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Accounting for Oil Price Variation and Weakening Impact of the Oil Crisis

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Abstract
Recent empirical studies reveal that the oil price-output relationship is weakening in the US. Oil price-output correlation is less negative, and output reduction in response to oil price rise is more moderate after mid 1980s. In contrast to the conventional view that there have been changes in the economic structures that have made output less responsive to oil price shocks, we show that what have changed are the sources of oil price variation. We develop a DSGE model where oil price and US output are endogenously determined by the exogenous movements of US TFP and the oil supply. Having no changes in economic structure, our model yields dynamics of the oil price and output that show a weakening in the oil price-output relationship. There are changes in the way that the exogenous variables evolve. Two changes are important. First, oil supply variation has become moderate in recent years. Second, oil supply shortage is no longer followed by a large decline in TFP. We show that less volatile oil supply variation results in less negative oil price-output correlations, and a smaller TFP decline during oil supply shortfall implies a smaller output decline during oil price increases.

Keywords: Oil Price Accounting; DSGE Model; Total Factor Productivity (TFP)
JEL classification: E32, E37, Q41

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1 Introduction

Many studies have reported that recessions in the US economy are related to political events in the Middle East, and subsequent rises in the oil price. For example, Hamilton (1983, 1996), Hooker (1996) show that most US recessions were preceded by increases in the oil price. It is not surprising, therefore, that the macroeconomic impact of the oil shock has been analyzed by many economists from several aspects. Kim and Loungani (1992) and Finn (2000) examine the role of exogenous oil price variation as a source of US business cycle fluctuations. Wei (2003) analyzes its implications for the US stock market. Bernanke et al. (1997), Lee et al. (2001) and Leduc and Sill (2004) discuss the oil price shocks in terms of monetary policy.

However, the claim that oil price shocks contribute to US recessions has become controversial, as several authors report that the relation between oil price and economic activity becomes weaker since the mid 1980s. Mork (1989) estimates a regression of US GDP on the real oil price and finds that the coefficients of the oil price become less significant (less negative), when the mid 1980s are used for the estimation. In addition, Hooker (1996a, b) and Hamilton (1996, 2003) also indicate that oil price fluctuations have had less impact on the US economy in the 1980s and 1990s than before.

This weakening of the impact of oil crises is summarized in Figure 1. Following McConnell and Perez-Quiros, (2000) and Blanchard and Galí (2007), we split the postwar periods (1973:Q1 to 2007:Q4) into two subsample periods, and choose 1984:Q1 as the break point. The upper panel shows the cross-correlation between the real oil price and US output for each of the two subsample periods. The line for the pre-1984 period lies below that for the post-1984 period, indicating that output tends to move in the opposite direction to oil price movements in the early in subsample period compared with recent subsample period. The lower panel displays the impulse responses of output to a unit rise in the real oil price for the two subsample periods. The responses are estimated by bivariate VAR\(^1\). In the two years average after the shock, a unit rise in the oil price generated a 3% decline in output from its trend value for the pre-1984 period, and it led to a less than 1% decline in output from its trend for the post-1984 period. Clearly, the output decline associated with the oil price rise has become moderate recently.

We present an explanation as to why the current oil price variation appears to be less important for the US economy than before. Several early papers address the same question. Blanchard and Galí (2007) propose that there has been three structural changes in the US economy between the two subsample periods, and that these changes made US output less responsive to oil price changes. Those are: the decrease in real wage rigidities, the increased credibility of monetary policy, and the decrease in the share of oil in consumption and in production. They point out that all three changes play important roles. Along the same lines, Katayama (2007) mentions that the deregulation of the transportation sector, the overall improvement of energy use of the economy, and the persistency of the oil price are the important determinants of the weakening impact of the oil price. We offer a different perspective on the same question. We focus on the fact that oil price increases occur for several reasons. Fernald and Trehan (2005) argue that the oil price should be highly correlated with output when oil price increases are driven by economic expansion. Killian (2005, 2008) also stresses the importance of identification as to the sources of oil price variation. In the current paper, we argue that the oil price variation is driven by the exogenous variables, and that they evolve differently in a significant way between the two subsample periods. This

\(^1\)Here we employ the residual of the oil price equation for oil price shock. Cholesky decomposition is one other way to identify the real oil price shocks used in the literature. See Burbidge and Harrison (1984) and Jimenez-Rodriguez and Sanchez (2004). Estimation results do not change by the use of Cholesky decomposition.
difference leads us to observe a weakening impact of the oil price rise on output.

We set up a simple DSGE model that accounts for oil price variation and US output variation over time. Both the oil price and output are determined endogenously by the exogenous changes in oil supply and US TFP\(^2\). Taking the actual time path of the exogenous variables in the post-war period as given, our model generates both oil price series and output series that are highly correlated with the historical movement of the actual oil price and output. Similar to the actual data, our model-generated series show a weakening impact of the oil price on output. That is, the cross-correlation between the two variables is less negative, and the output reduction in response to an oil price rise is smaller in the post-1984 period than in the pre-1984 period. A notable feature of our analysis is that no change in the economic structures between the two subsample periods are considered. The parameters are unaltered in the simulation. There is a weakening in the observed impact of the oil price on output, because the time series properties of the exogenous variables are not identical across the two subsample periods. Two key changes in the way that exogenous variables evolve are important for understanding the reason for this weakening. First, oil supply variation has decreased. Second, oil supply shortages are no longer followed by a large decline of TFP. With less-volatile oil-supply variation, the portion of oil price variation explained by oil supply variation decreases, leading to a less-negative oil price–output correlation. A smaller TFP decline during oil supply shortfalls implies a smaller output decline during oil price increases.

