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Discussion Paper No. 2010-E-9

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

2-1-1 NIHONBASHI-HONGOKUCHO

CHUO-KU, TOKYO 103-8660

JAPAN

IMES Discussion Paper Series 2010-E-9
June 2010

An Empirical Analysis of Equity Market Expectations in the Recent Financial Turmoil Using Implied Moments and Jump Diffusion Processes

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Abstract

This paper investigates market expectations of future equity prices using the probability distribution and the moments implied in equity option prices. We first conduct, without assuming a particular model, a nonparametric analysis of the development of market expectations in four major markets during the financial turmoil following the summer of 2007. We then conduct a parametric analysis to reconsider these expectations from the perspective of a stochastic process, assuming jump diffusion processes that configure the implied distribution. These analyses reveal that the possibility of discontinuous price jumps in each country increased downwards during the recent financial turmoil, while volatilities determining the dispersion of continuous price changes surged. Viewing the results from the perspective of a probability distribution, we find that kurtosis and the absolute value of skewness declined, while variance dramatically increased.

Keywords: implied distribution; implied moment; jump diffusion process; nonparametric method; GMM; characteristic function GMM
JEL classification: C13, C14, C16, G13, G15

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The authors wish to thank Kouichi Miyazaki (University of Electro-communications), Kenneth J. Singleton (Stanford University), participants in the December 2009 Finance Workshop held at the Bank of Japan, and research staff at the Bank of Japan for their helpful comments and suggestions. Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

I Introduction

Since the subprime mortgage loan problem was widely recognized in the summer of 2007, financial markets have become seriously concerned about the vulnerability of financial systems and the potential for recession in the global economy. Following the bankruptcy of Lehman Brothers in the fall of 2008 (the Lehman shock), the global financial and economic environment has deteriorated considerably. At the same time, global equity markets have become more turbulent, with major equity indices in the US, Europe and Japan falling to less than half of their highs since 2005, and the US implied volatility index (the VIX index, also known as the market's "fear gauge") surging up to 80%.

An important background for this turbulence is the dramatic change in market expectations of future price levels and uncertainty. Though equity prices aggregate market predictions for future economic and business conditions, the uncertainty associated with prediction is not well reflected in the price. In order to analyze this uncertainty, it is effective to incorporate the information implied in option prices; information such as the VIX index. Though the VIX measures the dispersion of uncertainty, we should also be careful about the asymmetry of uncertainty in such large market turbulence as the recent financial turmoil. In order to detect such a detailed change in expectations, the entire configuration of the implied distribution, that is, the markets' expected distribution of future equity returns implied in option prices, should be analyzed.

With the above motive, we analyze the development of the implied distribution of equity indices in Japan, Germany, the UK, and the US. Theoretically, we provide the nonparametric derivation of implied moments (or the moments of the implied distribution) by computing the expected value of the power of equity returns under a risk-neutral probability measure. We also show the characteristic function of the implied distribution (or the implied characteristic function) denoted analytically by a basket of plain European option prices. Empirically, we apply these methods to equity options in the four countries selected during the period from 2005 to the middle of 2009, and evaluate the daily implied moments and characteristic functions. We also estimate the implied parameters of the stochastic processes that underlie the implied distribution by assuming two types of jump diffusion process. Using these estimates, we analyze the magnitude and direction of the implied price jumps that make the implied distribution deviate from the normal distribution. We also analyze the development of two factors, a diffusion (or Brownian motion) factor and a jump factor, during the course of the recent financial turmoil. These analyses reveal that the possibility of discontinuous price jumps increased downwards during the turmoil, while the volatilities that determine the dispersion of the continuous part of the price process surged. Viewing the results from the perspective of a probability distribution, we found that the 2nd and 4th moments rose while the 3rd moment sharply declined. Examining the deviations of the implied distribution from the normal distribution, we detected that the kurtosis, as well as the absolute value of the skewness, decreased during the recent market turmoil.

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Study of the implied distribution was initially proposed by Breeden and Litzenberger [1978]. The implied distribution is also known as the risk-neutral distribution or the state-price density that reflects market expectations. There is a gap, however, between the option implied and actual distribution of the underlying asset returns. More particularly, the means of the distributions differ depending on the risk aversion of investors; the shapes may also be dissimilar. Based on financial theory, the gap of the mean is identifiable, and the shapes are known to coincide under the condition where the Girsanov theorem is satisfied. In the actual market, however, this condition is not always satisfied, thus the shapes may actually differ. This paper only focuses on the risk-neutral distribution and, unlike Ait-Sahalia and Brandt [2008] and some others, does not investigate further the difference in the shapes.

In the nonparametric estimation of the implied moments, this paper applies the method proposed by Bakshi, Kapadia, and Madan [2003]. They derived the analytical form of the implied moments by applying a replication method of European payoff products from plain options. Our method is almost the same, with an additional improvement explained in Section II 1. We also derive the implied characteristic function by generalizing the derivation of the implied moments. In the estimation of the implied parameters of the jump diffusion processes, this paper matches theoretical moments or characteristic functions to the corresponding estimates from market prices, while many earlier studies, such as Broadie, Chernov, and Johannes [2007], calibrate the theoretical prices of options (or the implied volatilities) to these market prices.

The additional feature of this paper is that it analyzes the Japanese market during the recent financial turmoil. To our best knowledge, such analyses are limited to Kobayashi, Miyazaki, and Tanaka [2009], though there are many earlier studies that consider the implied distribution or parameters of the Japanese equity market before the most recent financial turmoil, including Oda and Yoshiba [1998], Nakamura and Shiratsuka [1999], Hisata [2003], Nomura and Miyazaki [2005], and Kobayashi, Miyazaki, and Tanaka [2009].

The organization of the paper is as follows. Section II describes the theoretical scheme and estimation procedures of the nonparametric implied moments and characteristic functions. Section III explains the jump diffusion processes, purportedly the underlying processes that constitute the implied distributions in this analysis, with the estimation procedures of these parameters. Section IV empirically analyzes the implied moments and the parameters of the jump diffusion processes by applying the method in Section II and III to Japanese, German, UK, and US equity market data. We particularly focus on the contribution of the pure jump part of the jump diffusion process during the financial turmoil to investigate how it differs from that in the normal period. Section V summarizes the paper.

II Nonparametric Approach

This section provides the methodology used to nonparametrically analyze market expectations by implied moments and the implied characteristic function without assuming any particular model for the asset price process. In the following discussion, we assume a market where the returns are independent and identically distributed.

1. The Implied Moment

(a) Theoretical scheme

Bakshi, Kapadia, and Madan [2003] proposed the theoretical scheme of evaluating implied moments by applying a replication method of European-type products from plain options and forward prices. By partly improving their method, this paper evaluates the implied moments by the following method.

Let S_t denote an asset price underlying an option at time t and R_t denote the return from the present time 0 to time $t \quad (t > 0)$, i.e., $R_t = \ln(S_t/S_0)$. We also let $\Theta(0, t, K)$ denote the out-of-the-money (OTM) European-type plain option price at time 0 with a maturity t and a strike price K, while r denotes the risk-free rate (assumed constant over time for simplicity) and Q denotes the risk-neutral measure. Then, the 1st to 4th zero-centered moments of the risk-neutral probability density function of the return

R_t, $\kappa_n = \mathbf{E}_0^Q[R_t^n]$ $(n = 1, \cdots, 4)$, are expressed using $\Theta(0, t, K)$ as follows.

$$\begin{cases} \kappa_1 = e^{rt} - 1 - e^{rt} \displaystyle\int_0^\infty \frac{1}{K^2} \Theta(0, t, K) dK, \\ \kappa_2 = e^{rt} \displaystyle\int_0^\infty \frac{2\{1 + \ln(S_0/K)\}}{K^2} \Theta(0, t, K) dK, \\ \kappa_3 = e^{rt} \displaystyle\int_0^\infty \frac{6 \ln(K/S_0) - 3\{\ln(K/S_0)\}^2}{K^2} \Theta(0, t, K) dK, \\ \kappa_4 = e^{rt} \displaystyle\int_0^\infty \frac{12\{\ln(S_0/K)\}^2 + 4\{\ln(S_0/K)\}^3}{K^2} \Theta(0, t, K) dK. \end{cases} \tag{1}$$

See Appendix A for the derivation of Eqs.(1). Then, the nth moments m_n $(n = 1, \cdots, 4)$ around the mean are expressed by using Eqs.(1) as follows[1].

$$\begin{cases} m_1 = \kappa_1, \\ m_2 = \kappa_2 - (\kappa_1)^2, \\ m_3 = \kappa_3 - 3\kappa_1\kappa_2 + 2(\kappa_1)^3, \\ m_4 = \kappa_4 - 4\kappa_1\kappa_3 + 6(\kappa_1)^2\kappa_2 - 3(\kappa_1)^4. \end{cases} \tag{2}$$

In this study, the moments of the implied distribution, m_n in Eqs.(2), are referred to as the implied moments. m_1 and m_2 respectively correspond to the mean and variance of the distribution. Further, the variance-standardized 3rd and 4th moments, $m_3/m_2^{3/2}$ and m_4/m_2^2, are equivalent to the skewness and the kurtosis, respectively, of the implied distribution.

Bakshi, Kapadia, and Madan [2003] approximates the 1st moment by the weighted sum of the 2nd to 4th moments as:

$$\kappa_1 \simeq e^{rt} - 1 - \frac{e^{rt}}{2}\kappa_2 - \frac{e^{rt}}{6}\kappa_3 - \frac{e^{rt}}{24}\kappa_4. \tag{3}$$

In contrast, we evaluate the 1st moment rigorously as in the first equation in Eqs.(1) without using this approximation.[2]

[1] Eq.(2) is derived by expanding $m_n = \mathbf{E}_0^Q(R_t - \mathbf{E}_0^Q[R_t])^n$ and applying Eq.(1).

[2] The 1st moment κ_1 in Eqs.(1) approximates to $\kappa_1 \simeq (r - \sigma_{\mathrm{MFIV}}^2/2)t$ where σ_{MFIV}^2 is the model-free implied volatility:

$$\sigma_{\mathrm{MFIV}}^2 = \frac{2e^{rt}}{t} \int_0^\infty \frac{\Theta(0, t, K)}{K^2} dK.$$

This is equivalent to the risk-neutral condition of the geometric Brownian motion. See Sugihara [2010] for the model-free implied volatility.

(b) Estimation procedure

The implied moments are estimable from a basket of plain OTM option prices and the risk-free rate as the moments are represented by the weighted sum of the option prices in Eqs.(2). The computation is carried out numerically by discretizing, interpolating, and extrapolating the integral in Eqs.(1). We apply the same estimation procedures as the model-free implied volatility in Sugihara [2010] and Jiang and Tian [2007].

A brief description of the procedure is as follows. First, we convert all of the market-traded option prices into their Black=Scholes implied volatility (BSIV). Second, we interpolate and extrapolate the BSIVs using cubic spline functions to obtain BSIVs for any strike price. Third, we exponentially discretize Eqs.(1) in terms of the strike prices at-the-money.[3] Lastly, we compute the integral value in Eqs.(1) numerically using option prices computed back from the interpolated BSIVs with the grid of exponentially discretized strike prices.[4] See Section 3.(1) in Sugihara [2010] for details.

In order to enhance market liquidity, the maturity date of options traded in option exchanges is typically limited to one fixed date in every quarter.[5] In order to fix the term to maturity t irrespective of the evaluation day, we apply the following algorithm to compute the implied moments from the market-traded option prices.

1) Compute BSIV from all the OTM option prices with any maturity.

2) Interpolate and extrapolate the OTM BSIV smile curves in terms of strike prices using cubic spline functions.

3) Compute the BSIVs with fixed 1- to 12-month terms to maturity by linearly interpolating the BSIV spline functions computed in the previous item in terms of maturity using the market-traded strike-price grid of the nearest term to maturity.

4) Interpolate and extrapolate the BSIVs with fixed maturities evaluated in the previous item using cubic spline functions.

5) Discretize Eqs.(1).

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[3] Using this exponential discretization method, the approximated integrand is expressed as an exponential function of moneyness (K/S_0).

