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Abstract

After empirically showing imperfect financial integration among the euro countries, *i.e.*, bank loan market heterogeneities in stickinesses of loan interest rates and markups from policy interest rate to loan rates, we build a New Keynesian model where such elements of imperfect financial integration coexist within a single currency area. Our welfare analysis reveals characteristics of optimal monetary policy. A central bank should take these heterogeneities into consideration. The optimal monetary policy is tied to difference in the degree of loan rate stickiness, the size of the steady-state loan rate markup, and the share of the loan market. By calibrating our model to the euro, we present the raking of the euro countries in terms of monetary policy priority. Because of the heterogeneity in the loan markets among the euro area countries, this ordering is not equivalent to the size of the financial market.

Keywords: optimal monetary policy; financial integration; heterogeneous financial market; staggered loan contracts

JEL classification: E44, E52

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1 Introduction

As emphasized in Bernanke *et al.* (1999), Ravenna and Walsh (2006), Teranishi (2008a), and Cúrdia and Woodford (2008), the structure of financial markets is very important as monetary policy is implemented through the financial system. They show that financial market properties influence the implementation of monetary policy.

After the introduction of the euro on January 1st 1999, all banking activities in the euro area have started to be conducted under a single monetary authority. Under this integration, the formerly segmented financial markets have become more synchronized thanks to cross-border transactions (Adam *et al.*, 2002; Cabral *et al.*, 2002; Baele *et al.*, 2004; ECB, 2008). The bank loan market integration, however, has remained incomplete (Mojon, 2000; Adam *et al.*, 2002; Baele *et al.*, 2004; Sørensen and Werner, 2006; Gropp and Kashyap, 2008; ECB, 2008; van Leuvensteijn *et al.*, 2008). Bank loan interest rates in the euro area differ substantially in their levels and their responses to the policy rate from country to country. For example, Adam *et al.* (2002) and ECB (2008) report the presence of large cross-country difference in the levels of the bank loan rates. Mojon (2000), Sørensen and Werner (2006), and van Leuvensteijn *et al.* (2008) find statistically significant cross-country differences in terms of the bank loan rate stickinesses.[1]

Figure 1 shows the time path of bank loan rates in the euro area together with that of the policy rate. The bank loan rates are the ones for outstanding loans, lent from banks to nonfinancial businesses with contract lengths of up to one year.[2] We see general comovements between the bank loan rates and the policy rate, but, at the same time, sizable dispersions in the bank loan rates across countries. Bank loan rates differ in their

[1] Many existing studies show that the bank loan interest rate and the money market rate comove in the long run, but the former interest rate is sluggish compared with the latter interest rate in the short run; that is, bank loan rates adjust to the policy interest rate with some lags. See Mojon (2000), De Bomdt *et al.* (2002), Weth (2002), Gambacorta (2008), and De Graeve *et al.* (2007) for sticky bank loan rates in the euro area. Slovin and Sushka (1983) and Berger and Udel (1992) examine the short-run pass-through of US bank loans and conclude that there is a sizable stickiness in US bank loan rates. BOJ (2007, 2008) find similar results for Japanese banks. Mojon (2000) also discusses the possible causes of loan rate stickiness in the retail banking sector.

[2] The data are taken from the harmonized national MFI interest rate statistics (MIR), released from the ECB. The series are available only from January 2003.

levels and in their dynamics. Figure 2 focuses more on the heterogeneity in the dynamics and shows changes in the bank loan rates from November 2005.[3] We confirm that except for a few countries, the bank loan rates increase following an increase in the policy rate with different time lags.

In spite of a number of empirical studies on the heterogeneities of the bank loan rates, theoretical investigations on financial market heterogeneity in the context of monetary policy are still limited. In this paper, we incorporate heterogeneous features of the bank loan rates into the New Keynesian framework to offer a tractable tool to investigate the role of heterogeneous bank loan markets. We derive the optimal monetary policy under heterogenous bank loan markets, thereby showing significant changes in the way that monetary policy is implemented. This is because the optimal response to particular loan rate dynamics do not secure an optimal policy response to other loan rate dynamics because of this heterogeneity.

The contribution of the paper can be summarized into four points below. First, we empirically measure the levels and stickinesses of the bank loan rates in the euro area. There are substantial cross-country differences for both measures of the bank loan rates. For the first measure, the levels of the bank loan rates vary reflecting the differences in loan rate markup from country to country, while the banks in the euro area refer to the common policy rate. For the second measure, we examine the cross-country differences in stickinesses of the bank loan rates by estimating error correction model.

Second, we develop a New Keynesian model that captures the observed features of the heterogeneity in the bank loan rates. We explicitly incorporate the banking industry and the bank loan rate following Barth and Ramey (2000) and Ravenna and Walsh (2006). Our framework, however, crucially differs from Ravenna and Walsh (2006) in the bank loan market structure. While complete markets are assumed in Ravenna and Walsh (2006), we consider an economy where banks face monopolistic competition and friction associated

[3] We subtract the bank loan rates at each period from their own levels in November 2005, the date one month before the ECB started to raise its policy rate.

with bank loan rate adjustment as in Teranishi (2008a). Moreover, banks are not identical in their degree of nominal loan rate rigidity, the size of the loan rate steady-state markup, and their share of the lending volume. In our model, the wedge between the loan rate and the policy rate is due to the imperfect competition among banks following Sander and Kleimeier (2004), Gropp *et al.* (2006), van Leuvensteijn *et al.* (2008) and Gropp and Kashyap (2008) that show the importance of bank competitions on the staggered loan rate setting and the loan rate markup.

Third, we analyze a structure of the optimal monetary policy rule in this economy using a second-order approximated welfare function. Our welfare analysis reveals that the central bank should take account of the several measures of heterogeneity in the bank loan markets; *i.e.*, credit spread between bank loan rates and the variations in each bank loan rate. A central bank attaches weight to such measures according to the degree of bank loan rate stickiness, the degree of the steady-state loan rate markup, and the size of the loan market. With sufficiently large bank loan rate stickiness or a small steady-state loan rate markup, a central bank should weight more heavily the variations in bank loan rates rather than the credit spreads between them. Moreover, loan markets with larger loan rate stickiness, with smaller steady-state loan rate markups, or with a larger share of lending volume are given higher significance by the central bank.

Fourth, using the results of the welfare analysis we quantitatively investigate the optimal monetary policy in the euro area. With the estimated country-specific bank loan rate stickinesses, the country-specific steady-state markups on the loan rates, and the financial market share of the countries, the relative importance of each national bank loan market is derived from the viewpoint of the optimal monetary policy. The monetary policy priority ranking is not the same as the ordering according to the size of financial markets. As the financial integration deepens, however, it is predicted that the ordering will converge to those associated with the size of the financial markets.

The paper is organized as follows. Section 2 reports the size and diversity of the bank loan rate stickinesses and the markups on the bank loan rates in the euro area. Section 3 describes our model with heterogenous banks. Section 4 analyzes the welfare implication of the model. Section 5 is devoted to investigating the properties of the optimal monetary policy. In Section 6, we calibrate the economy to the data from the euro area and illustrate the optimal response of the ECB to the shock in the bank loan rates. Section 7 concludes.