This paper is organized as follows. In Section 2, we describe the model. Our model is a two-country model with the US and OPEC. In the model, US TFP and the oil supply are exogenous variables, and the oil price and US output are endogenous variables. In Section 3, we simulate the model using the actual time path of US TFP and the oil supply from 1973:Q1 to 2007:Q4. The model-generated series captures the feature of actual time path of data closely, and displays the weakening impact of the oil price between the two subsample periods. Section 4 is devoted to examining the link between the way the exogenous variables evolve over time and the observed oil price–output relationship. Section 5 concludes.

2 The economy

Two countries, the US and OPEC, are present in the model. There are three agents: the household, the final goods producer, and the oil producer in the US. OPEC produces and sells its oil to the US final goods producer in each period. The oil is used as a production input in the US, and the equilibrium oil price is determined endogenously so as to clear the market.

2.1 Final goods producer

The final goods producer solves the following optimization problem:

\[
\begin{align*}
&\max_{h_t, k_t, E_t} Y_t - w_t H_t - r_t K_t - p_t E_t, \\
&\text{subject to } Y_t = \left[\exp(z_t) H_t \right]^{\alpha} \left[\eta K_t^{1-\frac{1}{\alpha}} + (1 - \eta) E_t^{1-\frac{1}{\alpha}}\right]^{\frac{1}{1-\frac{1}{\alpha}}},
\end{align*}
\]

\(^2\)Several previous studies, such as Kim and Loungani (1992) and Finn (1995, 2000), assume that the oil price is exogenously determined, and analyze the equilibrium responses of the economy, given the realization of the oil price shock. In contrast, we follow Backus and Crucini (2000) and Leduc and Sill (2007), where the oil price is determined endogenously from a given sequence of exogenous oil supplies.
where labor hours $H_t$, capital $K_t$, and oil $E_t$ are used to produce the final good $Y_t$. Note that $Y_t$ is gross output, and it differs from output $y_t$ (value added), which we define below. The final goods producer acts as a price taker in both the product market and the input market, with the price of the final good, normalized to be unity, real wage $w_t$, real rental rate of capital $r_t$, and real price of oil $p_t$ taken as given. The production technology of the final goods producer is a nested Constant Elasticity of Substitution (CES) function with constant returns to scale. The parameter $v$ is the elasticity of substitution between capital $K_t$ and input of oil $E_t$, $\eta$ is capital share, and $\alpha$ is labor share. $z_t$ is the technology variable, which is exogenously given.

2.2 Oil supply

Oil is supplied from the two sources, from the US and OPEC. The quantity of oil supplied at each period obeys the following law of motion:

\[ E^D_t = E^D \exp(\gamma^ED_t), \]
\[ E^I_t = E^I \exp(\gamma^EI_t), \]

where $E^D_t$ and $E^I_t$ are the oil supply from the US and OPEC, respectively. For the US, $E^D$ and $E^I$ are the domestic oil supply and imported oil supply at the steady state. The sequence of $\gamma^ED_t$ and $\gamma^EI_t$ are exogenously given. Note that $E^I_t$ and $E^D_t$ are homogenous so that they are subject to the law of one price. All of the oil produced today is used today. The oil price adjusts so as to clear the oil market at each period:

\[ E_t = E^I_t + E^D_t. \]

2.3 Household

The representative household is infinitely lived with preferences over consumption and leisure and maximizes discounted utility (3). The household provides the final goods producer with capital service and labor and receives compensation for them. Domestically produced oil $E^D_t$ is owned by the household. The household solves the following optimization problem:

\[
\max_{C_{t+j}, K_{t+j}, H_{t+j}} \mathbb{E} \left[ \sum_{j=0}^{\infty} \beta^j [\theta \log C_{t+j} + (1 - \theta) \log(1 - H_{t+j})] \right],
\]

s.t. \[ C_{t+j} + K_{t+1+j} \leq w_{t+j} H_{t+j} + r_{t+j} K_{t+j} + (1 - \delta) K_{t+j} + p_{t+j} E^D_{t+j}, \]

where $C_t$ is consumption, $\beta \in (0, 1)$ is the discount factor, $\theta$ is the preference parameter, $\delta \in (0, 1]$ is the depreciation rate of the capital stock and the time endowment is normalized to unity. Our utility function has a unitary elasticity of substitution between consumption and leisure. Notice that for the household, the oil price $p_t$ appears as the return for lending the oil to the final goods producing firms.

\[ \eta \]


4Our model has vertical supply curves for oil production. This specification is similar to those used in Backus and Crucini (2000) and Leduc and Sill (2007).

5In the simulation below, we set the oil supply at the steady state equal to unity, and choose $E^D$ and $E^I$ based on the import share of the oil supply in the US.
Finally, we define output $y_t$ by the following equation:

$$y_t = C_{t+j} + K_{t+1+j} - (1 - \delta)K_{t+j}.$$

### 2.4 Equilibrium

An equilibrium consists of a set of allocations, $\{Y_t, C_t, K_t, H_t, E^I_t, E^P_t, E_t, y_t, p_t, w_t, r_t\}_{t=0}^{\infty}$ that satisfies the following conditions: (i) the household’s allocation solves its utility maximization problem; (ii) each producer’s allocations and the price solve its profit maximization problem, taking input prices as given; and (iii) all markets clear.

### 2.5 Parameter calibration

We choose conventional figures for many of the parameters. That includes the subjective discount factor $\beta$, the weight on leisure $(1 - \theta)$, the intertemporal elasticity of substitution $\sigma$, the depreciation rate of capital $\delta$, and the labor share $\alpha$. See Table 1 for details. Several views exist about the elasticity of substitution between capital and oil input, $\nu$. We set $\nu = \frac{1}{7}$, which falls between the figures used in two prior studies by Kim and Loungani (1992) and Backus and Crucini (2000). $\eta$ is selected so that the share of the expenditure for oil purchases over the US output is 1%. We set 0.4 for the import share of the oil supply at steady state.

