financially integrated, investors choose to own the worldwide portfolio of shares and we have a representative 'worldwide' investor.[7] Because domestic and foreign firms are owned by the same representative shareholder, they will use the same discount factor $m_{t+1} = \beta u_c(d_{t+1} + d^*_{t+1})/u_c(d_t + d^*_t)$, where investors' consumption is the sum of dividends paid by domestic firms, d_t, plus the dividends paid by foreign firms, d^*_t. From now on we will use the star superscript to denote variables pertaining to the foreign country.

Besides the common discount factor, firms continue to solve problem (1) and the first order conditions are given by equations (4) and (5). Let's focus on condition (5), which we rewrite

[7] A perfect diversification of portfolios is optimal because investors' utility depends only on consumption. If investors derived utility also from leisure, a perfect diversification would not be necessarily optimal.

here for both countries,

$$R_t E m_{t+1} = 1-\mu_t,$$
$$R_t^* E m_{t+1}^* = 1-\mu_t^*.$$

Since the discount factor is common to domestic and foreign firms, that is, $Em_{t+1} = Em_{t+1}^*$, and the interest rate is equalized across countries, $R_t = R_t^*$, the above conditions imply that the lagrange multiplier will also be equalized, that is, $\mu_t = \mu_t^*$. Therefore, independently of which country is hit by a shock, if the enforcement constraint is binding for domestic firms, it will also be binding for foreign firms. This also implies that the labor wedges are equalized across countries. In fact, condition (4) is still the optimality condition for the choice of labor in both countries, that is,

$$F_h(h_t) = w_t\left(\frac{1}{1-\mu_t}\right),$$
$$F_h(h_t^*) = w_t^*\left(\frac{1}{1-\mu_t^*}\right).$$

This property is crucial for understanding the cross-country impact of a financial shock as we will describe below. Later we will also consider an extension of the model where the labor wedge may respond differently in the two countries.

Households/workers: Although workers are still prevented from accessing the market for the ownership of firms, with capital mobility they can engage in international financial transactions with foreign workers. In addition to holding bonds issued by domestic firms, domestic workers can buy state-contingent claims from foreign workers. We still assume that firms borrow from domestic workers but they cannot sign state contingent contracts with workers. The assumption that firms borrow only from domestic workers is without loss of generality: whether they borrow domestically or in the foreign market is irrelevant in an integrated capital market. The unavailability of state-contingent claims between firms and workers is essential to retain market incompleteness.

Denote by $n_{t+1}(\mathbf{s}_{t+1})$ the units of consumption goods received at time $t+1$ by domestic workers if the aggregate states are $\mathbf{s}_{t+1}$. These are worldwide states, and therefore, they include the aggregates states of both countries as will be made precise below. Of course, in equilibrium, the consumption units received by workers in the domestic country must be equal to the consumption units paid by workers in the foreign country, that is, $n_{t+1}(\mathbf{s}_{t+1})+n^*_{t+1}(\mathbf{s}_{t+1})=0$. This must be satisfied for all possible realizations of the aggregate states $\mathbf{s}_{t+1}$.

The budget constraint of a worker in the domestic country is

$$w_t h_t + b_t + n_t = c_t + \frac{b_{t+1}}{R_t} + \int_{\mathbf{s}_{t+1}} n_{t+1}(\mathbf{s}_{t+1}) q(\mathbf{s}_{t+1})/R_t,$$

where $q_t(\mathbf{s}_{t+1})/R_t$ is the unit price of the contingent claims.

Given the specification of the utility function, the first order conditions for the choice of labor, h_t, next period bonds, b_{t+1}, and foreign claims, $n_{t+1}(\mathbf{s}_{t+1})$, are

$$\alpha h_t^{\gamma} c_t = w_t, \tag{8}$$

$$\delta R_t E_t \left(\frac{c_t}{c_{t+1}}\right) = 1, \tag{9}$$

$$\delta R_t \left(\frac{c_t}{c_{t+1}(\mathbf{s}_{t+1})}\right) p(\mathbf{s}_{t+1}) = q(\mathbf{s}_{t+1}), \qquad \text{for all } \mathbf{s}_{t+1}, \tag{10}$$

where $p(\mathbf{s}_{t+1})$ is the probability (or probability density) of the aggregate states in the next period for the world economy.

Since in equilibrium the prices and probabilities of the contingencies are the same for domestic and foreign workers, condition (10) implies that

$$\frac{c_t}{c^*_t} = \frac{c_{t+1}(\mathbf{s}_{t+1})}{c^*_{t+1}(\mathbf{s}_{t+1})} = \chi. \tag{11}$$

Therefore, the ratio of consumption of domestic and foreign workers remains constant over time. This is a well known property of environments with a full set of state-contingent claims. In our environment the constancy of the consumption ratio is among workers (and among investors) but not between workers and investors because of the assumption of market segmentation.

Before continuing we would like to clarify that the assumption of contingent claims among workers is not essential for the results of the paper. We could simply assume that workers can engage in international non-contingent lending and borrowing only. Or equivalently, that firms can engage in international borrowing. However, the availability of contingent claims greatly simplifies the characterization of the equilibrium because it allows us to reduce the number of 'sufficient' state variables. This property will be convenient once we extend the model with capital accumulation.

Aggregate states and equilibrium: We can now define the equilibrium for the open-economy version of the economy. The aggregate states $\mathbf{s}$ are given by the variables ξ and ξ^*, the financial liabilities of firms, B_t and B_t^*, and the net foreign asset position of the domestic country, N_t. Since in equilibrium the net foreign asset position of the domestic country is the negative of the foreign position, once we know B_t, B_t^* and N_t we also know the total wealth of domestic workers, $B_t + N_t$, and foreign workers, $B_t^* - N_t$. Therefore, $\mathbf{s}_t = (\xi, \xi^*, B_t, B_t^*, N_t)$.

Definition 3.2 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for: (i) households' policies $h_w(\mathbf{s})$, $c_w(\mathbf{s})$, $b_w(\mathbf{s})$, $n_w(\mathbf{s};\mathbf{s}')$, $h_w^*(\mathbf{s})$, $c_w^*(\mathbf{s})$, $b_w^*(\mathbf{s})$, $n_w^*(\mathbf{s};\mathbf{s}')$; (ii) firms' policies $h(\mathbf{s};b)$, $d(\mathbf{s};b)$, $b(\mathbf{s};b)$, $h^*(\mathbf{s};b)$, $d^*(\mathbf{s};b)$, $b^*(\mathbf{s};b)$; (iii) firms' values $V(\mathbf{s};b)$ and $V^*(\mathbf{s};b)$; (iv) aggregate prices $w(\mathbf{s})$, $w^*(\mathbf{s})$, $R(\mathbf{s})$, $m(\mathbf{s},\mathbf{s}')$, $q(\mathbf{s};\mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s})$. Such that: (i) household's policies satisfy the optimality conditions (6)-(10); (ii) firms' policies are optimal and satisfy the Bellman's equation (1) for both countries; (iii) the wages clear the labor markets; the interest rates and the price for contingent claims clear the worldwide financial markets; the discount rate used by firms satisfies $m(\mathbf{s},\mathbf{s}') = \beta u_c(d_{t+1} + d_{t+1}^*)/u_c(d_t + d_t^*)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic process for ξ and ξ^*.*

The only difference with respect to the equilibrium in the closed economy is that there is the additional market for foreign claims and the discount factor for firms is given by the worldwide representative investor. The market clearing condition for the foreign claims is $N(\mathbf{s}') + N^*(\mathbf{s}') = 0$. This is in addition to the clearing conditions for the domestic bond markets (lending to firms).

Although the general definition of the recursive equilibrium is based on the set of state variables $\mathbf{s}_t = (\xi_t, \xi_t^*, B_t, B_t^*, N_t)$, we can use some of the properties derived above and characterize the equilibrium with a smaller set of states. Let $W_t = B_t + B_t^*$ be the worldwide wealth of households/workers. This is the sum of bonds issued by domestic firms, B_t, and foreign firms, B_t^*. Then using the fact that the consumption ratio of domestic and foreign workers is constant at χ and the employment policy of firms does not depend on the individual debt, the recursive equilibrium can be characterized using the state variables $\mathbf{s}_t = (\xi_t, \xi_t^*, W_t)$. Essentially, the assumption of cross-country risk-sharing among workers and investors (but not between workers and investors) allows us to reduce the number of 'endogenous' states to only one variable.