2 Empirical facts about loan interest rates in the euro area

As discussed above, there is ample empirical evidence of the heterogeneity among the bank loan rates in the euro area. There are level differences among the loan rates, as suggested by ECB (2008) and Gropp and Kashyap (2008), and the diversity in the loan rate stickinesses, as suggested by Mojon (2000), Sørensen and Werner (2006), and van Leuvensteijn *et al.* (2008). We focus on the fact that cross-country level differences are equivalent to the cross-country markup differences of the bank loan rates and report the ratio between the country-specific bank loan rates over the policy rate. For the loan rate stickiness, we employ an error correction model, following Sørensen and Werner (2006).

Our data set contains monthly interest rate data on outstanding (stock) loans from banks to enterprises, taken from MFI Interest Rate Statistics released by the ECB. The sample period extends from January 2003 to May 2008. We have chosen the interest rates on outstanding loans rather than the interest rates on new loans so as to be consistent with our model.[4] MFI Interest Rate Statistics provides time series data of the loan interest rates for different lengths of loan maturity. We use loans up to one year.[5]

2.1 Heterogeneous loan rate markup

As we have noted above, the observed cross-country bank loan rates differ in levels as shown in Figure 1. One of the reasons for the difference in the loan rate level is that private banks

[4] In the model, we assume that all the firms' expenses associated with their production is financed by borrowings from their banks. From this point of view, we consider that the interest rates on outstanding loans capture more of the important features of our model than the loan interest rates on the newly contracted loans. Moreover, because our model does not assume financial intermediation among households, we focus on the bank loan interest rate between banks and enterprises.

[5] The estimation results for the loans over one year and up to five years are available on request.

in different countries have different markups from the ECB's policy rate to their loan rates. For the bank loan rate in each country, we consider the time average of the ratio of the bank loan rate over the policy rate, from January 2003 to May 2008, as the steady-state loan rate markup. We assume that the steady-state loan rate markup is time invariant. The second column of Table 1 shows the loan rate markup of each country. We see that there is enough heterogeneity in the loan rate markup. For example, the markup on the loan rate is much higher in Italy than in Finland.[6] The markup in Italy is 2.6%, but it is 1.4% in Finland.

2.2 Heterogeneous loan rate stickiness

Not many studies focus on the diversity of the bank loan rate stickiness across countries. Sørensen and Werner (2006) is one exception. Following Sørensen and Werner (2006), but with an extended sample period, we estimate the country-specific loan rate stickiness for 12 euro countries.

Our error correction model is described as follows:

$$\triangle \widehat{R}_{jt} = \alpha_j \left(\widehat{R}_{jt} - c_j - \beta_j \widehat{i}_t \right) + \sum_{k=1}^{p} \phi_{jk} \triangle \widehat{R}_{jt-k} + \sum_{k'=1}^{q} \varphi_{jk'} \triangle \widehat{i}_{t-k} + u_{jt},$$

where $\widehat{R}_{jt}$ denotes the bank loan interest rate of country j, $\widehat{i}_t$ denotes the ECB's policy interest rate, and c_j, α_j, β_j, ϕ_{jk}, $\varphi_{jk'}$ for $k = 1, ...p$ and $k' = 1, ...q$, are the estimated country-specific parameters. We assume $p = q = 2$ for the estimations. Among the estimated parameters, α_j governs the speed of the bank loan rate adjustment for country j by which we assess the country-specific stickiness of the bank loan rates.

The third column of Table 1 reports coefficients α_j for euro countries, where the numbers in parentheses are t values. The coefficient α_j being closer to -1.0 indicates that the bank loan rate of a country j adjusts quicker to a change in the policy rate. The first observation

[6] From Subsection 2.2, we can estimate the markups by c_j and β_j. However, in this case, the markups change according to the level of the policy interest rate. Thus, we use the average difference as an average measure of the markups.

from the table is that for all the countries, the bank loan rates show some degrees of stickiness. None of the coefficients α_j is below -0.5. The second observation is that there is a huge variety of loan rate stickiness across countries. For example in Germany, more than 40% of the deviation from the long-run relationship is canceled in the next period, while less than 10% of the deviation is canceled in Spain.

3 Model

Our model consists of five types of agents: two types of private banks, a consumer, firms, and a central bank. Banks finance firms' production by loan contracts. Existing studies of the cost channel, including Ravenna and Walsh (2006), investigate a case where the bank loan market is perfectly competitive. We assume that the bank loan market is monopolistically competitive and the loan interest rate contract is set under the Calvo pricing scheme following Teranishi (2008a). The loan rates in our model are thus sticky compared with the policy rate, as we observed from the data. Moreover, there are two types of loans, and each of them is set by the different types of banks. The two types of loans are distinct in their stickiness so that one loan rate adjusts quicker than the other does.

3.1 Demand for bank loans

We first describe how the volumes of the bank loans are determined in our model. Our economy includes the two types of banks, two types of workers, and one type of firm. Bank loan contracts are made between the banks and the firms. The loan contracts finance the firms' expenses for hiring workers. The interest rates on the bank loans are determined by banks in monopolistically competitive markets.

Private banks are differentiated and categorized into the two types, depending on the stickiness of their loan rates. The banks that provide more sticky loans populate over $h_M \in [0, n_M)$, and the banks that provide less sticky loans populate over $h_L \in [0, n_L]$. We assume that the sum of n_M and n_L is unity. We call the former type M banks and the latter type L banks. Workers are also differentiated. Similarly to the banks, they are categorized into two types, type M workers and type L workers. Type M workers populate over $h_M \in [0, n_M)$, and type L worker populate over $h_L \in [0, n_L]$.

Firm $f \in [0,1]$ optimally hires both types of workers as price takers and sells the differentiated final goods $y(f)$ as a monopolistically competitive producer. The types of workers and bank loans are linked. In order to finance the hiring cost of type M workers, firms need to borrow from a type M bank. Similarly, in order to finance the wages for type L workers, firms must borrow from type L banks. Firm f constructs the subcomposite of labor inputs for each type of worker, which we denote by $L_{M,t}(f)$ and $L_{L,t}(f)$, and aggregate them to $\widetilde{L}_t(f)$.[7] Firms employ both types of workers so that the two types of bank loans are both tied to the firm f.[8] One way to think about this specification is to consider that the two types of business units associated with the labor types in the firms are financed by the different loans. Note that the private banks monopolistically compete with each other both among the same type of bank and between the different types of bank in our model.

The first step in the cost-minimization problem for firm f with respect to the allocation of type j worker, for $j = L, M$, is given by:

$$\min_{l_{j,t}(h_j,f)} \int_0^{n_j} [1 + r_{j,t}(h_j)] \, w_{j,t}(h_j) \, l_{j,t}(h_j, f) dh_j,$$

subject to the size of the subcomposite of type j labor input to firm f, $L_{j,t}(f)$:

$$L_{j,t}(f) \equiv \left[\left(\frac{1}{n_j} \right)^{\frac{1}{\epsilon_j}} \int_0^{n_j} l_{j,t}(h_j, f)^{\frac{\epsilon_j - 1}{\epsilon_j}} dh_j \right]^{\frac{\epsilon_j}{\epsilon_j - 1}},$$

where $r_{j,t}(h_j)$ is the interest rate applied on the loan contract between firm f and bank h_j in type j banks, $l_{j,t}(h_j, f)$ is the labor h_j of type j workers hired by firm f, $w_{j,t}(h_j)$ is the nominal wage for hiring the labor h_j of type j workers, and ϵ_j is a preference parameter on

[7] The same structure is assumed for employment in Woodford (2003).