### 2.6 Model responses to changes in exogenous variables

In this subsection, we describe the responses of the endogenous variables in our model to the three exogenous variables. Figure 1 displays the impulse responses of the oil price and output to a change in each of the three exogenous variables. We assume that the exogenous variables jump at $t = 0$, by 1%, and that they gradually revert to their steady state values, with a quarterly autoregressive parameter of 0.9. We consider the cases of a positive jump for TFP and a negative jump for oil supplies.

The upper panel in Figure 2 indicates that the oil price increases in response to exogenous TFP increase, and to exogenous oil supply shortfalls. Provided that the oil supply is unaltered during the periods in which TFP deviates from its steady state value, higher TFP leads to higher demand for oil as an input. This results in an oil price rise. Declines in either OPEC oil production or US oil production reduces the availability of oil. At equilibrium, the oil price rises to dampen the demand for oil. Output rises in the wake of the TFP increase, and declines during oil supply shortages. It is important to note that TFP variation drives the oil price and output in the same direction, while oil supply variation drives them in the opposite directions. There is quantitative asymmetry between the impact of TFP and that of the oil supply on output. While a 1% increase in TFP increases output by 1.5% in the period of the shock, a 1% oil supply shortfall reduces output by only .02% at most. The effect of oil supply variation on output is limited, compared with that of TFP. This observation holds true even if we change the oil expenditure share from 1% to 4% (Barsky and Kilian, 2004), a percentage that is the maximum value among the empirically reasonable figures of oil expenditure. For the oil price variation, the quantitative effects are similar across the exogenous variables.

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\(6\) See the discussion in Apostolakis (1990) and Atkeson and Kehoe (1999) for details.
3 Simulation procedures

We conduct a deterministic simulation of the model. Using the actual time paths of OPEC oil production, US oil production and US TFP, from 1973:Q1 to 2007:Q4, $\gamma^E_t$, $\gamma^I_t$ and $z_t$ are derived. At 1973:Q1, the household knows all of the historical time paths of the exogenous variables up to 2007:Q4, and solves its optimization problem. The oil price and output are generated from the equilibrium conditions of the model. Earlier works such as Kim and Loungani (1992) and Backus and Crucini (2000) use a stochastic simulation rather than a deterministic simulation. However, the sources of oil supply changes are often episode specific. For example, the OPEC embargo of 1973–1974 and the OPEC meeting of 1999, were announced before they were realized, while abrupt oil shortfalls accompanied by the conflicts in the Middle East are regarded as innovations. Isolating the expected changes in oil production from the unexpected changes requires additional consideration for the identification. A deterministic simulation does not involve assumptions related to this issue.

The exogenous variables that are used for our simulation are linearly detrended. Early studies on oil price variation apply the Hodrick and Prescott (1997) filter with a smoothing parameter equal to 400 (Kim and Loungani, 1992; Finn, 1995) or 100 (Backus and Crucini, 2000), to the annual data. As Baxter and King (1999) discuss, the Hodrick and Prescott (HP) filter with the conventional smoothing parameters produces detrended series of the variables with a business cycle frequency of 2 to 32 quarters (Comin and Gertler, 2006). We use linear detrending so as to add the lower frequency variations into the analysis. In detrending the TFP series, we allow for a structural break in the linear trend in 1997:Q1, following Kahn and Rich (2007) and Fernald (2007).

4 Simulation results

Our simulation results are reported in this section. Our model replicates the empirical regularities associated with the time series of the oil price and output in the postwar period. The model-generated series captures a sizable portion of the oil price variation and output variation over the sample period. While all of the model parameters remain unaltered throughout the postwar period, the weakening impact of the oil price is also found in the model-generated series. The oil price–output cross-correlation is more negative, and the estimated output reduction in response to an oil price rise is larger in the first subsample period than in the second subsample period.

Figure 3 shows the time paths for the model-generated series of oil price (upper panel) and output (lower panel), from 1973:Q1 to 2007:Q4. All of the three exogenous variables are used in the simulation to calculate the endogenous variables. In what follows, we refer to this simulation as the benchmark simulation. We denote the actual data of the variables by lines with white circles, and the model-generated series by lines with blue circles. Our model captures the major ups and downs of the two variables that are examined in previous studies. For example, many of the oil crisis episodes such as the oil price shock of 1974 (OPEC embargo), 1979–1980 (Iranian revolution, Iran–Iraq war), 1990 (invasion of Kuwait) and 1999 (OPEC meeting), and the downturn of 1986 (OPEC collapse) are obtained from the simulation.

Again, we split the sample periods into two subsample periods. The first subsample period covers from 1973:Q1 to 1983:Q4, and the second subsample period covers from 1984:Q1 to 2007:Q4. The summary statistics of this simulation are reported in Table 2. As for the oil price variation, the contemporaneous correlations between the actual data and model-generated series for the two subsample periods are 0.69 and 0.56, respectively. Our model delivers a standard deviation that is almost the same size as that of the data for the first subsample period, and a lower figure than the actual value for the second subsample period. As for the output variation, the contemporaneous
correlations between the actual data and model-generated series for the two subsample periods are 0.92 and 0.75, respectively. The model yields a standard deviation that is almost the same size as that of the data for the first subsample period, and gives a larger standard deviation than the actual data for the second subsample period.