[4] The upper and lower bound of K in the integral are set to the level where the integrand is sufficiently small. We set $K \in [S_0 e^{-3}, S_0 e^{3}]$ roughly in a range between 1/20 times to 20 times the current underlying equity price. The interval of strike prices in the discretization of the integral, or the interval between K_j and K_{j+1}, is set to $\ln(K_{j+1}/K_j) = 10^{-4}$. This is equivalent to $\theta = 10^{-4}$ in Sugihara [2010].

[5] The maturity date can be set for every month in the US market.

6) Estimate the zero-centered moments using option prices computed from the BSIV in 4) and the risk-free rate.

7) Compute the implied moments with fixed terms to maturity using Eqs.(2).

In addition, we exclude the following option types from the implied moment computation: i) an option with a price outside of the no-arbitrage boundary range, [6] ii) an option with a price exactly the same as the minimum transaction price, and iii) an option with a term to maturity of less than 1 month (21 business days). This is because of the following reasons. First, the prices of options of type i) cannot be converted to BSIVs. Second, options of type ii) may not be correctly valued because they may be traded at the minimum transaction price, even if this is above the actual market value. Finally, estimates of the higher-order implied moments are sometimes very volatile if we include options of type iii) with nearer terms to maturity.[7]

2. The Implied Characteristic Function

(a) Theoretical scheme

The characteristic function of a random variable X is a function of frequency ω, obtained as the inverse Fourier transformation of the probability density function of X. Mathematically, this is defined as $\Phi_X(\omega) = \mathbf{E}[e^{i\omega X}]$ where i is an imaginary unit.

Extending the theoretical derivation of the implied moments shown in Section II 1.(a), we obtain the option implied characteristic function (ICF) of return R_t, denoted as $\Phi_{R_t}(\omega)$, as follows.

$$\Phi_{R_t}(\omega) = 1 + i\omega(e^{rt} - 1) - \omega\,(\omega + i)\,e^{rt} \int_0^\infty \frac{\Theta(0,t,K)}{K^2} \left(\frac{K}{S_0}\right)^{i\omega} dK. \qquad (4)$$

See Appendix B for the derivation of Eq.(4). Using the relationship $(K/S_0)^{i\omega} = \cos(\ln(K/S_0)\omega) + i\sin(\ln(K/S_0)\omega)$, the real and imaginary part of Eq.(4) is expressed

[6] The no-arbitrage range is set to be

$$\Theta(0,t,K) \in \begin{cases} [\max(0, S_0 - Ke^{-rt}), S_0] & (K < S_0), \\ [\max(0, Ke^{-rt} - S_0), Ke^{-rt}] & (K > S_0). \end{cases}$$

[7] Deep OTM options with closer terms to maturity boost the estimates of the higher-order moments.

as:

$$\begin{cases} \operatorname{Re}\Phi_{R_t}(\omega) = 1 - \omega e^{rt}\displaystyle\int_0^\infty \frac{\Theta(0,t,K)}{K^2}\left\{\omega\cos\left[\omega\ln\left(\frac{K}{S_0}\right)\right] - \sin\left[\omega\ln\left(\frac{K}{S_0}\right)\right]\right\}dK, \\ \operatorname{Im}\Phi_{R_t}(\omega) = \omega(e^{rt}-1) \\ \qquad\qquad -\omega e^{rt}\displaystyle\int_0^\infty \frac{\Theta(0,t,K)}{K^2}\left\{\omega\sin\left[\omega\ln\left(\frac{K}{S_0}\right)\right] + \cos\left[\omega\ln\left(\frac{K}{S_0}\right)\right]\right\}dK. \end{cases}$$

(b) Estimation procedure

Eq.(4) is represented as the weighted sum of the OTM option prices in Eqs.(1). This makes it possible to estimate the ICF by the same procedure as the implied moments explained in Section II 1.(b).

However, the following two points should be paid unique attention in the estimation of the ICF. The first is that the interval between the grids of the log strike prices should be chosen to be sufficiently small in the discretization of the integral in Eq.(4).[8] This is because the integrand in Eq.(4) is the periodic function of the log of the strike prices. The second is the choice of the step and the range of the frequency ω, in that the exponential step should differ from the 2π cycle. The choice of the range does not have a definite criterion, while evaluating the ICF only for a positive ω is sufficient because of the complex conjugate property. We determine the ICF where $\omega \leq 50$ represents the sufficient properties of the implied distribution based on the simulation of the theoretical ICF using a jump diffusion process with an appropriate parameter set.

III Parametric Approach

A Lévy process, a stochastic process representing a heavy-tailed distribution, is often used in financial theory. In turn, a jump diffusion process is a Lévy process commonly applied by financial practitioners.[9] This section explains the parametric analysis of market expectations using jump diffusion processes.

[8] We apply the same exponential grid interval, 10^{-4}, as in the estimation of the implied moments.

[9] Though the stochastic volatility model is widely applied in industry, the model does not fit the analysis of moments, as the higher-order moments in the model may diverge. See Andersen and Piterbarg [2007] for the mathematical background.

1. Jump Diffusion Processes

The jump diffusion process of equity return R_t is generally represented as follows.

$$R_t = \mu t + \sigma W_t + \sum_{j=1}^{N_t(\lambda)} Y_j, \tag{5}$$

where μ, σ ($\sigma > 0$) are parameters that indicate the drift and volatility of the return process, W_t is the standard Brownian motion, and $N_t(\lambda)$ is the number of jumps up to time t, assumed to grow following a Poisson process with intensity λ (> 0) independently from W_t. Y_j is another independent stochastic variable indicating the jth jump size. Here, we define the jump process J_t by the third term in Eq.(5) as:

$$J_t = \sum_{j=1}^{N_t(\lambda)} Y_j. \tag{6}$$

This indicates the pure jump component of the return process.

Let $\tilde{\Phi}_Y(\omega)$ denote the characteristic function of the jump size Y.[10] The characteristic function of the jump diffusion process $\tilde{\Phi}_{R_t}^{\mathrm{JD}}(\omega)$ is then represented as:

$$\tilde{\Phi}_{R_t}^{\mathrm{JD}}(\omega) = \exp\left[t\left\{i\mu\omega - \frac{\sigma^2}{2}\omega^2 + \lambda(\tilde{\Phi}_Y(\omega) - 1)\right\}\right]. \tag{7}$$

The drift μ is determined uniquely under the risk-neutral measure Q. Given the risk-neutral condition is expressed in terms of the characteristic function as $\tilde{\Phi}_{R_t}^{\mathrm{JD}}(-i) = e^{rt}$, μ is determined by the other parameters as:

$$\mu = r - \frac{\sigma^2}{2} - \lambda\left\{\tilde{\Phi}_Y(-i) - 1\right\}. \tag{8}$$

While several models are known for the jump size Y, this analysis employs two: the Gaussian jump diffusion process where Y obeys the Gaussian distribution, and the Laplacian jump diffusion process where Y obeys a Laplace distribution.

(a) The Gaussian jump diffusion process

The Gaussian jump diffusion process (GJD) was first put forward for financial analysis by Merton [1976]. Let Y obey the normal distribution with mean γ and standard deviation δ (> 0). Then, the characteristic function of GJD $\tilde{\Phi}_{R_t}^{\mathrm{GJD}}(\omega)$ is determined by

[10] We include a tilde (˜) with the parametric implied moments or parametric functions hereafter in order to distinguish them from the nonparametric variables.

Eq.(7) as:

$$\tilde{\Phi}_{R_t}^{\text{GJD}}(\omega) = \exp\left[t\left\{i\mu\omega - \frac{\sigma^2}{2}\omega^2 + \lambda(e^{i\gamma\omega-\delta^2\omega^2/2} - 1)\right\}\right]. \tag{9}$$

μ is determined by the risk-neutral condition in Eq.(8) as $\mu = r - \sigma^2/2 - \lambda(e^{\gamma+\delta^2/2} - 1)$. The 1st to 4th moments $\tilde{m}_n^{\text{GJD}}$ $(n = 1, \cdots, 4)$ are determined by:

$$\begin{cases} \tilde{m}_1^{\text{GJD}} = \lambda t\gamma + \mu t, \\ \tilde{m}_2^{\text{GJD}} = \lambda t(\gamma^2 + \delta^2) + \sigma^2 t, \\ \tilde{m}_3^{\text{GJD}} = \lambda t(\gamma^3 + 3\gamma\delta^2), \\ \tilde{m}_4^{\text{GJD}} = \lambda t(\gamma^4 + 6\gamma^2\delta^2 + 3\delta^4) + 3(\tilde{m}_2^{\text{GJD}})^2. \end{cases} \tag{10}$$

GJD generates larger jumps as γ departs from zero or as δ increases.

(b) The Laplacian jump diffusion process

We next propose another type of jump diffusion process, namely, the Laplacian jump diffusion process (LJD), where the jump size Y obeys the Laplace distribution. The probability density function of the Laplace distribution is known as $f_Y^{\text{Laplace}}(x) = \exp(-|x-\xi|/\zeta)/(2\zeta)$ with two parameters ξ, ζ $(\zeta > 0)$, and its characteristic function is derived as $\tilde{\Phi}_Y^{\text{Laplace}}(\omega) = \exp(i\xi\omega)/(1+\zeta^2\omega^2)$. The mean and the variance are ξ and $2\zeta^2$, respectively. Compared with a normal distribution, the Laplace distribution has a larger density around the mean and in its tails.

The characteristic function of the LJD is derived from Eq.(7) as:

$$\tilde{\Phi}_{R_t}^{\text{LJD}}(\omega) = \exp\left[t\left\{i\mu\omega - \frac{\sigma^2}{2}\omega^2 + \lambda\left(\frac{e^{i\xi\omega}}{1+\zeta^2\omega^2} - 1\right)\right\}\right], \tag{11}$$

where $\mu = r - \sigma^2/2 - \lambda\{e^{\xi}/(1-\zeta^2) - 1\}$ from Eq.(8). The moments $\tilde{m}_n^{\text{LJD}}$ are:

$$\begin{cases} \tilde{m}_1^{\text{LJD}} = \lambda t\xi + \mu t, \\ \tilde{m}_2^{\text{LJD}} = \lambda t(\xi^2 + 2\zeta^2) + \sigma^2 t, \\ \tilde{m}_3^{\text{LJD}} = \lambda t(\xi^3 + 6\xi\zeta^2), \\ \tilde{m}_4^{\text{LJD}} = \lambda t(\xi^4 + 12\xi^2\zeta^2 + 24\zeta^4) + 3(\tilde{m}_2^{\text{LJD}})^2. \end{cases} \tag{12}$$

See Appendix C for the derivation of Eqs.(12). Similarly to the GJD, the LJD generates larger jumps as the jump-size-mean parameter ξ departs from zero or as the jump-deviation parameter ζ increases. Comparing Eq.(12) with Eq.(10), the coefficients for δ or ζ are larger in the LJD moments than the GJD moments, thereby indicating that the tail of the LJD distribution decays slower than that of the GJD.

The asymmetric double exponential jump diffusion process (DEJD) proposed by Kou [2002] is another well-known process similar to the LJD. Although the LJD has the constraint that the jump-size distribution is symmetric, the LJD has fewer parameters than the DEJD and thus the parameters in the LJD can be estimated from the moments.[11]

2. Parameter Estimation of the Jump Diffusion Processes

An efficient method has not yet been established for the parameter estimation of jump diffusion processes, though various approaches have been proposed. Although the maximum likelihood estimation is one of the more effective methods, the likelihood function of jump diffusion processes is not generally written in a closed form. In addition, the log-likelihood function is not necessarily a concave function and may overshoot for some parameter sets. This makes it difficult to apply the maximum likelihood approach directly for jump diffusion processes. Several modifications have been proposed to overcome these difficulties: Honoré [1998] estimated jump-related parameters separately from the diffusion parameters, Ramezani and Zeng [2007] applied Gaussian quadrature in the likelihood function evaluation, and Nakajima and Omori [2009] used a Markov Chain Monte Carlo method to sample the parameters. In addition to these methods, a calibration that matches the theoretical and empirical option prices (or implied volatilities) is widely applied in the literature, including Bakshi, Cao, and Chen [1997], Carr and Wu [2003a], Cont and Tankov [2004], and Miyazaki [2009, in Japanese]. The generalized method of moments (GMM) that matches the theoretical and empirical moments is also applied in Pan [2002].[12]

This analysis estimates the parameters by the GMM and spectral GMM, or the characteristic function GMM, which extends GMM to the frequency domain. More specifically, we match the theoretical moments or characteristic functions with the implied moments or ICFs estimated by the nonparametric method explained in Section II.

