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## Measuring International Business Cycles by Saving for a Rainy Day

Mario J. Crucini\* and Mototsugu Shintani\*\*

### Abstract

We examine the business cycles of the member countries of the G-7 and Australia based on the cyclical measure considered by Cochrane (1994). The measure is motivated by the prediction that the representative consumer changes savings in response to temporary deviations of income from its stochastic trend, while satisfying a long-run budget constraint. We also compare Cochrane's original cyclical measure and an alternative simple saving-based measure and show that they track each other. Our analysis reveals that the extent of international business cycle comovement and the Great Moderation are significantly altered when the saving-based measures are employed in place of commonly used univariate business cycle filters.

**Keywords:** Error correction model; the Great Moderation; international comovement puzzle; permanent income hypothesis; stochastic trends; trend cycle decomposition

**JEL classification:** C32, E21, E32, F44

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# 1 Introduction

Quantitative macroeconomics typically begins with a decomposition of aggregate output into a trend and a cycle. The two most common approaches are statistical filters which extract cycles from the raw data with the goal of matching official business cycle dating and estimates of transitory deviations from a growth path based on a dynamic stochastic general equilibrium (DSGE) model. The filtering approach is subject to the ‘measurement without theory’ critique for failing to impose any meaningful economic restrictions on the data, while the DSGE approach can make the decomposition totally invalid if the structural model is misspecified. Middle ground between a purely statistical filter and an overly restrictive economic model was advocated by Cochrane (1994) who argued that researchers should rely on actual consumption responses to infer a representative agent’s view of the trend and cycle, while imposing only minimal economic restrictions. Later Cogley (1997) confirmed that Cochrane’s business cycle measure was able to capture the dynamics of a standard flexible price DSGE model and conformed closely to the dates of NBER recessions when applied to the U.S. data.

While there is a large amount of literature on trend-cycle decompositions of national output, the usefulness and broader implications of Cochrane’s approach in the context of international business cycles has not yet been established. This is the void filled by this paper. Specifically, we make two contributions. First, we revisit Cochrane’s approach and provide evidence that a computationally much simpler saving-based measure of the business cycle tracks the more elaborate measure very closely in each of the G-7 countries and Australia. Second, we show that the business cycle emerging from our method contrasts significantly with what other methods produce, altering key moments of the data that have been the focus of international business cycle research. In particular, relative to the Hodrick-Prescott (1997, HP) filter benchmark, we find that: i) the standard deviation is about 30% higher; ii) dividing the sample at 1983:Q4, the standard deviation falls by 20% rather than 40%; and 3) the average correlation of international business cycles falls from 0.47 to approximately

zero. In words: business cycles are more volatile, the Great Moderation is less dramatic and international output comovement is much lower, than previously thought.

Our saving-based business cycle measure is closely related to the ‘saving for a rainy day’ implication of the permanent income model discussed by Campbell (1987).<sup>1</sup> In the rational expectation-permanent income model, the representative consumer’s long-run budget constraint requires cointegration of consumption and total income in the presence of a stochastic trend. However, consumption is less volatile and contains better information about the long-run trend than does income alone. This motivates a multivariate approach to trend-cycle decompositions using consumption as a proxy for the trend component and implies that the cyclical component of output is essentially national saving. Using the permanent income model as a benchmark, along with a cointegrating relationship between consumption and income, Cochrane (1994) extracted the cyclical component of U.S. output based on a bivariate error correction model (ECM).<sup>2</sup> In particular, he considered a multivariate Beveridge-Nelson (1981, BN) decomposition using an ECM of gross national product (GNP) and consumption with a log consumption/GNP ratio as the error correction term.<sup>3</sup> Unlike the ECM/BN cycle, our saving-based measure of business cycles does not require a parametric specification of short-run business cycle dynamics.

In our analysis, we examine Cochrane’s original ECM/BN method in detail and compare the performance of saving-based measure and the ECM/BN decomposition using international data.<sup>4</sup> We first point out that it is important to incorporate an additional zero coefficient

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<sup>1</sup>This testable implication of high savings when income is expected to fall has been examined by Campbell (1987) for U.S., Campbell and Clarida (1987) for Canada and U.K., MacDonald and Kearney (1990) for Australia, and Shintani (1994) for Japan.

<sup>2</sup>Instead of using the ECM, Morley (2007) considers a bivariate unobserved components (UC) model of GDP and consumption while imposing a cointegrating restriction for the purpose of estimating the stochastic trend component of the output.

<sup>3</sup>Lettau and Ludvigson (2004) extend Cochrane’s ECM/BN approach by including asset wealth as an additional covariate. While not in ECM format, Rotemberg and Woodford (1996) and Ravn (1997) also consider the multivariate BN decomposition using consumption/output ratio.

<sup>4</sup>Cogley (1997) also argues similarity between ECM/BN cycles and saving in the sense that both measures outperform standard filtered cycles in terms of their correlation to structural cycles.

restriction when estimating the ECM. We then show our saving-based measure nests the ECM/BN cycle with the cointegrating restriction in many cases. In the case of random walk consumption, the saving measure would work without the need to smooth consumption at all.<sup>5</sup> Even in the presence of transitory variation in consumption, we show that the ECM/BN cycle is well approximated by saving when raw consumption is replaced by a moving average of current and past consumption or HP filtered consumption.

Throughout the paper,  $y_t$  denotes log gross domestic product (GDP),  $c_t$  denotes log aggregate private consumption,  $\Delta$  denotes first differences,  $\Delta y_t = y_t - y_{t-1}$ .

## 2 The trend-cycle decomposition of GDP

### 2.1 A saving-based measure of the business cycle

In this paper, we conduct trend-cycle decompositions of aggregate output using one of the most basic conditions imposed in almost all macroeconomic models – the representative consumer’s long-run budget constraint. Consider the additive trend-cycle decomposition of output  $y_t$ ,

$$y_t = y_t^g + y_t^c,$$

where  $y_t^g$  is the ‘growth’ or ‘trend’ component and  $y_t^c$  is the ‘cyclical’ or ‘transitory’ component. Instead of relying on researcher’s preconceptions about business cycle frequencies, the idea is to obtain the cyclical component such that a representative agent would view it as a transitory deviation from the stochastic trend in output. The long-run budget constraint identity implies that income and consumption share a common long-run trend. In other words, income and consumption are cotrended. Since almost every modern macroeconomic model embodies this sensible economic restriction, it is useful to impose it at the outset.

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<sup>5</sup>If consumption is an exact random walk: “the Beveridge-Nelson trend would exactly equal consumption less the mean log GNP/consumption ratio” (Cochrane, 1994, page 252).

If aggregate output is used in place of total disposable income, the simple permanent income model suggests that the common stochastic trend is permanent income,  $y_t^g = c_t$ , and the transitory component is savings,  $y_t^c = s_t = y_t - c_t$ . In practice, consumption does not necessarily follow a pure random walk process as suggested by the model. The point, however, is that consumption is less volatile than output and therefore contains better information about the trend as a result of thoughtful consumption decisions made by rational consumers. Thus, consumption is expected to be a good proxy for the trend after removing its transitory component, sometimes referred to as ‘transitory consumption.’ This motivates the following simple measure of the cyclical component in GDP based on a modified aggregate saving:

$$y_t^c = y_t - \bar{c}_t^g - \alpha - \beta t, \quad (1)$$

where  $\bar{c}_t^g$  is the estimated trend based on smoothed consumption. If  $\bar{c}_t^g = c_t$  and if there is no deterministic trend component ( $\alpha = 0$  and  $\beta = 0$ ), the measure reduces to just aggregate saving itself,  $s_t = y_t - c_t$ . For the consumption trend, one may simply use the moving average of current and lagged consumption as,  $\bar{c}_t^g = (1/M) \sum_{j=0}^{M-1} c_{t-j}$  where  $M$  is the moving average length.

