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### **Central Bank Communication and Multiple Equilibria**

Kozo Ueda\*

#### Abstract

We construct a simple model in which a central bank communicates with money market traders. We demonstrate that there exist multiple equilibria. In one equilibrium, traders truthfully reveal their own information, and by learning this, the central bank can make better forecasts. Another equilibrium is a "dog-chasing-its-tail" equilibrium in Blinder (1998). Traders mimic the central bank's forecast, so the central bank simply observes its own forecast from traders. The latter equilibrium is socially worse in that inflation variability becomes larger. We also demonstrate that too high transparency of central banks is bad because it yields the "dog-chasing-its-tail" equilibrium, and that central banks should conduct continuous monitoring or emphasize that their forecasts are conditional because doing so eliminates the "dog-chasing-its-tail" equilibrium.

**Keywords:** Transparency; disclosure; coordination **JEL Classification:** C72, D83, E52

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#### 1. Introduction

Monetary policy transparency plays an essential role in conducting monetary policy. It serves not only for legitimacy, but also enhances the effectiveness of monetary policy. High transparency helps anchor inflation expectations (e.g. Faust and Svensson [2001]), and influences the path of future interest rates (e.g. Woodford [2005]). For these reasons, central banks around the globe have greatly increased monetary policy transparency in recent years.

However, too high transparency is costly, and no central bank in the real world discloses all the information it has. This may partly reflect the fear of losing credibility (e.g. Mishkin [2007]) or of political pressure in a federal state like Canada (Chant [2003]). From a different perspective, and in a theoretical manner, Morris and Shin (2002) illustrate that a coordination problem makes too high transparency harmful. They argue that public information disclosed by central banks is focal, which plays an excessive coordination role among private agents.<sup>1</sup> They, however, consider central bank transparency only in one direction – from the central bank to private agents – and neglect the feedback in the other direction. In short, their analyses simply focus on central bank transparency, not communication.

Blinder (1998) examines this feedback process and calls it the "dog-chasing-its-tail" problem. He argues that if a central bank tries to "please the markets" too much, the markets stop functioning and the central bank only observes its own image in the markets. Although by "please the markets", Blinder (1998) does not necessarily mean high transparency, Morris and Shin (2005) formulate Blinder's idea in the context of central bank communication.<sup>2</sup> With a simple but theoretical model, they demonstrate the risk of informational inefficiency such that transparency prevents the central bank from obtaining potentially valuable market information and reduces the precision of its forecasts.

As the communication counterpart of a social planner, all the above literature looks at private firms which engage in capital investment (Angeletos and Pavan [2004]), price-setting (Hellwig [2005] and Demertzis and Viegi [2008]), or zero-sum trades (Morris and Shin [2002, 2005]), and regards these firms as consumers ultimately, so the sum of their utility equals a social welfare function which social planners aim to maximize. We admit that it is very important to study central bank communication with consumers, particularly in terms of anchoring inflation expectations (Demertzis and Viegi [2008]). However, regarding central bank

<sup>&</sup>lt;sup>1</sup> Morris and Shin (2002)'s paper has attracted much attention and has been challenged by, for example, Angeletos and Pavan (2004), Hellwig (2005), Svensson (2006) and Demertzis and Viegi (2008). Svensson (2006) argues that the coordination induced by high transparency is good provided that the public signal is more precise than the private signal. In reply to Svensson (2006), Morris, Shin and Tong (2006) argue that, if the public signal is correlated with the private signal, then quantitative evaluation supports their original papers.

<sup>&</sup>lt;sup>2</sup> Blinder's idea is partly formulated by Woodford (1994) and Bernanke and Woodford (1997) who point out the risk of indeterminacy arising from using not a structural model but private forecasts.

communication, traders in the financial market should not be neglected. Traders are keen observers of central bank disclosure, and also provide information on financial market conditions. Further, these traders account for only a fraction of the total population, so the sum of the traders' utility does not represent social welfare. The "dog-chasing-its-tail" problem raised by Blinder (1998) seems to be related to traders in the financial market, and so it is very important to consider more explicitly what can happen in central bank communication with traders.