[11] All of the jump-related parameters in the DEJD are shown as a triple product in the moments; this makes it impossible to identify each parameter.

[12] In addition, Miyahara [2003, in Japanese] proposed a two-stage estimation where the volatility parameter is firstly estimated using the asymptotic feature of the jump diffusion characteristic functions as

$$\hat{\sigma}^2 = -2 \lim_{\omega \to \infty} \operatorname{Re} \ln \Phi_{R_t}(\omega)/(\omega^2 t) = - \lim_{\omega \to \infty} \ln\{(\operatorname{Re} \Phi_{R_t}(\omega))^2 + (\operatorname{Im} \Phi_{R_t}(\omega))^2\}/(\omega^2 t),$$

and the other parameters are estimated by GMM. Although we attempted this method, we do not present the results here as the volatility estimator was fairly unstable.

(a) GMM

While we have mainly focused on the moments m_n, we use cumulants c_n in the GMM estimation because cumulants are written in a simpler form than the moments in jump diffusion models.

Let $\boldsymbol{\theta}$ denote a vector of parameters, and let $\mathbf{c}(t)$ and $\tilde{\mathbf{c}}(\boldsymbol{\theta})$ denote the market-implied 1st to nth cumulant vector on day t and the theoretical cumulant vector given the parameter vector $\boldsymbol{\theta}$. The difference is denoted as $\mathbf{g}^{\mathrm{M}}(t,\boldsymbol{\theta}) = \mathbf{c}(t) - \tilde{\mathbf{c}}(\boldsymbol{\theta})$. Then, the GMM estimator satisfies $\mathbf{E}[\mathbf{g}^{\mathrm{M}}(t,\boldsymbol{\theta})] = \mathbf{0}$.[13]

More specifically, suppose we use U samples for each GMM estimation and let $\mathbf{h}^{\mathrm{M}}_{t_0}(U,\boldsymbol{\theta}) = 1/U \sum_{u=-U+1}^{0} \mathbf{g}^{\mathrm{M}}(t_u,\boldsymbol{\theta})$ denote the mean of the samples on days t_u ($u = -U+1, -U+2, \ldots, 0$). Then, the GMM estimator on day t_0 is defined as:

$$\hat{\boldsymbol{\theta}}_{\mathrm{GMM}}(t_0) = \arg\min_{\boldsymbol{\theta}} \mathbf{h}^{\mathrm{M}}_{t_0}(U,\boldsymbol{\theta})^{\top} {\mathbf{W}^{\mathrm{M}}_{t_0}}^{-1} \mathbf{h}^{\mathrm{M}}_{t_0}(U,\boldsymbol{\theta}), \tag{13}$$

where the optimal choice of the weight matrix $\mathbf{W}^{\mathrm{M}}_{t_0}$ is known to be the asymptotic sample covariance $\mathbf{h}^{\mathrm{M}}_{t_0}(U,\boldsymbol{\theta}_{\mathrm{GMM}})$. Here, we apply the consistent covariance estimator computed from past U day samples $\mathbf{g}^{\mathrm{M}}(t_u,\hat{\boldsymbol{\theta}}_{\mathrm{GMM}})$ ($u = -U+1, -U+2, \cdots, 0$).[14] In addition, we adjust the sample autocorrelation in the weight matrix using the method proposed by Newey and West [1987].[15]

The GMM estimator is known to be efficient with a sufficiently large number of samples and a consistent number of moment conditions. We set four moment conditions ($n = 4$) corresponding to the number of parameters in the GJD or the LJD. We also set the sample period to be 1 month ($U = 21$) after considering the trade-off between the sensitivity of the estimates and the efficiency of the estimation.

(b) Spectral GMM

In the GMM estimation explained in the previous section, we neglect moment information higher than the fifth order. On the other hand, the spectral GMM (SGMM) estimation employs characteristic functions that take higher-order information into consideration. The SGMM was proposed by Feuerverger and McDunnough [1981] and Feuerverger [1990]. Singleton [2001] first applied the method to finance.

[13] $\mathbf{E}$ in this section indicates the expectation with respect to the samples.

[14] We first set the weight to be the unit matrix, then the consistent estimator is computed from the result.

[15] We apply a Bartlett kernel with a bandwidth set to the optimal period where $\mathbf{g}^{\mathrm{M}}(t_u,\boldsymbol{\theta})$ obeys an AR(1) process, as based on Andrews [1991].

According to these studies, the SGMM estimator $\boldsymbol{\theta}$ satisfies

$$\mathbf{E}|\exp(i\omega R_t) - \tilde{\Phi}_{R_t}(\omega, \boldsymbol{\theta})| = 0, \qquad \forall \omega \in \mathbb{R}.$$

Jiang and Knight [2002], Chacko and Viceira [2003], and Yu [2004] apply the SGMM method in empirical studies. The present analysis considers an SGMM estimator that satisfies:

$$\mathbf{E}|\Phi_{R_t}(\omega) - \tilde{\Phi}_{R_t}(\omega, \boldsymbol{\theta})| = 0, \quad \forall \omega \in \mathbb{R}. \tag{14}$$

However, we develop a method to obtain the ICF, $\Phi_{R_t}(\omega)$, as explained in Section II 2. More specifically, we consider the vector to be composed of the real and imaginary part of the gap between the implied and theoretical characteristic functions on day t_u $(u = -U + t, -U + 2, \dots, 0)$ given the appropriate frequency grids $\boldsymbol{\omega}$ as follows.

$$\mathbf{g}^{\mathrm{S}}(t_u, \boldsymbol{\theta}) = \begin{bmatrix} \mathrm{Re}\,\Phi_{R_{t_u+t}}(\boldsymbol{\omega}) - \mathrm{Re}\,\tilde{\Phi}_{R_t}(\boldsymbol{\omega}, \boldsymbol{\theta}) \\ \mathrm{Im}\,\Phi_{R_{t_u+t}}(\boldsymbol{\omega}) - \mathrm{Im}\,\tilde{\Phi}_{R_t}(\boldsymbol{\omega}, \boldsymbol{\theta}) \end{bmatrix}.$$

Let $\mathbf{h}^{\mathrm{S}}_{t_0}(U, \boldsymbol{\theta}) = 1/U \sum_{u=-U}^{0} \mathbf{g}^{\mathrm{S}}_{t_0}(t_u, \boldsymbol{\theta})$ be the sample mean of $\mathbf{g}^{\mathrm{S}}(t_u, \boldsymbol{\theta})$ in the past U days. Then, the SGMM estimator on day t_0 is defined as:

$$\hat{\boldsymbol{\theta}}_{\mathrm{SGMM}}(t_0) = \arg\min_{\boldsymbol{\theta}} \mathbf{h}^{\mathrm{S}}_{t_0}(U, \boldsymbol{\theta})^{\top} \mathbf{W}^{\mathrm{S}\,-1}_{t_0} \mathbf{h}^{\mathrm{S}}_{t_0}(U, \boldsymbol{\theta}), \tag{15}$$

where the weighted matrix $\mathbf{W}^{\mathrm{S}}_{t_0}$ is set to be the consistent covariance estimate of the past U day samples $\mathbf{g}^{\mathrm{S}}(t_u, \hat{\boldsymbol{\theta}}_{\mathrm{SGMM}})$ $(u = -U + 1, -U + 2, \cdots, 0)$. The adjustment in Newey and West [1987] is also applied to the SGMM.

The SGMM estimator is asymptotically normal, as is the GMM estimator. More specifically, let $\boldsymbol{\theta}_0$ denote a true parameter set. Then, the SGMM estimator satisfies:

$$\sqrt{U}(\hat{\boldsymbol{\theta}}_{\mathrm{SGMM}} - \boldsymbol{\theta}_0) \xrightarrow{p} N(\mathbf{0}, \mathbf{V}^{\mathrm{S}}), \tag{16}$$

$$\mathbf{V}^{\mathrm{S}} = \left(\mathbf{D}^{\mathrm{S}}_{t_0} \mathbf{W}^{\mathrm{S}\,-1}_{t_0} \mathbf{D}^{\mathrm{S}\,\top}_{t_0}\right)^{-1}, \quad \mathbf{D}^{\mathrm{S}\,\top}_{t_0} = \frac{\partial \mathbf{h}^{\mathrm{S}}_{t_0}(U, \hat{\boldsymbol{\theta}}_{\mathrm{SGMM}})}{\partial \boldsymbol{\theta}^{\top}}.$$

The SGMM estimator is also known to be consistent with the maximum likelihood estimator when we take ω continuously and the weight matrix appropriately.[16]

The frequency grids and the range should be determined carefully. Whereas the SGMM estimator is known to become efficient by increasing the number of frequency grids, the weight matrix $\mathbf{W}^{\mathrm{S}}_{t_0}$, substituted for by the consistent covariance estimator,

[16] See Singleton [2001] for details.

tends to be singular. This makes it quite difficult to invert the matrix.[17] This problem, unique to the SGMM, is also pointed out in Carrasco *et al.* [2003] and Yu [2004]; however, neither provides a definitive solution. This analysis set the grids as $\boldsymbol{\omega} = (2^n)^\top$, $(n = 0, 1, 2, \ldots)$,[18] and the range as $|\omega| \leq 50$ for most of the examined period,[19] based on the analysis of the characteristic function computed by the simulation of a jump diffusion process with appropriate parameters. In addition, the bandwidth of the Newey and West [1987] correction is set the same as 2.(a) in Section II.

IV Empirical Analysis

1. Data

This paper analyzes the distribution implied in equity index options in Japan, Germany, the UK, and the US during the period from the beginning of 2005 to the end of September 2009. While we focus on the market expectations formed during the financial turmoil after the summer of 2007, we also include the preceding data, regarded as an ordinary period, for comparison. Our surveillance period includes: i) the period from 2005 to July 2007 when global financial markets were relatively calm, ii) the period from September 2007 to September 2008 when equity prices gyrated against the backdrop of the prevailing subprime mortgage crisis in the US, and iii) the period from October 2008 to September 2009 when global equity prices dropped sharply following the Lehman shock and the serious deterioration of macroeconomic indicators.

We employ exchange-traded liquid options on stock indices, thereby selecting the Nikkei 225 in Japan, the DAX in Germany, the FT100 in the UK, and the S&P500 in the US. The last prices of each trading day are obtained from Bloomberg, except for the S&P500 where we employ daily summary data from the Chicago Board Options

[17] This may stem from the values of characteristic functions with different frequencies being too close, or from the variances of samples with high- and low-frequency regions being too far apart.

[18] However, if the determinant of $\mathbf{W}^{\mathrm{S}}_{t_0}$ is lower than 10^{-15}, we reset the grids as $\boldsymbol{\omega} = (3^n)^\top$ $(n = 0, 1, 2, \ldots)$.

[19] In some countries, the information in the ICF extends to the lower-frequency region because of the expansion of the implied distribution. Using information densely on the lower-frequency region stabilizes the estimators. More particularly, we set $\omega \leq 15$ in the GJD process to the period after August 2008 in the US data, after December 2008 in the Japanese data, and after January 2009 in the UK data. We also set $\omega \leq 10$ for the period from September to December 2009 in the UK data. In the LJD process, we set $\omega \leq 15$ during the period from March to August and from the middle of October to November 2008 in the Japanese data, we set $\omega \leq 20$ during the period from September to the middle of October 2008 in the Japanese data, after January 2009 in the UK data, and from August to September 2008 and from January to September 2009 in the US data. We lastly set $\omega \leq 20$ for the period after June 2009 in the US data.