In contrast, most trend-cycle decompositions involve applying the HP filter to output and consumption by minimizing

$$\sum_{t=1}^T (y_t^c)^2 + \lambda \sum_{t=1}^T [(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2. \quad (2)$$

and

$$\sum_{t=1}^T (c_t^c)^2 + \lambda \sum_{t=1}^T [(c_{t+1}^g - c_t^g) - (c_t^g - c_{t-1}^g)]^2 \quad (3)$$

separately, where  $\lambda$  is a smoothing parameter chosen by the researcher and  $c_t^c = c_t - \bar{c}_t^g$  is the cyclical component of consumption. Obviously, when income and consumption are filtered

separately, there is no guarantee that they are cointegrated. To impose cointegration and a common trend structure, a convenient multivariate HP filtering procedure was proposed by Kozicki (1999). With a cointegrating vector  $(1, -1)$ , her constrained multivariate HP filter applied to output and consumption can be obtained by minimizing

$$\omega_1 \sum_{t=1}^T (y_t^c)^2 + \omega_2 \sum_{t=1}^T (c_t^c)^2 + \lambda \sum_{t=1}^T [(c_{t+1}^g - c_t^g) - (c_t^g - c_{t-1}^g)]^2 \quad (4)$$

where  $c_t^g (= y_t^g)$  is a common stochastic trend,  $\omega_1$  and  $\omega_2$  are weights controlling the relative importance of the two variables. In our paper, we claim that consumption contains more information about the trend than the income, thus our weight should satisfy  $\omega_1 < \omega_2$ . In an extreme case, we can simply set  $\omega_1 = 0$  and  $\omega_2 = 1$  so that the minimization of (4) reduces to the minimization of (3). In such a case, the HP trend component of consumption which minimizes (3), denoted,  $c_t^g$ , can be used as the measure of smoothed consumption. Thus simply setting  $\bar{c}_t^g = c_t^g$  and substituting this into (1) defines the HP-version of our saving-based business cycle measure.<sup>6</sup>

If  $y_t$  and  $c_t$  are cointegrated,  $y_t$  and  $\bar{c}_t^g$  are also cointegrated. In the presence of a linear deterministic trend, the cointegrating vector that eliminates a common stochastic trend may or may not eliminate a common deterministic trend at the same time. Using the terminology of Ogaki and Park (1997), two variables are deterministically cointegrated if the cointegrating relationship also eliminates the deterministic trend. The long-run budget constraint is more closely related to this notion of deterministic cointegration, suggesting  $\beta = 0$  in the definition of the saving-based measure of the cycle. However, leaving the room for the possibility of  $\beta \neq 0$  may be useful in some cases as it can proxy for missing additional covariates (e.g., an asset variable), a gradual shift in the preference for the precautionary saving, or the presence of measurement errors. In applications, the residual from the regression of  $y_t - \bar{c}_t^g$  on a linear

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<sup>6</sup>Alternatively, we could replace the HP filter by the Baxter and King (1999, BK) filter and introduce the BK-version of our saving-based business cycle measure.



trend allows for such possibilities.<sup>7</sup>

## 2.2 A bivariate Beveridge-Nelson decomposition of Cochrane's (1994) error correction model

Cochrane (1994) extracts the information on the trend in GNP from consumption using a bivariate error correction model (ECM) of GNP and consumption combined with a multivariate Beveridge-Nelson (BN) decomposition.

The specification of the ECM employed here is,

$$\Delta c_t = \alpha_{c1} + \alpha_{c2}t + \beta_{c1}\Delta c_{t-1} + \beta_{c2}\Delta c_{t-2} + \beta_{c3}\Delta y_{t-1} + \beta_{c4}\Delta y_{t-2} + \varepsilon_t^c \quad (5)$$

$$\begin{aligned} \Delta y_t = & \alpha_{y1} + \alpha_{y2}t + \gamma_y(c_{t-1} - y_{t-1}) \\ & + \beta_{y1}\Delta c_{t-1} + \beta_{y2}\Delta c_{t-2} + \beta_{y3}\Delta y_{t-1} + \beta_{y4}\Delta y_{t-2} + \varepsilon_t^y \end{aligned} \quad (6)$$

where  $\varepsilon_t^c$  and  $\varepsilon_t^y$  are zero-mean and mutually correlated shocks in the reduced-form ECM. The fixed lag length of 2 follows Cochrane.

Let us first consider the role of the loading coefficient  $\gamma_y$ . A characteristic equation of a cointegrated bivariate system should have one unit root and the other root outside the unit circle. Thus, in the absence of an error correction term in (5), cointegration of consumption and income requires  $\gamma_y \in (0, 2)$  in (6). For this reason, the estimate of  $\gamma_y$  has a direct implication for the long-run budget constraint. If  $\gamma_y = 0$ , both consumption and income are difference stationary and they are not cointegrated.<sup>8</sup> Since the  $t$ -statistic follows a nonstandard distribution under the null hypothesis  $\gamma_y = 0$ , Cochrane employs a bootstrap method to claim that the coefficient is significantly different from zero in his analysis. As long

<sup>7</sup>If trend breaks are observed in savings, they can be incorporated by running the kinked trend regression in place of linear regression in this final stage of extracting cyclical component.

<sup>8</sup>If  $\gamma_y < 0$  or  $\gamma_y > 2$ , the system is an explosive process.

as the cointegrating restriction  $\gamma_y \in (0, 2)$  is satisfied, however, the ordinary least squares (OLS) estimator remains asymptotically normal.

To identify the impulse responses to permanent and transitory shocks, Cochrane employs a recursive orthogonalization of shocks with the order consumption and then income. The orthogonalized shock in income growth equation (6) can be interpreted as the transitory shock because it has zero (contemporaneous) impact on consumption and thus cannot be a permanent shock, according to the permanent income model. To be more specific, the permanent shock, denoted by  $\nu_t^P$ , and the transitory shock, denoted by  $\nu_t^T$ , are identified as

$$\begin{bmatrix} \nu_t^P \\ \nu_t^T \end{bmatrix} = R^{-1} \begin{bmatrix} \varepsilon_t^c \\ \varepsilon_t^y \end{bmatrix}$$

where  $R$  is the lower triangular matrix which satisfies  $RR' = E(\varepsilon_t \varepsilon_t') = \Sigma$  and  $E(\nu_t \nu_t') = I$  where  $\varepsilon_t' = [\varepsilon_t^c \ \varepsilon_t^y]$  and  $\nu_t' = [\nu_t^P \ \nu_t^T]$ . In addition to the permanent response to  $\nu_t^P$ , the presence of cointegration (i.e., the error correction term) restricts the impulse responses of consumption and income to converge to a common level in the long-run in response to either shock, consistent with the long-run budget constraint.<sup>9</sup>

Note that equations (5) and (6) involve two modifications of Cochrane's original ECM: including time trends in both equations and omitting the error correction term,  $c_{t-1} - y_{t-1}$ , from the consumption growth equation. The trend terms are included to reflect the elimination of any deterministic trend from our saving-based cyclical component and the absence of error correction term in consumption growth equation is related to shock identification. Without this exclusion restriction, what Cochrane identifies as a transitory (GNP) shock, will have a permanent effect on both income and consumption (see Figure 1, p. 245 of Cochrane (1994)). This is inconsistent with the permanent income model, where transitory shocks have no long-run effect on either income or consumption.

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<sup>9</sup>See King, Plosser, Stock and Watson (1991) on this point.