Intuitively, by knowing W_t, we know the worldwide liability of firms, but not the distribution between domestic and foreign firms. However, to characterize the firms' policies, we only need to know the worldwide debt, which is equal to W_t. Since investors own an internationally diversified portfolio of shares, effectively there is only one representative global investor. It is as if there is a representative firm with two productive units: one unit located in the domestic country and the other in the foreign country. Since both units have a common owner, it does not matter how the debt is distributed between the two units. What matters from the perspective of the investor is the total debt and the total payment of dividends. This has some similarity with the problem solved by a multinational firms that faces demand uncertainty in different countries as studied in Goldberg and Kolstad (1995). There is also some similarity with the problem faced by multinational banks that own subsidiaries in different countries. Cetorelli and Goldberg (2010) provide evidence that multinational banks do reallocate financial resources internally in response to country specific shocks.

Total workers' wealth is also a sufficient statistic for the characterization of the workers' policies since the consumption ratio between domestic and foreign households remains constant at χ. This property limits the computational complexity of the model, making feasible the use of non-linear approximation methods. We will come back to this point after the description of the general model with capital accumulation.

We are not ready to state the following proposition about the impact of a financial shock.

Proposition 3.1 *An unexpected change in ξ_t (domestic credit shock) has the same impact on employment and output of domestic and foreign countries.*

Proof 3.1 *We have already shown that the Lagrange multiplier μ_t is common for domestic and foreign firms. If the wage ratio in the two countries does not change, the first order conditions imply that all firms choose the same employment. To complete the proof we have to show that the cross-country wage ratio stays constant. Because firms in both countries have the same demand for labor and the ratio of workers' consumption remains constant, the first order condition for the supply of labor from workers implies that the wage ratio between the two countries does not change.*

Therefore, independently of whether a credit shock hits the domestic or foreign markets, both countries experience the same macroeconomic consequences. Notice though that although exogenous credit shocks can explain co-movement in GDP and other real variables there are two problems with this approach. The first, and more general, is that treating shocks as exogenous limits our ability to understand their source and possible policy intervention to mitigate their adverse effects. The second, and more specific, is that although a country-specific exogenous shock can generate macroeconomic co-movement, it will also induce financial flows tend to move in opposite directions. To show this, consider an initial equilibrium in which the enforcement constraints are not binding in either countries. Starting from this equilibrium suppose that the domestic economy is hit by a credit contraction (reduction in ξ_t) inducing binding enforcement constraints in both countries. Since ξ_t is lower only in the domestic country, the outstanding debt of domestic firms contracts but the debt of foreign firms actually increases. Actually the foreign firm increase their debt so they can pay more dividends to their shareholder, now that the domestic firm is constrained. Therefore, the model with 'exogenous' credit shocks generates negative cross-country co-movement in debt.

This feature of the model is inconsistent with the data in the right panel of 4 showing a high degree of cross-country co-movement also in the flows of financing. However, as we will see in the next section, once we make fluctuations in ξ_t and ξ_t^* endogenous, the model also generates a high degree of co-movement in financial flows, introducing a second source of real macroeconomic synchronization.

4 Endogenous credit shocks

After illustrating how a credit shock propagates to the real sector of the economy, we now provide a micro foundation for endogenous fluctuations in ξ_t. We proceed first with the closed-economy model and then we extend it to a two-country set-up.

Financial autarky: Suppose that in case of liquidation, physical capital $\bar{k}$ can be sold either to households or firms. In the first case one unit of capital is transformed in $\underline{\xi}$ units of consumption. Alternatively, the capital can be sold to other firms for productive uses. In this case one unit of capital can be transformed in $\overline{\xi}$ units of reinstalled capital. The reallocation in other firms is more efficient than its transformation in consumption goods, that is, $\underline{\xi} < \overline{\xi} \leq 1$. However, in order for non-defaulting firms to buy additional capital, they need liquid funds. In this sense our model shares some features of the model studied in Kiyotaki and Moore (2008).

Since all firms face the enforcement constraint

$$\xi_t \bar{k} \geq \frac{b_{t+1}}{R_t} + y_t, \tag{12}$$

a non-defaulting firm can buy additional capital only if the firm has previously chosen not to borrow up to the limit, that is, constraint (12) is not binding. Therefore, if at the beginning of the period firms choose not to borrow up to the limit, ex-post there will be firms that are capable of purchasing the capital of a defaulting firm. In this case the market price of the liquidated capital is $\overline{\xi}$. On the other hand, if at the beginning of the period all firms choose to borrow up to the limit, ex-post there will not be firms with liquidity. Then the capital of a defaulting firm can only be sold to households and the market price is $\underline{\xi}$.

Since the value of liquidated capital depends on the financial choices of firms, which in turn depends on the expected liquidation value, the model could generate multiple (self-fulfilling) equilibria.

Suppose that the expected liquidation price is $\xi_t = \underline{\xi}$. The low price makes the enforcement constraint (12) tighter, which may induce firms to borrow up to the limit in order to contain the cut in dividends and/or employment. Then, if all firms borrow up to the limit, ex-post there will not be any firm that has liquidity to purchase the capital of a defaulting firm. Thus, the ex-post liquidation price is $\underline{\xi}$, fulfilling the market expectation.

Now suppose that the expected liquidation price is $\overline{\xi}$. Because the enforcement constraint (12) is not tight in the current period but could become tighter in the future, firms may choose not to borrow up to the limit. But then, in case of liquidation, there will be firms capable of purchasing the liquidated capital and the market price is $\overline{\xi}$. So also in this case we have that the expectation of a high liquidation value is fulfilled by the firms' borrowing choice.

Whether multiple equilibria could arise depends on the particular states of the economy. Three cases are possible:

1. The liquidation price is $\underline{\xi}$ with probability 1. This arises if we are in a state in which firms choose to borrow up to the limit independently of the expectation over ξ_t.

2. The liquidation price is $\overline{\xi}$ with probability 1. This arises if we are in a state in which firms do not borrow up to the limit independently of the expectation over ξ_t.

3. The liquidation price is $\underline{\xi}$ with some probability $p \in (0, 1)$. This arises if we are in a state in which firms choose to borrow up to the limit when the expectation for the liquidation

value is $\xi_t = \underline{\xi}$ but they do not borrow up to the limit when the expectation for the liquidation price is $\xi_t = \overline{\xi}$.

The third case is the most interesting because it generates multiple sunspot equilibria, and therefore, potential fluctuations in ξ_t. In this case the low liquidation price $\underline{\xi}$ could arise with any probability p. In general we can denote by $p_t(\mathbf{s}_t)$ the probability of $\xi_t = \underline{\xi}$. Besides the fact that the probability distribution of ξ_t could be time variant, the properties of the model characterized in previous sections do not change.

Financial integration: As in the closed economy, different values of ξ_t are associated to self-fulling expectations. Although each country could have different liquidation values of capital, we want to show that ξ_t cannot be different from ξ_t^* once the two countries become financially integrated.

As we have seen in the previous section, if the enforcement constraint is binding in one country, it must also be binding in the other country, that is, $\mu_t = \mu_t^* > 0$. This eliminates equilibria where $\xi_t = \underline{\xi}$ and $\xi_t^* = \overline{\xi}$. We state this property formally in the next proposition.

Proposition 4.1 *In equilibria with integrated financial markets, ξ_t is always equal to ξ_t^**

Proof 4.1 *Suppose that the equilibrium is characterized by $\xi_t = \underline{\xi}$ and $\xi_t^* = \overline{\xi}$. In order to have $\xi_t = \underline{\xi}$ we need that $\mu_t > 0$ and to have $\xi_t^* = \overline{\xi}$ we need that $\mu_t^* = 0$. But in an equilibrium with integrated financial markets μ_t is always equal to μ_t^*. Therefore, this cannot be an equilibrium. Using the same argument we can exclude the possibility of an equilibrium with $\xi_t = \overline{\xi}$ and $\xi_t^* = \underline{\xi}$. Thus, the only possible equilibria are characterized by $\xi_t = \xi_t^*$.*

Therefore, financial integration implies perfect cross-country co-movement in ξ_t, which introduces a second channel of real macroeconomic synchronization: not only a change in one country ξ affects the real sector of the other country but movements in ξ become perfectly correlated across countries. This also implies international co-movement in financial flows.