Our paper constructs a simple model in which a central bank and traders in the financial market communicate. Communications are made in both directions, from the central bank to the traders and vice versa. The traders take positions in relation to short-term interest rate expectations, to finance their portfolios, to hedge their positions and to square liquidity imbalances. This paper therefore assumes that the money market traders aim to forecast interest rates set by a central bank as accurately as possible. In our model, central bank information is, by construction, focal, and we demonstrate that coordination among traders arises. This is opposite to Morris and Shin (2002, 2005) where a coordination motive is introduced as an assumption, and public information is proved to be focal.

The second and somewhat minor contribution of our paper is that we explicitly introduce a continuous transparency parameter when a central bank discloses its information. In the above papers except for Angeletos and Pavan (2004), the choice of transparency is either full transparency or perfect secrecy. However, this assumption is unrealistic and may lead to an extreme conclusion that supports perfect secrecy. Angeletos and Pavan (2004) argue that there is a transparency parameter, but it is merely a central bank's forecast accuracy. There have been relatively few papers on the optimal extent of transparency, for instance, Morris and Shin (2007), Walsh (2007), and Cornand and Heinemann (2008).<sup>3</sup> In this paper, we explicitly model central bank transparency, which represents the clearness of central bank disclosure and whose degree is continuous in a much simpler and broader context. We then examine how the central bank's and traders' utility change depending on the transparency parameter and if there is an optimal degree of transparency.

By constructing a simple model and solving it analytically, we find that there exist multiple equilibria regarding the action of the central bank and the traders. In one equilibrium, traders truthfully reveal their own information, and by learning this, the central bank can make a better forecast. Another equilibrium is the "dog-chasing-its-tail" equilibrium in Blinder (1998). Traders mimic the central bank's forecast, so the central bank simply observes its own forecast from traders. This equilibrium is always worse for the central bank and ultimately for

<sup>&</sup>lt;sup>3</sup> Morris and Shin (2007) investigate the optimal number of signals to be disclosed, and the latter two papers investigate how many private agents should be informed by a central bank.

consumers. The existence of multiple equilibria differs from Morris and Shin (2002, 2005), where the equilibrium is unique. Morris and Shin (2002, 2005) and following literature such as Svensson (2006) disagree as to whether transparency yields better or worse coordination, but because of multiple equilibria, our paper integrates both the bad coordination outcome as in Morris and Shin (2002, 2005) and Blinder (1998) and the good coordination outcome as in Svensson (2006).

Our analyses yield policy implications. Firstly, too high transparency of central banks is bad because it increases the incentive for traders to mimic central banks and yields the "dog-chasing-its-tail" equilibrium. Secondly, central banks should conduct continuous monitoring or emphasize that their forecasts are conditional because this eliminates the "dog-chasing-its-tail" equilibrium by making a central bank's disclosed information less valuable for the guidance of future monetary policy.

The structure of this paper is as follows. Section 2 introduces our model. Section 3 solves the model, and shows that there arise multiple equilibria. Section 4 concludes the paper.

#### 2. Model

We assume that inflation  $\pi$  is given by

$$\pi = i + u,\tag{1}$$

where i is an interest rate set by a central bank (hereafter CB) and u is a shock. Neither CB nor money market traders (hereafter T) know the amount of the shock u in advance. By setting an appropriate interest rate, CB aims to minimize inflation variability

$$L^{CB} = \pi^2, \tag{2}$$

which results in ultimately minimizing the loss of consumers.

As to CB's counterpart, we consider a sufficiently large number of money market traders. The traders take positions in relation to short-term interest rate expectations, to finance their portfolios, to hedge their positions and to square liquidity imbalances, so they aim to forecast i set by CB as accurately as possible. In other words, trader j's loss function is written as

$$L_{i}^{T} = (i_{i}^{T} - i)^{2}, (3)$$

where  $i_j^{T}$  is trader *j*'s forecast about *i*. Two remarks are worth making. First, CB's information is, by construction, focal. Instead, a coordination motive is not explicit in the above setup. However, as will become clear, traders have a coordination motive because CB's decision of *i* depends on the average of traders' forecasts. Second, we explicitly model money market traders who are different from consumers, so the sum of the traders' utility does not represent social

welfare.