To see why this is so, let  $\gamma_c$  be the loading coefficient on the error correction term in consumption growth equation in Cochrane's specification. Then, the long-run responses to the identified transitory shock are:

$$\lim_{h \rightarrow \infty} \frac{\partial E_t(y_{t+h})}{\partial \nu_t^T} = \lim_{h \rightarrow \infty} \frac{\partial E_t(c_{t+h})}{\partial \nu_t^T} = \frac{-\gamma_c}{\gamma_y - \gamma_c}.$$

From Table 1 of Cochrane (1994, p. 243)  $\hat{\gamma}_c = -0.02$  and  $\hat{\gamma}_y = 0.08$ , and this formula gives a long-run impulse response of 0.2 ( $= 0.02/0.1$ ) consistent with Figure 1 of Cochrane's paper. In contrast, provided  $\gamma_y \neq 0$ , the transitory shock,  $\nu_t^T$ , identified by a recursive scheme has zero long-run effect if and only if  $\gamma_c = 0$ .<sup>10</sup>

For this reason, the zero restriction on the loading coefficient  $\gamma_c$  in the consumption growth equation is imposed when estimating the ECM in what follows. Due to the fact that the two equations no longer have common regressors, OLS becomes inefficient since the equivalence of OLS and generalized least squares (GLS) no longer holds. To achieve efficiency, we employ a restricted multivariate GLS method. The GLS estimator is asymptotically normal as long as the cointegrating restriction  $\gamma_y \in (0, 2)$  is satisfied as shown in the Statistical Appendix.

Having established the appropriateness of the ECM and the estimation method, the next technical detail we discuss is the BN decomposition in the ECM context. The multivariate BN decomposition in a cointegrated system was first proposed by Stock and Watson (1988) and has been used in many applied studies, including Cochrane (1994), Evans and Reichlin (1994), and Lettau and Ludvigson (2004). As in the case of a univariate BN decomposition, both the trend and cycle components are generated from a common vector error component. The trend component follows a (multivariate) random walk process, while the cyclical component is serially correlated. This feature contrasts to an alternative decomposition based on the

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<sup>10</sup>An alternative is to directly impose a zero long-run impulse response assumption as Blanchard and Quah (1989) do. However, a simple application of Blanchard-Quah method cannot incorporate the contemporaneous impact of the permanent shock implied by the permanent income model.

ECM, often referred to as Granger-Gonzalo decomposition (Gonzalo and Granger, 1995), where the trend and cycle components are orthogonal, but the trend component is generated from serially correlated errors.<sup>11</sup>

What is the relationship between the BN decomposition ( $y_t^g$  and  $y_t^c$ ) and shocks identified by the recursive scheme ( $\nu_t^P$  and  $\nu_t^T$ )? When there is an error correction term in consumption growth equation (5), the bivariate cointegrated system generally implies that the random-walk trend component  $y_t^g$  is generated by a linear combination of current  $\nu_t^P$  and  $\nu_t^T$ . However, if we impose  $\gamma_c = 0$ , the long-run impulse response to identified shocks becomes lower triangular, and thus the random-walk trend component  $y_t^g$  is generated only from the permanent shock  $\nu_t^P$ . In contrast, the cyclical component  $y_t^c$  consists of current and past values of both type of shocks,  $\nu_t^P$  and  $\nu_t^T$ .

What does this imply about the relationship between our saving-based business cycle measure (1) and the ECM/BN cycle?<sup>12</sup> Cochrane points out that, if consumption follows a pure random walk, as predicted by the simple permanent income model, the ECM/BN trend becomes (log) consumption less the mean of savings,  $y_t^g = c_t - \bar{s}_t$ . Thus the ECM/BN cycle is simply demeaned savings,  $y_t^c = y_t - y_t^g = s_t - \bar{s}_t$ . In the presence of a deterministic trend, the cyclical component becomes detrended savings,  $y_t^c = s_t - \alpha - \beta t$ , which corresponds to (1) with a choice of  $\bar{c}_t^g = c_t$ . What happens if consumption growth is serially correlated? Such an extension can be considered by imposing  $\beta_{c3} = \beta_{c4} = 0$  in (5). The Statistical Appendix shows that the ECM/BN cycle once again corresponds to (1) provided consumption is smoothed according to:

$$\bar{c}_t^g = \sum_{i=0}^2 w_i c_{t-i} \quad (7)$$

where  $w_0 = \frac{1}{1-(\beta_{c1}+\beta_{c2})}$ ,  $w_1 = \frac{-\beta_{c1}}{1-(\beta_{c1}+\beta_{c2})}$ , and  $w_2 = \frac{-\beta_{c2}}{1-(\beta_{c1}+\beta_{c2})}$ . It is important to note that

<sup>11</sup>See also Gonzalo and Ng (2001) for identification of shocks combined with Granger-Gonzalo decomposition, and Levchenkova, Pagan and Robertson (1998) for a broader discussion.

<sup>12</sup>See Appendix A.2 for our definition of the BN cycle in a multivariate context. We adopt the opposite sign convention for the transitory component, making it procyclical.

the moving average weights,  $w_i$ 's, depend only on the coefficients in (5), not on those in (6). It is straightforward to obtain a similar result for more generalized cases beyond two lags in the ECM by adding more lags in the moving average of (7). In summary, even in the presence of transitory consumption variation, the ECM/BN cycle takes the form of (1) with the moving average weights for consumption determined by the parameters which capture the short-run dynamics of consumption growth.

### 3 Estimation of ECMs in the G-7 and Australia

The bivariate error correction models of (5) and (6) are estimated on a country-by-country basis. Quarterly series of GDP and total consumption are used for  $y_t$  and  $c_t$ , respectively.<sup>13</sup> Both were obtained from the OECD database. The countries are: Australia, Canada, France, Italy, Germany, Japan, the United Kingdom, and the United States. The starting dates of the samples vary, all end in the first quarter of 2005 (see Table 1).

Table 1 reports the estimation results obtained using a restricted multivariate GLS method. The coefficient  $\gamma_y$  on the error correction term is positive in all cases supporting the saving for a rainy day implication of Campbell (1987). Furthermore, all the point estimates fall in the range of cointegrating restriction  $\gamma_y \in (0, 2)$  mostly with a tight confidence interval (exceptions are Italy and the United Kingdom). Thus, the results are consistent with long-run budget constraints.<sup>14</sup>

In contrast, the short-run dynamics are imprecisely estimated: the coefficients on lagged consumption and income growth are statistically significant in only 18 of 56 cases. Fourteen of

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<sup>13</sup>We can exclude durable consumption from  $c_t$  since nondurable and services better match the notion of consumption in the model. It turns out using alternative measure of consumption does not alter the main results. Here, we report the results based on total consumption because (i) the national saving (rate) can be conveniently used as a business cycle measure, and (ii) total expenditure is more appropriate to impose the long-run budget constraint .

<sup>14</sup>For several countries, including Japan, significantly negative trend coefficients are found in output growth equations. This reflects the fact that long-run negative trend in saving which is included as an error correction term.

these statistically significant coefficients are accounted for by only three countries: Japan (6), Canada (4) and Italy (4). There is no tendency for lagged growth rates to be more significant in the consumption equation than in the income equation. Thus, transitory consumption may not be as important as the cointegrating relationship, which is the robust feature of the empirical model.

### 3.1 Decomposition of variance

Table 2 reports variance decompositions at forecast horizons of one quarter, one year and infinity. As the Table shows, virtually all of the variance in consumption growth gets attributed to the permanent shock. In sharp contrast, the variance of output growth is split almost exactly 50-50 between permanent and transitory shocks for the United States and this is robust across forecast horizons. Cochrane (1994) attributed 85% of the 1-quarter ahead forecast to the transitory component compared to only about 60% here. Most of this difference is likely due to the fact that his bivariate specification also included an error-correction term in (5), thereby allowing transitory shocks to alter the long-run level of consumption and income and elevating their importance in the variance decomposition of output. Germany and the United Kingdom are similar to the U.S. with about 60% of output variation attributed to the transitory shock. Japan is an outlier with a small fraction of variance attributed to the transitory shock (40%).<sup>15</sup> What is interesting about the remaining countries – Australia, Canada, France and Italy – is that transitory shocks are even more important than is true of the U.S.. Thus, the international evidence against the pure random walk model of output growth seems even more compelling in other countries than it is for the United States.