Also in the case of financial integration the probability of $\xi_t = \underline{\xi}$ can be expressed as a function of the aggregate states, that is, $p(\mathbf{s}_t)$. Now, however, one of the two equilibria can be induced by changes in expectations in one of the two countries. For simplicity suppose that in states with multiple equilibria the domestic country expects $\xi_t = \underline{\xi}$ with probability $\overline{p}$. The same for the foreign country. Based on this assumption we have that $p(\mathbf{s}_t) \in \{0, 2\overline{p}(1 - \overline{p}) +$

$\bar{p}^2, \bar{p}^2, 1\}$. The probability is zero when firms choose not to borrow up to the limit ($\mu_t = \mu_t^* = 0$) even if the expectation is $\xi_t = \xi_t^* = \underline{\xi}$. The probability is $2\bar{p}(1-\bar{p}) + \bar{p}^2$ if firms choose to borrow up to the limit ($\mu_t = \mu_t^* > 0$) when either ξ_t or ξ_t^* are equal to $\underline{\xi}$. The probability is $\bar{p}^2$ if firms choose to borrow up to the limit ($\mu_t = \mu_t^* > 0$) only if both ξ_t and ξ_t^* are equal to $\underline{\xi}$. Finally, the probability is 1 if firms choose to borrow up to the limit ($\mu_t = \mu_t^* > 0$) independently of the values of ξ_t and ξ_t^*.

The general definition of equilibrium is analogous to the definition provided for the model with exogenous ξ_t. We simply need to add the probability function $p(\mathbf{s}_t)$ which must be consistent with the optimal decisions of firms as described above.

5 Model with capital accumulation

We now relax the assumption that the input of capital is fixed. This introduces additional state variables that increase the computational complexity of the model. Since the enforcement constraint is only occasionally binding, we need to use global approximation techniques. Unfortunately, these techniques are computationally intensive and become quickly impractical when we have a large numbers of state variables. Therefore, in order to reduce the sufficient set of state variables, we will make some special assumptions about the production technology.

Investors-firms: The production function takes the form

$$y_t = (K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu \equiv F(K_t + K_t^*, k_t, h_t),$$

where K_t is the 'aggregate' capital in the domestic country and K_t^* in the foreign country, k_t is the 'individual' input of capital and h_t is the 'individual' input of labor. We assume that $\theta + \nu < 1$.

The dependence of the production function from the worldwide stock of capital, $K_t + K_t^*$, captures positive externalities. The purpose of the externalities is to have constant returns in reproducible factors (AK technology), without loosing the competitive structure of the model, that is, each producer runs a production technology with non-increasing returns.

Given i_t the flow of investment, the stock of capital evolves according to

$$k_{t+1} = (1-\tau)k_t + \Upsilon\left(\frac{i_t}{k_t}\right)k_t,$$

where τ is the depreciation rate and the function $\Upsilon(.)$ is strictly increasing and concave, capturing adjustment costs in investment. The assumption of capital adjustment costs is common in international macro models and it is made to prevent excessive volatility of investments.

The budget constraint of the firm is

$$b_t + d_t + i_t = F(K_t, k_t, h_t) - w_t h_t + \frac{b_{t+1}}{R_t},$$

and the enforcement constraint

$$\xi_t k_{t+1} \geq F(k_t, h_t) + \frac{b_{t+1}}{R_t}.$$

We will now take advantage of the AK structure and normalize the model by the worldwide stock of capital $K_t + K_t^*$. Using the tilde sign to denote normalized variables, we can rewrite the budget constraint, law of motion for capital and enforcement constraint as

$$\tilde{b}_t + \tilde{d}_t + \tilde{i}_t = F(\tilde{k}_t, h_t) - \tilde{w}_t h_t + \frac{g_t \tilde{b}_{t+1}}{R_t}, \tag{13}$$

$$g_t \tilde{k}_{t+1} = (1-\tau)\tilde{k}_t + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\tilde{k}_t, \tag{14}$$

$$\xi_t g_t \tilde{k}_{t+1} \geq F(\tilde{k}_t, h_t) + \frac{g_t \tilde{b}_{t+1}}{R_t}. \tag{15}$$

The variable $g_t = (K_{t+1}+K_{t+1}^*)/(K_t+K_t^*)$ is the gross growth rate of worldwide capital and $\tilde{k}_t = k_t/(K_t + K_t^*)$ is the normalized individual capital. We will denote by $s_t = K_t/(K_t + K_t^*)$ the aggregate share of capital owned by domestic firms. Since in equilibrium $k_t = K_t$, we also have that $\tilde{k}_t = s_t$.

As in the simpler model without capital accumulation, investors hold an internationally diversified portfolio of shares, and firms use the common discount factor $m_{t+1} = \beta[(d_{t+1} + d_{t+1}^*)/(d_t + d_t^*)]^{-\sigma}$. In terms of variables normalized by the worldwide capital, the discount factor can be rewritten as

$$m_{t+1} = g_t^{-\sigma}\beta\left(\frac{\tilde{d}_{t+1} + \tilde{d}_{t+1}^*}{\tilde{d}_t + \tilde{d}_t^*}\right)^{-\sigma} = g_t^{-\sigma}\tilde{m}_{t+1}.$$

The optimization problem solved by an individual firm can be rewritten as

$$\tilde{V}(\tilde{\mathbf{s}};\tilde{k},\tilde{b}) \quad = \quad \max_{\tilde{d},\tilde{h},\tilde{i},\tilde{b}'} \left\{ \tilde{d} + g^{1-\sigma} E\tilde{m}'\tilde{V}(\tilde{\mathbf{s}}';\tilde{k}',\tilde{b}') \right\} \tag{16}$$

$$\text{subject to (13), (14), (15),}$$

where $\tilde{V}$ is the firm's value normalized by aggregate worldwide capital $K + K^*$, and $\tilde{\mathbf{s}}$ denotes the normalized aggregate states as specified below.

We can now see the analytical convenience of having the capital externality. Thanks to the AK structure, we can write the firm's value function as $V_t = (K_t + K_t^*) \cdot \tilde{V}_t$ and rescale the problem of the firm by worldwide capital. By doing so, we do not need to keep track of the aggregate stock of capital as a state variable. Of course, because we are looking at a general equilibrium, we also need to make sure that the supply of labor does not grow over time. This will be the case with the worker's utility specified earlier.

Appendix B derives the first order conditions for the firm's problem. After imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, the first order conditions can be written as

$$F_h(s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t}, \tag{17}$$

$$g_t^{-\sigma} R_t E\tilde{m}_{t+1} = 1 - \mu_t, \tag{18}$$

$$Q_t \Upsilon'\left(\tilde{i}_t\right) = 1, \tag{19}$$

$$Q_t = \xi_t \mu_t + \bar{g}_t^{-\sigma} E\tilde{m}_{t+1} \Big\{ (1 - \mu_{t+1}) F_k(s_{t+1}, h_{t+1}) - \tilde{i}_{t+1} + \Big[1 - \tau + \Upsilon\left(\tilde{i}_{t+1}\right)\Big] Q_{t+1} \Big\}. \tag{20}$$

Here μ_t is the Lagrange multiplier associated with the enforcement constraint and Q_t is the Lagrange multiplier associated with the law of motion for the stock of capital (Tobin's q). We can verify that capital does not enter these equations.

Notice that the property established in the simpler model for which the Lagrange multiplier is common across domestic and foreign firms, also applies to this extended model. In fact, from condition (18) we can see that the common discount factor and the equalization of the interest rates across countries imply $\mu_t = \mu_t^*$. Therefore, if the enforcement constraint is binding in one country, it must also be binding in the other. The labor wedge in the demand of labor, $1/(1-\mu_t)$, is also equalized across countries.

Aggregate states and equilibrium: Denote by $\tilde{W}_t = \tilde{B}_t s_t + \tilde{B}_t^*(1-s_t)$ the normalized worldwide wealth of households/workers. Thanks to the AK technology and the normalization described above, we only need to keep track of two 'endogenous' state variables: $\tilde{W}_t$ and s_t. Therefore, compared to the simpler model considered earlier, the introduction of capital accumulation adds only one state variable, that is, the share of worldwide capital owned by domestic firms, s_t.[8] Therefore, having only two continuous states variables, it becomes manageable to solve the model numerically using global approximation methods. Appendix C reports the list of equilibrium conditions and describes the computational procedure.

5.1 Extension with productivity shocks

Since a large body of literature in international macroeconomics has developed from the International Real Business Cycle, it would be informative to investigate how our model performs with productivity shocks and compare it with credit shocks.