[8] In the current paper, we assume that firms finance all of their expenditure for their inputs by bank loans. Teranishi (2008a) relaxes this assumption and develops a setting where firms finance part of the production cost by loans.

differentiated laborers. ϵ_j governs both the wage markup and the markup from the policy rate to the loan rate in our model.

The relative demand for worker $l_{j,t}(h_j, f)$ is given as follows:

$$l_{j,t}(h_j, f) = \frac{1}{n_j} L_{j,t} \left\{ \frac{[1 + r_{j,t}(h_j)] \, w_{j,t}(h_j)}{\Omega_{j,t}} \right\}^{-\epsilon_j}, \tag{1}$$

where:

$$\Omega_{j,t} \equiv \left\{ \frac{1}{n_j} \int_0^{n_j} \{[1 + r_{j,t}(h_j)] \, w_{j,t}(h_j)\}^{1-\epsilon_j} \, dh_j \right\}^{\frac{1}{1-\epsilon_j}}. \tag{2}$$

As a result, the total cost of hiring h_j of type j workers for firm f is expressed by:

$$\int_0^{n_j} [1 + r_{j,t}(h_j)] \, w_{j,t}(h_j) \, l_{j,t}(h_j, f) \, dh_j = \Omega_{j,t} L_{j,t}(f).$$

The two optimal conditions above ensure the optimal allocation of $l_{j,t}(h_j, f)$ for $j = M, L$ with $L_{j,t}(f)$ for $j = M, L$ provided. For $j = M, L$, the firm f's optimal allocations associated with $L_{j,t}(f)$ are obtained by the second step of the cost-minimization problem described below:

$$\min_{L_{M,t}, L_{L,t}} \sum_{j=M,L} \Omega_{j,t} L_{j,t}(f),$$

subject to the quantity of total labor input, which we assume as:

$$\widetilde{L}_t(f) \equiv \prod_{j=M,L} \frac{[L_{j,t}(f)]^{n_j}}{n_j^{n_j}}.$$

Then, the relative demand functions for each labor composite are derived as follows:

$$L_{j,t}(f) = n_j \widetilde{L}_t(f) \left(\frac{\Omega_{j,t}}{\widetilde{\Omega}_t} \right)^{-1}, \tag{3}$$

where:

$$\widetilde{\Omega}_t \equiv \prod_{j=M,L} \Omega_{j,t}^{n_j}.$$

Using the relationships above, the following equations are obtained:

$$\Omega_{M,t} L_{M,t}(f) + \Omega_{L,t} L_{L,t}(f) = \widetilde{\Omega}_t \widetilde{L}_t(f), \tag{4}$$

$$l_{j,t}(h_j, f) = \left\{ \frac{[1 + r_{j,t}(h_j)]\, w_{j,t}(h_j)}{\Omega_{j,t}} \right\}^{-\epsilon_j} \left(\frac{\Omega_{j,t}}{\widetilde{\Omega}_t} \right)^{-1} \widetilde{L}_t(f), \text{ for } j = M, L. \tag{5}$$

Equation (4) denotes the cost share of each type of worker in the total cost expenditure of firm f. $\widetilde{\Omega}_t \widetilde{L}_t(f)$ stands for the total cost, and $\Omega_{j,t} L_{j,t}(f)$ stands for the cost of hiring type j workers. Equation (5) indicates the demand function for the differentiated labor input $l_{j,t}(h_j, f)$. Note that the demand for each differentiated worker depends on wages $w_{j,t}(h_j)$, loan interest rates $r_{j,t}(h_j)$, and the relative price of the subcomposite of labor input $\Omega_{j,t}$, given the total demand for labor $\widetilde{L}_t(f)$.

Finally, we can derive the demand function for bank loans associated with hiring of differentiated labor h_j of type j, borrowed by firm f as follows:

$$\begin{aligned} q_{j,t}(h_j, f) &= w_{j,t}(h)\, l_{j,t}(h_j, f) \\ &= w_{j,t}(h_j) \left\{ \frac{[1 + r_{j,t}(h_j)]\, w_{j,t}(h_j)}{\Omega_{j,t}} \right\}^{-\epsilon_j} \left(\frac{\Omega_{j,t}}{\widetilde{\Omega}_t} \right)^{-1} \widetilde{L}_t(f), \text{ for } j = M, L. \end{aligned}$$

This condition demonstrates that the demand for each differentiated loan $q_{j,t}(h_j, f)$ depends on the wages, loan interest rates, and relative price of the labor subcomposite given the total labor demand. With the two-step cost minimization, the private banks monopolistically compete both against the same type of bank and between the two different types.

For aggregate labor demand conditions, we obtain following expression:

$$\widetilde{L}_t = \int_0^1 \widetilde{L}_t(f)\, df.$$

3.2 Consumer

A representative consumer derives utility from consumption, and disutility from the supply of work. The consumer maximizes the following utility function:

$$UT_t = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T) - \sum_{j=M,L} \int_0^{n_j} V(l_{j,T}(h_j)) dh_j \right] \right\},$$

where E_t is an expectation conditional on the state of nature at data t. The function U and V are increasing and concave in the consumption index and the labor supply, respectively. The budget constraint of the consumer is given by:

$$\begin{aligned} P_t C_t + \mathrm{E}_t \left[X_{t,t+1} B_{t+1} \right] + D_t &\leq B_t + (1 + i_{t-1}) D_{t-1} \\ &\quad + \sum_{j=M,L} \frac{1}{\tau_j} \int_0^{n_j} w_{j,t}(h_j) l_{j,t}(h_j) dh_j + \Pi_t^B + \Pi_t^F + T_t, \end{aligned} \tag{6}$$

where B_t is a risky asset, D_t is the amount of bank deposits, τ_j is the tax for wage income, T_t is the lump-sum tax, and i_t is the nominal deposit rate set by a central bank from t to $t+1$. $\Pi_t^B = \int_0^1 \Pi_{t-1}^B(h) dh$ is the nominal dividend stemming from the ownership of the banks, $\Pi_t^F = \int_0^1 \Pi_{t-1}^F(f) df$ is the nominal dividend from the ownership of the firms, and $X_{t,t+1}$ is the stochastic discount factor. We assume a complete financial market for risky assets. Thus, we can hold a unique discount factor and can characterize the relationship between the deposit rate and the stochastic discount factor as:

$$\frac{1}{1+i_t} = \mathrm{E}_t \left[X_{t,t+1} \right]. \tag{7}$$

We assume that the household has Dixit–Stiglitz preferences (Dixit and Stiglitz, 1977), where aggregate consumption C_t is linked to the differentiated goods $c_t(f)$ produced by a firm $f \in [0,1]$ by the following equation:

$$C_t \equiv \left[\int_0^1 c_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the elasticity of substitution across goods. The aggregate consumption-based price index P_t is defined as:

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}},$$

where $p_t(f)$ is the price of differentiated good $c_t(f)$. The demand function for $c_t(f)$ is derived from the cost-minimization behavior of the consumer as:

$$c_t(f) = C_t \left[\frac{p_t(f)}{P_t}\right]^{-\theta}. \tag{8}$$

Given the optimal allocation of consumption expenditure across the differentiated goods, the consumer must choose the total amount of consumption, the optimal amount of risky assets to hold, and an optimal amount to deposit in each period. Necessary and sufficient conditions are given by:

$$U_C(C_t) = \beta(1+i_t)\mathrm{E}_t\left[U_C(C_{t+1})\frac{P_t}{P_{t+1}}\right], \tag{9}$$

$$\frac{U_C(C_t)}{U_C(C_{t+1})} = \frac{\beta}{X_{t,t+1}}\frac{P_t}{P_{t+1}}.$$

Equations (7) and (9) express the intertemporal optimal allocation on aggregate consumption. Assuming that the goods market clears for all $f \in [0,1]$, the standard New Keynesian IS curve is derived by log-linearizing equation (9):

$$x_t = E_t x_{t+1} - \sigma(\widehat{i}_t - E_t\pi_{t+1}), \tag{10}$$

where x_t and π_{t+1} are the output gap and inflation, respectively. The definition of the output gap is given in the next subsection. We use $\widehat{m}_t$ to denote the percentage deviation of the variable m_t around the nonstochastic steady state. σ is defined by $\sigma \equiv -\frac{U_Y}{U_{YY}\overline{Y}} > 0$.

A consumer provides differentiated types of labor to firms, holding the power to decide the wage of each type of labor as assumed in Erceg *et al.* (2000). Given the labor demand function of firms discussed earlier, a consumer sets each wage $w_{j,t}(h_j)$ for any type j worker in every period to maximize its utility subject to the budget constraint (6).[9] This yields the following relations for the type j worker:

$$\frac{w_{j,t}\left(h_j\right)}{P_t} = \frac{1}{\tau_j}\frac{\epsilon_j}{\epsilon_j - 1}\frac{V_l\left[l_{j,t}\left(h_j\right)\right]}{U_C\left(C_t\right)}. \tag{11}$$

3.3 Firms

Given the cost of hiring labor inputs $\widetilde{\Omega}_T \widetilde{L}_T\left(f\right)$ discussed above, firms set the price of their products optimally. We assume that firms face a monopolistically competitive market as in Calvo (1983) and Yun (1996) (henceforth Calvo-Yun setting). That is, facing downward-sloping demand curves, firms set differentiated goods prices in a staggered manner a la Calvo-Yun setting.

A firm $f \in [0,1]$ maximizes the present discounted value of profit, which is given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T}\left[p_t\left(f\right) y_{t,T}\left(f\right) - \widetilde{\Omega}_T \widetilde{L}_T\left(f\right)\right], \tag{12}$$

where $(1-\alpha)$ is the probability that the firm can reset its price. We assume the production function of the firm f as $y_t(f) = F(\widetilde{L}_T\left(f\right))$, where $F\left(\right)$ is increasing and concave. The Dixit–Stiglitz preferences implies that equation (12) can be written as:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T}\left\{p_t\left(f\right)\left[\frac{p_t\left(f\right)}{P_T}\right]^{-\theta} C_T - \widetilde{\Omega}_T \widetilde{L}_T\left(f\right)\right\}.$$

The optimal prices $\overline{p}_t\left(f\right)$ set by the active firms are given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \left(\alpha\beta\right)^{T-t} \frac{U_C\left(C_T\right)}{P_T} y_{t,T}\left(f\right)\left[\frac{\theta-1}{\theta}\overline{p}_t\left(f\right) - \widetilde{\Omega}_T \frac{\partial \widetilde{L}_T\left(f\right)}{\partial y_{t,T}\left(f\right)}\right] = 0, \tag{13}$$

where we substitute equation (8). Further substituting equation (11) into equation (13)

[9] In contrast to Erceg *et al.* (2000), however, we assume no sticky wages so that the consumers can change their wages in every period.

leads to:

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T)\, y_{t,T}(f) \left[\frac{\theta-1}{\theta} \frac{\overline{p}_t(f)}{P_t} \frac{P_t}{P_T} - \left(\frac{\epsilon_M}{\epsilon_M - 1}\right)^{n_M} \left(\frac{\epsilon_L}{\epsilon_L - 1}\right)^{n_L} Z_{t,T}(f) \right] = 0, \tag{14}$$

where:

$$Z_{t,T}(f) = \prod_{j=M,L} \left\{ \left(\frac{1}{n_j}\right) \int_0^{n_j} \left[1 + r_{j,t}(h_j)\right]^{1-\epsilon_j} \left\{ \frac{V_l\left[l_{j,T}(h_j)\right]}{U_C(C_t)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)} \right\}^{1-\epsilon_j} dh_j \right\}^{\frac{n_j}{1-\epsilon_j}}.$$

By log-linearizing equation (14), we derive:

$$\frac{1}{1-\alpha\beta} \widehat{\widetilde{p}}_t(f) = \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\sum_{\tau=t+1}^{T} \pi_{H,\tau} + n_M \widehat{R}_{M,T} + n_L \widehat{R}_{L,T} + \widehat{mc}_{t,T}(f) \right]. \tag{15}$$

We define the real marginal cost as:

$$\widehat{mc}_{t,T}(f) \equiv \sum_{j=M,L} \int_0^{n_j} \widehat{mc}_{j,t,T}(h_j, f)\, dh_j,$$

where:

$$mc_{j,t,T}(h_j, f) \equiv \frac{V_l\left[l_{j,T}(h_j)\right]}{U_Y(C_T)} \frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)} \text{ for } j = M, L.$$

We also define for $j = L, M$:

$$R_{j,t} \equiv \frac{1}{n_j} \int_0^{n_j} r_{j,t}(h_j)\, dh_j, \tag{16}$$

$$\widetilde{p}_t(f) \equiv \frac{\overline{p}_t(f)}{P_t} \text{ and } \pi_t \equiv \frac{P_t}{P_{t-1}}.$$

Then, equation (15) can be transformed into:

$$\frac{1}{1-\alpha\beta} \widehat{\widetilde{p}}_t(f) = \mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1+\omega_p\sigma)^{-1} \left(\widehat{mc}_T + n_M \widehat{R}_{M,T} + n_L \widehat{R}_{L,T} \right) + \sum_{\tau=t+1}^{T} \pi_\tau \right], \tag{17}$$

where we make use of the relationship:

$$\widehat{mc}_{t,T}(f) = \widehat{mc}_T - \omega_p \theta \left[\widehat{\overline{p}}_t(f) - \sum_{\tau=t+1}^{T} \pi_\tau \right],$$

where ω_p is the elasticity of $\frac{\partial \widetilde{L}_{t,T}(f)}{\partial y_{t,T}(f)}$ with respect to y. We further denote the average real marginal cost as:

$$\widehat{mc}_T \equiv \sum_{j=M,L} \int_0^{n_j} \widehat{mc}_{j,T}(h_j)\, dh_j,$$

where for $j = M, L$:

$$mc_{j,T}(h_j) \equiv \frac{V_l [l_{j,T}(h_j)]}{U_Y(C_T)} \frac{\partial \widetilde{L}_T}{\partial Y_T}.$$

The point is that the unit marginal cost is the same for all firms because each firm pays for all the types of labor and associated loans with the same proportion. Thus, all the firms set the same price if they have a chance to reset their prices at time t.