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<sup>15</sup>This observation seems to be consistent with the dominance of the stochastic trend in explaining the rapid economic growth in Japan during 1960s.

## 3.2 Impulse responses

To evaluate Cochrane’s business cycle measure, Cogley (1997) used artificial structural cycle data generated from a standard flexible price DSGE model of Christiano and Eichenbaum (1992). When technology shocks have permanent effects, he found that both saving and ECM/BN cycle perform better than HP and BK filters in matching structural cycles.<sup>16</sup> Here, we employ a different approach and evaluate estimated impulse responses from eight countries by comparing their shape with those of the impulse responses obtained from a standard flexible price DSGE model of King, Plosser and Rebelo (1988).

The estimated impulse responses to transitory and permanent shocks are shown in Figure 1 and are broadly consistent with the qualitative predictions of structural models. First, the half-life of the income response to a transitory shock is 7 quarters according to the benchmark parameterization of King, Plosser and Rebelo (1988), while the average half-life across countries using the estimated impulse responses is 8 quarters. The path back to the steady-state is predicted to be monotonic in the benchmark theory, typical of our estimates. Consumption responds less to a transitory shock as predicted by the permanent income theory. Recall, the impact response of consumption to a transitory shock is zero due to the identification assumption.

Turning to the case of a permanent productivity shock (i.e., a permanent increase in productivity of 1%), the standard flexible price DSGE model predicts a monotonically rising profile of both consumption and income. Empirically, both variables rise toward a new higher steady state from below. Theory attributes the response path to capital accumulation such that the marginal product of capital is eventually re-equated with the steady-state real interest rate in the long-run. In a closed economy this is accomplished by an investment boom and a transitory increase in hours of work, in an open economy some of the capital accumulation

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<sup>16</sup>The saving-based measure considered by Cogley (1997) is based on the residuals obtained from the regression of output on consumption, thus is different from our saving-based measure.

may be financed by running a current account deficit. Most of the empirical impulses indicate rapid adjustment toward the new higher steady state with half of the adjustment achieved in the first few quarters.<sup>17</sup> The response of the calibrated DSGE model (not shown) is considerably slower.

The role of savings as a cyclical measure is evident in the impulse response functions. Income and consumption move very closely together in response to a permanent shock and thus savings and the current account move only modestly. In the case of the transitory shock savings and the current account move dramatically and in a very persistent fashion.

## 4 Saving cycles and ECM/BN cycles

This section reports comparisons of our saving-based business cycle measure for each nation with the BN cycle obtained from the restricted bivariate ECM estimates as well as some commonly used univariate business cycle measures. We denote Cochrane's ECM/BN cycle by *ECM-BN*. The simplest version of our saving-based measure, namely, the detrended savings with the choice of  $\bar{c}_t^g = c_t$  in (1) is denoted by *SV*. The saving-based measure derived using a moving average of consumption  $\bar{c}_t^g = (1/4) \sum_{j=0}^{4-1} c_{t-j}$ , is denoted by *SV-MA*. The saving-based measure with the HP-filtered consumption obtained by minimizing (3) with  $\lambda = 1600$  is denoted by *SV-HP*. The univariate BN business cycle measure obtained from the OLS estimate of a second-order autoregressive (AR) model of output growth is denoted by *BN*, and the standard HP cycle obtained by minimizing (2) with  $\lambda = 1600$  is denoted by *HP*.

Figure 2 plots ECM-BN and SV cycles of eight countries, along with HP cycle. Recession episodes for each country, based on OECD Composite Leading Indicators (CLIs), are shown as shaded areas. Overall, recession periods seem to be associated with the time when ECM-BN and SV tend to decrease. For example, at the end of each recession, both ECM-BN and

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<sup>17</sup>Only Australia, Italy and the United Kingdom take more than one year to account for one-half of the distance between the impact response and the new higher steady-state level of output.



SV have been reduced from the beginning of recession in the United States for all recession episodes, which is consistent with a claim made by Cogley (1997).<sup>18</sup>

## 4.1 Comparisons of alternative cyclical measures

Table 3 examines the time series correlations of the various estimates of the business cycle. We first point out the low correlation between the multivariate measures and commonly used univariate measures, BN and HP.<sup>19</sup> The BN cycle has an average (across countries) correlation of near zero with multivariate measures, such as ECM-BN and SV. The HP cycle is positively correlated with multivariate measures for all countries, with the highest correlation being 0.72 with SV-HP.

Next, as expected, correlations among multivariate measures are found to be positive and very high. The most notable observation is that correlation between ECM-BN cycle and the SV cycle is very close to one, except for Italy. Thus, one might suspect that the two follow a similar time path. Figure 2, in fact, shows that the two estimates are virtually identical. Simply put: to a very close approximation, savings is the cycle in GDP and consumption is the trend.

## 4.2 Cyclical variation

Table 4 reports the standard deviations of the cyclical measures. The notable finding here is that the magnitude of volatility in general is much larger for the multivariate measures than for the univariate measures. For the multivariate measures, ECM-BN cycle and saving-based cycles, SV in particular, are remarkably similar to each other. For the univariate measures,

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<sup>18</sup>Note that turning points based on OECD CLI do not necessarily match turning points in NBER recession episodes. See OECD website ([http://www.oecd.org/document/29/0,2340,en\\_2649\\_34349\\_35725597\\_1\\_1\\_1\\_1,00.html](http://www.oecd.org/document/29/0,2340,en_2649_34349_35725597_1_1_1_1,00.html)) for the construction method of CLI in detail.

<sup>19</sup>While not reported, we also have examined the correlation of multivariate and univariate measures to other available business cycle indicators, such as unemployment rates. It turns out, for some countries including Australia and Japan, correlation between saving-based measures and unemployment rates is higher than the correlation between HP cycle and unemployment.

the largest (across country) cyclical volatility for the BN cycle barely reaches a standard deviation of 0.5%, the mean across countries is a mere 0.34%. The standard deviation of the HP cycle is always intermediate between the univariate BN model and the multivariate models. Moreover, cyclical variability of output according to the univariate BN model is trivial.

The obvious difference between the HP cycle and the SV cycle (see Figure 2) is that the SV typically produces business cycles longer in duration and greater in amplitude than the HP cycle (for the U.S., the two are more comparable). The average duration of these cycles would push the upper limits of what business cycle theorists (or the NBER dating committee, for that matter) would consider reasonable. And yet, if the goal of the exercise is to decompose the data into trend and cycle for purposes of applications to growth theory and business cycle theory, it seems logical to infer the stochastic trend from permanent income behavior of the mythical representative agent rather than relegate the task to a purely statistical procedure.<sup>20</sup>

Taking the view that non-inflationary output growth is tracked by the growth component, the implication of our cyclical measure is dramatically different from the conventional view of the output gap using the HP filter. The differences involve many policy relevant aspects: i) the extent of variance around the growth trend, ii) the duration of cycle; and iii) the timing of turning points. The welfare implications of business cycles is also altered given the greater persistence and volatility of the deviations from the growth path.

### 4.3 The Great Moderation

Stock and Watson (2005) documented a decline in the size of common, international, shocks in recent decades. In this sense business cycle volatility may be sensitive not only to the choice of the method of the trend-cycle decomposition but also to the time period.

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<sup>20</sup>While the original permanent income model assumes a quadratic utility function, it is known that the introduction of more reasonable utility functions leads to the buffer-stock saving. The observed large variation of saving-based measure may partly be explained by the importance of the saving as a buffer stock.

Table 5 shows the standard deviations of alternative business cycle measures computed for two subsamples. We follow Stock and Watson and divide the sample into a period that ends in 1983 and a period starting in 1984. On average, we observe declines in all the business cycle measures as we move from the first period to the second period. However, volatility reductions are not so obvious when multivariate measures are employed in place of univariate measures. In fact, standard deviations of some countries are largely unchanged in the second subsample when SV and SV-MA cycles are employed.