Exchange. Data for all expirations are used, fixed at the last month of each quarter.[20] The data for all strike prices obtained are used in the following estimation.[21]

For the equity index prices, we use the last price of each trading day from Bloomberg, and the London Interbank Offered Rate (LIBOR) as a proxy for the risk-free interest rate.[22]

Hereafter, we set a 3-month term to maturity ($t = 1/4$) for the analysis. This is because 3 months is a suitable period for the analysis of market expectations given the trade-off between the reliability of the data and the stability of the estimators. While option liquidity is generally concentrated in the nearest term to maturity options (those with less than 3-month terms), the estimates of the higher-order implied moments lose stability with maturities of less than 2 months.

2. The Results of the Nonparametric Analysis

(a) The implied moments

We first evaluate the implied moments estimated using the method in Section II 1. Figure 1 displays the daily estimation result of the moments with 1st to 4th orders from top down. Each panel contains the implied moments of Japan, Germany, the UK, and the US with the line types specified in the legend.

Each time series develops stably before July 2007. The levels of the 1st moment vary among countries based on the differences in the risk-free rates and volatilities.[23] The levels of the 2nd moment are largely similar, except for when Japanese volatility surged when the fraud of Livedoor, an Internet service company, became known in early 2006. The levels of the 3rd and 4th moments are also closer to one another. Here, the 3rd moment moves in a slightly negative range and the 4th moment shifts

[20] Although options with other maturities are traded in the US market, we do not use this additional data so as to maintain consistency with the other three markets. The maturity date in the quarter-end months differs by country; the second Friday is the maturity date for Japanese options, and the third Friday is the last trading day for the German, UK, and US options.

[21] The options market system has changed in Japan since September 2008 in that options with a maturity date of less than 3 months have strike prices with a 250-yen tick while all other options have 500-yen tick strikes. To maintain consistency, we only use the 500-yen tick of strike prices for Japanese options.

[22] While the LIBOR rate surged alongside the risk premiums of major money market players during the financial turmoil, short-term government-bond rates declined. We employ LIBOR by assuming that financial institutions, the major option market players, hedge their option risk using the money markets.

[23] The 1st moment is, as defined in Section II 1.(a), equivalent to the risk-free rate minus half of the squared model-free implied volatility or volatility drag. The Japanese 1st moment is estimated to be negative, as the risk-free rate in Japan during the period examined is relatively lower than in other countries, and this exceeds the level of the volatility drag.

in an extremely low range before July 2007.

After August 2007, however, the 2nd moment distinctively rises, and the 4th moment hikes slightly in every country. The fluctuations in the 1st and 3rd moments scale up, alongside the expanding negative level of the 3rd moment. Further, after the Lehman shock in September 2008, every moment jumps to an extreme level; the odd-order moments bounce intensively in a negative direction, while the even-order moments display a dramatic surge in the positive direction. Afterwards, each moment gradually reduces toward the level just before the Lehman shock.

In addition, it is notable that the levels and directions of the estimators in each country are close to each other for most of the period, particularly during the period of the financial turmoil. In fact, in each country the moments of any order jump in the same direction at almost the same scale. This implies that the market expectations formed during the financial turmoil are similar in each country. This evidences a link between not only equity prices, but also market recognition of the uncertainty of future price levels, in these countries.

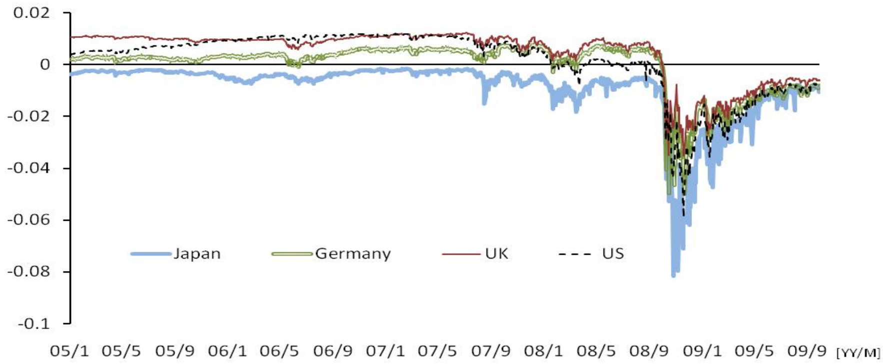
(b) Skewness and kurtosis

Next, we reconsider the implied moments from the perspective of the deviation from the normal distribution. Figure 2 plots the skewness and kurtosis of the implied distribution. As shown, the estimate of skewness is negative, while the kurtosis exceeds 3 throughout the period examined in each country. This indicates that the implied distribution is leptokurtic with a fat tail of negative values. This is consistent with earlier findings, such as Oda and Yoshiba [1998], Bakshi, Kapadia, and Madan [2003], or Carr and Wu [2003b].

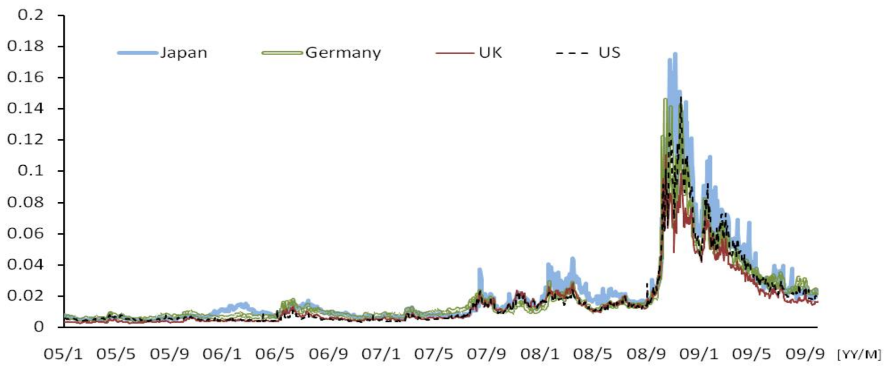
Analyzing the development of skewness and kurtosis, we detect that the negative skewness scales down to some degree, while the kurtosis declines after the middle of 2007 in all countries except Japan (Figure 2). After the Lehman shock, these values gradually return to the level just before the Lehman shock. This feature in Europe and the US indicates that large price moves, recognized in ordinary times as fluctuations of the 3rd or higher order, are recognized at the 2nd order during the financial turmoil because of the dramatic increase in the 2nd moments. In other words, the implied distribution stretches or flattens out during the turmoil and becomes closer to the normal distribution with a very large variance. This weakens the leptokurtic feature appearing in more ordinary times. Putting this in the perspective of option prices, OTM options are traded at extraordinarily expensive prices, particularly deep OTM options.

Figure 1 Estimation results of implied moments in Japan, Germany, the UK and the US (3-month term)

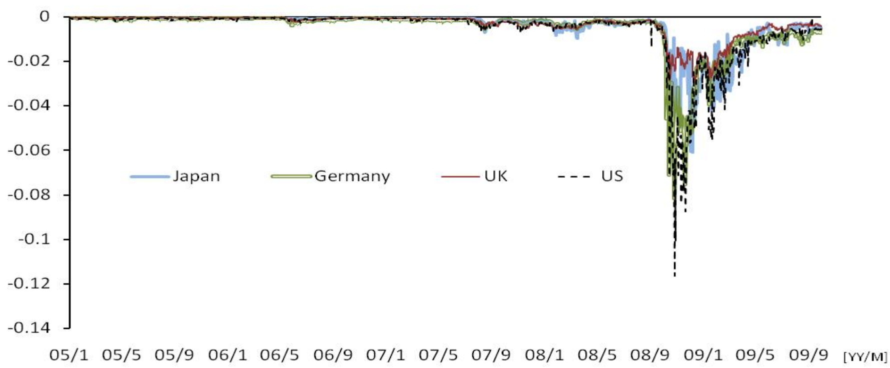
i) 1st moment



ii) 2nd moment



iii) 3rd moment



iv) 4th moment

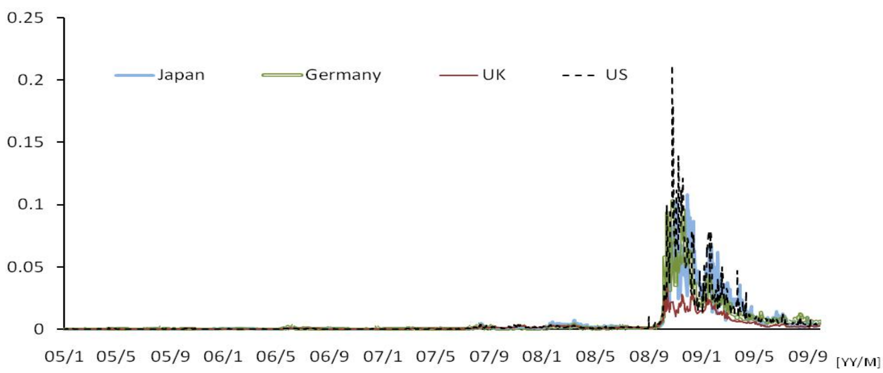
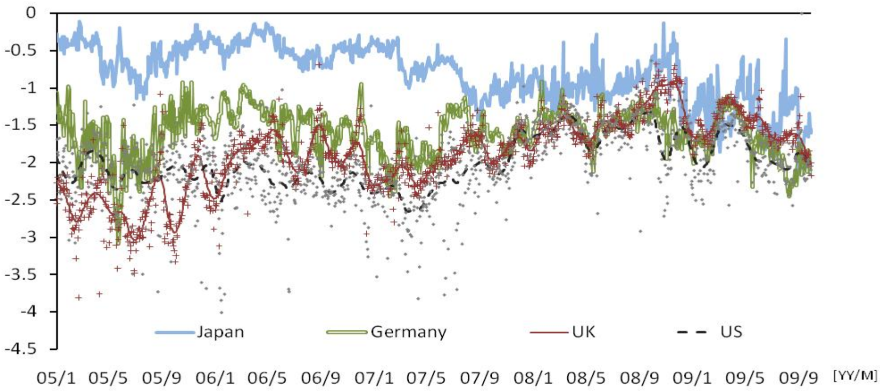
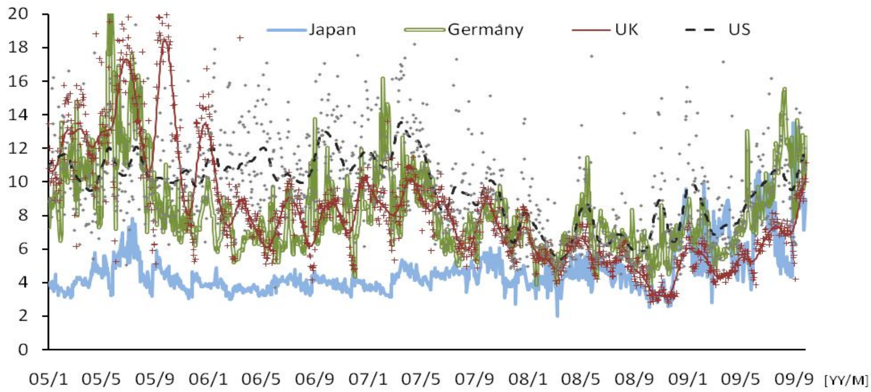


Figure 2 Skewness and kurtosis of the implied distribution in Japan, Germany, the UK, and the US (3-month term)

i) Skewness



ii) Kurtosis



Notes: The solid lines for UK and US data are smoothed series filtered on the basis of Hodrick and Prescott [1997] with smoothing parameter 2,430 after excluding abnormal values. Dots, + and ⧫, indicate the raw UK and US data, respectively.

In contrast, the negative skewness and kurtosis in Japan broadens after the middle of 2007, particularly after the Lehman shock. This is accounted for by a technical background unique to Japan. That is, there is an increase in the number of OTM strike prices traded in the market or available in the examined period, and this may lead to an increase in the estimated 3rd and 4th moments in Japan.[24,25]

Additionally, we can see that the skewness and kurtosis vary extensively when compared with the mean and variance (the 1st and 2nd moments) in Figure 2. This is because of technical difficulties in the estimation of implied moments from traded

[24] In addition, the relatively small losses suffered by Japanese financial institutions from the financial turmoil may be involved. We reject this possibility, however, because of the fact that the Japanese 2nd moment surges to a level similar to those of Europe and the US during the turmoil.