In the communication between CB and T, we consider the following four steps. At step 1, CB receives some information about the shock u, that is,  $u_1^{CB}$ , and discloses a not perfectly transparent forecast about the interest rate i, that is,  $i^{CB}$ . The actual interest rate i is set at step 3, by which CB gathers new information about the shock by itself and through communication with T, so at step 1, CB does not perfectly commit to the disclosed interest rate at step 1. In other words, CB's disclosed forecast is conditional. At step 2, we assume that trader *i* can gather its own information about the shock  $u_i^{T}$  by paying fixed costs,  $cE(u^2)$ . Public information  $i^{CB}$ disclosed by CB is free. This assumption is motivated by Grossman and Stiglitz (1976, 1980), who argue that when there is the cost of information, a price system cannot perfectly convey information to the public. In our model, as we will soon see, this assumption yields an incentive for T to mimic CB's disclosed information.<sup>4</sup> Using T's information set, T tries to forecast the interest rate set by CB at step 3. At step 3, CB observes the average of T's forecasts  $i^{T}$ , and also receives new information about the shock  $u_2^{CB}$ . Using CB's information set, CB refines its forecast about the shock, and sets the optimal interest rate *i* so as to stabilize inflation. Finally at step 4, the inflation rate  $\pi$  is realized, by which the true amount of the shock u is known to everyone.

Forecast precisions are formulated in a standard way following Morris and Shin (2002, 2005). CB and T cannot observe an incoming u, but observes  $u_1^{\text{CB}}$ ,  $u_j^{\text{T}}$ , and  $u_2^{\text{CB}}$  given by

$$u_{1}^{CB} = u + e_{1}^{CB}, \qquad E(e_{1}^{CB^{2}}) / E(u^{2}) = 1/\alpha_{1}^{CB}, u_{j}^{T} = u + e_{j}^{T} + e^{T}, \qquad E(e_{j}^{T^{2}}) / E(u^{2}) = 1/\alpha_{i}^{T} \text{ and } E(e_{j}^{T^{2}}) / E(u^{2}) = 1/\alpha^{T}, \quad (4)$$
$$u_{2}^{CB} = u + e_{2}^{CB}, \qquad E(e_{2}^{CB^{2}}) / E(u^{2}) = 1/\alpha_{2}^{CB},$$

where  $\alpha_1^{CB}$  and  $\alpha_2^{CB}$  represent CB's forecast precisions at step 1 and at step 3, respectively. At step 2, trader *j* encounters idiosyncratic uncertainty  $e_j^T$  and aggregate uncertainty  $e^T$ , and the inverse of its amplitude is given by  $\alpha_{i}^T$  and  $\alpha^T$ . It is convenient to transform these parameters into:

$$x_{1}^{CB} = \frac{\alpha_{1}^{CB}}{1 + \alpha_{1}^{CB}}, x_{2}^{CB} = \frac{\alpha_{2}^{CB}}{1 + \alpha_{1}^{CB} + \alpha_{2}^{CB}}, x_{i}^{T} = \frac{\alpha_{i}^{T}}{1 + \alpha_{i}^{T}}, x^{T} = \frac{\alpha^{T}}{1 + \alpha^{T}}.$$
(5)

Moreover, different from early literature discussed in the introduction, we explicitly introduce a continuous transparency parameter  $\tau$ . CB's disclosure is described as:

$$i^{CB} = -E(u \mid u_1^{CB}) + v, \qquad E(v^2) = E[(u - E(u \mid u_1^{CB}))^2]/\tau.$$
(6)

In our model, it is optimal for CB to set an interest rate so as to cancel a coming shock, so if CB is perfectly transparent,  $\nu$  becomes zero. Therefore,  $\nu$  represents the obscurity of CB's

<sup>&</sup>lt;sup>4</sup> Appendix considers whether multiple equilibria arise without introducing fixed costs.

disclosure. As  $\tau$  becomes larger,  $\nu$  becomes smaller, which suggests higher transparency.

#### 3. Equilibrium

This section solves the above model, and shows that multiple equilibria can exist. The solution of this model is derived by answering the following two questions. The first question is how T discloses its own forecast about the interest rate at step 2. We ask if T tries to make the best guess about the shock u and truthfully reveals its own forecast, or if T mimics CB's forecast made at step 1. The second question is how CB evaluates the shock u, and sets the optimal interest rate i. We ask how CB refines its forecast by combining its own information and T's forecast, and if CB can learn anything valuable from T's forecasts.