#### 4.4 The international comovement puzzle

Given the analysis above, it should not be surprising that the simple saving-based measure of business cycles has dramatic implications for some key business cycle *facts*. Consider the most cited of these: the international correlation of GDP. The correlation of U.S. and foreign business cycles, is uniformly positive using the HP cycle, averaging about 0.5 across countries. This is higher than the correlation of 0.02 produced in the two-sector, two-country benchmark model of Backus, Kehoe and Kydland (1994).<sup>21</sup> The contrast of the observed and simulated moments has been dubbed the comovement puzzle: pointing to the observation that models have difficulty producing the high international output correlations observed in the data. Table 6 shows that the SV measure of the cycle in U.S. output has an average correlation of nearly zero with its foreign counterpart. Aiming for this empirical target seems likely to change the relative merit of alternative international business cycle models.

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<sup>21</sup>Baxter and Crucini (1995) explore the role of incomplete markets in a one-sector, two-country business cycle model and find significantly positive output correlations arise only when productivity are near random walks with modest international correlations in the innovations and no dynamic spillovers.

## 5 Conclusions

This paper considers a class of business cycle measures derived by imposing a minimal economic restriction: the long-run budget constraint. To some extent, this circumvents the ‘measurement without theory’ critique often directed at purely statistical filters. As a practical matter the stochastic trend and cycle estimates derived from the bivariate error correction model turn out to be very well approximated by consumption and savings. The saving-based measure of business cycles greatly simplifies updates of the decomposition as national statistical agencies release preliminary NIPA estimates.

Applying this method to the G-7 countries and Australia, we find the three most-scrutinized business cycle properties are fundamentally altered: business cycles are less volatile, the reduction of variance due to the Great Moderation is significantly reduced and the international correlation of business cycles falls toward zero. This is not surprising since, as emphasized by Canova (1998), alternative detrending methods extract different types of business cycle information from the data. However, our results point to the need to reconsider the relative merits of alternative detrending methods and their implications for DSGE models of business cycles. In addition, since the stochastic trends are likely to be playing more important role in emerging market economies than in developed countries (Aguiar and Gopinath, 2007), applying our procedure to developing countries would be an important direction of extending our analysis.<sup>22</sup> Much remains to be done.

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<sup>22</sup>Aguiar and Gopinath (2007) employed a fully specified DSGE model estimation approach.

# Appendix

## A1. Multivariate GLS Estimation of Restricted ECMs

An unrestricted bivariate ECM with lag two (with constant and trend term),

$$\begin{aligned} \begin{bmatrix} \Delta c_t \\ \Delta y_t \end{bmatrix} &= \begin{bmatrix} \alpha_{c1} \\ \alpha_{y1} \end{bmatrix} + \begin{bmatrix} \alpha_{c2} \\ \alpha_{y2} \end{bmatrix} t + \begin{bmatrix} \gamma_c \\ \gamma_y \end{bmatrix} (c_{t-1} - y_{t-1}) \\ &+ \begin{bmatrix} \beta_{c1} & \beta_{c3} \\ \beta_{y1} & \beta_{y3} \end{bmatrix} \begin{bmatrix} \Delta c_{t-1} \\ \Delta y_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{c2} & \beta_{c4} \\ \beta_{y2} & \beta_{y4} \end{bmatrix} \begin{bmatrix} \Delta c_{t-2} \\ \Delta y_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^c \\ \varepsilon_t^y \end{bmatrix} \end{aligned}$$

can be written in a matrix form

$$\Delta X = AZ + E$$

where  $\Delta X = [\Delta X_1, \dots, \Delta X_T]$ ,  $\Delta X_t = (\Delta c_t, \Delta y_t)'$ ,  $Z = [Z_0, \dots, Z_{T-1}]$ ,

$$Z_t = \begin{bmatrix} 1 \\ t \\ c_{t-1} - y_{t-1} \\ \Delta X_{t-1} \\ \Delta X_{t-2} \end{bmatrix},$$

$$A = \begin{bmatrix} \alpha_{c1} & \alpha_{c2} & \gamma_c & \beta_{c1} & \beta_{c3} & \beta_{c2} & \beta_{c4} \\ \alpha_{y1} & \alpha_{y2} & \gamma_y & \beta_{y1} & \beta_{y3} & \beta_{y2} & \beta_{y4} \end{bmatrix}$$

and  $E = [\varepsilon_1, \dots, \varepsilon_T]$ . A linear restriction  $\gamma_c = 0$  on  $A$  can be expressed by  $a = \text{vec}(A) = Sr + s$ , where  $\text{vec}(A) = (\alpha_{c1}, \alpha_{y1}, \alpha_{c2}, \alpha_{y2}, \gamma_c, \gamma_y, \beta_{c1}, \beta_{y1}, \beta_{c3}, \beta_{y3}, \beta_{c2}, \beta_{y2}, \beta_{c4}, \beta_{y4})'$ ,

$$S = \begin{bmatrix} I_4 & 0 \\ 0 & 0 \\ 0 & I_9 \end{bmatrix},$$

$r = (\alpha_{c1}, \alpha_{y1}, \alpha_{c2}, \alpha_{y2}, \gamma_y, \beta_{c1}, \beta_{y1}, \beta_{c3}, \beta_{y3}, \beta_{c2}, \beta_{y2}, \beta_{c4}, \beta_{y4})'$  and  $s = 0$ . Since

$$\text{vec}(\Delta X) = (Z' \otimes I_K) \text{vec}(A) + \text{vec}(E) = (Z' \otimes I_K) Sr + \text{vec}(E),$$

a restricted GLS estimator is given by

$$\begin{aligned} \hat{a} &= \text{vec}(\hat{A}) = S\hat{r} \\ &= S [S'(ZZ' \otimes \Sigma_\varepsilon^{-1})S]^{-1} S'(Z \otimes \Sigma_\varepsilon^{-1}) \text{vec}(\Delta X) \end{aligned}$$

with its limit distribution

$$\sqrt{T}(\hat{a} - a) \xrightarrow{d} N \left[ 0, S [S'(Q \otimes \Sigma_\varepsilon^{-1})S]^{-1} S' \right]$$

where  $Q = \text{plim } ZZ'/T$ .

## A2. Beveridge-Nelson Decomposition of Restricted ECMs

To simplify the derivation, we omit constant and trend term without the loss of generality. Suppose a vector ECM

$$\Delta X_t = \gamma\beta'X_{t-1} + \sum_{i=1}^p B_i\Delta X_{t-i} + \varepsilon_t$$

where  $X_t$  is an  $n \times 1$  vector of variables,  $\beta$  is an  $n \times r$  matrix representing cointegrating vectors,  $\gamma$  is an  $n \times r$  loading coefficients,  $B_i$ 's are  $n \times n$  coefficient matrices and  $\varepsilon_t$  is an  $n \times 1$  zero mean error vector. Its VAR(1) representation is given by

$$W_t = AW_{t-1} + u_t$$

where  $W_t = [\Delta X_t', \dots, \Delta X_{t-p+1}', \beta'X_{t-1}']$ ,  $A$  is an  $(np+r) \times (np+r)$  coefficient matrix and  $u_t$  is an  $(np+r) \times 1$  error vector. A multivariate version of Beveridge and Nelson (1981) decomposition for the  $i$ -th element of  $X_t$  yields its cyclical component given by

$$\tilde{X}_{it}^c = \lim_{k \rightarrow \infty} \sum_{j=1}^k e_i' \widehat{W}_{t+j|t} = \lim_{k \rightarrow \infty} \sum_{j=1}^k e_i' A^j W_t = e_i' A(I-A)^{-1} W_t$$

and trend component given by

$$X_{it}^g = X_{it} + \tilde{X}_{it}^c = X_{it} + e_i' A(I-A)^{-1} W_t$$

where  $e_i$  is a selection vector for  $i$ -th element. Note that we can simply flip the sign of the original BN cycle to match the definition of cyclical component in the main text as  $X_{it}^c = -\tilde{X}_{it}^c$ . For a restricted bivariate ECM with transitory consumption ( $\gamma_c = 0$  and  $\beta_{c3} = \beta_{c4} = 0$ ), its