[25] See Figure 3 in Sugihara [2010] for the number of Nikkei options traded in the market.

option prices. To be more specific, deep OTM options are not always traded every day, and, if traded, may be priced at a higher level than the actual market price because of market constraints, such as the fixed tick size or the minimum transaction price. These restrictions generate noise in the estimated higher-order moments. This noise is prominent in the UK and US data where the market-traded range of strike prices is relatively broad. Therefore, Figure 2 plots the smoothed series filtered using Hodrick and Prescott [1997] after excluding abnormal values[26] as references for higher-order moments in the UK and the US.[27]

Similar features are observed in the implied moments with more than 3 months to maturity.[28] We further analyze this in Section IV 3. by considering the stochastic processes that configure the implied distribution.

3. The Results of the Parametric Analysis

This section analyzes the implied distribution in a parametric way in order to further comprehend the change in the implied moments evaluated in Section IV 2.(a). Using the Gaussian and Laplacian jump diffusion processes defined in Section III 1., we examine the development of these parameters in each country and for each period. We also investigate the discrepancy between market expectations and normal distributions using the magnitude of the implied jumps indicating the deviation from a Brownian motion, while we consider the discrepancy in skewness and kurtosis in Section IV 2.(b).

(a) Implied parameters of the jump diffusion processes

Table 1 summarizes the estimated parameters of the GJD and the LJD based on the GMM and SGMM explained in Section III 2. We divide the examined period (from January 2005 to September 2009) into three subperiods, as the entire sample period includes the large shock of the financial turmoil, as noted in Section IV 2.

A: Ordinary period: the period from January 2005 to August 8, 2007 when global markets were relatively calm.

B: Rising instability period: the period from August 9, 2007 when global equity prices plunged in response to the announcement by BNP Paribas that it was sus-

[26] We set the abnormal value thresholds of skewness at less than -5 and for kurtosis larger than 20.

[27] A 3-month cycle is observed in the time series of skewness and kurtosis. This is mainly because we apply composite 3-month term interpolated option prices in the moment evaluation. The prices of deep OTM options with closer terms to maturity are theoretically very small, though these are sometimes traded at prices much higher than the theoretical price. In turn, this leads to spikes in the estimated higher-order implied moment.

[28] We detect similar findings for the 1-, 3-, and 6-month terms in the US data.

Table 1 Estimation result for parameters

i) The Gaussian jump diffusion (GJD)

	procedure	GMM				SGMM			
	parameter	σ	λ	γ	δ	σ	λ	γ	δ
a) Ordinary period	Japan	0.146	1.117	-0.068	0.077	0.148 ***	0.710 ***	-0.086 ***	0.093 ***
(1/4/05 ~ 8/8/08)	Germany	0.102	0.954	-0.094	0.124	0.109 ***	1.105 ***	-0.100 ***	0.079 ***
	UK	0.056	1.104	-0.086	0.093	0.053 ***	1.053 ***	-0.091 ***	0.080 ***
	US	0.071	0.623	-0.115	0.115	0.071 ***	0.885 ***	-0.104 ***	0.070 ***
b) Rising instability	Japan	0.151	1.371	-0.202	0.019	0.144 ***	1.027 ***	-0.218 ***	0.011
period	Germany	0.106 ***	1.897 ***	-0.108 ***	0.121 ***	0.116 ***	1.796 ***	-0.123 ***	0.096 ***
(8/9/07 ~ 9/14/08)	UK	0.089	1.113	-0.230	0.029	0.094 ***	0.856 ***	-0.239 ***	0.020 ***
	US	0.125	1.103	-0.162	0.121	0.099 ***	1.098 ***	-0.197 ***	0.051 ***
c) Post Lehman shock	Japan	0.294	1.099	-0.291	0.185	0.125 ***	3.990 ***	-0.086 ***	0.180 ***
period	Germany	0.223	0.725	-0.324	0.202	0.132 ***	3.200 ***	-0.100 ***	0.201 ***
(9/15/08 ~ 9/30/10)	UK	0.000	1.697	-0.301	0.000	0.095 ***	1.066 ***	-0.273 ***	0.024 ***
	US	0.208 *	0.911	-0.277	0.305 ***	0.081 ***	3.368 ***	-0.097 ***	0.210 ***

ii) The Laplacian jump diffusion (LJD)

	procedure	GMM				SGMM			
	parameter	σ	λ	ξ	ζ	σ	λ	ξ	ζ
a) Ordinary period	Japan	0.149	0.690	-0.103	0.044	0.152 ***	0.227 ***	-0.200 ***	0.019
(1/4/05 ~ 8/8/08)	Germany	0.109	0.803	-0.131	0.070	0.108 ***	0.999 ***	-0.113 ***	0.056 ***
	UK	0.091	1.865	-0.115	0.076	0.047 ***	1.075 ***	-0.090 ***	0.057 ***
	US	0.066	0.819	-0.114	0.062	0.069 ***	0.848 ***	-0.110 ***	0.052 ***
b) Rising instability	Japan	0.152	1.330	-0.206	0.008	0.144 ***	1.028 ***	-0.218 ***	0.008
period	Germany	0.060 *	2.924 ***	-0.104 ***	0.067	0.115 ***	1.694 ***	-0.130 ***	0.075 ***
(8/9/07 ~ 9/14/08)	UK	0.135	1.019	-0.195	0.080	0.094 ***	0.856 ***	-0.239 ***	0.014 ***
	US	0.110	1.107	-0.182	0.071	0.100 ***	1.074 ***	-0.199 ***	0.038 ***
c) Post Lehman shock	Japan	0.290	0.989	-0.337	0.104	0.166 ***	1.242 **	-0.206 ***	0.039
period	Germany	0.192	1.011	-0.288	0.128	0.164 ***	1.369 ***	-0.265 ***	0.007
(9/15/08 ~ 9/30/10)	UK	0.116	1.756	-0.171	0.065	0.106 ***	1.038 ***	-0.206 ***	0.015
	US	0.219	0.718	-0.385	0.170 ***	0.121 ***	1.276 ***	-0.270 ***	0.013

Notes: Estimators with *, **, and *** indicate rejection of the null hypothesis at the 90%, 95%, and 99% significance level, respectively (one-sided test for γ, ξ; otherwise two-sided test).

pending three of its asset-backed securities funds (the so-called "Paribas shock"), to September 14, 2008, the day before the Lehman shock.

C: Post-Lehman shock period: the period from September 15, 2008 when Lehman Brothers filed for Chapter 11 bankruptcy, to September 30, 2009.

First, we analyze the difference in the estimation procedures. In Table 1, most of the SGMM estimators are statistically significant at the 99% significance level, unlike most of the GMM estimators. This indicates that the SGMM estimators are more efficient. For this reason, we employ SGMM estimators in the following analyses of the differences in models, countries, and periods.

Second, we can note some differences in the models. While the estimated volatility σ is higher in the GJD than in the LJD during the ordinary period for most countries except Japan, the estimated volatility in the LJD is higher than in the GJD during the post-Lehman shock period in all countries. Further, while the estimated jump intensities of λ in the GJD and the LJD are of a similar level during the ordinary period and the rising instability period, the estimate for the GJD is higher than in the LJD during the post-Lehman shock period.

Third, we examine the differences across countries. As shown, the estimated volatility is higher in Japan than in the other countries. The levels of the other parameters vary at different periods and do not appear to display any distinct difference across countries. The hypothesis test shows that the GJD appears to fit the Japanese data better in each period. On the other hand, the European and US data appear to fit the LJD better than the GJD because of the higher t-values for ξ, ζ than γ, δ, even though both the GJD and the LJD fit well based on the hypothesis test. After the Lehman shock however, the GJD parameter δ is significant, while ζ is not, indicating that the GJD fits the European or US data better during the post-Lehman period. Given the fact that the distribution of the LJD is more leptokurtic with fatter tails than the GJD, those observations are consistent with the results in Section IV 2.(b), where the kurtosis is higher in the European and US distribution than in Japan during the ordinary period, while it declines after the Lehman shock. These considerations indicate that the decay of the implied distribution before the summer of 2007 is slower in Europe and the US than in Japan.

Finally, we investigate the difference across periods. Though both the jump intensity λ and its deviation δ increase significantly during both the rising instability period and the post-Lehman shock period, the other parameters do not provide any clear direction. Any distinctive differences in short periods are also difficult to observe in the estimates from the divided samples, particularly during the period of financial turmoil, as the shape of the implied distribution may change dramatically in the short period of time after the Lehman shock. Therefore, the next section analyzes the time development of the parameters more precisely by applying a rolling estimation.

(b) Time series of estimated parameters

This section presents rolling estimates with a 1-month (21-business day) window of past samples in order to more precisely analyze the time development of market expectations.

(i) The Gaussian jump diffusion process

Figure 3 displays rolling estimates of the parameters in the GJD. The panels in the left-hand side column plot the GMM estimates from the implied moments, while those in the right-hand side column plot the SGMM estimates from the ICFs. Both the GMM and SGMM estimates generally move closer with a similar direction. As shown, the SGMM estimators have fewer outliers and are more stable than the GMM estimators as a whole. We detect the following four aspects of each parameter.

First, the diffusion volatility σ is estimated roughly in the 0.1 to 0.4 range. This is somewhat smaller than the estimate of the model-free implied volatility in Sugihara [2010].[29] This rises just after the Lehman shock, before gradually declining.

Second, the jump-size mean γ is negative in all countries. The absolute value slowly increases and then abruptly grows with the Lehman shock, once again in each country. This indicates that the global financial markets are aware of the uncertainty of price moves in a negative direction even in ordinary times, and this grows much stronger during the financial turmoil.

Third, the jump intensity λ is estimated roughly in the 0.5 to 3 range with relatively large swings. This implies that financial markets incorporate the possibility of zero to a few jumps in the coming one-year period. Though the level appears to increase after the middle of 2007, we do not observe a distinctive change as seen in the other parameters. This suggests that markets prepare for possible jumps, even in ordinary periods.

Fourth, the estimated levels of the jump's deviation δ vary country by country. The feature whereby it drops just after the Lehman shock, and then gradually recovers afterwards, is common to the countries examined.

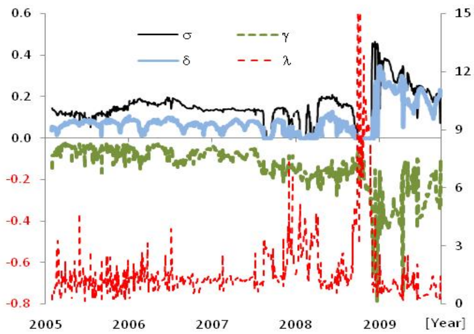
(ii) The Laplacian jump diffusion process

Figure 4 displays the estimators of the parameters in the LJD. The panels on the left-hand side plot the GMM estimates while those on the right-hand side plot the SGMM estimators, as in Figure 3. Compared with the GJD case, while the overall direction of the LJD parameters is similar, the jump-mean parameter in the LJD falls more substantially during the turmoil, and recovers more slowly afterwards, than in the GJD. This implies that market expectations of large price fluctuations were relatively cautiously formed after the Lehman shock. In addition, while the estimated parameters in the GJD are stable in Japan, those in the LJD are relatively more stable in the other

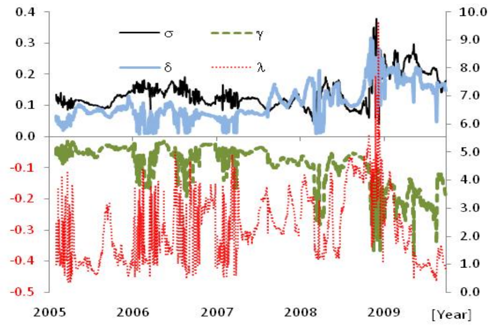
[29] This result is consistent, as the model-free implied volatility is generally higher than σ in this paper. This is because σ is an estimate of a pure diffusion volatility, while the model-free implied volatility is the estimate of quadratic variation, including the 2nd-order effects of jumps.