Since the model is linear quadratic, the actions of CB and T are written in the following linear form:

Step 2 
$$i_j^T = E(i | I_j^T) = a_1^j i^{CB} + a_2^j u_j^T,$$
  
Step 3  $E(u | I^{CB}) = -i = b_1 i^T + b_2 u_2^{CB} + b_3 u_1^{CB} + b_4 v.$ 
(7)

Here we use the fact that trader *j*'s information set at step 2 is  $I_j^{T} = \{i^{CB}, u_j^{T}\}$  and that CB's information set at step 3 is  $I^{CB} = \{i^{T}, u_2^{CB}, u_1^{CB}, v\}$ . The coefficients *a* and *b* are obtained from

$$(a_{1}^{j}, a_{2}^{j}) = \arg\min E[i - E(i \mid I_{j}^{T})]^{2} \quad \text{given} (a_{1}^{k}, a_{2}^{k}, b_{1}, b_{2}, b_{3}, b_{4})$$
  

$$(b_{1}, b_{2}, b_{3}, b_{4}) = \arg\min E[u - E(u \mid I^{CB})]^{2} \quad \text{given} (a_{1}, a_{2}),$$
(8)

where  $k \neq j$ .

The above problem can be solved analytically, which leads to our first proposition. Hereafter, we limit our attention to the equilibrium where all traders have the same strategy.

PROPOSITION 1: There exist multiple equilibria "M" and "R" when fixed costs c satisfies:

$$c > F(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau).$$
(9)

Otherwise, only equilibrium "R" exists. In equilibrium "M" (or "dog-chasing-its-tail"), T mimics CB's forecast. In equilibrium "R", T truthfully reveals its own forecast.

Proof: We can show that there can exist two equilibria, where the coefficients a and b are respectively written as

$$a_{1} = \left\{ 1 - x_{1}^{CB} + \frac{1 - x_{1}^{CB}}{\tau x_{1}^{CB}} + \left(\frac{1}{\tau} \frac{1 - x_{1}^{CB}}{x_{1}^{CB}} + 1\right) \left(\frac{1 - x_{i}^{T}}{x_{i}^{T}} + \frac{1 - x^{T}}{x^{T}}\right) \right\}^{-1} \left(\frac{1 - x_{i}^{T}}{x_{i}^{T}} + \frac{1 - x^{T}}{x^{T}}\right),$$

$$a_{2} = -1 + a_{1} \left(\frac{1}{\tau} \frac{1 - x_{1}^{CB}}{x_{1}^{CB}} + 1\right),$$

$$b_{1} = \frac{1}{a_{2}} \frac{(1 - x_{1}^{CB})(1 - x_{2}^{CB})}{(1 - x_{1}^{CB})(1 - x_{2}^{CB}) + \frac{1 - x^{T}}{x^{T}}}, \quad b_{2} = x_{2}^{CB}(1 - a_{2}b_{1}),$$

$$b_{3} = x_{1}^{CB}(1 - a_{2}b_{1} - x_{2}^{CB} + x_{2}^{CB}a_{2}b_{1} + a_{1}b_{1}), \quad b_{4} = -a_{1}b_{1},$$
(10)

or

$$a_{1} = \left(\frac{1 - x_{1}^{CB}}{\pi x_{1}^{CB}} + 1\right)^{-1}, \quad a_{2} = 0,$$

$$b_{1} = \text{arbitrary}, \quad b_{2} = x_{2}^{CB}, \quad b_{3} = x_{1}^{CB} (1 - x_{2}^{CB} + a_{1}b_{1}), \quad b_{4} = -a_{1}b_{1}.$$
(11)

In equation (11),  $a_2$  is zero which suggests that T does not reveal its own information and mimics CB's forecast. CB therefore sees its own image through communication with T. This is the "M" or "dog-chasing-its-tail" equilibrium. The parameter  $b_1$  is arbitrary. This is because, in "M", CB's reaction function in equation (7) becomes redundant. Using equation (11), we can easily show that equation (7) is transformed into:

$$E(u | I^{CB}) = b_1 i^T + b_2 u_2^{CB} + b_3 u_1^{CB} + b_4 v$$
  
=  $x_2^{CB} u_2^{CB} + x_1^{CB} (1 - x_2^{CB}) u_1^{CB}.$  (12)

In other words, in "M", CB has to rely on its own information. On the other hand, in equation (10),  $a_2$  is non-zero, which suggests that T truthfully reveals its own forecast.