VAR(1) representation is simplified to

$$\begin{bmatrix} \Delta c_t \\ \Delta y_t \\ \Delta c_{t-1} \\ \Delta y_{t-1} \\ c_t - y_t \end{bmatrix} = \begin{bmatrix} \beta_{c1} & 0 & \beta_{c2} & 0 & 0 \\ \beta_{y1} & \beta_{y3} & \beta_{y2} & \beta_{y4} & \gamma_y \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \beta_{c1} - \beta_{y1} & -\beta_{y3} & \beta_{c2} - \beta_{y2} & -\beta_{y4} & -\gamma_y + 1 \end{bmatrix} \begin{bmatrix} \Delta c_{t-1} \\ \Delta y_{t-1} \\ \Delta c_{t-2} \\ \Delta y_{t-2} \\ c_{t-1} - y_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ 0 \\ 0 \\ \varepsilon_{1t} - \varepsilon_{2t} \end{bmatrix}.$$

Thus, for the transitory component of  $y_t$ , we have

$$\begin{aligned} y_t^c &= -\tilde{y}_t^c = -[0 \ 1 \ 0 \ 0 \ 0]A(I-A)^{-1}W_t \\ &= -[0 \ 1 \ 0 \ 0 \ 0] \begin{bmatrix} \beta_{c1} & 0 & \beta_{c2} & 0 & 0 \\ \beta_{y1} & \beta_{y3} & \beta_{y2} & \beta_{y4} & \gamma_y \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \beta_{c1} - \beta_{y1} & -\beta_{y3} & \beta_{c2} - \beta_{y2} & -\beta_{y4} & -\gamma_y + 1 \end{bmatrix} \\ &\quad \times \begin{bmatrix} 1 - \beta_{c1} & 0 & -\beta_{c2} & 0 & 0 \\ -\beta_{y1} & 1 - \beta_{y3} & -\beta_{y2} & -\beta_{y4} & -\gamma_y \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ \beta_{y1} - \beta_{c1} & \beta_{y3} & \beta_{y2} - \beta_{c2} & \beta_{y4} & \gamma_y \end{bmatrix}^{-1} \begin{bmatrix} \Delta c_t \\ \Delta y_t \\ \Delta c_{t-1} \\ \Delta y_{t-1} \\ c_t - y_t \end{bmatrix} \\ &= - \begin{bmatrix} \frac{\beta_{c1} + \beta_{c2}}{1 - (\beta_{c1} + \beta_{c2})} & 0 & \frac{\beta_{c2}}{1 - (\beta_{c1} + \beta_{c2})} & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta c_t \\ \Delta y_t \\ \Delta c_{t-1} \\ \Delta y_{t-1} \\ c_t - y_t \end{bmatrix} \\ &= y_t - c_t - \frac{\beta_{c1} + \beta_{c2}}{1 - (\beta_{c1} + \beta_{c2})} \Delta c_t - \frac{\beta_{c2}}{1 - (\beta_{c1} + \beta_{c2})} \Delta c_{t-1}. \end{aligned}$$

The trend component of  $y_t$  can be obtained as

$$\begin{aligned} y_t^g &= y_t - y_t^c = y_t + c_t - y_t + \frac{\beta_{c1} + \beta_{c2}}{1 - (\beta_{c1} + \beta_{c2})} \Delta c_t + \frac{\beta_{c2}}{1 - (\beta_{c1} + \beta_{c2})} \Delta c_{t-1} \\ &= \frac{1}{1 - (\beta_{c1} + \beta_{c2})} c_t + \frac{-\beta_{c1}}{1 - (\beta_{c1} + \beta_{c2})} c_{t-1} + \frac{-\beta_{c2}}{1 - (\beta_{c1} + \beta_{c2})} c_{t-2}. \end{aligned}$$

Consistent with the argument of Cochrane, in the case of random walk consumption with  $\beta_{c1} = \beta_{c2} = 0$ , the results above implies that the cyclical component reduces to  $y_t^c = y_t - c_t = s_t$  (saving) and the trend component reduces to  $y_t^g = c_t$  (consumption).

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Table 1. – Bivariate Vector ECM Estimates

Country		const.	trend	$c_{t-1} - y_{t-1}$	$\Delta c_{t-1}$	$\Delta c_{t-2}$	$\Delta y_{t-1}$	$\Delta y_{t-2}$
Australia								
	$\Delta c_t$	0.81	0.00	0.00	0.04	0.01	0.09	0.05
1959:3 to 2005:1		(5.24)	(0.78)	(-)	(0.48)	(0.13)	(1.62)	(0.94)
T = 183	$\Delta y_t$	4.82	0.00	0.08	0.09	0.06	-0.06	0.01
		(2.44)	(0.70)	(1.91)	(0.70)	(0.47)	(0.67)	(0.15)
Canada								
	$\Delta c_t$	0.90	0.00	0.00	-0.19	0.03	0.29	0.05
1961:1 to 2005:1		(4.72)	(1.67)	(-)	(2.17)	(0.30)	(3.21)	(0.54)
T = 177	$\Delta y_t$	6.02	0.00	0.10	0.15	0.19	0.18	-0.05
		(3.23)	(0.19)	(2.84)	(1.66)	(2.15)	(1.97)	(0.55)
France								
	$\Delta c_t$	0.67	0.00	0.00	-0.13	0.22	-0.02	0.09
1970:1 to 2005:1		(3.84)	(1.60)	(-)	(1.21)	(2.17)	(0.16)	(0.64)
T = 141	$\Delta y_t$	3.18	0.00	0.05	0.11	0.15	0.12	0.13
		(2.32)	(0.32)	(2.05)	(1.45)	(2.07)	(1.07)	(1.35)
Germany								
	$\Delta c_t$	1.19	-0.01	0.00	-0.33	-0.14	0.04	0.14
1970:1 to 2005:1		(5.17)	(2.71)	(-)	(3.13)	(1.36)	(0.35)	(1.24)
T = 141	$\Delta y_t$	9.04	0.00	0.15	-0.14	-0.05	0.07	0.13
		(3.87)	(2.11)	(3.58)	(1.46)	(0.55)	(0.61)	(1.24)

Notes: The regressions are of the form:

$$\begin{aligned} \Delta c_t &= \alpha_{c1} + \alpha_{c2}t + \beta_{c1}\Delta c_{t-1} + \beta_{c2}\Delta c_{t-2} + \beta_{c3}\Delta y_{t-1} + \beta_{c4}\Delta y_{t-2} + \varepsilon_t^c \\ \Delta y_t &= \alpha_{y1} + \alpha_{y2}t + \gamma_y(c_{t-1} - y_{t-1}) \\ &\quad + \beta_{y1}\Delta c_{t-1} + \beta_{y2}\Delta c_{t-2} + \beta_{y3}\Delta y_{t-1} + \beta_{y4}\Delta y_{t-2} + \varepsilon_t^y \end{aligned}$$

where  $c_t$  and  $y_t$  are total consumption and GDP in logs, respectively. Restricted multivariate generalized least squares estimates. Numbers in parentheses are  $t$ -values.