The model-generated series of the oil price and output exhibit a weakening impact of the oil price in the post-1984 period, compared with the pre-1984 period. Similar to Figure 1, but using the model-generated series instead of the actual data, we calculate the cross-correlation and the impulse responses of output, for the two subsample periods. The upper panel in Figure 4 exhibits the cross-correlation, and the lower panel shows the estimated output response. The lines with blue circles are the estimates based on the first subsamples. The lines with white circles are the estimates based on the second subsamples. It appears from the figure that the oil price–output cross-correlation is significantly lower, and the estimated output reduction in response to the oil price increase is larger in the pre-1984 period than in the post-1984 period.

5 Accounting for the weakening of the oil price impact

We see above that the observed relationship between the oil price and output have changed over the two subsample periods. In the model, the two variables are endogenously determined and all of their variations are equilibrium responses to changes in exogenous variables. The observations associated with the weakening of oil price impact imply that the statistical properties of the exogenous variables, or their joint relationships, have changed over the two subsample periods. To see this more clearly, we decompose the variations of the endogenous variables into a portion explained by oil supply variation and one explained by TFP variation, using the following approximations:

\[ dp_t \approx \frac{\partial p}{\partial o} do_t + \frac{\partial p}{\partial A} dA_t, \]  
\[ dy_t \approx \frac{\partial y}{\partial o} do_t + \frac{\partial y}{\partial A} dA_t. \]  

\( dp_t, dy_t, do_t, \) and \( dA_t \) are deviations in oil price, output, oil supply and TFP, from their trend values. Oil supply is the sum of OPEC and US oil production. \( \frac{\partial p}{\partial o} \) and \( \frac{\partial y}{\partial o} \) are the partial derivatives of the oil price and output, with respect to changes in oil supply. Similarly, \( \frac{\partial p}{\partial A} \) and \( \frac{\partial y}{\partial A} \) are the partial derivatives of the oil price and output, with respect to changes in TFP. Roughly speaking, these terms represent the impulse responses of the two endogenous variables to the exogenous variables, which are illustrated in Figure 2. Figure 2 suggests that \( \frac{\partial y}{\partial o}, \frac{\partial p}{\partial A}, \) and \( \frac{\partial y}{\partial A} \) are positive, and \( (\frac{\partial p}{\partial o}) \) is negative. In addition, the term \( \frac{\partial y}{\partial o} \) is considered to take a small value, compared with the other terms.

In the subsections below, using the approximations (5) and (6), we show that our two measures of the oil price–output relation are expressed by the variations in the exogenous variables \( do_t \) and \( dA_t \). Changes in the way that the exogenous variables evolve between the two subsample periods, yield the changes in the observed oil price–output relationship over the two periods.

5.1 Accounting for the correlation

We observe from Figure 1 and Figure 4 that the oil price–output cross-correlation became less negative in the post-1984 subsample period, than in the pre-1984 subsample period. We show that the change is attributable to a change in the statistical properties of the exogenous variables during
these periods. That is, oil supply variation is reduced in the second period, making a negative oil price–output relation less likely.

As equations (5) and (6) demonstrate, the variations of the endogenous variables are tied to the variations in the exogenous variables. Our next step is to see how the correlation among the endogenous variables is tied to the variations of exogenous variables. For convenience, we use the covariance instead of the correlation. Using equations (5) and (6), the oil price–output covariance is linked to variations in the exogenous variables, in the following way:

\[
\text{Cov}(dp_t, dy_t) \approx \text{Cov} \left( \frac{\partial p}{\partial o} do_t + \frac{\partial p}{\partial A} dA_t, \frac{\partial y}{\partial o} do_t + \frac{\partial y}{\partial A} dA_t, \right)
\]

\[
= \frac{\partial p \partial y}{\partial o \partial o} \left( T_s^{-1} \sum_{t=0}^{T_s} (do_t)^2 \right) + \frac{\partial p \partial y}{\partial A \partial A} \left( T_s^{-1} \sum_{t=0}^{T_s} (dA_t)^2 \right) \\
+ \left[ \frac{\partial p \partial y}{\partial o \partial A} + \frac{\partial p \partial y}{\partial A \partial o} \right] \left( T_s^{-1} \sum_{t=0}^{T_s} (do_t dA_t) \right) \\
\approx \frac{\partial y}{\partial A} \sum_{t=0}^{T_s} \left\{ \left( \frac{\partial p}{\partial A} dA_t + \frac{\partial p}{\partial o} do_t \right) dA_t \right\} \approx \frac{\partial y}{\partial A} \sum_{t=0}^{T_s} \{dp_t dA_t\},
\]

where \( T_s \) for \( s = 1, 2 \) is the number of samples in the first period and the second period. As for the second equality, we apply the argument that \( \frac{\partial y}{\partial o} \) is close to zero\(^7\). The covariance is expressed as the product of term \( (\partial p/\partial AdA_t + \partial p/\partial do_t) \), multiplied by \( dA_t \). Equation (5) indicates that \( (\partial p/\partial AdA_t + \partial p/\partial do_t) \) is equal to \( dp_t \). Note that Figure 2 suggests that \( \partial p/\partial A \) is positive, while \( \partial p/\partial o \) is negative. Thus, the sign of this product term takes a positive value when the two exogenous variables move in the opposite directions. When they move together and oil supply variation is sufficiently larger than TFP variation, the sign is negative. We see below that in general, the oil supply and TFP vary in the same direction, over the full sample period. What breaks the two periods is the change in the relative size of the variations of each exogenous variable. As variation of oil supply \( do_t \) decreases compared with the variation of TFP \( dA_t \), the observed oil price–output correlation takes a less negative value.