In the Calvo-Yun setting, the aggregate price index P_t evolves by:

$$P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(\overline{p}_t)^{1-\theta}, \tag{18}$$

where:

$$P_t^{1-\theta} \equiv \int_0^1 p_t(f)^{1-\theta} df \text{ and } \overline{p}_t^{1-\theta} \equiv \int_0^1 \overline{p}_t(f)^{1-\theta} df.$$

Log-linearizing equation (18) and equation (17) leads to the following New Keynesian Phillips curve:

$$\pi_t = \chi \left(\widehat{mc}_t + n_M \widehat{R}_{M,t} + n_L \widehat{R}_{L,t} \right) + \beta \mathrm{E}_t \pi_{t+1}, \tag{19}$$

where the slope coefficient $\chi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega_p\theta)}$ is a positive parameter. The equation is similar to the standard New Keynesian Phillips curve, but it contains the terms associated with the loan interest rates.

Here, according to the discussion in Woodford (2003), we define the natural rate of output Y_t^n from equation (14) as:

$$\frac{\theta-1}{\theta} = \prod_{j=M,L}\left(\frac{\epsilon_j}{\epsilon_j-1}\right)^{n_j}\left[1+\overline{R}_j\right]^{n_j} \times\left\{\left(\frac{1}{n_j}\right)\int_0^{n_j}\left\{\frac{V_l\left[l_{j,t}^{n_j}(h_j)\right]}{U_C(C_t)}\frac{\partial\widetilde{L}_t^n(f)}{\partial Y_t^n(f)}\right\}^{1-\epsilon j} dh_j\right\}^{\frac{n_j}{1-\epsilon_j}},$$

where we assume a flexible price setting, $p_t^*(f)=P_t$, and assume no impact of monetary policy, $r_{L,t}(h_L)=r_{M,t}(h_M)=\overline{R}$, and so hold $y_t(f)=Y_t^n$ under the natural rate of output. $l_{j,t}^{n_j}(h_j)$ for $j=L,M$ and $\widetilde{L}_t^n(f)$ are the amount of labor corresponding to Y_t^n, respectively. Then, we have:

$$\widehat{mc}_t=(\omega+\sigma^{-1})(\widehat{Y}_t-\widehat{Y}_t^n),$$

where $\widehat{Y}_t\equiv\ln(Y_t/\overline{Y})$, $\widehat{Y}_t^n\equiv\ln(Y_t^n/\overline{Y})$, and ω is the sum of the elasticity of the marginal disutility of work with respect to output increase and the elasticity of $\frac{1}{F'(F^{-1}(y))}$ with respect to output increase.[10] Then, by defining $x_t\equiv\widehat{Y}_t-\widehat{Y}_t^n$, we finally have:

$$\pi_t=\kappa x_t+\chi\left(n_M\widehat{R}_{M,t}+n_L\widehat{R}_{L,t}\right)+\beta\mathrm{E}_t\pi_{t+1}, \tag{20}$$

where $\kappa\equiv\chi(\omega+\sigma^{-1})$.

3.4 Private banks

The pricing strategy for the private banks is described in this subsection. Two types of banks collect deposits from consumers at the interest rate $1+i$, and each bank h_j of type $j\in[M,L]$ makes bank loan contracts of volume $q_{j,t}(h_j,f)$ for firm f. The loans are differentiated and supplied in a monopolistically competitive manner. The banks are subject to a nominal rigidity in adjusting their bank loan rates. We assume that bank h_j

[10] $\omega\equiv\omega_p+\omega_w$, where ω_w is the elasticity of marginal disutility of work with respect to output increase in $\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)}$. A more detailed derivation is provided in Woodford (Ch. 3, 2003).

of type j can reset loan rates with probability $\left(1-\varphi^{j}\right)$ following the Calvo-Yun setting.

Bank h_j of type j chooses the loan interest rate $r_{j,t}\left(h_j\right)$ to maximize the present discounted value of profit:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \left(\varphi^{j}\right)^{T-t} X_{t,T} q_{j,t,T}\left(h_j, f\right) \left\{\left[1+r_{j,t}\left(h_j\right)\right]-\left(1+i_T\right)\right\}.$$

The optimal loan rate condition is now given by:

$$\mathrm{E}_t \sum_{T=t}^{\infty} \left(\varphi^{j}\beta\right)^{T-t} \frac{P_t}{P_T}\frac{U_C\left(C_T\right)}{U_C\left(C_t\right)} q_{j,t,T}\left(h_j\right) \left\{\left[1+r_{j,t}\left(h_j\right)\right]-\epsilon_j\left\{\left[1+r_{j,t}\left(h_j\right)\right]-\left(1+i_T\right)\right\}\right\}=0. \tag{21}$$

Because the active banks set the same loan interest rate, the aggregate loan interest rate index $R_{j,t}$ evolves as follows:

$$1+R_{j,t}=\varphi^{j}\left(1+R_{j,t-1}\right)+\left(1-\varphi^{j}\right)\left(1+\overline{r}_t\right). \tag{22}$$

Log-linearizing equations (21) and (22) gives the following loan rate curve for the type j banks:

$$\widehat{R}_{j,t}=\lambda_1^{j}\mathrm{E}_t\widehat{R}_{j,t+1}+\lambda_2^{j}\widehat{R}_{j,t-1}+\lambda_3^{j}\widehat{i}_t, \tag{23}$$

where $\lambda_1^{j}\equiv\frac{\varphi^{j}\beta}{1+\left(\varphi^{j}\right)^2\beta}$, $\lambda_2^{j}\equiv\frac{\varphi^{j}}{1+\left(\varphi^{j}\right)^2\beta}$, and $\lambda_3^{j}\equiv\frac{1-\varphi^{j}}{1+\left(\varphi^{j}\right)^2\beta}\frac{\epsilon_j}{\epsilon_j-1}\frac{\left(1-\beta\overline{\varphi}^{j}\right)\left(1+\overline{i}\right)}{1+\overline{R}_j}$ for $j=M,L$ are all positive.

Finally, the market clearing conditions for bank loans j for $j=M,L$ are:

$$q_{j,t,T}(h_j)=\int_0^1 q_{j,t,T}(h_j,f)df\ ,$$

$$\int_0^{n_j} q_{j,t,T}(h_j)dh_j=n_jD_T \text{ for } j=M,L.$$

3.5 System of equations

The linearized system of equations consists of five equations: (10), (20), (23), and an optimal monetary policy derived in the following sections for five endogenous variables: x,

π, $\widehat{i}$, $\widehat{R}_M$, and $\widehat{R}_L$.

4 Optimal monetary policy with heterogeneous loans

4.1 Approximated welfare function

In this section, we derive a second-order approximation to the welfare function following Woodford (2003). The detail of derivation is shown in Appendix A.