Table 1 (continued). – Bivariate Vector ECM Estimates

Country		const.	trend	$c_{t-1} - y_{t-1}$	$\Delta c_{t-1}$	$\Delta c_{t-2}$	$\Delta y_{t-1}$	$\Delta y_{t-2}$
Italy								
	$\Delta c_t$	0.40	0.00	0.00	0.42	0.16	0.14	-0.08
1970:1 to 2005:1		(3.30)	(1.98)	(-)	(4.56)	(1.75)	(2.27)	(1.34)
T = 141	$\Delta y_t$	7.83	-0.01	0.13	0.31	0.15	0.30	-0.05
		(3.24)	(3.09)	(3.16)	(2.20)	(1.09)	(3.17)	(0.54)
Japan								
	$\Delta c_t$	2.51	-0.01	0.00	-0.48	-0.20	0.26	0.32
1955:2 to 2005:1		(7.72)	(6.48)	(-)	(4.42)	(1.80)	(2.28)	(2.77)
T = 200	$\Delta y_t$	4.57	-0.01	0.05	-0.26	-0.10	0.22	0.32
		(4.21)	(2.80)	(2.45)	(2.56)	(0.98)	(2.06)	(2.96)
United Kingdom								
	$\Delta c_t$	0.55	0.00	0.00	-0.08	0.10	-0.01	0.01
1955:1 to 2005:1		(3.09)	(0.82)	(-)	(0.86)	(1.06)	(0.12)	(0.06)
T = 201	$\Delta y_t$	2.27	0.00	0.03	0.11	0.11	-0.14	0.02
		(2.24)	(1.32)	(1.70)	(1.31)	(1.30)	(1.58)	(0.20)
United States								
	$\Delta c_t$	0.60	0.00	0.00	0.12	0.10	0.10	0.01
1955:1 to 2005:1		(4.71)	(0.17)	(-)	(1.30)	(1.03)	(1.41)	(0.15)
T = 201	$\Delta y_t$	5.15	-0.01	0.10	0.44	0.20	0.02	0.00
		(3.20)	(2.77)	(3.06)	(3.76)	(1.62)	(0.23)	(0.05)

Table 2. – Consumption and Income Variance Decompositions

Variance of:		$\Delta c_t$		$\Delta y_t$	
Country	Horizon	Accounted for by		Accounted for by	
		Permanent shocks	Transitory shocks	Permanent shocks	Transitory shocks
Australia	1	100.0	0.0	22.5	77.5
	4	98.3	1.7	22.6	77.4
	$\infty$	98.2	1.8	22.3	77.7
Canada	1	100.0	0.0	29.5	70.5
	4	94.9	5.1	38.1	61.9
	$\infty$	94.6	5.4	37.2	62.8
France	1	100.0	0.0	37.3	62.7
	4	99.7	0.3	47.0	53.0
	$\infty$	99.7	0.3	46.8	53.2
Germany	1	100.0	0.0	38.3	61.7
	4	99.1	0.9	38.4	61.6
	$\infty$	99.1	0.9	37.5	62.5
Italy	1	100.0	0.0	18.1	81.9
	4	97.7	2.3	35.4	64.6
	$\infty$	97.6	2.4	36.1	63.9
Japan	1	100.0	0.0	58.4	41.6
	4	96.0	4.0	58.4	41.6
	$\infty$	95.7	4.3	57.3	42.7
United Kingdom	1	100.0	0.0	38.9	61.1
	4	100.0	0.0	39.2	60.8
	$\infty$	100.0	0.0	39.1	60.9
United States	1	100.0	0.0	41.2	58.8
	4	99.1	0.9	51.5	48.5
	$\infty$	99.0	1.0	50.8	49.2

Note: See Table 1 for the sample period.

Table 3. – Correlation of Alternative Business Cycle Measures

	Multivariate measures				Univariate	
	Cochrane's	Saving-based			measures	
	measure	measures			BN	HP
	ECM-BN	SV	SV-MA	SV-HP		
(1) Correlation with ECM-BN						
Australia	1.00	1.00	0.94	0.87	0.27	0.49
Canada	1.00	0.99	0.85	0.75	-0.05	0.26
France	1.00	0.99	0.83	0.76	-0.03	0.23
Germany	1.00	0.98	0.91	0.80	-0.18	0.42
Italy	1.00	0.83	0.50	0.43	0.14	0.07
Japan	1.00	0.99	0.92	0.88	-0.09	0.39
United Kingdom	1.00	1.00	0.95	0.88	0.03	0.08
United States	1.00	0.97	0.75	0.72	0.06	0.39
Averages	1.00	0.97	0.83	0.76	0.02	0.29
(2) Correlation with SV						
Australia	–	1.00	0.95	0.89	0.34	0.51
Canada	–	1.00	0.88	0.77	-0.18	0.30
France	–	1.00	0.90	0.81	-0.16	0.25
Germany	–	1.00	0.85	0.77	-0.20	0.40
Italy	–	1.00	0.90	0.66	-0.24	0.22
Japan	–	1.00	0.92	0.87	-0.14	0.39
United Kingdom	–	1.00	0.95	0.89	0.01	0.08
United States	–	1.00	0.88	0.79	-0.16	0.45
Averages	–	1.00	0.90	0.81	-0.09	0.33
(3) Correlation with HP						
Australia	–	–	0.58	0.74	0.39	1.00
Canada	–	–	0.38	0.77	-0.31	1.00
France	–	–	0.29	0.66	-0.34	1.00
Germany	–	–	0.49	0.79	-0.36	1.00
Italy	–	–	0.36	0.79	-0.30	1.00
Japan	–	–	0.51	0.72	-0.38	1.00
United Kingdom	–	–	0.18	0.43	-0.34	1.00
United States	–	–	0.50	0.86	-0.34	1.00
Averages	–	–	0.41	0.72	-0.25	1.00

Note: See Table 1 for the sample period.

Table 4. – Standard Deviations of Alternative Business Cycle Measures

	Multivariate measures				Univariate	
	Cochrane's	Saving-based			measures	
	measure	measures			BN	HP
	ECM-BN	SV	SV-MA	SV-HP		
Australia	2.49	2.21	2.50	2.31	0.09	1.44
Canada	2.15	1.57	1.77	1.85	0.34	1.39
France	1.49	1.38	1.48	1.49	0.44	0.91
Germany	1.73	1.58	1.59	1.77	0.07	1.34
Italy	2.06	1.53	1.76	1.69	0.57	1.43
Japan	3.22	2.15	2.34	2.54	0.52	1.50
United Kingdom	3.30	3.31	3.30	3.37	0.10	1.45
United States	1.69	1.40	1.60	1.94	0.55	1.55
Averages	2.27	1.89	2.04	2.12	0.34	1.38

Note: See Table 1 for the sample period.

Table 5. – Standard Deviations of Alternative Business Cycle Measures: Subsamples

	Multivariate measures				Univariate	
	Cochrane's	Saving-based			measures	
	measure	measures			BN	HP
	ECM-BN	SV	SV-MA	SV-HP		
(1) 1970Q4 - 2005Q1						
Australia	2.57	2.27	2.55	2.24	0.08	1.34
Canada	1.96	1.45	1.78	1.81	0.34	1.50
France	1.48	1.37	1.48	1.50	0.44	0.91
Germany	1.68	1.54	1.59	1.75	0.06	1.34
Italy	2.05	1.52	1.76	1.68	0.58	1.43
Japan	2.70	1.79	1.84	2.11	0.45	1.32
United Kingdom	3.05	3.11	3.11	3.18	0.10	1.50
United States	1.68	1.31	1.29	1.87	0.51	1.61
Averages	2.15	1.80	1.93	2.02	0.32	1.37
(2) 1970Q4 - 1983Q4						
Australia	3.36	2.97	3.28	2.77	0.11	1.68
Canada	2.00	1.47	1.73	1.71	0.43	1.75
France	1.61	1.44	1.53	1.49	0.51	0.97
Germany	2.12	1.87	2.01	2.20	0.07	1.60
Italy	1.86	1.34	1.81	2.08	0.78	2.05
Japan	2.66	1.78	2.08	2.23	0.50	1.57
United Kingdom	2.05	2.05	2.21	2.29	0.14	1.96
United States	2.00	1.42	1.14	2.41	0.72	2.29
Averages	2.21	1.79	1.97	2.15	0.41	1.74
(3) 1984Q1 - 2005Q1						
Australia	1.63	1.44	1.68	1.65	0.05	1.08
Canada	1.95	1.45	1.80	1.87	0.28	1.31
France	1.40	1.34	1.45	1.49	0.40	0.87
Germany	1.34	1.28	1.25	1.41	0.06	1.16
Italy	2.13	1.61	1.71	1.35	0.40	0.87
Japan	2.02	1.38	1.46	1.63	0.37	1.16
United Kingdom	2.02	2.07	2.33	1.96	0.06	1.13
United States	1.41	1.19	1.30	1.33	0.31	0.96
Averages	1.74	1.47	1.62	1.59	0.24	1.07