Figure 3 Time series of the estimated parameters in the Gaussian jump diffusion process

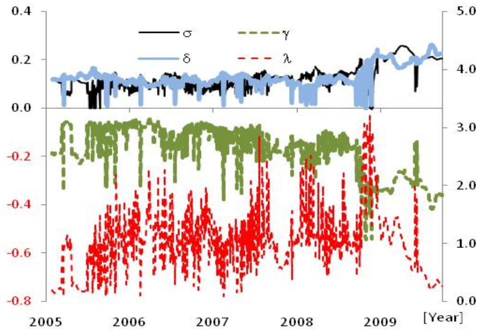
i) Japan: A) GMM estimates



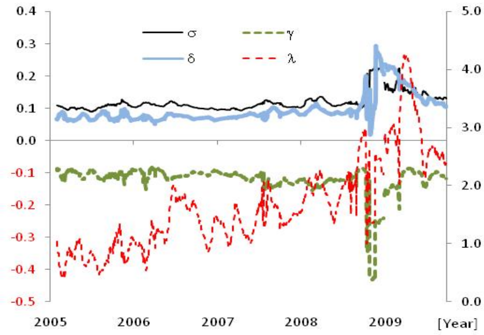
B) SGMM estimates



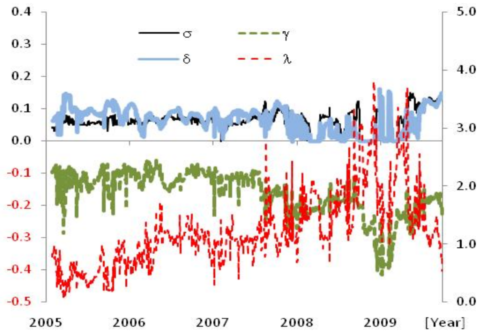
ii) Germany: A) GMM estimates



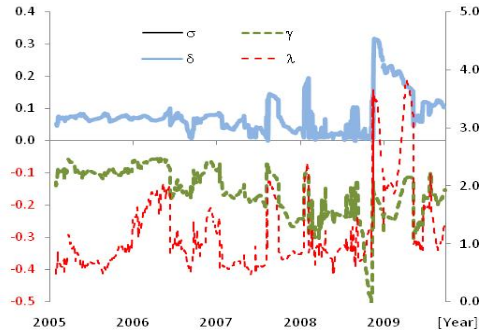
B) SGMM estimates



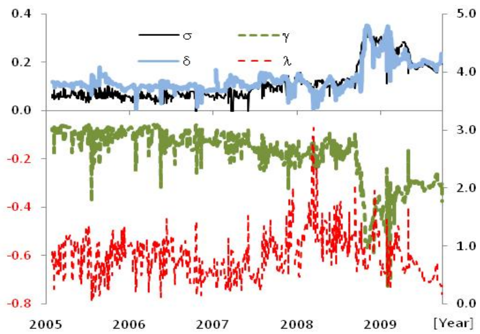
iii) UK: A) GMM estimates



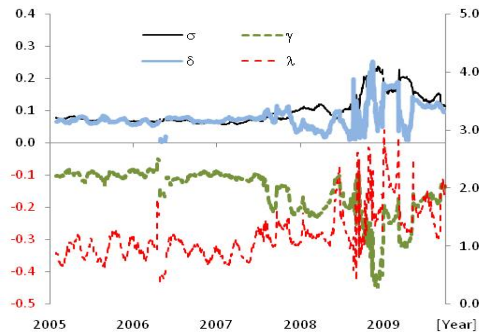
B) SGMM estimates



iv) US: A) GMM estimates



B) SGMM estimates



Note: Right axis for λ; left axis otherwise.

countries. This is consistent with the results in Section IV 3.(a).

(c) The implied jump

The implied jump is defined as the pure jump component in a jump diffusion process supposed to configure the implied distribution. Mathematically, this is defined as J_t in Eq.(6) in Section III 1. The implied jump traps large and discontinuous price changes not represented by the Brownian motion or a normal distribution. Given the means of J_t in GJD and LJD are denoted as $\lambda\gamma t,\ \lambda\zeta t$, and the variances are $\lambda(\gamma^2+\delta^2)t,\ \lambda(\xi^2+2\zeta^2)t$, respectively, its development or its effects on moments can be analyzed by applying the parameters estimated in Section IV 3.(b). In what follows, we present the implied jump using the SGMM estimators, as these are more efficient and stable than the GMM estimators, as explained in Sections IV 3.(a) and (b).

(i) Levels of the implied jump

This section analyzes the mean of the implied jump, $\mathbf{E}_0^Q J_t$. Figure 5 displays the mean of the implied jumps in the GJD and the LJD. The mean value indicates the market-expected level of increase or decrease in equity returns through discontinuous and large price changes. The following four points are proved.

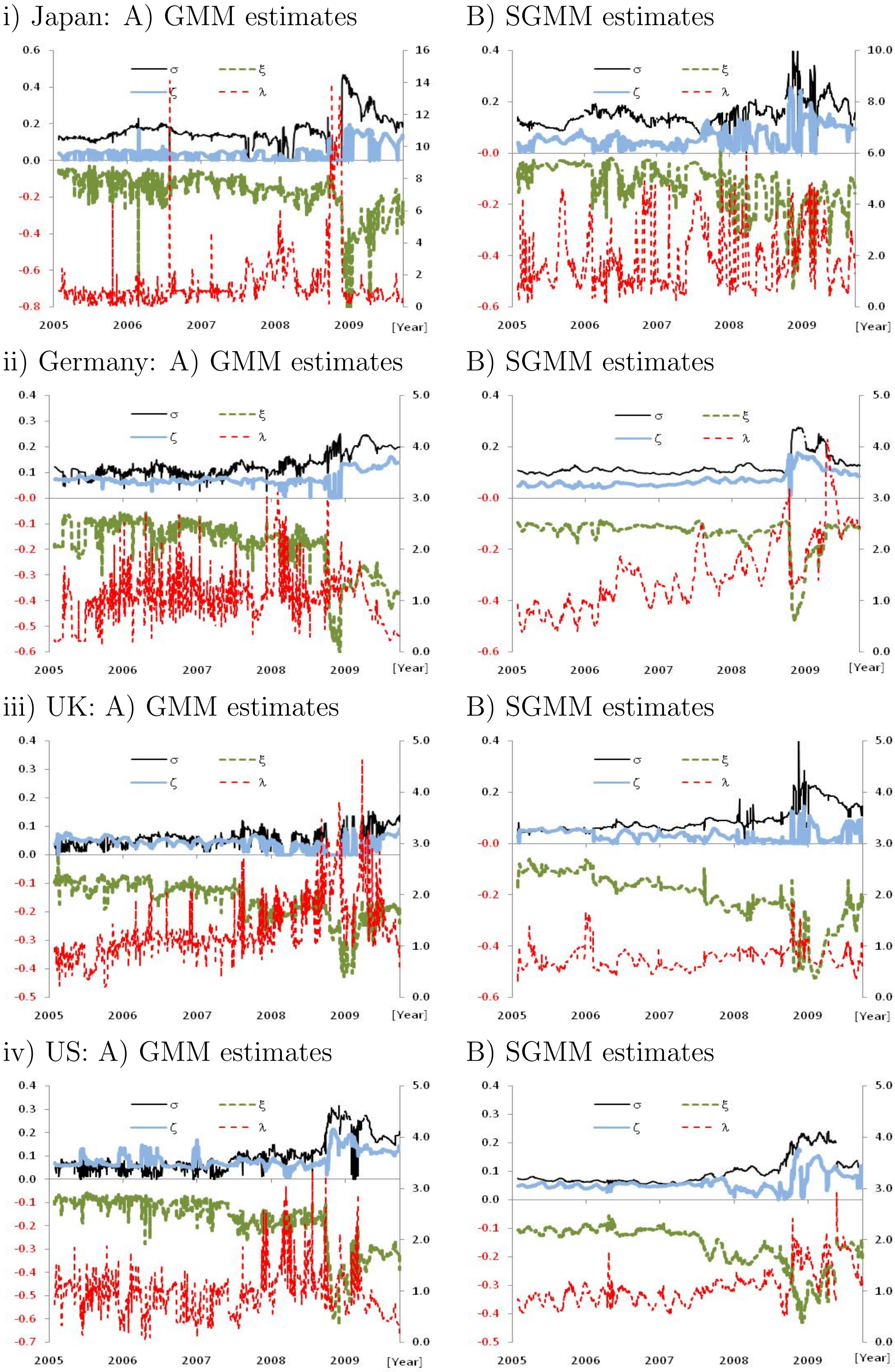
First, the mean jump is always estimated to be negative throughout the period examined. This indicates that market participants expect that their returns are likely to deteriorate if the price jumps. This is consistent with the negative skewness analyzed in Section IV 2.(a).

Second, the implied jump in each country moves, on average, in the range of -1% to -5% in the ordinary period before the middle of 2007. This indicates that market participants expect their equity returns are likely to deteriorate a few percent in the following 3 months, even in ordinary periods.

Third, the implied jump in each country drops after the middle of 2007. Moreover, it plunges dramatically after the Lehman shock. This indicates that market participants expect their equity returns to sharply decline by more than -15% through discontinuous price jumps. In particular, Japanese market participants expect precipitous falls in their returns by as much as -25%; this is much larger than European and US participants' expectations of about -15%.

Fourth, the sharp dip gradually recovers to the level before the Lehman shock, and almost returns to the level immediately before the Lehman shock by the end of September 2009.

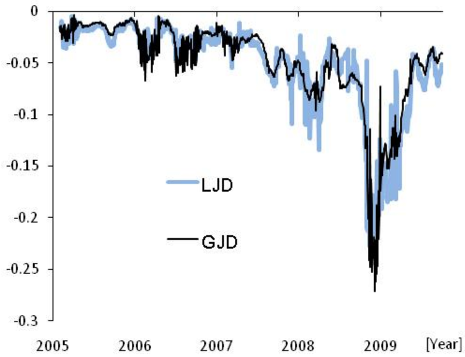
Figure 4 Time series of the estimated parameters in the Laplacian jump diffusion process



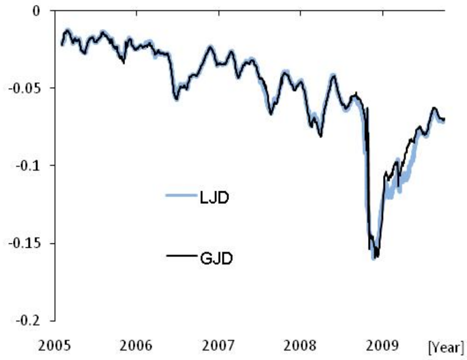
Note: Right axis for λ; left axis otherwise.

Figure 5 Time series of the means of the implied jumps in Japan, Germany, the UK, and the US (3-month term)

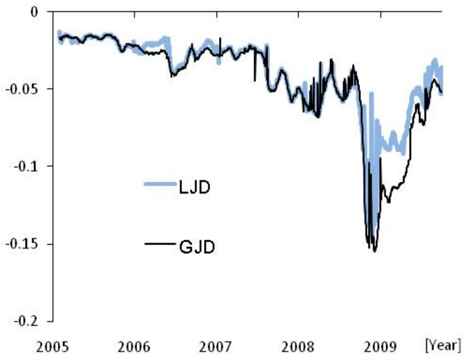
i) Japan



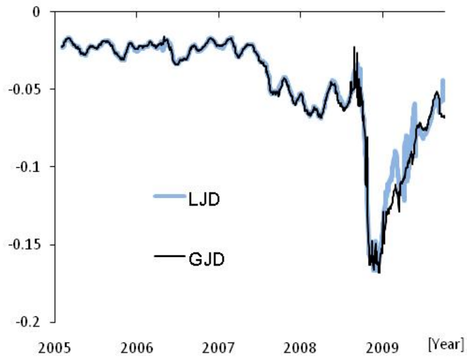
ii) Germany



iii) UK



iv) US



(ii) Contributions of the implied jump

This section analyzes the variance of the implied jump. We examine its contribution to the 2nd implied moment, or the implied variance, by comparing it with the diffusion volatility.[30] Figure 6 and Figure 7 display the estimated jump component (the left-hand side column) and its contribution (the right-hand side column) based on the GJD and the LJD, respectively.[31] The following two aspects are proved, though there are subtle differences in countries and models.