We next examine the condition that no trader has an incentive to deviate from "M". If we neglect fixed costs, and when all the other traders choose "M", a trader *j* who deviates from "M" can minimize its expected loss by choosing:

$$a_{2}^{j} = \left(\frac{1}{x_{i}^{T}} + \frac{1 - x_{1}^{CB}}{\pi x_{1}^{CB} x_{i}^{T}} - x_{1}^{CB} + \frac{1 - x^{T}}{x^{T}} \left(1 + \frac{1 - x_{1}^{CB}}{\pi x_{1}^{CB}}\right)\right)^{-1} \\ \cdot \left(x_{2}^{CB} - x_{1}^{CB} x_{2}^{CB} + \frac{1 - x_{1}^{CB}}{\pi x_{1}^{CB}} (x_{1}^{CB} + x_{2}^{CB} - x_{1}^{CB} x_{2}^{CB})\right),$$
(13)  
$$a_{1}^{j} = \frac{1}{x_{1}^{CB}} \left(a_{2}^{j} \left(\frac{1}{x_{i}^{T}} + \frac{1 - x^{T}}{x^{T}}\right) + x_{1}^{CB} + x_{2}^{CB} - x_{1}^{CB} x_{2}^{CB}\right).$$

Comparing the expected loss including fixed costs in this case with the expected loss in "M"

suggests that the equilibrium "M" exists when:

$$c > F(x_{1}^{CB}, x_{2}^{CB}, x^{T}, x_{i}^{T}, \tau)$$

$$\equiv -x_{1}^{CB} \left\{ \left( 1 + \frac{1 - x_{1}^{CB}}{\alpha x_{1}^{CB}} \right)^{-1} - 1 + x_{2}^{CB} \right\} - 2x_{1}^{CB} x_{2}^{CB} \left\{ \left( 1 + \frac{1 - x_{1}^{CB}}{\alpha x_{1}^{CB}} \right)^{-1} - 1 + x_{2}^{CB} \right\} - x_{2}^{CB^{2}}$$

$$- \frac{1 - x_{1}^{CB}}{\tau} \left( 1 + \frac{1 - x_{1}^{CB}}{\alpha x_{1}^{CB}} \right)^{-2}$$

$$+ \frac{1 - x_{1}^{CB}}{x_{1}^{CB}} x_{2}^{CB^{2}} + \frac{1 - x_{1}^{CB}}{\tau} \left( \frac{x_{1}^{CB} + x_{2}^{CB} - x_{1}^{CB} x_{2}^{CB}}{x_{1}^{CB}} \right)^{2}$$

$$- \frac{(1 - x_{1}^{CB})^{2}}{x_{1}^{CB}} \left( \frac{1}{x_{i}^{T}} + \frac{1 - x^{T}}{x^{T}} \right) \left( x_{2}^{CB} + \frac{x_{1}^{CB} + x_{2}^{CB} - x_{1}^{CB} x_{2}^{CB}}{\alpha x_{1}^{CB}} \right)^{2}$$

$$\cdot \left\{ \frac{1}{x_{i}^{T}} - x_{1}^{CB} + \frac{1 - x_{1}^{CB}}{x_{1}^{CB}} \frac{1}{x_{i}^{T}} + \frac{1 - x^{T}}{x^{T}} \left( 1 + \frac{1 - x_{1}^{CB}}{x_{1}^{CB}} \right) \right\}^{-1}.$$
(14)

Q.E.D.

This equilibrium "M" in our model is much closer to the "dog-chasing-its-tail" problem raised by Blinder (1998) than that of early literature. Woodford (1994) and Bernanke and Woodford (1997) point out the risk of indeterminacy, but their papers do not necessarily explain Blinder's argument that, if a central bank tries to "please the markets" too much, the markets stop functioning and the central bank only observes its own image from the markets. In Morris and Shin (2005), transparency makes private agents' disclosed information less valuable to a central bank, but it is still valuable. On the other hand, our paper demonstrates that, in the "dog-chasing-its-tail" equilibrium, CB simply observes its own forecast, so private agents' disclosed information has no value.