Consumer welfare is given by:

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t UT_t = \mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^{T-t}\left[U(C_t) - \int_0^{n_M} V(l_{M,t}(h_M))dh_M - \int_0^{n_L} V(l_{L,t}(h_L))dh_L\right]\right\}.$$

Then, we have a second-order approximated loss function expressed as follows:

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t UT_t \simeq -\Lambda \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t \left(\begin{array}{c} \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_{ML}\left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right)^2 \\ +\lambda_M\left(\widehat{R}_{M,t} - \widehat{R}_{M,t-1}\right)^2 + \lambda_L\left(\widehat{R}_{L,t} - \widehat{R}_{L,t-1}\right)^2 \end{array}\right), \quad (24)$$

where λ_π, λ_x, λ_M, λ_L, and λ_{ML} are positive parameters.[11]

It is important to note that equation (24) indicates that the central bank should take account of the heterogeneity in the bank loan market as well as the aggregate variables such as inflation rate π_t and the output gap x_t. The central bank minimizes the variations of each bank loan rate, the last two terms in equation (24), and the credit spread between them, the third term in equation (24). Apart from the aggregate variables, the relative significance among them depends on the sizes of λ_j for $j = M, L$ and λ_{ML}. Because of the heterogeneity in the financial market, the stabilization policy for one loan rate acts as the destabilization policy for another loan rate. This is a trade-off in the financial market in conducting monetary policy. Details of this trade-off are discussed in the next subsection. For illustrative purposes, consider an economy where bank types are homogenous, where only one type of loan is available, so that their loan interest rates respond to a policy

[11] Note that we assuem the wage tax as $\frac{1}{\tau_j} = \left(\frac{\epsilon_j}{\epsilon_j - 1}\right)^2$ to realize $\overline{l_L}(h_L) = \overline{l_L}(h_L) = \overline{\overline{L}}$ from an optimization by a social planner in the steady state. See the detail in Appendix A.

interest rate uniformly. Then it is the case that $\widehat{R}_{M,t} = \widehat{R}_{L,t} = \widehat{R}_t$ and $\lambda_M = \lambda_L = \lambda$, and the loss function is reduced to:

$$\sum_{t=0}^{\infty} \beta^t UT_t \simeq -\Lambda \sum_{t=0}^{\infty} \beta^t \left(\lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda \left(\widehat{R}_t - \widehat{R}_{t-1} \right)^2 \right). \tag{25}$$

In this equation, neither the credit spread term nor the term of the heterogeneous bank loan variations is present. No trade-off regarding the financial market emerges in conducting the monetary policy. Comparing this economy, the optimal monetary policy becomes more complicated in an economy with heterogenous banks.

4.2 Priority in monetary policy

With heterogenous banks, an optimal response to a cost shock in one loan interest rate curve does not secure an optimal response to another loan interest rate curve. Thus, there is a clear trade-off in conducting the optimal monetary policy, and a central bank needs to determine the ordering among the bank loan curves.

To see this point, consider a response of a central bank to a cost-rise shock that hits the bank loan rate. We assume that the shock $u_{j,t}$ enters in the laws of motion for the bank loan rates curve in the following way:

$$\widehat{R}_{j,t} = \lambda_1^j \mathrm{E}_t \widehat{R}_{j,t+1} + \lambda_2^j \widehat{R}_{j,t-1} + \lambda_3^j \widehat{i}_t + u_{j,t} \text{ for } j = M, L.$$

For illustrative purposes, we assume that $u_{M,t}$ and $u_{L,t}$ are not correlated.

Before showing details of the simulation, we analytically demonstrate the trade-off in the financial market. When we observe a positive idiosyncratic shock $u_{M,t} > 0$ ($u_{L,t} = 0$) in the bank loan market of type M, equation (24) leads to a decline in the policy interest rate $\widehat{i}_t$, so as to accommodate $u_{M,t}$. It, however, operates as a disturbance in the type L bank loan curve. In particular:

$$(\widehat{R}_{M,t} - \widehat{R}_{M,t-1})^2 = \left\{ \left(1 - \lambda_1^M F - \lambda_2^M L\right)(1 - L) \left[\lambda_3^M \widehat{i}_t + u_{M,t}\right] \right\}^2,$$

$$(\widehat{R}_{L,t} - \widehat{R}_{L,t-1})^2 = \left\{ \left(1 - \lambda_1^L F - \lambda_2^L L\right)(1-L)\left[\lambda_3^L \widehat{i}_t\right] \right\}^2 .$$

The equations illustrate the trade-off originating from the central bank's welfare function. As we see in equation (24), a central bank adjusts its policy rate to a shock in the bank loan rate curve so as to reduce the LHS of these equations. We learn from the first equation that a central bank lowers the policy rate i_t in order to offset the variations caused by $u_{M,t}$. Lowering $\widehat{i}_t$, however, leads to larger variations in the LHS in the second equation. The relative significance of those two LHSs is dependent on the welfare weight λ_j for $j = M, L$.[12] It should be noted that even for the common shock, *i.e.*, $u_{M,t} = u_{L,t} \neq 0$, there is a trade-off in conducting monetary policy when the heterogeneous bank loan markets are present.

In this subsection, we quantitatively investigate the relative significance of each term that appears in the equation above. As we discussed above, the banks in the current model differ from each other in terms of three parameters: the degree of loan rate stickiness φ^j, the size of the steady-state loan rate markup $\frac{\epsilon_j}{\epsilon_j - 1}$, and the financial market share n_j of the lending volume. We consider the commitment of optimal monetary policy under a timeless perspective in the sense of Woodford (2003).[13] Appendix B shows the optimal targeting rule in this model. For the parameterization on the stickiness of the bank loan rates, we follow the results obtained in Slovin and Sushka (1983). From the time series analysis of the US commercial loans, they obtain that, on average, bank loan rates need at least two quarters and perhaps more to adjust to a change in the market interest rate. Thus we set the average contract duration of less sticky loan interest rates $R_{L,t}$ as $\varphi^L = 0.5$ (two quarters) and set $\varphi^M = 0.6$ for the more sticky loan interest rate $R_{M,t}$. Other parameters are borrowed from Rotemberg and Woodford (1997), and reported in Table 2.

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[12] As we discussed in the previous section, when the bank loan rates are nearly flexible, the response of the central bank to the credit spread shock is ambiguous because of the credit spread term. This point is summarized in Teranishi (2008b) in terms of the spread-adjusted Taylor rule.

[13] In this setting, as shown in Woodford (2003), the central bank conducts the monetary policy in a forward-looking way by paying attention to future economic variables and by taking the effect of the monetary policy on those future variables into account.

4.2.1 Sensitivity to loan rate stickiness

Figure 3 reports the values of relative weights $\frac{\lambda_L}{\lambda_x}$ and $\frac{\lambda_{ML}}{\lambda_x}$ for various sizes of bank loan rate stickiness φ^L.[14] The upper panel displays the case when φ^L is sufficiently small, and the lower panel shows the case when φ^L is large. It is clear from the figure that the welfare weight on the variation in the bank loan rate set by the type L bank, $\frac{\lambda_L}{\lambda_x}$, is monotonically increasing in φ^L, while $\frac{\lambda_{ML}}{\lambda_x}$ is invariant to changes in φ^L. A reason for this monotonic increase is that higher stickiness of the loan rate leads to larger dispersion among labor inputs when a shock hits the economy. Our welfare function indicates that higher labor dispersion is tied to lower quantity of goods production accompanied by lower utility from consumption and higher labor disutility.[15] When the bank loan interest rate φ^L is sufficiently small, the prime concern for the central bank is the spread between the more sticky bank loan rate and the less sticky bank loan rate rather than the variations of each bank loan rate. As φ^L becomes larger, it is more relevant for the central bank to concentrate on the variations of the individual loan rate rather than the spread.