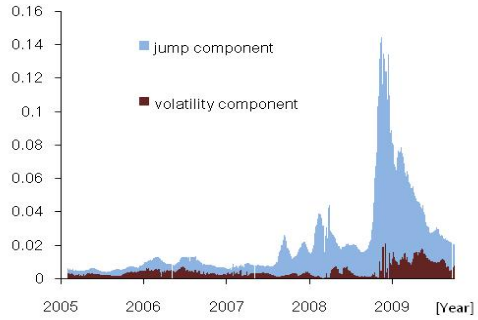
First, the jump contribution exists to a certain degree at ordinary times; about 40% in Japan, 60% in Germany, 80% in the UK, and 70% in the US of the 2nd moment are considered to stem from the implied jumps. The estimated shares are

[30] The contributions of the volatilities and jumps to the implied variance are considered respectively to be $\sigma^2 t$, and the variance of J_t.

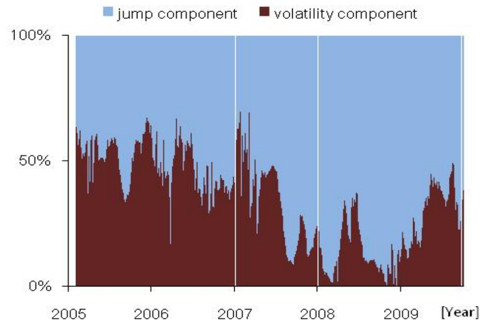
[31] The sum of the diffusion and the jump components is not always exactly equal to the second moment, as this section evaluates these components using the parameters estimated from the past 21 days of the ICFs.

Figure 6 Time series of the contribution of the implied jump to the 2nd moment (the Gaussian jump diffusion process)

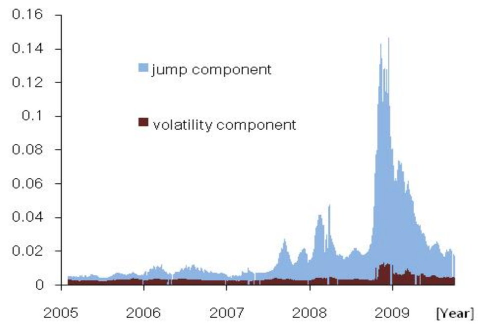
i) Japan: A) components



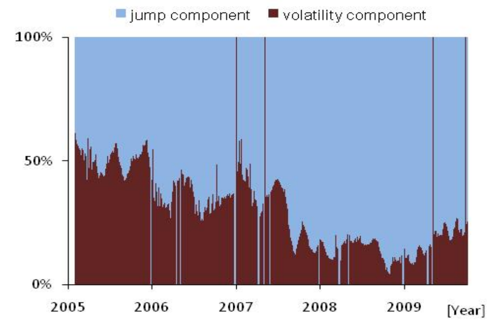
B) contributions



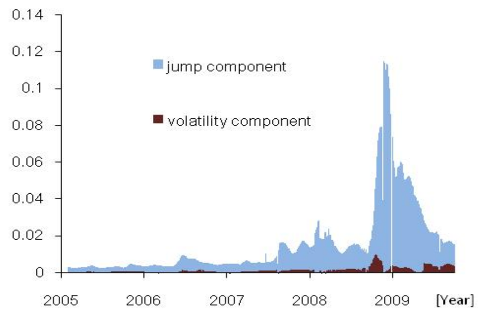
ii) Germany: A) components



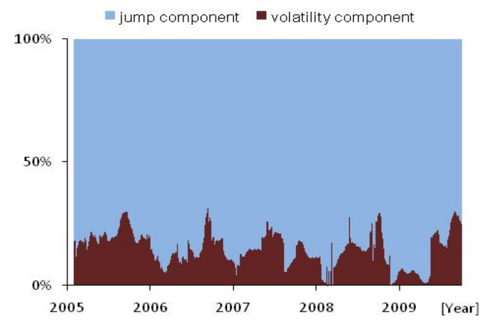
B) contributions



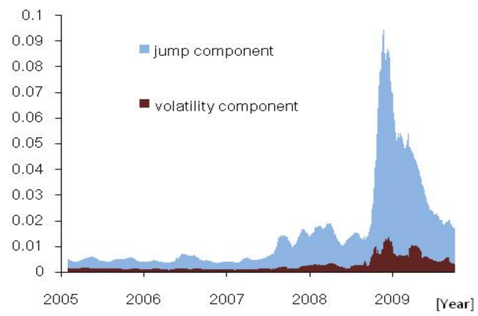
iii) UK: A) components



B) contributions



iv) US: A) components



B) contributions

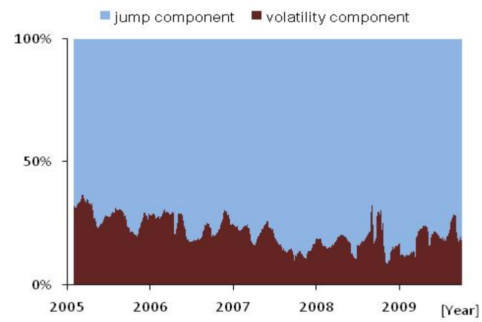
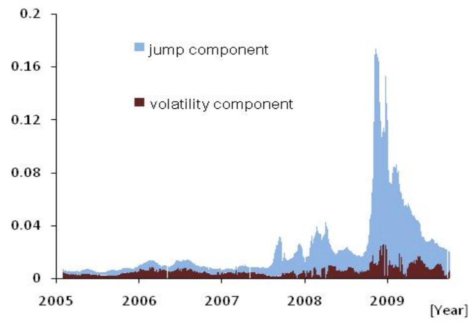
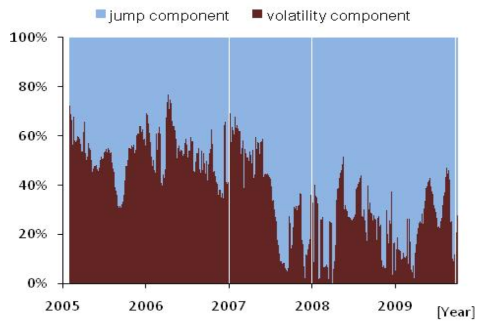


Figure 7 Time series of the contribution of the implied jump to the 2nd moment (the Laplacian jump diffusion process)

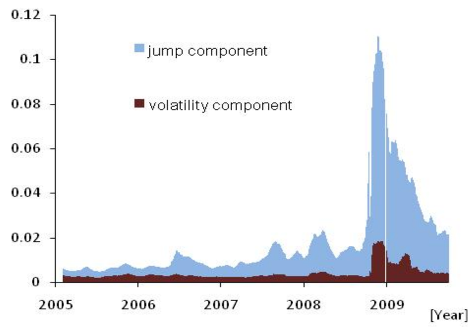
i) Japan: A) components



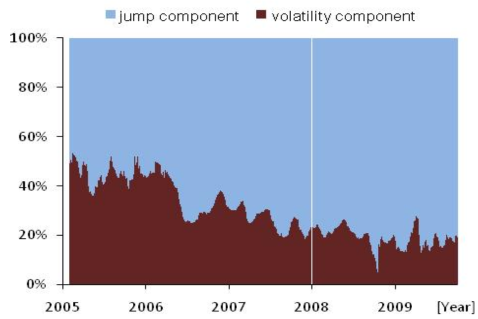
B) contributions



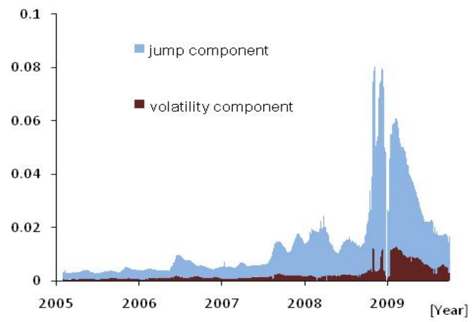
ii) Germany: A) components



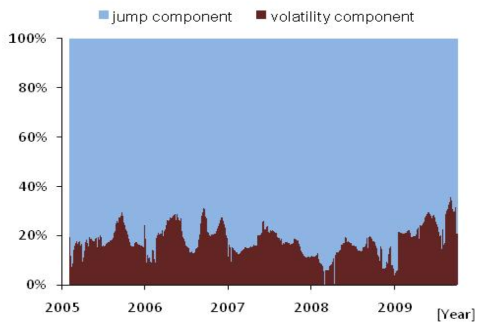
B) contributions



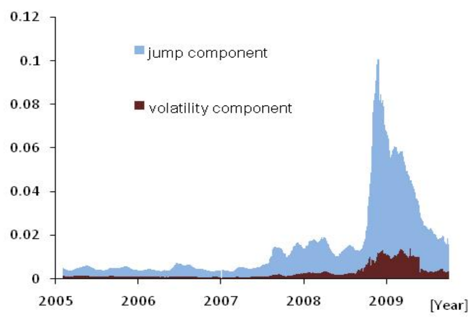
iii) UK: A) components



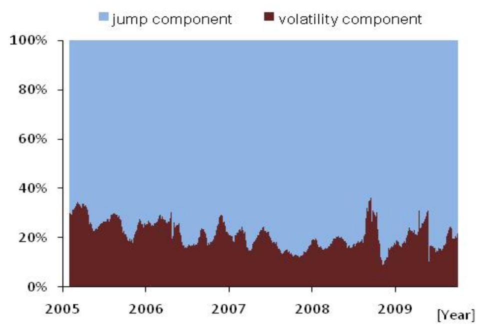
B) contributions



iv) US: A) components



B) contributions



relatively large because all of the factors that generate volatility smiles are attributable to jumps in a jump diffusion model such as the GJD or the LJD.[32] The difference in jump contributions may stem from the differences in the degree of maturity in these option markets. In particular, it would appear that in the very mature UK and US option markets, deep OTM options are traded more frequently than in the Japanese or German markets, even in ordinary times. This demonstrates the existence of fat tails in the implied distribution, and this leads to the larger contribution of implied jumps.

Second, the contribution of the jumps increases with the financial turmoil. Though the extent varies, it is larger in Japan and Germany where the contribution is relatively smaller in ordinary times than in the UK or the US.

V Summary

This paper investigated how market expectations formed during the financial turmoil following the summer of 2007 with both nonparametric analysis using implied moments and implied characteristic functions and parametric analysis using jump diffusion processes. First, we improved the method of the implied moments derivation proposed by Bakshi, Kapadia, and Madan [2003] and derived the implied characteristic function through generalization. We applied the method to equity options in Japan, Germany, the UK, and the US for the period from 2005 to the middle of 2009, and precisely analyzed the development of the implied distribution without assuming any particular model. Then, supposing two types of jump diffusion processes configure the implied distribution, we further estimated parameters from the implied moments or characteristic functions. Using these estimations, we considered the development of market expectations, particularly focusing on the magnitude and direction of price jumps that made the implied distribution depart from the normal distribution. We also analyzed how the contribution of two factors—the diffusion or Brownian motion factor and the jump factor—changed during the course of the recent financial turmoil.

Those analyses revealed that the possibility of discontinuous price jumps increased downwards during the financial turmoil, while the volatility that determined the dispersion of the continuous price process increases. Viewing the situation from the perspective of the implied distribution, we showed that the second and fourth moments increased while the third moment sharply declined. Taking these results as the deviation from a normal distribution, we detected the weakening negative skewness

[32] For instance, the jump share declines if we apply a stochastic volatility type of jump diffusion model because other factors such as the correlation between the price process and the volatility process also contribute to the volatility smile.

and the declining kurtosis of the implied distribution in the financial turmoil.