To this end we specify the production function as

$$y_t = z_t(K_t + K_t^*)^{1-\theta} k_t^{\theta} h_t^{\nu} \equiv F(z_t, K_t + K_t^*, k_t, h_t),$$

where z_t denotes the stochastic level of productivity. The variable z_t is country-specific and follows a first order Markov process.

6 Quantitative analysis

This section studies the properties of the model quantitatively after the calibration of the parameters. The model is solved numerically using the procedure described in Appendix C.

We think of country 1 as the US and country 2 as the other countries in the group of the seven largest industrialized economies, that is, Canada, Japan, France, Germany, Italy, UK. We refer to this group as G6 countries. The discount factor for workers, δ, and the discount factor

[8] This additional state is necessary because of the adjustment cost in investment. In absence of adjustment costs, we could also ignore s_t.

for investors, β, are set to target an average interest rate of 1.6 percent and an average return on equity of 7 percent. In the deterministic steady state the interest rate is equal to $1/\delta - 1$ and the return on equity is equal to $1/\beta - 1$. In the stochastic economy the relations between the intertemporal discount factors and the average returns are more complex. Therefore, to choose δ and β we have to follow an iterative procedure where we fix these two parameters together with all other parameters, solve the model and check whether the average returns match the targets. The required values are $\delta = 0.996$ and $\beta = 0.984$. Therefore, there is a 1 percent difference between the two discount factors. This is smaller than the equity premium, $5.4\%/4 = 1.35\%$. The difference is the risk premium.

The utility function takes the form $U(c,h) = \ln(c) - \alpha h^{1+1/\eta}/(1+1/\eta)$ where η is the Frisch elasticity of labor supply. We set the elasticity to 0.75 which is between the micro and macro estimates. The parameter α is set so that working hours are 0.3 on average.

Next we parameterize the production function. The parameter ν is chosen to have a steady state labor income share of 0.7. Without uncertainty, the fraction of output going to workers in the form of wages is equal to $\nu\beta/\delta$.[9] Given the values of δ and β, we choose ν so that this fraction is equal to 0.7. Of course, in the stochastic economy the average labor share is not exactly 0.7 but the difference is small. Next we set the return to scale for an individual firm to $\theta + \nu = 0.9$. Given the value of ν we derive the value of $\theta = 0.9 - \nu$.

The stock of capital evolves according to $k' = (1-\tau)k + \Upsilon(i/k)k$ with

$$\Upsilon\left(\frac{i}{k}\right) = \frac{\phi_1}{1-\zeta}\left(\frac{i}{k}\right)^{1-\zeta} + \phi_2.$$

This functional form for the capital adjustment cost is widely used in the literature (see, for example, Jermann (1998)). The parameters ϕ_1 and ϕ_2 are chosen so that in the deterministic steady state $Q = 1$ and $I = \tau K$. This requires $\phi_1 = \tau^{\zeta}$ and $\phi_2 = -\zeta\tau/(1-\zeta)$. Therefore, we need to choose two parameters, τ and ζ. The first is the depreciation rate which we set to $\tau = 0.02$. The second determines the sensitivity of the adjustment cost and we set it to $\zeta = 0.5$.

At this point we are left with the parameters for the stochastic properties of the shocks. Let's start with the productivity series. After constructing Solow residuals series for the US

[9]From the first order condition of labor, equation (4), we derive $wh/F(z,k,h) = \nu(1-\mu)$, which provides an expression for the labor share. We now use condition (5) to derive an expression for μ. Taking into account that in a deterministic steady state $m' = \beta$ and $R = 1/\delta$, this condition becomes $\beta/\delta = 1 - \mu$. Substituting in the labor share $\nu(1-\mu)$, we get the expression reported in the main text.

and for the aggregate of the G6 countries, we estimate the dynamic system

$$\left(\begin{array}{c}\log(z_{t+1}^{US})\\ \log(z_{t+1}^{G6})\end{array}\right)=\left[\begin{array}{cc}\rho_z & \psi_z\\ \psi_z & \rho_z\end{array}\right]\left(\begin{array}{c}\log(z_t^{US})\\ \log(z_t^{G6})\end{array}\right)+\left(\begin{array}{c}\epsilon_{t+1}^{US}\\ \epsilon_{t+1}^{G6}\end{array}\right).$$

The log series are linearly detrended and ϵ_{t+1}^{US} and ϵ_{t+1}^{G6} are mean zero white noises with standard deviation σ_z^{US} and σ_z^{G6} respectively. The estimation returns $\rho_z = 0.98$, $\psi_z = -0.008$, $\sigma_z^{US} = 0.0059$, $\sigma_z^{G6} = 0.0065$. The correlation between residuals is 0.15.

The estimation shows that there is very low cross-country comovement between the Solow residuals of the two countries and they have very similar standard deviations. Therefore, the process for the productivity variables can be well approximated by symmetric and independent first order autoregressive processes with autocorrelation parameter $\rho = 0.98$ and standard deviation of residuals $\sigma = 0.0062$.

Let's turn now to the financial shocks. The variable ξ takes only two values. In addition to the choice of these two values we have to pin down $\bar{p}$, that is, the probability with which each country form pessimistic expectations ($\xi = \underline{\xi}$) in states with multiple equilibria. We choose $\underline{\xi}$, $\overline{\xi}$ and $\bar{p}$ to match three targets: (i) the average leverage (debt over capital), which we set to 0.5); (ii) the standard deviation of debt over output; (iii) the frequencies of crisis, which we set to about 4%.[10] The full list of parameter values are reported in Table 1.

Table 1: List of parameters

Discount factor for households/workers, δ	0.996
Discount factor for entrepreneurs, β	0.986
Utility parameter, α	16.293
Production technology, θ	0.200
Production technology, ν	0.700
Depreciation rate, τ	0.020
Capital adjustment cost, ζ	0.050
Productivity persistence, ρ_z	0.980
Productivity volatility, σ_z	0.006
Low liquidation value, $\underline{\xi}$	0.550
High liquidation value, $\overline{\xi}$	0.650
Frequency of low liquidation value, $\bar{p}$	0.200

Appendix C describes the computational procedure which is based on the discretization

[10] Although the three parameters are chosen jointly, we can identify the primary parameter that affects each of the three targets. The average leverage is mostly determined by the average ξ. The standard deviation of debt is mostly determined by the difference between $\overline{\xi}$ and $\underline{\xi}$. The frequency of crisis is mostly determined by $\bar{p}$.

of the state space. The exogenous states z_t and z_t^* are each approximated with a three-state Markov chain using Tauchen (1986). The endogenous states $\tilde{b}_t$ and s_t are each discretized on a grid with eleven points. Values outside the grids are determined through bi-linear interpolation.

6.1 Results

Our first result follows simply by noticing that proposition 4.1 extends to this more general environment so that with endogenous credit disturbances credit markets conditions (changes in ξ_t and ξ_t^*) are perfectly correlated so, since countries are perfectly symmetric, in presence of credit shocks alone all variables (real and financial) are perfectly correlated across countries. Hence a large credit shock can induce very strong co-movement in real and financial variables like the one documented in section 2.

In presenting additional results we outline four main properties: (i) the asymmetric response to shocks; (ii) the counter-cyclicality of labor productivity in response to credit shocks; (iii) the severity of crisis after long periods of credit and macroeconomic booms; (iv) the importance of credit shocks for the volatility of labor and asset prices.

Asymmetry: Figure 5 plots the impulse responses to a credit expansion and a credit contraction. We report only the responses for one country since they are symmetric. A credit expansion is generated starting from the limiting equilibrium in which the economy converges after a long series of draws $\xi_t = \underline{\xi}$. From this equilibrium we consider a sequence of draws $\xi_t = \overline{\xi}$ starting at $t = 1$. Therefore, a credit expansion is generated by a permanent switch from $\underline{\xi}$ to $\overline{\xi}$. Similarly, the impulse responses to a credit contraction are generated starting from the limiting equilibrium in which the economy converges after a long series of draws $\xi_t = \overline{\xi}$. Starting at $t = 1$ the economy experiences a sequence of draws $\xi_t = \underline{\xi}$. The draws of productivity are assumed to be $z_t = \bar{z}$, the mean value.

Two remarks are in order. First, the impulse responses take place in a range of states that admit multiple equilibria. Therefore, the draws of ξ_t are possible equilibrium outcomes. Second, agents do not know in advance the actual draws of ξ_t and z_t, and therefore, they take into account the uncertainty induced by the stochastic distribution of ξ_t and z_t.