The existence of multiple equilibria makes a clear contrast with the case of Morris and Shin (2002, 2005) where the equilibrium is unique. Morris and Shin (2002, 2005) and following literature such as Svensson (2006) disagree as to whether transparency yields better or worse coordination, but because of multiple equilibria, our paper integrates both the bad coordination outcome as in Morris and Shin (2002, 2005) and Blinder (1998) and the good coordination outcome as in Svensson (2006).<sup>5</sup>

As we noted in the previous section, CB's information is, by construction, focal, and a

<sup>&</sup>lt;sup>5</sup> In Angeletos and Pavan (2004) and Demertzis and Viegi (2008), the possibility of multiple equilibria is demonstrated, but their characteristics are very different from ours. For instance, the "dog-chasing-its-tail" equilibrium in Blinder (1998) does not appear.

coordination motive is not explicit but implicit. This is understood by rearranging equation (3) using equation (7):

$$L_{j}^{T} = (i_{j}^{T} - i)^{2}$$

$$= \left\{ i_{j}^{T} - (-b_{1}) \lim_{N \to \infty} \frac{1}{N} \sum_{k} i_{k}^{T} \right\}^{2} + \dots \quad .$$
(15)

From this, it is clear that trader j who aims to forecast CB's action attempts to evaluate other traders' forecasts. This is a beauty contest, which induces a coordination motive among agents. The remaining terms in the above depend on the fundamental u.<sup>6</sup> Therefore, T's loss function is similar to that in Morris and Shin (2002, 2005). However, a difference is that, in our model, central bank information is, by construction, focal, and we demonstrate that coordination among traders arises. In Morris and Shin (2002, 2005), a coordination motive is introduced as an assumption, and public information is proved to be focal.

Which equilibrium is better for CB? To answer this question, we make the second proposition.

#### PROPOSITION 2: For CB, "R" is better than "M".

Proof: CB's loss,  $L^{CB} = \pi^2$ , is given by:

(M) 
$$L^{CB(M)} = (1 - x_1^{CB})(1 - x_2^{CB}),$$
  
(R)  $L^{CB(R)} = \left(\frac{1}{(1 - x_1^{CB})(1 - x_2^{CB})} + \frac{x^T}{1 - x^T}\right)^{-1} < L^{CB(M)}$  Q.E.D. (16)

As simple new-Keynesian economics states, if we consider that minimizing inflation variability leads to maximizing social welfare, then this proposition suggests that "R" is socially more desirable than "M". This proposition is intuitive. In "R", CB can learn T's information about the shock u, so CB can improve its forecast accuracy for u, and set a better interest rate to stabilize inflation than in "M" where CB has to rely on only its own information. T's information, even if its precision is low, is always useful for CB if CB extracts the signal appropriately. It is also worth noting that the transparency parameter,  $\tau$ , does not make any difference to CB's loss.

<sup>&</sup>lt;sup>6</sup> We can show that even though T's loss depends on a fundamental described by inflation variability as well as T's forecast error, the same multiple equilibria can arise. This implies that this result is robust even though traders are not of the money market but of long-term bond or equity markets However, the following discussion on traders' welfare will be modified.

We next examine the condition of multiple equilibria in more detail. It is obvious that the larger the fixed costs, the more likely multiple equilibria are to arise. Furthermore, we can find clear properties on other parameters such as transparency. Proposition 2 states that equilibrium "M" is undesirable, and the following lemma provides a way to eliminate "M".

#### LEMMA 3:

$$dF(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau) / dx_2^{CB} \ge 0, dF(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau) / dx^T > 0,$$
  
$$dF(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau) / dx_i^T > 0, dF(x_1^{CB}, x_2^{CB}, x^T, x_i^T, \tau) / d\tau \le 0.$$

This lemma includes many valuable policy implications. Firstly, the fact that  $dF/d\tau$  is non-positive suggests that, as CB's transparency becomes higher, "M" becomes more likely to arise. The reason is simple. As transparency increases, T can infer more accurately CB's valuation, which increases the incentive to mimic CB and not to give CB any additional information. This finding has the clear policy implication that too high transparency is bad because it yields socially bad equilibrium "M".

Secondly, the fact that  $dF/dx_2^{CB}$  is non-negative suggests that, as CB's forecast after disclosure becomes more accurate, "M" becomes less likely to arise. If  $x_2^{CB}$  is high, CB weighs its own information after disclosure more than that before disclosure, so CB's disclosed information does not provide T with a good sign for future interest rate decisions. Therefore, mimicking CB becomes less valuable to T than the case of truthfully revealing. This result implies that CB should conduct continuous monitoring or emphasize that CB's forecasts are conditional. This is because these actions increase CB's forecast precision after the disclosure and make CB less committed to future policy, which can reduce the incentive for T to choose "M".