One of the remaining issues is the extension of the examined period. Analyzing and comparing our findings with other periods of financial distress, such as the asset-price bubble in Japan, the Long-Term Capital Management (LTCM) crisis, or the information technology stock bubble in the US, would be interesting. Applying other models, such as the stochastic volatility jump diffusion model, and comparing model fit would also be challenging. In addition, pursuing research on the difference or conversion from the risk-neutral distribution to the underlying price distribution is essential. Our analytical scheme is applicable to a wide range of asset classes other than equities. In time, we believe the method will be further enhanced so that it becomes one of the more versatile methods of financial analysis.

Appendix A Derivation of the Implied Moments

Appendix A explains the derivation of the zero-centered implied moments shown in Eq.(1) in Section II 1.(a).

Let $f(x)$ be a payoff function that is twice differentiable. It is readily shown that

$$f(y) = f(x) + f'(x)(y-x) + \int_x^\infty (y-v)^+ f''(v)dv + \int_0^x (v-y)^+ f''(v)dv,$$

where x^+ denotes the positive part of x. See Appendix 1(2) in Sugihara [2010] for the proof. When $x = S_0,\ y = S_t,\ v = K$, taking expectation under the risk-neutral probability yields,

$$\begin{aligned}
\mathbf{E}_0^Q[f(S_t)] &= f(S_0) + f'(S_0)(\mathbf{E}_0^Q S_t - S_0) \\
&\quad + e^{rt}\mathbf{E}_0^Q \int_0^{S_0} f''(K)e^{-rt}(K-S_t)^+ dK + e^{rt}\mathbf{E}_0^Q \int_{S_0}^\infty f''(K)e^{-rt}(S_t-K)^+ dK \\
&= f(S_0) + S_0 e^{rt} f'(S_0) - S_0 f'(S_0) + e^{rt}\int_0^\infty f''(K)\Theta(0,t,K)dK, \qquad \text{(A-1)}
\end{aligned}$$

since $\mathbf{E}_0^Q S_t = S_0 e^{rt}$. Let $f(S_t) = R_t^n = [\ln(S_t/S_0)]^n$ $(n = 1, 2, \cdots)$, then,

$$\frac{d^2}{dS_t^2}\left[\ln\left(\frac{S_t}{S_0}\right)\right]^n = \begin{cases} \dfrac{1}{S_t^2}\left\{ n(n-1)\left[\ln\left(\frac{S_t}{S_0}\right)\right]^{n-2} - n\left[\ln\left(\frac{S_t}{S_0}\right)\right]^{n-1}\right\}, & (n \geq 2) \\ -1/S_t^2. & (n = 1) \end{cases} \qquad \text{(A-2)}$$

Given $f(S_0) = 0$ and $f'(S_0) = 0$ when $n \leq 2$, applying Eq.(A-2) to Eq.(A-1) yields:

$$\mathbf{E}_0^Q[R_t^n] = e^{rt}\int_0^\infty \frac{\Theta(0,t,K)}{K^2}\left\{ n(n-1)\left[\ln\left(\frac{K}{S_0}\right)\right]^{n-2} - n\left[\ln\left(\frac{K}{S_0}\right)\right]^{n-1}\right\} dK, \quad (n \geq 2) \qquad \text{(A-3)}$$

Since $f(S_0) = 0$ and $f'(S_0) = 1/S_0$ when $n = 1$, applying Eq.(A-2) to Eq.(A-1) yields

$$\mathbf{E}_0^Q[R_t] = e^{rt} - 1 - e^{rt}\int_0^\infty \frac{\Theta(0,t,K)}{K^2}dK. \qquad \text{(A-4)}$$

■

Appendix B Derivation of the Implied Characteristic Function

Appendix B explains the derivation of the implied characteristic function shown in Eq. (4).

By Maclaurion expansion of $\Phi_{R_t}(\omega) = \mathbf{E}_0^Q e^{i\omega R_t}$ in terms of R_t, we obtain:

$$\begin{aligned}
\Phi_{R_t}(\omega) &= \mathbf{E}_0^Q e^{i\omega R_t} \\
&= \sum_{n=0}^{\infty} \frac{1}{n!} (i\omega)^n \mathbf{E}_0^Q R_t^n \\
&= 1 + i\omega(e^{rt} - 1) - i\omega e^{rt} \int_0^\infty \frac{\Theta(0,t,K)}{K^2} dK \\
&\quad + e^{rt} \sum_{n=2}^{\infty} \frac{(i\omega)^n}{n!} \int_0^\infty \frac{\Theta(0,t,K)}{K^2} \left\{ n(n-1) \left[\ln\left(\frac{K}{S_0}\right)\right]^{n-2} - n \left[\ln\left(\frac{K}{S_0}\right)\right]^{n-1} \right\} dK \\
&= 1 + i\omega(e^{rt} - 1) - \omega(\omega + i) e^{rt} \int_0^\infty \frac{\Theta(0,t,K)}{K^2} \left(\frac{K}{S_0}\right)^{i\omega} dK, \qquad \text{(A-5)}
\end{aligned}$$

using Eq.(A-3) and Eq.(A-4). ■

Eq.(A-5) is also derived from the implied density introduced by Breeden and Litzenberger [1978]. According to their analysis, the implied density of the equity price S_t at time 0, denoted by $f_{S_t}(x)$ $(x > 0)$ is expressed in terms of a call option price $C(0,t,K)$ with strike price K:

$$f_{S_t}(x) = e^{rt} \left. \frac{\partial^2 C(0,t,K)}{\partial K^2} \right|_{K=x}. \qquad \text{(A-6)}$$

This can be transformed into the density of return $f_{R_t}(\bar{x})$ as:

$$f_{R_t}(\bar{x}) = x e^{rt} \left. \frac{\partial^2 C(0,t,K)}{\partial K^2} \right|_{K=\bar{x}}. \qquad \text{(A-7)}$$

By Eq.(A-7), the characteristic function of returns $\Phi_{R_t}(\omega)$ is computed from the definition as:

$$\begin{aligned}
\Phi_{R_t}(\omega) &= \int_{-\infty}^{\infty} e^{i\omega\bar{x}} f_{R_t}(\bar{x}) d\bar{x} \\
&= e^{rt} \int_0^\infty e^{i\omega \ln(K/S_0)} \frac{\partial^2 C(0,t,K)}{\partial K^2} dK \\
&= i\omega(i\omega - 1) e^{rt} \int_0^\infty \frac{e^{i\omega \ln(K/S_0)}}{K^2} C(0,t,K) dK,
\end{aligned}$$

where we use the asymptotic value of the call option prices and their derivatives, such as $C_K(0,t,0) = -e^{-rt},\ C_K(0,t,\infty) = C(0,t,\infty) = 0$. Rearranging the above equation in terms of OTM option prices $\Theta(0,t,K)$ using put-call parity yields:

$$\begin{aligned}\Phi_{R_t}(\omega) = i\omega(i\omega-1)e^{rt}\int_0^{\infty}\frac{e^{i\omega\ln(K/S_0)}}{K^2}\Theta(0,t,K) \\ + i\omega(i\omega-1)e^{rt}\int_0^{S_0}\frac{e^{i\omega\ln(K/S_0)}}{K^2}(S_0-Ke^{-rt})dK \\ = 1 + i\omega(e^{rt}-1) - \omega(\omega+i)e^{rt}\int_0^{\infty}\frac{\Theta(0,t,K)}{K^2}\left(\frac{K}{S_0}\right)^{i\omega}dK.\end{aligned}$$

This is equivalent to Eq.(A-5).

Appendix C Derivation of Moments for Laplacian Jump Diffusion Process

Appendix C explains the derivation of implied moments of the Laplacian jump diffusion process shown in Eq.(12) in Section III 1.(b).

By taking derivatives of the characteristic function $\tilde{\Phi}^{\text{Laplace}}(\omega) = e^{i\xi\omega}/(1+\zeta^2\omega^2)$,

$$\begin{aligned}\frac{d}{d\omega}\tilde{\Phi}^{\text{Laplace}}(\omega) &= \frac{i(\xi+2i\zeta^2\omega+\xi\zeta^2\omega^2)e^{i\xi\omega}}{(1+\zeta^2\omega^2)^2}, \\ \frac{d^2}{d\omega^2}\tilde{\Phi}^{\text{Laplace}}(\omega) &= \frac{8\zeta^4\omega^2e^{i\xi\omega}}{(1+\zeta^2\omega^2)^3} - \frac{2\zeta^2(1+2i\xi\omega)e^{i\xi\omega}}{(1+\zeta^2\omega^2)^2} - \frac{\xi^2e^{i\xi\omega}}{1+\zeta^2\omega^2}, \\ \frac{d^3}{d\omega^3}\tilde{\Phi}^{\text{Laplace}}(\omega) &= -\frac{48\zeta^6\omega^3e^{i\xi\omega}}{(1+\zeta^2\omega^2)^4} + \frac{24\zeta^4\omega(1+i\xi\omega)e^{i\xi\omega}}{(1+\zeta^2\omega^2)^3} \\ &\quad -\frac{6i\xi\zeta^2(1+i\xi\omega)e^{i\xi\omega}}{(1+\zeta^2\omega^2)^2} - \frac{i\xi^3e^{i\xi\omega}}{1+\zeta^2\omega^2}, \\ \frac{d^4}{d\omega^4}\tilde{\Phi}^{\text{Laplace}}(\omega) &= \frac{384\zeta^8\omega^4e^{i\xi\omega}}{(1+\zeta^2\omega^2)^5} - \frac{96\zeta^6\omega^2(3+2i\xi\omega)e^{i\xi\omega}}{(1+\zeta^2\omega^2)^4} \\ &\quad +\frac{24\zeta^4(1+4i\xi\omega-2\xi^2\omega^2)e^{i\xi\omega}}{(1+\zeta^2\omega^2)^3} + \frac{4\xi^2\zeta^2(3+2i\xi\omega)e^{i\xi\omega}}{(1+\zeta^2\omega^2)^2} + \frac{\xi^4e^{i\xi\omega}}{1+\zeta^2\omega^2}.\end{aligned}$$

By setting $\omega = 0$ in the above equations,

$$\begin{cases} \dfrac{d}{d\omega}\tilde{\Phi}^{\text{Laplace}}(0) = i\xi, \\ \dfrac{d^2}{d\omega^2}\tilde{\Phi}^{\text{Laplace}}(0) = -(2\zeta^2 + \xi^2), \\ \dfrac{d^3}{d\omega^3}\tilde{\Phi}^{\text{Laplace}}(0) = -i(\xi^3 + 6\xi\zeta^2), \\ \dfrac{d^4}{d\omega^4}\tilde{\Phi}^{\text{Laplace}}(0) = \xi^4 + 12\xi^2\zeta^2 + 24\zeta^4. \end{cases} \tag{A-8}$$

From the characteristic function of LJD in Eq.(11), the n-th cumulants $\tilde{c}_n^{\text{LJD}}$ $(n = 1, \ldots, 4)$ are calculated by applying Eq.(A-8) as:

$$\begin{aligned} \tilde{c}_n^{\text{LJD}} &= \frac{1}{i^n}\frac{d^n}{d\omega^n}\ln\tilde{\Phi}_{R_t}^{\text{LJD}}(0) \\ &= \begin{cases} \mu t - i\lambda t\dfrac{d}{d\omega}\tilde{\Phi}_Y^{\text{Laplace}}(0) & (n=1) \\ \sigma^2 t - \lambda t\dfrac{d^2}{d\omega^2}\tilde{\Phi}_Y^{\text{Laplace}}(0) & (n=2) \\ \lambda t\dfrac{1}{i^n}\dfrac{d^n}{d\omega^n}\tilde{\Phi}_Y^{\text{Laplace}}(0) & (n \geq 3) \end{cases} \\ &= \begin{cases} \lambda t\xi + \mu t, & (n=1) \\ \lambda t(\xi^2 + 2\zeta^2) + \sigma^2 t, & (n=2) \\ \lambda t(\xi^3 + 6\xi\zeta^2), & (n=3) \\ \lambda t(\xi^4 + 12\xi^2\zeta^2 + 24\zeta^4). & (n=4) \end{cases} \end{aligned}$$

From the relationship between the cumulants and moments, the moments are computed as in Eq.(12). ■

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