As we will see below, for all the countries in the euro area, the estimated size of the bank loan rate stickiness φ^J is far above the level under which the credit spread term has a larger significance for the central bank. The same is true for other developed countries, such as the US (Slovin and Sushka, 1983; Berger and Udell, 1995) and Japan (BOJ, 2007; BOJ, 2008). Therefore, in implementing monetary policy in the current environment, a central bank should attach higher weight to the variations of the loan interest rates in bank loan markets. Our results for the credit spread term have, however, important implications for the future. Existing studies about the banking sectors agree with the view that bank loan rate stickiness is diminishing. For example, Ito and Ueda (1981), Sander and Kleimeier (2004), Gropp *et al.* (2006), and van Leuvensteijn *et al.* (2008) argue that the deregulation of bank loan markets accompanied by higher competition among banks leads to lower loan rate stickiness. Thus, with a competitive financial market, a central bank should pay more

[14] We show the relative weights of λ_M and λ_{ML} to λ_x because λ_π changes according to the price stickiness.

[15] See Appendix A for details.

attention to the credit spread rather than the variation in bank loan rates.

Figure 4 illustrates the simulation where we impose idiosyncratic shocks on the bank loan interest rate curves. We assume a unit cost rise for the bank loan rates, u_j with persistence of 0.9. In the upper panel, we show the impulse responses of the policy rate and the bank loan rate set for two types of banks, to a positive shock in the type M loan interest rate, u_M, maintaining $u_L = 0$. Similarly, the lower panel displays the impulse responses of the variables to a positive shock in the type L loan u_L, maintaining $u_M = 0$. Two features are observed from the panels. First, even with the trade-off problem originating from the heterogeneity of the loan interest rate stickiness, the central bank lowers the policy rate when a positive shock hits the economy.[16] Second, a central bank lowers the policy rate more in response to a shock in a type M bank loan rate than to a shock in a type L bank loan rate. The larger monetary easing implies a larger weight on the bank loan markets hit by the shock because it induces a smaller positive response in the loan rate that is directly hit by the shock but a larger negative response in the loan rate that is not directly hit by the shock. This is a trade-off to the financial market in conducting monetary policy. The simulation result shows that the central bank should put its monetary policy priority on the more sticky loan rate dynamics than the less sticky loan rate dynamics, even when the same size shocks hit the economy. As we discussed above, one reason for this is that the welfare weight increases as the loan rate stickiness increases. The central bank has an incentive to avoid a large fluctuation in bank loans with a higher λ_j. Another reason is that the same size shock creates a larger fluctuation in the loan rate curve with more stickiness because the inertia in the loan interest rate dynamics increases as the loan rate stickiness increases in the loan rate curves.[17]

[16] This optimal monetary policy response to a shock in the bank loan interest rate curve is consistent with the actual behavior of the central banks. For example, the FRB has lowered the policy interest rate in the subprime mortgage crisis from the fall of 2007, in order to mitigate the rise in the premium (see Taylor, 2008).

[17] As φ^J increases, the parameter related to inertia in equation (23) increases.

4.2.2 Sensitivity to steady-state markup from policy rate to loan rate

Figure 5 reports the values of relative weights $\frac{\lambda_L}{\lambda_x}$ and $\frac{\lambda_{ML}}{\lambda_x}$ for various sizes of bank loan rate stickiness φ^L, changing the size of ϵ_L. Because the steady-state markup is $\frac{\epsilon_L}{\epsilon_L - 1}$, this parameter governs a markup from the policy rate to the loan rate. As ϵ_L increases (the labor difference decreases), the steady-state markup decreases. As the degree of the steady-state markup decreases, the weight of $\frac{\lambda_L}{\lambda_x}$ increases. $\frac{\lambda_{ML}}{\lambda_x}$, however, does not change with the steady-state markups. We can think of the same reason in the relation between $\frac{\lambda_L}{\lambda_x}$ and φ^L as discussed above. In economic shocks associated with the loan rate, smaller differences in labor, *i.e.,* higher ϵ_j, yield a larger substitution effect among loans and lead to higher economic fluctuations. In other words, we interpret this point as indicating that greater similarity among businesses increases loan rate fluctuations to the shocks thanks to high substitution among loans in the loan demand function. Then, a smaller steady-state markup induces both more labor disutility and more production dispersions associated with more disutility from consumption to the shocks. Moreover, the importance of the credit spread decreases as the size of the steady-state markup decreases. As shown in later sections, for all the countries in the euro area, the estimated size of the steady-state markup is above the level under which the credit spread term becomes more important to the central bank.

Figure 6 illustrates the simulation where we impose idiosyncratic shocks on the bank loan market with different steady-state markups. We assume an asymmetric degree of labor difference such that $\epsilon_M = 7.66$ and $\epsilon_L = 10$. Thus, the steady-state markup on the loan rate is larger in bank loan M than in bank loan L. For illustrative purposes, we set a symmetric situation except for ϵ_j as $\varphi^j = 0.6$ and $n_j = 0.5$ for $j = M, L$. We impose these shocks and show the results. First, even under the presence of the trade-off problem because of heterogeneity in the steady-state markup, the central bank lowers the policy rate when the shock hits the economy. Second, a central bank lowers the policy interest rate more in response to the shock in the smaller steady-state loan rate markup, *i.e.* less labor difference, than to the shock in the larger loan rate markup, *i.e.* more labor difference.

Thus, the central bank should put its monetary policy priority on the loan rate with the smaller steady-state markup rather than that with the larger steady-state markup for the same size shock. The reason for this is that the weight in the welfare function increases as the steady-state markup in the interest rate decreases as implied in Figure 5. Note that the markups associated with ϵ_j capture the banks' markups at the steady state. Thus, in our model, the central bank can affect the variation in the loan rate around its steady state through monetary policy, but it cannot alter the markup in the steady state itself. Our view is in line with that of Adam *et al.* (2002). They stress the importance of the institutional differences in accounting for the loan rate differences across countries. Cross-country heterogeneity in the tax systems and legal systems is present in the euro area after 1999, and the related coordination has progressed only gradually (Adam *et al.*, 2002). Those factors are beyond the influence of monetary policy. We consider an economy where the central bank concentrates on the loan rate dynamics, taking the size of the markup at the steady state as exogenously given. Under this environment, it is optimal for the central bank to attach higher weight to the loan rate variations with a smaller steady-state loan rate markup than to those with a larger steady-state loan rate markup.