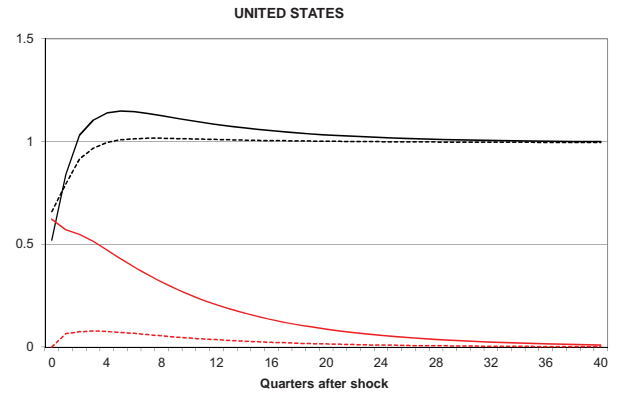
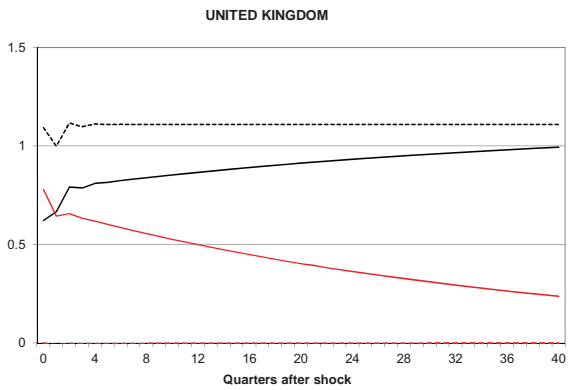
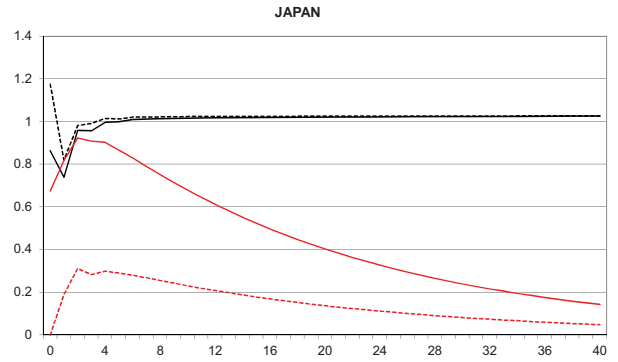
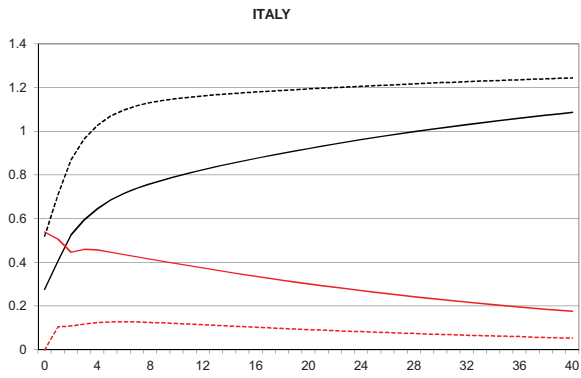
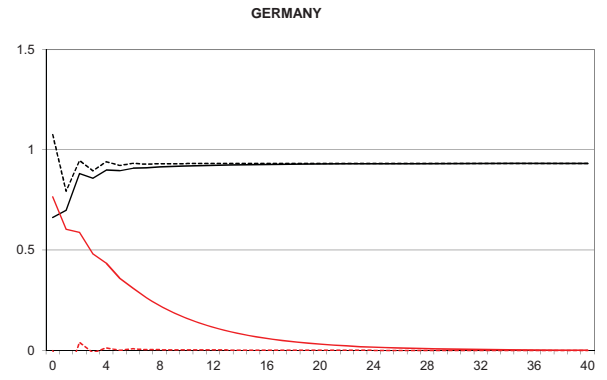
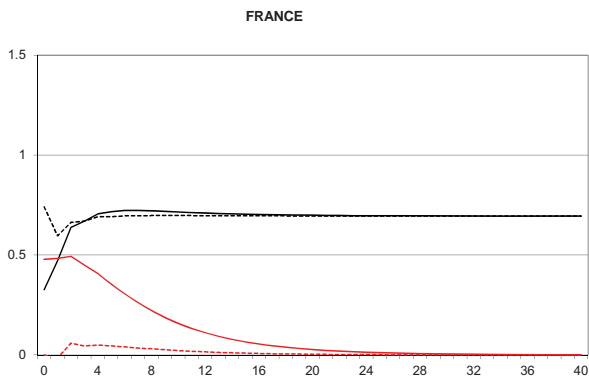
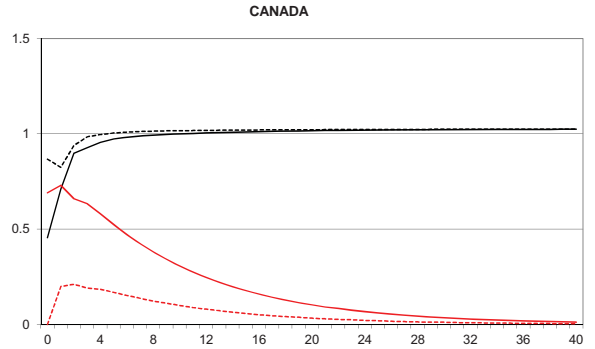
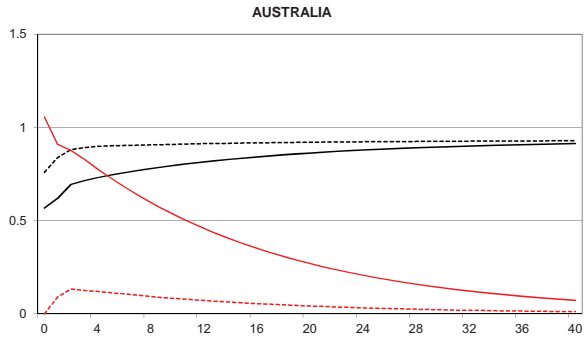


Table 6. – International Business Cycle Comovement with the U.S.

	Multivariate measures				Univariate	
	Cochrane's	Saving-based			measures	
	measure	measures			BN	HP
	ECM-BN	SV	SV-MA	SV-HP		
Australia	0.24	0.27	0.31	0.40	-0.33	0.43
Canada	0.01	0.00	0.06	0.45	0.57	0.72
France	0.25	0.24	0.20	0.23	0.29	0.36
Germany	-0.05	-0.06	-0.09	0.15	0.17	0.43
Italy	-0.17	-0.15	-0.04	0.19	0.15	0.33
Japan	-0.04	-0.04	-0.07	0.15	0.30	0.40
United Kingdom	-0.10	-0.11	-0.04	0.20	0.34	0.65
United States	1.00	1.00	1.00	1.00	1.00	1.00
Averages	0.02	0.02	0.05	0.25	0.21	0.47

Note: Sample period is 1970:Q4 to 2005:Q1.

Figure 1. Estimated Impulses Response Functions



- - - Permanent shock → consumption      — Permanent shock → output  
 - - - Temporary shock → consumption      — Temporary shock → output

- - - Permanent shock → consumption      — Permanent shock → output  
 - - - Temporary shock → consumption      — Temporary shock → output

Figure 2. Business Cycle Measures