In response to the credit expansion we see a gradual increase in the stock of debt and a persistent expansion in labor and output. The magnitude of the macroeconomic expansion, however, is not large at impact.The macroeconomic expansion induced by the credit boom arises through the following mechanism. At impact the firm becomes unconstrained which

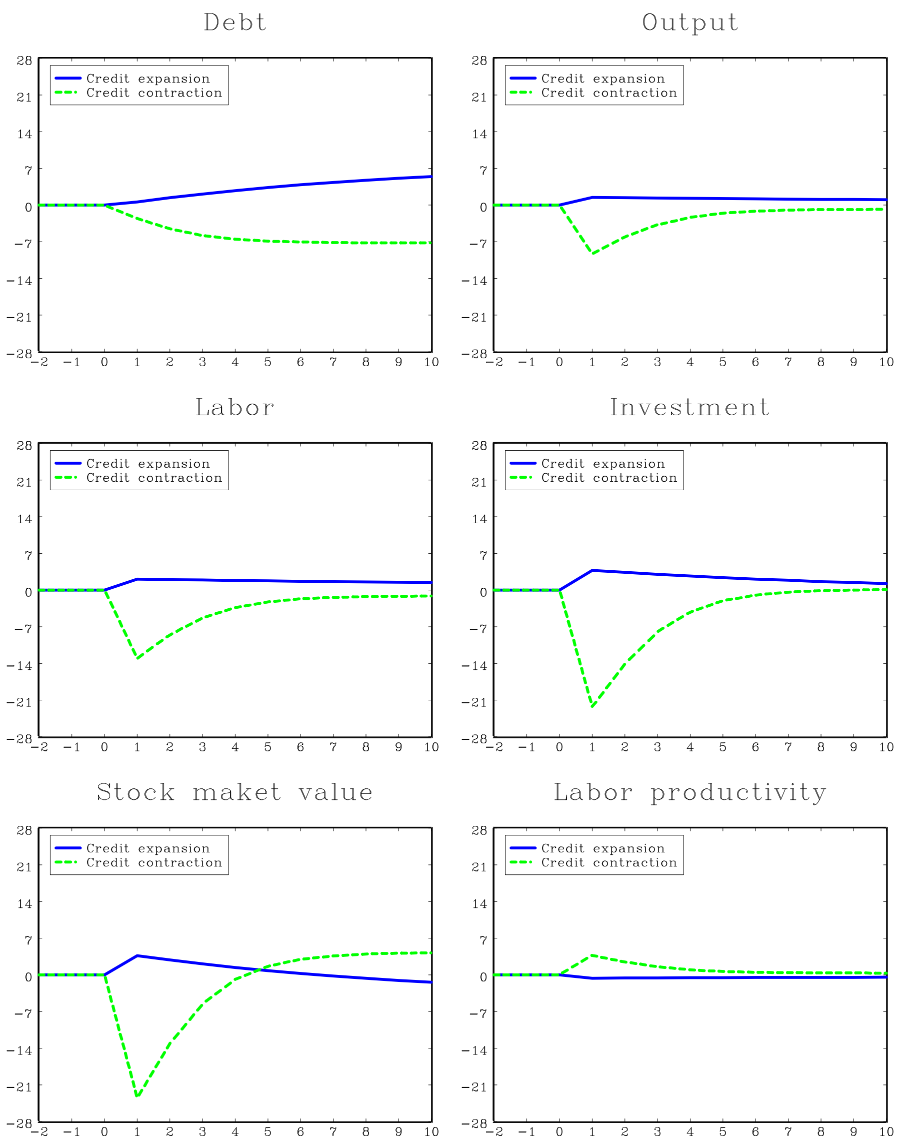


Figure 5: Impulse responses to credit expansions and contractions.

eliminates the labor wedge. In addition to that and after the initial period, there is a second mechanism. As firms take on more debt, they pay more dividends, increasing the discount factor m'. Thanks to the lower discounting, firms invest more. At the same time, the higher borrowing from firms increases equilibrium interest rate that increases equilibrium labor supply and output. The responses to a credit contraction displays a different pattern. The stock of debt declines more quickly and the responses of labor, output and investment are much larger at impact. Therefore, the model generates a strong asymmetry in the responses to credit expansions and contractions. The intuition for the asymmetry is best understood starting from a situation in which the enforcement constraint is not binding; if the constraint gets relaxed the Lagrange multiplier cannot fall below zero so the expansionary effect on unemployment will be mild (only through the general equilibrium discussed above). If instead the constraints get tighter the Lagrange multiplier goes from 0 to being positive and that causes, through equation 17, equilibrium employment and output to fall. As we discussed in section 2 this asymmetry is consistent with macroeconomic patterns observed during the period 2005-2010.

Countercyclical labor productivity: The last panel of Figure 5 plots the impulse responses of labor productivity, that is, the ratio between output and hours. As in the previous figure we see an asymmetry between credit expansions and credit contractions. More importantly, a credit expansion inducing an increase in hiring without any change in TFP causes a decline in labor productivity while a credit contraction, inducing a decline in hiring, generates an increase in labor productivity. This is important for capturing one of the counter-cyclical nature of labor productivity during the crisis in US documented in section 2.

Credit booms and severity of recessions: Figure 6 plots the impulse responses to a credit expansion that later reverts back to the pre-expansion level. A credit boom is generated as described above. Starting from an equilibrium to which the economy converges after a long series of $\xi_t = \underline{\xi}$, we assume that at time 1 the economy experiences a switch to $\xi_t = \overline{\xi}$ (credit expansion). The value of ξ stays at the higher value for several periods and then it reverts back to $\underline{\xi}$ permanently. We consider credit booms with duration of 4 quarters (left panels) and 20 quarters (right panels).

The key finding is that the macroeconomic impact of the credit contraction increases with the duration of the credit expansion. After a protracted credit boom, the economy accumulates large leverages. When the credit reversal arrives, the required de-leveraging is more severe and