Thirdly, we find that both  $dF/dx^{T}$  and  $dF/dx_{i}^{T}$  are positive. This suggests that, as T's forecast precision is lower, "M" becomes more likely to arise. This is because, if T's forecast precision is low, then T's payoff by truthful revelation becomes small, which lowers the incentive for T to deviate from "M". Since policymakers cannot adjust the parameters  $x^{T}$  and  $x_{i}^{T}$ , this finding does not necessarily give them a direct policy implication. However, it gives them warning of "M" when T's ability is low. In this respect, it is important for CB to evaluate T's ability.

To check these results visually, we implement numerical evaluations by changing some parameters. As a baseline, we assume the values of  $x_1^{CB} = 0.5$ ,  $x_2^{CB} = 0.5$ ,  $x_1^{T} = 0.2$ , and  $\tau = 10$ . The forecast precision of each trader is worse than that of CB. The aggregate forecast accuracy of traders,  $x^{T}$ , is chosen to be 0.8 so that traders' aggregate information has the same accuracy as

CB's information. Fixed costs, *c*, are chosen to be 0.05 so that our numerical calculation clearly demonstrates this model's characteristics. By changing one parameter while keeping the other parameters fixed, we examine how T's loss changes depending on forecast precisions and transparency.



Figure 1: Trader's Loss

Figure 1 shows the results. The solid line shows T's loss in "R", while the dotted line with circles shows T's loss in "M". We find that the dotted line disappears in some regions. This suggests that, as Proposition 1 and Lemma 3 state, the bad equilibrium "M" is eliminated when  $x_2^{CB}$  is high,  $x_i^{T}$  (so as  $x^{T}$ ) is high, or  $\tau$  is low.

The bottom-right panel suggests that high transparency is always good for T in two senses. Firstly, high transparency is good given the same equilibrium "M" or "R". Secondly, high transparency is good because it yields socially bad but privately good equilibrium "M" for T. This implies that there may exist an optimal degree of transparency. On the one hand, too high transparency is bad because, as transparency becomes higher, the socially bad equilibrium "M" is more likely to arise. But, on the other hand, too low transparency appears bad because, when there is only "R", as transparency becomes lower, T's loss increases while the central bank's loss stays constant.

#### 4. Concluding Remarks

This paper constructed a very simple model in which a central bank communicates with money market traders in the financial market. We assume that a central bank aims to stabilize inflation, while traders aim to forecast the interest rate set by the central bank, and that traders have to pay fixed costs when gathering information. With these assumptions, we demonstrate that there may exist multiple equilibria. In one equilibrium "R", traders truthfully reveal their own information, and by learning this, the central bank can make a better forecast. Another equilibrium "M" is a "dog-chasing-its-tail" equilibrium in Blinder (1998): traders mimic the central bank's forecast, so the central bank simply observes its own forecast from traders. The latter equilibrium is worse for the central bank and ultimately for consumers in that inflation variability increases. We also demonstrate that too high transparency of central banks is bad because it yields "M", and that central banks should conduct continuous monitoring or emphasize that their forecasts are conditional because it eliminates "M".

There are remaining tasks, however. In particular, it is important to extend this model into a dynamic model with consideration of the inflation target and inflation expectations. Our paper neglects this aspect although many people emphasize its importance (e.g. Faust and Svensson [2001], Woodford [2005], Aoki and Kimura [2008], and Demertzis and Viegi [2008]). This may well reestablish the importance of greater transparency.

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#### Appendix: Model without Fixed Costs

In this paper, we assume that T has to pay fixed costs when gathering information. This is clearly one of the key assumptions to yield multiple equilibria, in particular, "M". This appendix considers another way of obtaining similar multiple equilibria without introducing fixed costs.

We make four modifications in our model. Firstly, we do not assume fixed costs in gathering information. Secondly, we consider a limited number of money market traders given by N. Thirdly and most importantly, we assume that CB cannot observe the deviation of each strategy of T  $(a_1^{j}, a_2^{j})$  from a certain aggregate equilibrium strategy  $(a_1, a_2)$ . At step 3, CB observes  $i^T$  but CB considers that its change comes from not the deviation of each strategy of T  $(a_1^{j}, a_2^{j})$  but from a change in the shock traders receive  $u_j^T$ . Therefore, each trader's strategy does not influence CB's strategy, and the trader optimizes its strategy with CB's strategy fixed. Fourthly, for simplicity we neglect aggregate uncertainty  $e^T$ . In other respects, our model setup is the same as before.