**Time paths of exogenous variables**

Figure 5 presents the time paths of the exogenous variables from 1973:Q1 to 2007:Q4. The upper panel displays oil supply variation, the middle panel displays TFP variation, and the lower panel displays the product of the two exogenous variables. The variations are shown in absolute value of the deviation of each variable from their trend values. Three observations are important. The first observation is the decline in oil supply variation. In terms of sample means, the absolute value of oil supply variation declines by 66% in the second period, compared with the first period. The second observation is about the direction of the variations. Oil supply and TFP tend to vary in the same direction. For both periods, the sample mean of the product \( do_t dA_t \) takes a positive value. In terms of timing, too, quarters they move in the same direction is more frequent than those they go in opposite directions, in both periods. The third observation is the relative size of oil

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\(^7\)Figure 6 below indicates that this assumption is reasonable. Using only the time path of oil supply variation, the model generates output series that hardly varies over time.
supply variation, compared with TFP variation. For both subsample periods, oil supply variation is greater than TFP variation. Because oil supply variation decreases by 66% in the second period while TFP variation only decreases by 2%, their relative size has changed. On average, oil supply variation is 4.3 times larger than TFP variation in the first period and 1.5 times larger in the second period.

Role of exogenous variables over the two periods

The three observations together suggest that the change in the relative size of the variations in the exogenous variables decreases the covariance. According to the last term of equation (7), provided that the oil supply and TFP comove, as the portion of oil price variation explained by oil supply variation \( \frac{\partial p}{\partial o_d} \) decreases, the oil price–output covariance tends to be less negative. To see that this statement holds in the actual data, we conduct two more simulations. In simulation I, we only use the actual oil supply variation to generate the endogenous variables, setting TFP variation equal to zero throughout the sample period. In simulation II, only the actual TFP variation is used for the simulation. The model-generated oil price and output are shown in Figures 6 and 7. In both figures, the upper panel displays the result of simulation I, and the lower panel displays that of simulation II.

The figures illustrate the relative significance of each exogenous variable in explaining variations in the oil price and output during the two periods. Oil supply variation is responsible for the majority of oil price variation in the first period. The time path generated from simulation I follows closely the time path generated from the benchmark simulation. Meanwhile, TFP variation only produces small fluctuations in the oil price. In the second period, it appears that TFP variation has a relatively larger effect than before, especially after the late 1990s. On average, 87% of oil price variation \( dp_t \) is originated from oil supply variation in the first period, and this number declines to 47% in the second period. Oil supply variation has become less important in explaining oil price variation in recent years. As equation (7) states, this change yields the change in the oil price–output correlation. On the other hand, for output variation, TFP variation explains almost all of the variation over the two periods. Whereas the time paths generated from simulation II and the benchmark simulation coincide, the impact of oil supply variation is hardly detectable. All of the recessions\(^8\) that are discussed along with the oil price increases, are also due to TFP declines. Rotemberg (2007) states that a small response of the economy to the oil price is more consistent with “standard macroeconomic models.” The share of oil expenditure is small and any decline in oil input associated with the oil price rise has a limited effect on aggregate output. Our result is consistent with their findings\(^9\).

5.2 Accounting for the impulse response

Our second observation is about the estimated impulse response of output to an oil price rise. Following the oil price rise, a larger decrease in output is observed for the pre-1984 period, compared with that in the post-1984 period (Figures 1 and 4). We show below that this unresponsiveness


\(^9\)In the model, US recessions during oil price upsurge is therefore the outcome of that oil supply shortages and TFP decline occur simultaneously. The productivity declines in the US economy during the past oil crisis episodes are reported in studies such as those of Bruno and Sachs (1985), Barsky and Kilian (2004), and Blanchard and Galí (2007). For example, Barsky and Kilian (2004) point out that during the oil crisis from 1974 to 1985, the average TFP growth rate was 0.31, which is exceptionally low compared with the other periods. Blanchard and Galí (2007) find the presence of “large shocks of a different nature” which coincide with the oil price shock in the 1970s.
of output is due to the change in the joint realization of the exogenous variables. In the earlier subsample period, an oil supply shortage is accompanied by a larger drop in TFP. In the latter subsample period, this observation does not hold. Moderation of the output response to an oil price rise is brought about by the moderation of TFP declines, conditional on the realization of oil supply shortages.

To see this clearly, we focus on the two impulse response functions (IRFs) estimated from two VARs. One is the output response to the oil price rise, and the other is the TFP response to the oil supply shortage. While we do not have a theoretical grounding as to the law of motion for oil supply and TFP, the latter IRF gives the description as to how they evolve in the two subsample periods. Given the relationship between the endogenous variables and exogenous variables proposed in equations (5) and (6), the two IRFs are directly related, and changes in the former IRFs are explained by changes in the latter IRFs.

The joint evolution of TFP and oil supply are described by the following bivariate VAR framework:

\[
\begin{bmatrix}
dA_t \\
do_t
\end{bmatrix}
= \begin{bmatrix}
\sum_{j=1}^{q} \alpha_{j} dA_{t-j} + \sum_{j=1}^{q} \alpha_{j} do_{t-j} + u_{t}^{A} \\
\sum_{j=1}^{q} \gamma_{j} dA_{t-j} + \sum_{j=1}^{q} \gamma_{j} do_{t-j} + u_{t}^{o}
\end{bmatrix}.
\] (8)