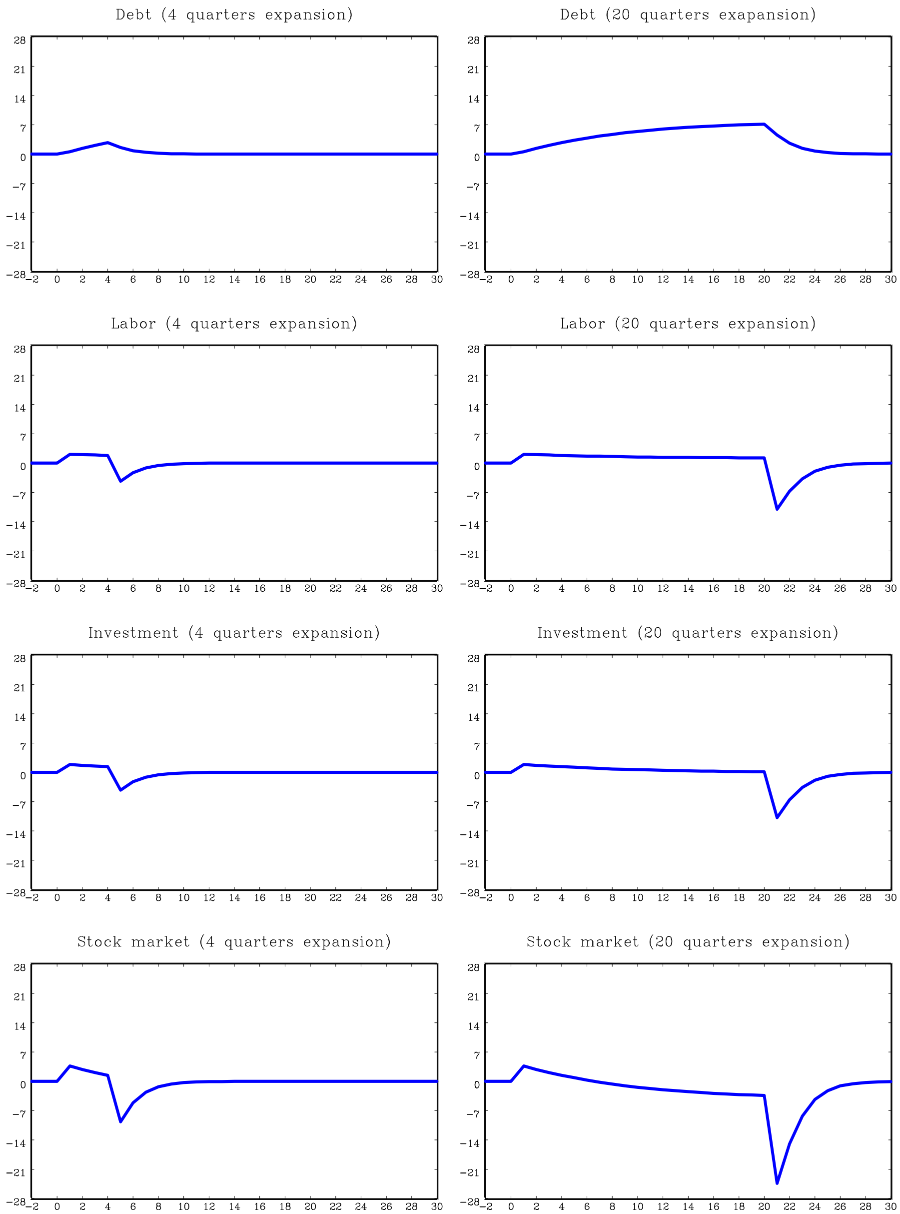


Figure 6: Duration of credit expansions and severity of contractions.

that forces firm to reduce hiring more generating a stronger macroeconomic contraction. In this way the model captures why recessions that arise after long periods of financial and macroeconomic expansions tend to be more damaging for the real sector of the economy.

Volatility of labor and asset prices: Table 2 reports the standard deviations of various variables. Three versions of the economy are considered: the economy with productivity shocks only; the economy with credit shocks only; and the economy with both shocks. The statistics are computed after detrending the simulated series with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999)).

Table 2: Business cycle statistics of key variables from detrended simulated series.

	Productivity shocks only	Credit shocks only	Both shocks
	Standard deviations		
Output	0.76	0.88	1.16
Consumption	0.44	0.68	0.77
Labor	0.26	1.26	1.26
Investment	0.77	2.27	2.36
Tobin's q	0.38	1.14	1.18
Stock market value	0.54	2.46	2.45
Interest rate	0.25	0.48	0.48
Return on equity	0.37	5.82	5.82
	Expected returns (% annualized)		
Interest rate	1.56	1.40	1.40
Return on equity	5.62	6.96	6.96
Equity risk premium	0.06	1.56	1.56
Nonbinding constraints, %	99.99	96.44	96.04

Two properties are especially noticeable. First, the model with credit shocks can generate much higher volatility of labor, bringing the model closer to the US data for the crisis where employment fell more than output (see figure 2). The reason is simply that credit shocks cause, through the lagrange multiplier on the enforcement constraints, autonomous movements in employment that, due to decreasing returns, drive smaller movements in output. Second, credit shocks also generate a high volatility of asset prices. In particular, in the version of the model with only credit shocks, the stock market value (equity value of firms) is almost three times more volatile than output. This can also be seen in the bottom panel of Figure

5 which plots the impulse responses of the market value of equity to a credit expansion and contraction. In contrast, with only productivity shocks, the value of the firm is less volatile than output. The reason for the higher asset prices volatility is mainly that credit shocks can sharply change the stochastic discount factor of investors (see equation 17 above) who hold stocks and hence large fluctuations in stock prices emerge in equilibrium. This suggests that credit shocks can contribute to explain at least part of the large volatility of stock prices we have observed during the crisis (see figure 4).

As a result of the higher volatility of asset prices and discount factor of investors, the model can also generate a non-negligible equity risk-premium.[11] This is about 1.56 percent yearly and it mainly generated by credit shocks. We also observe that the volatility of equity returns is quite high in the model but the volatility of the interest rate is small.[12]

7 Global financial crisis and heterogeneous labor markets

In section 2 we pointed out that the pattern of employment during the crisis is different between US and the other G6. This point is also shown using the idea of the 'labor wedge', that is, the difference between the marginal rate of substitution in consumption and leisure and the marginal product of labor. Formally, this is defined as $U_h(c_t, h_t)/U_c(c_t, h_t) - F_h(k_t, h_t)$, where U_h and U_c are the marginal utilities of leisure and consumption respectively and F_h is the marginal product of labor. In a standard Real Business Cycle model with CES utility and Cobb-Douglas production function, the labor wedge is equal to

$$Wedge = \frac{\phi c_t}{1 - h_t} - (1 - \theta)\frac{y_t}{h_t}. \tag{21}$$

Using this formula, Ohanian and Raffo (2011) find that while in the US the labor wedge dropped dramatically during the recent crisis, the average wedge in other G7 counties experienced a modest drop. In few countries like Germany it even increased. The goal of this section is to show that the different responses of the labor market can be reconciled with the view

[11]We should be careful in defining the equity risk-premium. Since bond holders (workers) have a higher discounting than equity holders (investors), the difference between the expected return on equity (for investors) and on bonds (for workers) is not the risk premium. In fact, even in absence of risk, the return on equity will be higher than the return on bonds. Given the calibration of $\delta = 0.996$ and $\beta = 0.986$, the return differential in absence of risk would be about 4 percent yearly. Given this, we define the equity risk-premium as the difference between the return differential between equity and bonds and the difference in discount rates between investors and workers.

[12]The standard deviations for the returns are calculated on unfiltered data.

of a global financial crisis when the characteristics of the labor markets are different across countries.

In order to show this point we extend our model by adding two elements: variable labor utilization and heterogeneous labor rigidities. The role of variable labor utilization is to allow for a more powerful mechanism for endogenous fluctuations in measured labor productivity. The role of labor rigidities is to allow for a different response of labor utilization and measured labor input to shocks. By further assuming that labor rigidities differ across countries the model can generate different responses of macroeconomic and labor market variables. This last assumption is motivated by the widespread view that there are significant cross countries heterogeneity in labor markets rigidities. Ohanian and Raffo (2011) refer to indicators from the OECD Employment Outlook (2008) and report that the US is the country with the most flexible labor market. On the other hand, many of the countries in continental Europe and Japan are placed at the opposite end in the scale of labor market flexibility.

Let's start with labor utilization. The production function is specified as

$$F(K_t, k_t, n_t),$$

where n_t is the 'effective' input of labor resulting from the combination of (measured) hours, h_t, and (unmeasured) utilization, e_t, according to

$$n_t = \left[h_t^{\frac{\varrho-1}{\varrho}} + e_t^{\frac{\varrho-1}{\varrho}} \right]^{\frac{\varrho}{\varrho-1}}.$$

The parameter ϱ is the elasticity of substitution between hours spent in the working place and the actual utilization of labor. When $\varrho = 1$ we have $n_t = h_t \cdot e_t$, which is used often in the literature. The cost of utilization comes from workers disutility. Given the utility function $U(c_t, h_t+e_t)$, workers face higher disutility not only when they spent more hours in the working place but also when their services are utilized more. An implication of this specification is that the utilization cost is equal to the wage rate w_t and the total cost of labor for the firm is $(h_t + e_t)w_t$.

So far the addition of labor utilization is inconsequential for the properties of the model. Given the CES aggregation and the fact that the wage rate is the price for both h_t and u_t, firms always choose $e_t = h_t$. Thus, we can simply focus on h_t as in the original model and abstract from utilization. This equivalence no longer holds once we add labor market rigidities on working hours h_t but not on utilization u_t. Some authors interpret labor market rigidities as constraining the extensive margin (employment) rather than intensive margin (per-worker hours). However, since the model does not distinguish the extensive from the intensive margin, we interpret labor market rigidities as restrictions on total hours h_t. More specifically we assume that firms incur the convex cost

$$\kappa(h_t - \bar{h})^2 w_t,$$

where $\bar{h}$ is exogenous.

Ideally we would like to use a more standard adjustment cost. For example something like $\kappa(h_t - h_{t-1})^2 w_t$. This alternative formulation, however, would introduce an additional state variable, h_{t-1}, which increases the computational complexity of the model. To avoid this, we have specified the cost as deviation from a fixed target. The multiplication by the wage rate is motivated by economic and technical considerations. From an economic point of view it is likely that the direct cost of labor, which depends on the wage, also affects the cost of changing employment. An example is severance payments. From a technical point of view the presence of the wage allows us to apply the same normalization procedure used in the version of the model with capital accumulation.