With this setup, we can obtain the following two propositions.

**PROPOSITION A1:** There always exist multiple equilibria. In equilibrium "M" (or "dog-chasing-its-tail"), T mimics CB's forecast. In equilibrium "R", T truthfully reveals its own forecast.

Proof: We can show that there are two equilibria, where the coefficients a and b are written as

$$a_{1} = -\left\{x_{1}^{CB} - \frac{1}{x^{T}}\left(\frac{1}{\tau}\frac{1-x_{1}^{CB}}{x_{1}^{CB}} + 1\right)\right\}^{-1}\left(\frac{1-x_{i}^{T}}{x_{i}^{T}}\right), \quad a_{2} = -1 + a_{1}\left(\frac{1}{\tau}\frac{1-x_{1}^{CB}}{x_{1}^{CB}} + 1\right),$$

$$b_{1} = \frac{1}{a_{2}}\frac{(1-x_{1}^{CB})(1-x_{2}^{CB})}{(1-x_{1}^{CB})(1-x_{2}^{CB}) + \frac{1-x_{i}^{T}}{Nx_{i}^{T}}}, \quad b_{2} = x_{2}^{CB}(1-a_{2}b_{1}),$$

$$b_{3} = x_{1}^{CB}(1-a_{2}b_{1} - x_{2}^{CB} + x_{2}^{CB}a_{2}b_{1} + a_{1}b_{1}), \quad b_{4} = -a_{1}b_{1},$$
(A1)

or

$$a_1 = -b_4 / b_1, \quad a_2 = 0,$$
  
 $b_1 = -N, \quad b_2 = x_2^{CB}, \quad b_3 = x_1^{CB} (1 - x_2^{CB} + a_1 b_1), \quad b_4 = \text{arbitrary}.$ 
(A2)

In equation (A2),  $a_2 = 0$ , which suggests that T mimics CB's forecast. A parameter  $b_4$  is arbitrary, but we can easily show that equation (7) is transformed into:

$$E(u | I^{CB}) = b_1 i^T + b_2 u_2^{CB} + b_3 u_1^{CB} + b_4 v$$
  
=  $x_2^{CB} u_2^{CB} + x_1^{CB} (1 - x_2^{CB}) u_1^{CB}$ . (A3)

On the other hand, in equation (A1),  $a_2$  is non-zero, which suggests that T truthfully reveals its own forecast. Q.E.D.

PROPOSITION A2: For CB, "R" is better than "M".

Proof: CB's loss,  $L^{CB} = \pi^2$ , is given by:

(M) 
$$L^{CB(M)} = (1 - x_1^{CB})(1 - x_2^{CB}),$$
  
(R)  $L^{CB(R)} = \left(\frac{1}{(1 - x_1^{CB})(1 - x_2^{CB})} + \frac{Nx_i^T}{1 - x_i^T}\right)^{-1} < L^{CB(M)}$  Q.E.D. (A4)

These multiple equilibria are highly similar to those in our main model. A notable difference is that, in this modified model, multiple equilibria always exist irrespective of the forecast precision parameters  $\alpha_1^{CB}$ ,  $\alpha^T$ , and  $\alpha_2^{CB}$  and the transparency parameter  $\tau$ .

As this model demonstrates, fixed costs are not necessarily needed to yield multiple equilibria, in particular, "M". For T, deviating from "M" is not worthwhile because CB can respond to the deviation of a single trader's strategy by choosing  $b_1 = -N$ , which lowers T's profit by deviating. However, as we noted at the beginning of this appendix, this result crucially depends on the assumption that CB observes  $i^T$  but CB considers that its change comes from not the deviation of each T's strategy  $(a_1^{j}, a_2^{j})$  but from a change in the shock traders receive  $u_j^T$ .

Finally, in order to evaluate T's loss, we assume the values of  $x_1^{CB} = 0.5$ ,  $x_2^{CB} = 0.5$ ,  $x_i^T = 0.2$ ,  $\tau = 10$ , and N = 16. The number of traders, N, is set so that T's pooled forecast precision is the same as CB's forecast precision. Figure A demonstrates the results. Although multiple equilibria always exist, these panels closely resemble Figure 1. Comparing two losses in "M" and "R" implies that high transparency increases the attractiveness of "M" over "R" for T. On the other hand, CB's high forecast precision after disclosure decreases the attractiveness of "M" for T.



Figure A: Trader's Loss