Here, \( \alpha_{j}, \alpha_{j}^{o}, \gamma_{j}, \gamma_{j}^{o} \) for \( j = 1,..q \) are scalars that give the law of motion to exogenous variables \( dA_{t} \) and \( do_{t} \). \( q \) is the number of lags. \( u_{t}^{A} \) and \( u_{t}^{o} \) are innovations in TFP and oil supply, respectively. For analytical convenience, we transform (8) to MA(\( \infty \)) form as follows:

\[
\begin{bmatrix}
dA_t \\
do_t
\end{bmatrix}
= \begin{bmatrix}
\sum_{k=1}^{\infty} \beta_{k}^{11} u_{t-k}^{A} + \sum_{k=1}^{\infty} \beta_{k}^{12} u_{t-k}^{o} \\
\sum_{k=1}^{\infty} \beta_{k}^{21} v_{t-k}^{A} + \sum_{k=1}^{\infty} \beta_{k}^{22} v_{t-k}^{o}
\end{bmatrix},
\] (9)

where \( \beta_{k}^{11}, \beta_{k}^{12}, \beta_{k}^{21}, \) and \( \beta_{k}^{22} \) for \( k = 1,..\infty \), are the IRFs for the innovations \( u_{t-k}^{A} \) and \( u_{t-k}^{o} \) for \( k = 1,..\infty \). Note that our VAR estimation for output and the oil price are also described by an MA(\( \infty \)) form such as:

\[
\begin{bmatrix}
dy_t \\
dp_t
\end{bmatrix}
= \begin{bmatrix}
\sum_{k=1}^{\infty} \varphi_{k}^{11} v_{t-k}^{y} + \sum_{k=1}^{\infty} \varphi_{k}^{12} v_{t-k}^{p} \\
\sum_{k=1}^{\infty} \varphi_{k}^{21} v_{t-k}^{y} + \sum_{k=1}^{\infty} \varphi_{k}^{22} v_{t-k}^{p}
\end{bmatrix},
\] (10)

where \( v_{t-k}^{y} \) and \( v_{t-k}^{p} \) for \( k = 1,..\infty \) are the innovations in the equation for output and the oil price, respectively. The IRFs for \( v_{t-k}^{y} \) and \( v_{t-k}^{p} \) are denoted by \( \varphi_{k}^{11}, \varphi_{k}^{12}, \varphi_{k}^{21}, \) and \( \varphi_{k}^{22} \) for \( k = 1,..\infty \). Our second measure of the weakening effect of the oil price, response of output to an oil price rise, is captured by \( \varphi_{k}^{12} \) for \( k = 1,..\infty \). Empirical analysis conducted in Figure 4 tells us that \( \varphi_{k}^{12} \) has become considerably smaller in the second subsample period compared with the first subsample period, at least for \( k = 1,..12 \). We show in the proposition below that changes in the dynamic law of motions in the endogenous variables \( \varphi_{k}^{12} \) reflect changes in the dynamic motions of exogenous variables \( \beta_{k}^{12} \).

**Proposition.** Provided that equations (5) and (6) hold, and that \( \partial y / \partial o \) is approximately equal to zero, the IRFs of \( \varphi_{k}^{12} \) for \( k = 1,..\infty \) are linked with the IRFs of \( \beta_{k}^{12} \) for \( k = 1,..\infty \), through the following equality:

\[
\varphi_{k}^{12} = \frac{\partial y}{\partial A} \frac{\partial p^{-1}}{\partial o} \beta_{k}^{12}.
\] (11)

**Proof.** See Appendix A.
According to equation (11), a larger $\beta_k^{12}$ implies a smaller $\varphi_k^{12}$. For example, if we suppose that there is a larger decline in TFP following an oil supply shortage, then the observed output response to the oil price becomes more negative. Equation (11) is consistent with our findings discussed in earlier sections. We saw that TFP variation is responsible for almost all of the output variation (Figures 2 and 7). The oil price rise is, on the other hand, accompanied by oil supply shortages if it is co-occurring with an output decline (Figure 2). An observed larger output drop in response to an oil price rise, in the first sub-sample period, comes from a larger decline of the TFP in the first subsample period than in the second subsample period, upon the oil supply shortages.

To see if there is a change in $\beta_k^{12}$, we estimate VARs (9) for the two subsample periods. Figure 8 reports the IRFs of TFP to a unit decrease in the oil supply, $\beta_k^{12}$ for $k = 1, \ldots, 12$. The lines with blue circles are the estimates based on the first subsamples. The lines with white circles are the estimates based on the second subsamples. For both subsample periods, TFP declines following an oil supply shortfall, but there is a substantial difference in the size of the decline. In the earlier subsample period, TFP decreases more than in the latter subsample period. As equation (11) states, a larger $\beta_k^{12}$ leads to a larger negative value of $\varphi_k^{12}$. In other words, the larger output decline in response to an oil price rise observed in the early subsample period is produced by a larger TFP decline following the oil supply shortages that occurred in the early subsample period.

6 Conclusion

Existing empirical works regarding the co-occurrence of soaring oil prices and a decline in US output suggest that this link has weakened in recent years. We estimate (i) the cross-correlation between the oil price and output, and (ii) the impulse response of output to the oil price rise, for the pre-1984 period and for the post-1984 period. Our two measures agree that the oil price–output link has weakened in recent years. Early studies, including that of Blanchard and Galí (2007), stress the role of the structural changes in the US economy that have made US output less responsive to oil price shocks. For example, changes in the wage rigidity, US production function or changes in policy are claimed to lead to the weakening of the oil price effect. Our paper offers an alternative view.

We focused on the causes of the variations in the oil price and output. Both the oil price and output are determined by the exogenous sequence of oil supply and TFP. The changes in the way that these exogenous variables evolve over time cause changes in the observed relationship between the oil price and output. We developed a DSGE model with the US and OPEC, where oil is traded between the two countries. Both the oil price and US output are endogenously determined as equilibrium responses to exogenous variations in oil supply and TFP. With the actual time paths of oil supply and TFP variation used in the simulation, our model yields the time series of the oil price–output relationship of the model-generated series has become weaker in recent years. The oil price–output cross-correlation is less negative, and the output decline in response to the oil price rise has decreased in recent years.