The key parameter is κ. With a positive value of κ, the response of utilization e_t to shocks is bigger than the response of total hours h_t. This generates a decline in measured TFP and, potentially, to a decline in measured labor productivity y_t/h_t. These effects increase with the value of κ (labor market rigidity). Therefore, if in our model the first country (the US) is characterized by lower labor market rigidities than the second country (the other G7), the model could generate very different responses of the labor markets to a financial shock. This will also be reflected in the responses of the labor wedge.

7.1 Simulation results

We describe here only the calibration of the parameters that need to be re-calibrated or were not present in the baseline model.

We start with the elasticity of substitution between hours and utilization, the parameter ϱ, which we set to 5. This value implies a high degree of substitutability between hours and utilization. To show the importance of this parameter we also report the results for lower values. The utility parameter α is chosen to have average working hours of 0.33 in the equilibrium without labor rigidities.

At this point we are left with the parameters $\bar{h}$, κ^1 for country 1 and κ^2 for country 2. Given the values of κ^1 and κ^2, we could choose $\bar{h}$ to have the desired differential in average employment between the two countries. We choose total hours in the US to be 5 percent higher than in other G7 countries. However, this is not important for the quantitative properties of the model. The important parameters are κ^1 and κ^2. Unfortunately we are not aware of statistics that can be used directly to pin down these two parameters. Because of this we take a more pragmatic approach. We pick the values of $\kappa^1 = 0.3$ and $\kappa^2 = 1.5$ so that the model generates heterogeneous drops of labor wedges in response to a negative financial shock similar to the drops observed during the recent crisis. Of course, the relevance of the exercise is only to show that the model 'could' in principle generate the heterogeneous responses of the labor market observed in the US and the G6 countries. Nevertheless we think that the exercise is helpful in clarifying that the idea of a global financial crisis as a driver of the recent recession cannot be written down by the observation of cross country heterogeneity in labor market dynamics.

Figure 7 plots the impulse responses of several variables to a permanent credit contraction. The impulse responses are constructed using the same methodology as in Figure 5. As can be seen from the figure, the responses of investment and output are very similar between the two countries. However, the responses of hours and the labor wedge are significantly smaller in country 2.[13] We also observe strong heterogeneity in the response of labor productivity which falls only slightly in country 1 but experiences a large drop in country 2. Therefore, the model could replicate the different dynamics of the labor market between the US and other G6 countries even if the dynamics of other macroeconomic variables are very different.

Before closing, we would like to make some remarks about the concept of labor rigidities. These are typically interpreted as the consequence of institutional factors such as regulations and union power. Here, instead, we would like to give a broader interpretation. For example, it is well known that labor market rigidities are different across sectors. To the extent that in certain countries the crisis has impacted sectors with greater labor market flexibility, we

[13] In our model, the labor wedge is slightly different from equation (21) because the production function is not constant returns. Furthermore, there is labor utilization and c_t is only the consumption of workers, not aggregate consumption. However, we measure the labor wedge as if the true model was the standard RBC since this is the way it has been measured in the literature. After simulating our model and generating the series for c_t, h_t and y_t, we compute the wedge by plugging the series in equation (21). The values of the parameters are the same values used in Ohanian and Raffo (2011), that is, $\beta = 0.99$, $\theta = 0.36$, $\delta = 0.0175$, $g = 0.005$ and ϕ is chosen so to have steady hours of 0.33.

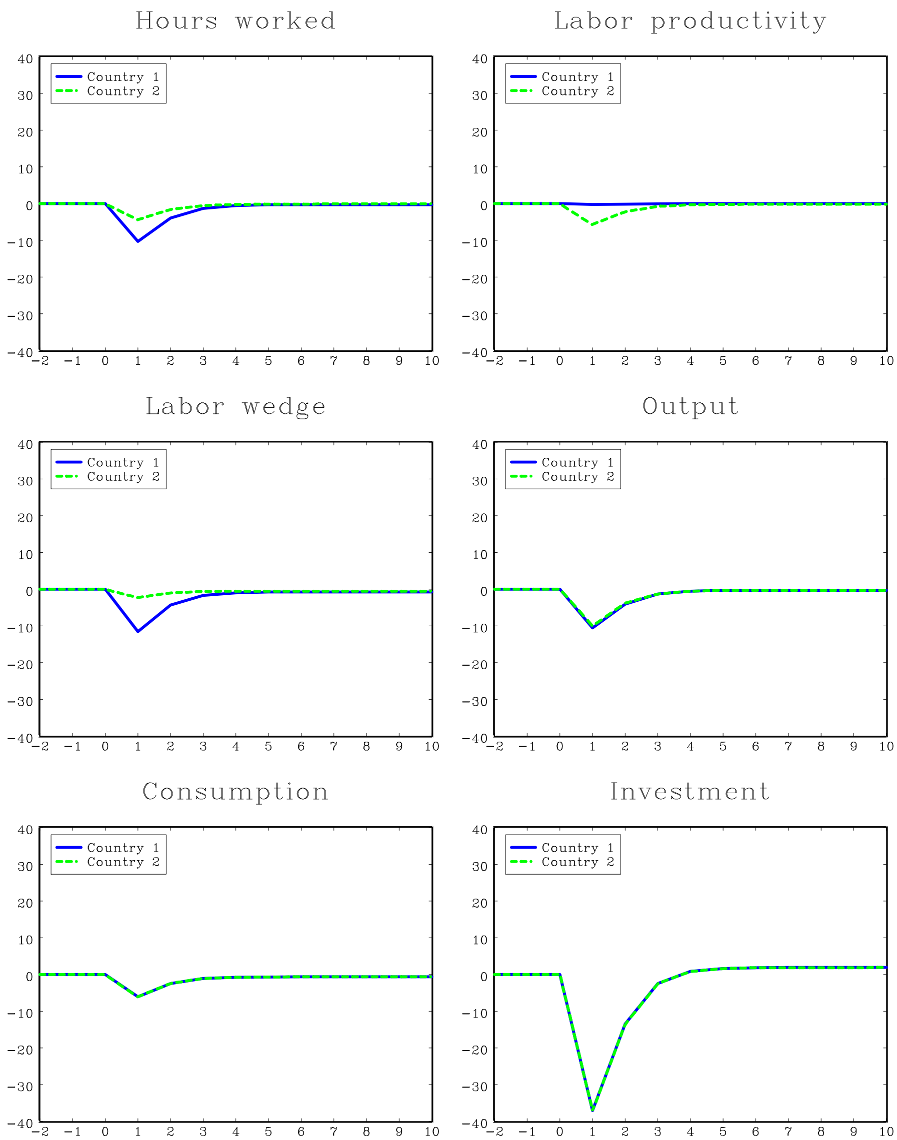


Figure 7: Impulse responses to a credit contraction with asymmetric labor markets.

may observe larger declines in employment and hours. For example, it is well know that construction tends to have more flexible hiring as a result of its high cyclicality. Then, countries that experience large contractions in the real estate sector are also likely to experience large drops in employment. This is the case, for example, for Spain, a country where the real estate sector experienced an abnormal boom before the crisis. Because of the previous boom it was not surprising that this sector was hit hard by the crisis. Since during the boom this sector was hiring a significant fraction of temporary workers, many of which immigrants, it was relatively easy for this sector to downsize employment. In this sense, a country like Spain could be considered a country with a flexible labor market, simply because the sectors with greater labor market flexibility are relatively more important. On the other hand, it is well known that Germany is one of the few countries that did not experience a real estate boom before the crisis and, because of this, the sector was not hit as hard as in other countries. Instead, the German economy was adversely affected in industries like automotive. But this is one of the industries where labor rigidities are especially high. So it is not surprising that Germany cut production without firing. In some cases this was encouraged by government support policies.

8 Conclusion

The recent financial crisis has been characterized by an historically high degree of international synchronization in real and financial variables. We have proposed a theoretical framework in which endogenous fluctuations in credit market conditions result from self-fulfilling expectations. These fluctuations affect the real sector of the economy through a credit channel: booms enhance the borrowing capacity of firms and in the general equilibrium they lead to higher employment and production. The opposite arises after a credit contraction. Interestingly, business cycle fluctuations induced by movements in credit markets are highly asymmetric, *i.e.*, contractions are sharper than expansions and they generate large fluctuations in asset prices.

When countries are financially integrated, movements in credit markets also generate large spillovers to the real and financial sectors of other countries. There are two channels of international transmission. The first is through the cost of capital which in an integrated financial market is equalized across countries. The second channel is based on the endogenous nature of credit market conditions. These conditions change when the economy switches from one self-fulfilling equilibrium to another self-fulfilling equilibrium. But in an integrated world market the shift in one country can only arise if the shift takes place also in the other. Therefore, changing financial market conditions are highly synchronized when financial markets are internationally integrated.

This study does not exclude the possibility that other sources of business cycle fluctuations also generate international co-movement in real variables. Our interest in changing credit market conditions as a source of business cycle is motivated by their ability to generate large cross-country co-movements in the real sector of the economy together with the large international co-movements in the flows of financing and asset prices.

Appendix

A First order conditions

Consider the optimization problem (1) and let λ and μ be the Lagrange multipliers associate with the two constraints. Taking derivatives we get:

$$\begin{aligned} d: &\quad 1-\lambda=0 \\ h: &\quad \lambda[F_h(h)-w]-\mu F_h(h)=0 \\ b': &\quad Em'V_{b'}(\mathbf{s}';b')+\frac{\lambda}{R}-\frac{\mu}{R}=0 \end{aligned}$$

The envelope condition is:

$$V_b(\mathbf{s};b) \quad = \quad -\lambda$$

The above conditions can be re-arranged as in (4) and (5).

B First order conditions for the model with capital

Differentiating the firm's problem (16) with respect to h_t, $\tilde{b}_{t+1}$, $\tilde{i}_t$, $\tilde{k}_{t+1}$, we get:

$$F_h(z_t,\tilde{k}_t,h_t)=\frac{\tilde{w}_t}{1-\mu_t} \tag{22}$$

$$\frac{1-\mu_t}{R_t}+g_t^{-\sigma}E\tilde{m}_{t+1}\tilde{V}_b(\tilde{\mathbf{s}}_{t+1};\tilde{k}_{t+1},\tilde{b}_{t+1})=0 \tag{23}$$

$$Q_t\Upsilon'\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)=1 \tag{24}$$

$$Q_t=\xi_t\mu_t+g_t^{-\sigma}E\tilde{m}_{t+1}\tilde{V}_k(\tilde{\mathbf{s}}_{t+1};\tilde{k}_{t+1},\tilde{b}_{t+1}) \tag{25}$$

where μ_t is the lagrange multiplier associated with the enforcement constraint and Q_t is the lagrange multiplier associated with the law of motion of capital (Tobin's q). The multiplier associated with the budget constraint is 1. For the foreign country we have the same conditions but with country specific variables denoted with the start superscript.

The envelope conditions are:

$$\tilde{V}_b(\tilde{\mathbf{s}}_t; \tilde{k}_t, \tilde{b}_t) = -1 \tag{26}$$

$$\tilde{V}_k = (1-\mu_t)F_k(z_t, \tilde{k}_t, h_t) + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right) - \Upsilon'\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\frac{\tilde{i}_t}{\tilde{k}_t}\right]Q_t \tag{27}$$

Substituting the envelope conditions and imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, we obtain (17)-(20).

C Dynamic system and solution approach

We will use the bar sign to denote aggregate worldwide variables normalized by the worldwide stock of capital. For example, $\bar{d}_t$ is the normalized worldwide dividend, defined as

$$\bar{d}_t = \frac{d_t + d_t^*}{K_t + K_t^*} \equiv \tilde{d}_t + \tilde{d}_t^*.$$

The full list of equilibrium conditions are:

$$1 = \delta g_t^{-1} R_t E_t \left(\frac{\bar{c}_{t+1}}{\bar{c}_t} \right)^{-1} \tag{28}$$

$$\tilde{c}_t^* = \chi \tilde{c}_t \tag{29}$$

$$\tilde{w}_t h_t + \tilde{w}_t^* h_t^* + \bar{b}_t = \bar{c}_t + \frac{g_t \bar{b}_{t+1}}{R_t} \tag{30}$$

$$\bar{b}_t + \bar{d}_t + \bar{i}_t = F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) - \tilde{w}_t h_t - \tilde{w}_t^* h_t^* + \frac{\bar{g}_t \bar{b}_{t+1}}{R_t} \tag{31}$$

$$g_t(\xi_t s_{t+1} + \xi_t^* s_{t+1}^*) \geq \frac{g_t \bar{b}_{t+1}}{R_t} + F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) \tag{32}$$

$$(1 - \mu_t)\bar{d}_t^{-\sigma} = \beta g_t^{-\sigma} R_t E \bar{d}_{t+1}^{-\sigma} \tag{33}$$

$$\alpha h_t^{\gamma} = \frac{\tilde{w}_t}{\tilde{c}_t} \tag{34}$$

$$\alpha (h_t^*)^{\gamma} = \frac{\tilde{w}_t^*}{\tilde{c}_t^*} \tag{35}$$

$$g_t s_{t+1} = (1 - \tau) s_t + \Upsilon \left(\frac{\tilde{i}_t}{s_t} \right) s_t \tag{36}$$

$$g_t s_{t+1}^* = (1 - \tau) s_t^* + \Upsilon \left(\frac{\tilde{i}_t^*}{s_t^*} \right) s_t^* \tag{37}$$

$$F_h(z_t, s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t} \tag{38}$$

$$F_h(z_t^*, s_t^*, h_t^*) = \frac{\tilde{w}_t^*}{1 - \mu_t} \tag{39}$$

$$Q_t \Upsilon' \left(\frac{\tilde{i}_t}{s_t} \right) = 1 \tag{40}$$

$$Q_t^* \Upsilon' \left(\frac{\tilde{i}_t^*}{s_t^*} \right) = 1 \tag{41}$$

$$Q_t = \xi_t \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\} \tag{42}$$

$$Q_t^* = \xi_t^* \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\} \tag{43}$$

Equations (28)-(43) form a dynamic system composed of 16 equations. Given the states $z_t, \xi_t, z_t^*, \xi_t^*, \bar{b}_t, s_t$, the unknown variables are h_t, h_t^*, c_t, c_t^*, w_t, w_t^*, i_t, i_t^*, Q_t, Q_t^*, g_t, μ_t, R_t, $\bar{d}_t$, $\bar{b}_{t+1}$, s_{t+1}. Therefore, we have a dynamic system of 16 equations in 16 unknowns.

The computational procedure is based on the approximation of four functions:

$$
\begin{aligned}
\Gamma_1(\mathbf{s}_{t+1}) &= \bar{c}_{t+1}^{-1} \\
\Gamma_2(\mathbf{s}_{t+1}) &= \bar{d}_{t+1}^{-\sigma} \\
\Gamma_3(\mathbf{s}_{t+1}) &= \bar{d}_{t+1}^{-\sigma}\left\{(1-\mu_{t+1})F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_{t+1}}{s_{t+1}}\right)\right] Q_{t+1}\right\} \\
\Gamma_4(\mathbf{s}_{t+1}) &= \bar{d}_{t+1}^{-\sigma}\left\{(1-\mu_{t+1})F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*}\right)\right] Q_{t+1}^*\right\}.
\end{aligned}
$$

In addition to these four function, we need to guess the function $p(\mathbf{s}_{t+1})$, that is, the probability of $\xi_{t+1} = \underline{\xi}$. This is necessary to compute the next period expectation.

The procedure starts with a guess for the values of the approximated functions $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$. We first form a two dimensional grid for the endogenous states $\bar{b}$ and s. Then for each realization of the exogenous shocks—$z_t, \xi_t, z_t^*, \xi_t^*$—we guess the values taken by the above functions over the grid points. Values outside the grid are obtained through bi-linear interpolation. Next we guess $p(\mathbf{s}_{t+1})$ for each grid point. Once we know the approximated functions and probabilities for ξ_{t+1}, we can solve for the 16 unknowns of the system (28)-(43) at each grid point and for each possible values of $z_t, \xi_t, z_t^*, \xi_t^*$. In finding the solutions we check whether the enforcement constraint is binding ($\mu_t > 0$) or not binding ($\mu_t = 0$). We then use the solutions found at each grip point to update the guesses for the four functions $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$ and the probabilities $p(\mathbf{s}_{t+1})$. To update these probabilities we need to check whether multiple equilibria are feasible at any possible states. Effectively we check this on the grid points of the states. We keep iterating until the guesses for $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$, $\Gamma_4(\mathbf{s}_{t+1})$ and $p(\mathbf{s}_{t+1})$ at each grid point for the states are equal to the values obtained by solving the dynamic system.

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