We found that nearly all of the output variation can be explained by TFP variation, and the oil price variation is explained by both oil supply variation and TFP variation over the postwar periods. There are, however, changes in the way that these exogenous variables evolve between the early period and the later period. Changes in the statistical properties of the exogenous variables cause changes in the observed oil price–output relationship. Two changes in the realized pattern of exogenous variables are important. First, oil supply variation has decreased. Second, oil supply
shortages are no longer followed by a large decline in TFP. Less-volatile oil supply variation leads to a less-negative oil price–output correlation, as the portion of oil price variation explained by oil supply variation decreases. A smaller TFP decline during oil supply shortfalls implies a smaller output decline during the oil price increase because TFP is responsible for output variation.
A Appendix A

This section provides a proof of the proposition claimed in Section 5.2. Provided that approximations (5) and (6) hold, we first show that the residuals of the bivariate VAR estimation for output and the oil price, denoted by $v_t^y$ and $v_t^o$, are expressed by a linear combination of the residuals of the bivariate VAR estimation for TFP and oil supply, denoted by $u_t^A$ and $u_t^o$.

Recall that the residual series $v_t^y$ and $v_t^o$ are written as:

$$
\begin{bmatrix}
  v_t^y \\
  v_t^o \\
\end{bmatrix}
= \begin{pmatrix}
  I - X (X'X)^{-1} X' & 0 \\
  0 & I - X (X'X)^{-1} X' \\
\end{pmatrix}
\begin{bmatrix}
  y \\
  p \\
\end{bmatrix},
$$

where $t$ is the sample period, $I$ is a $t \times t$ identity matrix and $X$ is a matrix that contains lagged output and oil price. $X'$ is the transpose of matrix $X$. Consider matrix $Z$, which is composed of lagged TFP and oil supply series with the same number of lags as those of $X$. The relationship expressed in equations (5) and (6) ensures that there is an invertible matrix $\Gamma$ that links the lagged endogenous variables to the lagged exogenous variables, such that $X = Z\Gamma$. The equations above are reduced to the expression that ties the residuals of the two VAR estimations:

$$
\begin{pmatrix}
  I - Z \Gamma (\Gamma' Z' Z\Gamma)^{-1} \Gamma' Z' & 0 \\
  0 & I - Z \Gamma (\Gamma' Z' Z\Gamma)^{-1} \Gamma' Z' \\
\end{pmatrix}
\begin{bmatrix}
  \frac{\partial y}{\partial A} u_t^A \\
  \frac{\partial y}{\partial o} u_t^A \\
  \frac{\partial y}{\partial A} u_t^o \\
  \frac{\partial y}{\partial o} u_t^o \\
\end{bmatrix}
= \begin{pmatrix}
  \frac{\partial y}{\partial A} \sum_{k=1}^{\infty} \varphi_{k,11} u_{t-k}^A \\
  \frac{\partial y}{\partial o} \sum_{k=1}^{\infty} \varphi_{k,12} u_{t-k}^o \\
  \frac{\partial y}{\partial A} \sum_{k=1}^{\infty} \varphi_{k,21} u_{t-k}^A \\
  \frac{\partial y}{\partial o} \sum_{k=1}^{\infty} \varphi_{k,22} u_{t-k}^o \\
\end{bmatrix}
+ \begin{pmatrix}
  \frac{\partial y}{\partial A} \sum_{k=1}^{\infty} \varphi_{k,11} u_{t-k}^A \\
  \frac{\partial y}{\partial A} \sum_{k=1}^{\infty} \varphi_{k,21} u_{t-k}^A \\
  \frac{\partial y}{\partial o} \sum_{k=1}^{\infty} \varphi_{k,12} u_{t-k}^o \\
  \frac{\partial y}{\partial o} \sum_{k=1}^{\infty} \varphi_{k,22} u_{t-k}^o \\
\end{pmatrix}.
$$

Using this equality, the RHS of (10) is expressed by the linear combination of $u_{t-k}^A$ and $u_{t-k}^o$ as follows:

$$
RHS \approx \begin{pmatrix}
  \frac{\partial y}{\partial A} \sum_{k=1}^{\infty} \varphi_{k,11} u_{t-k}^A + \sum_{k=1}^{\infty} \varphi_{k,12} \left( \frac{\partial p}{\partial A} u_{t-k}^A + \frac{\partial p}{\partial o} u_{t-k}^o \right) \\
  \frac{\partial y}{\partial o} \sum_{k=1}^{\infty} \varphi_{k,21} u_{t-k}^A + \sum_{k=1}^{\infty} \varphi_{k,22} \left( \frac{\partial p}{\partial A} u_{t-k}^A + \frac{\partial p}{\partial o} u_{t-k}^o \right) \\
\end{pmatrix}.
$$

Note that we apply our observation that $\partial y/\partial o$ is negligible. Equations (5) and (6) provide an alternative expression of the LHS of (10) as follows:

$$
LHS \approx \begin{pmatrix}
  \frac{\partial y}{\partial A} \left( \sum_{k=1}^{\infty} \beta_{k,11} u_{t-k}^A + \sum_{k=1}^{\infty} \beta_{k,12} u_{t-k}^o \right) + \frac{\partial p}{\partial o} \left( \sum_{k=1}^{\infty} \beta_{k,21} u_{t-k}^A + \sum_{k=1}^{\infty} \beta_{k,22} u_{t-k}^o \right) \\
  \frac{\partial y}{\partial o} \left( \sum_{k=1}^{\infty} \beta_{k,11} u_{t-k}^A + \sum_{k=1}^{\infty} \beta_{k,12} u_{t-k}^o \right) + \frac{\partial p}{\partial o} \left( \sum_{k=1}^{\infty} \beta_{k,21} u_{t-k}^A + \sum_{k=1}^{\infty} \beta_{k,22} u_{t-k}^o \right) \\
\end{pmatrix}.
$$

Combining equations (12) and (13) provides the relation between the two estimated IRFs obtained in the separate VARs:

$$
\frac{\partial p}{\partial o} \varphi_{k,12} = \frac{\partial y}{\partial A} \beta_{k,12} \quad \text{for} \quad k = 1, \ldots, \infty.
$$

\footnote{Note that $v_t^y, v_t^o, u_t^A, u_t^o, y$ and $p$ are all $t \times 1$ column vectors.}

\footnote{\(X\) is a \(t \times (2q+1)\) matrix, where \(q\) is the number of lags employed in the VAR estimation.}
B Data appendix

Quantity series of oil supply
OPEC oil production series, US oil production series and US oil import series are taken from “World Crude Oil Production: OPEC Members,” “US Crude Oil Field Production” and “US Crude Oil Imports from All Countries,” released by the Energy Information Administration, Department of Energy. The three series are seasonally adjusted.

Real oil price
The nominal oil price is from “Crude Oil Composite Acquisition Cost by Refiners,” released by the Energy Information Administration, Department of Energy. Acquisition cost series is available only from January 1974. Following Mork (1989), we extend the series forward to 1973. The series is converted to quarterly values, using the quarterly GDP deflator.

Real gross output
Real gross output $Y_t$ is calculated using the following equation:

$$Y_t = \frac{P_{oil}^t \times E_l^t + GDP_t}{P_{GDP}^t},$$

where $P_{oil}^t$ is the nominal crude oil composite acquisition cost, $E_l^t$ is the quantity of oil imported to the US, $GDP_t$ is nominal gross domestic product and $P_{GDP}^t$ is the GDP deflator.

Total factor productivity (TFP)
TFP $A_t$ is estimated so as to be consistent with our production function:

$$Y_t = A_t H_t^{1-\alpha} (\eta K_{t-1}^{1-v} + (1-\eta) E_t^{1-v})^{\frac{\alpha}{1-v}}.$$

Working hours $H_t$ is calculated from total Current Population Survey (CPS) hours worked divided by noninstitutional population from age 16 to 64 reported in Cociuba (2008). $K_t$ is constructed from real gross domestic product, real net stock of fixed assets taken from the NIPA tables. $E_t$ is the sum of the quantity of oil imported into the US and the quantity of oil produced in the US.
References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Quarterly subjective discount rate</td>
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<tr>
<td>$\sigma$</td>
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<td>Intertemporal elasticity of substitution</td>
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<tr>
<td>$\delta$</td>
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<td>Quarterly depreciation rate of capital</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>Labor Share</td>
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<tr>
<td>$\nu$</td>
<td>$1/7$</td>
<td>Elasticity of substitution between capital and oil input</td>
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<td>$E^I/(E^D + E^I)$</td>
<td>.4</td>
<td>Import share of crude oil in the U.S.</td>
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</table>
Table 2: Correlation between the model-generated series and the actual data.

Std reports the standard deviations of the model-generated series normalized by those of the actual data during the corresponding periods.
Figure 1: Upper panel shows the cross-correlation between oil price at $t$ and output at $t+j$. The lower panel shows the response of output to a unit increase in oil price. The Y-axis denotes the deviation of the variables from the trend, and the X-axis denotes the quarter after the shock.

Confidence intervals are shown by dotted lines.
Figure 2: Responses of oil price (upper panel) and output (lower panel) to a change in each of the exogenous variable. For the OPEC oil production and the U.S. oil production, 1% drop from the trend at period zero, and subsequent declines by 90% at each period are considered. For TFP, 1% rise at period zero and subsequent declines is feeded. The right scale is used for the output response to change in TFP (lower panel). The left scale is used for other all lines.
Figure 3: Upper panel shows time path of the oil price. Lower panel shows that of the U.S. output. For both panels, Y-axis indicates the deviation of the variable from the trend. The period from 1973:1Q to 1983:4Q is shaded.
Figure 4: The upper panel shows the cross-correlation between oil price at t and output at t+j. The lower panel shows the response of output to a unit increase in oil price. The Y-axis denotes the deviation of the variables from the trend, and the X-axis denotes the quarter after the shock. The model-generated series are used for estimation. Confidence intervals are shown by dotted lines.
Figure 5: Upper panel indicates time path of $|d_o_t|$ where $d_o_t$ stands for the sum of the variations in the OPEC oil production and the U.S. oil production. The middle panel indicates that of $|d_A_t|$ where $d_A_t$ stands for the TFP variation. The lower panel indicates $(d_o_t)(d_A_t)$. Average of the time series for pre-1984 period is denoted by the dotted line, and that for post-1984 period is denoted by solid lines.
Figure 6: The time path of the oil price from simulation I (upper panel) and simulation II (lower panel). Actual data and the result from the benchmark simulation are depicted for comparison. Note that in benchmark simulation, all of the three exogenous variables are used for the simulation.
Figure 7: The time path of the output from simulation I (upper panel) and simulation II (lower panel). Actual data and the result from the benchmark simulation are depicted for comparison. Note that in benchmark simulation, all of the three exogenous variables are used for the simulation.
Figure 8: Impulse response of TFP to a unit decrease in the oil supply (sum of OPEC oil production and U.S. oil production). The Y-axis denotes the deviation of the variables from the trend, and the X-axis denotes the quarter after the shock. Confidence intervals are shown by dotted lines.