4.2.3 Sensitivity to market share

Figure 7 reports the values of relative weights $\frac{\lambda_L}{\lambda_x}$ and $\frac{\lambda_{ML}}{\lambda_x}$ for various levels of bank loan interest rate stickiness φ^L, with a changing market share of loans n_L. We see that $\frac{\lambda_L}{\lambda_x}$ increases as n_L increases. This is just because the impact of the loan market L from the shock to the economy increases because the market share of the loan M increases. $\frac{\lambda_{ML}}{\lambda_x}$ takes the maximum value at $n_L = 0.5$, and it decreases as n_L moves away from 0.5 (see Appendix A for details).

Figure 8 illustrates the simulation where we impose idiosyncratic shocks on the bank loan market with a larger share of bank loan type L, *i.e.*, $n_L = 0.8$, in a symmetric situation with $\varphi^j = 0.6$ and $\epsilon_j = 7.66$ for $j = M, L$. Two observations should be noted. First, under the presence of the trade-off problem stemming from the heterogeneity in the market share, the central bank lowers the policy rate when the shock hits the economy. Second, a central bank lowers the policy rate more in response to the shock in the loan rate with the larger share than to the shock in the loan rate with the smaller share. Thus, the central bank should put its monetary policy priority on the loan rate with the larger market share rather than that with the smaller market share. One reason for this is that the weight on the more sticky loan in the welfare function increases when n_L increases as shown in Figure 7.[18] Another reason is that banks with a higher market share have a greater effect on the economic dynamics as shown in equation (20). This implies that the market share, *i.e.* the impact of the share on the economic dynamics, is also an important element in the determination of monetary policy.

5 Monetary policy in the euro area

In this section, we conduct numerical analyses using the euro area data. Our model is very suitable for an analysis of financial integration because the model includes the international competition among heterogeneous banks that is a key element in the financial integration in the euro area. The arguments about different bank types hold exactly for those banks with distinct country-specific loan rate stickiness and steady-state markups on the loan rate. As we have seen earlier, the cross-sectional heterogeneity in both the bank loan rate stickiness and steady-state markups on the loan rate remains observable even a decade after the debut of the new currency. The optimal monetary policy for a central bank is then formulated distinctively given the observed size of bank loan stickiness for each country, the size of steady-state markup on the loan rate for each country, and the size of their financial markets.

5.1 Optimal monetary policy in the euro area

We first provide the relative significance of each country, *i.e.,* the current monetary policy priority ranking of each country, for the ECB based on the parameters obtained from the

[18] See Appendix A.

current euro area data. The ECB responds differently to each country shock as member countries differ in their bank loan stickiness, their steady-state markup on the loan rate, and their market size. We set the parameter values for the euro area as follows. The main parameters $\sigma = 0.72$, $\theta = 6$, and $\alpha = 0.91$ are from Smets and Wouters (2003).[19] The other parameters are reported in Table 2. For the loan stickiness for each country, we calculate the Calvo parameter from our estimation result in Table 1. For the steady-state markup on the loan rate, we also use the values in Table 1. Table 3 provides the Calvo parameters φ^j corresponding to the estimated stickinesses in Table 1 and ϵ_j corresponding to the estimated markup in Table 1.[20] j belongs to one of 12 countries. We maintain the two-types bank analysis used earlier. We divide the euro area into country j and the rest. The lending volume n_j for country j is set according to the GDP share of the country in the euro area as in Table 3.[21] The bank loan stickiness φ^R, where R denotes the rest of the euro area, and the markup for the rest $\frac{\epsilon_R}{\epsilon_R - 1}$ are calculated by the weighted average of those parameters of the 11 countries in the euro area other than country j using their GDP as weights.

In the simulation, we calculate the time path of the policy rate in response to a positive innovation in the bank loan rate in country j. We measure the ordering of country j by the size of the policy rate response to a country-specific shock in the loan rate in country j relative to the other countries. A larger response of the policy rate to the shock for country j indicates higher significance of the bank lending channel of country j for the central bank. As shown above, the bank loan rate stickiness φ^j, the steady-state loan rate markup on the loan rate $\frac{\epsilon_j}{\epsilon_j - 1}$, and the share of lending volume n_j play an important role

[19] We pick these values from the means in Table 1 of Smets and Wouters (2003). For calculation of σ, we assume that the consumption habit is zero.

[20] We use the transformation:

$$\varphi^j = 1 - x,$$

where x is the absolute values in the first and second lines in Table 1. Then we change this monthly value to a quarterly value.

[21] In the third column of Table 3 below, we show the GDP share of each euro country, based on the 2006 GDP series in the Total Economy Database, released by The Conference Board and Groningen Growth and Development Centre.

in our model.

Figure 9 shows the impulse responses of the policy rate to a positive innovation in the bank loan rate in each country. In response to the innovations, the central bank lowers the policy rate, but the size of the decline in the policy rate differs across countries. Table 4 reports the cumulative impulse responses (denoted as CIR in the table) of the policy rates and size of GDP for 12 countries by ranking. The numbers in parentheses are the cumulative impulse response values. More negative values in the cumulative impulse response indicate that the central bank responds more to the shock originated from the country. An important implication of this simulation is that the ECB includes inequality, which is not captured by the financial market sizes, in its monetary policy priorities under imperfect financial integration.

5.2 Optimal monetary policy toward perfect financial integration

Another important feature is the ongoing processes of integration. While several studies including Adams *et al.* (2002) and Gropp and Kashyap (2008) stress the prominent delays of financial integration in the bank loan market, several legislative initiatives have been conducted by the ECB membership countries to unify truly the euro loan market.[22] Empirical studies, such as Mojon (2000) and van Leuvensteijn *et al.* (2008), indicate some degree of convergence in the bank loan market. We show several possible scenarios associated the financial integration in the euro area and offer a description of the optimal monetary policy in each stage.

One plausible view on bank loan market convergence in the next decade is that cross-country heterogeneity of the bank loan market may become less prominent. Mojon (2000) argues that as a consequence of integration of the money market, the presence of alternative ways to finance investments makes the banks exposed to a higher degree of competition. More severe competition brings homogenous and faster responses of the cross-country bank

[22] For example, the 2nd Banking Directive of 1989 legally permitted the establishment of subsidiaries and branches of any bank residing in the EU in any other EU countries (Gropp and Kashap, 2008).

loan rates to the policy rate.[23] Gropp and Kashyap (2008) imply that banks' markups on the loan rates decrease as the competition in the loan market increases. This study suggests that bank loan rates respond to the policy rate quicker in a homogenous manner and the steady-state loan rate markup becomes smaller in all countries, as the bank loan markets approach perfect financial integration where we assume that strong bank competition exists.

First, we consider a scenario in which the bank loan stickinesses of all countries approach that of Germany, which has the lowest stickiness in the bank loan rate. For a given size of the bank loan stickiness φ^j_{data} for country j, we use the difference in stickiness from that of Germany $\varphi^{Germany}_{data}$ as a measure of the degree of financial integration. We conduct this hypothetical exercise by changing the difference in the country-specific bank loan stickiness relative to that of Germany.

Table 5 displays the cumulative impulse response (denoted as CIR in the table) of the policy rate to a shock in the loan rate at various levels of the financial integration. The numbers in parentheses are the cumulative impulse response values. The second column indicates the cumulative impulse response and the monetary policy priority ranking of the countries when $\varphi^j = \left(\varphi^j_{data} - \varphi^{Germany}_{data}\right) \times \tau + \varphi^{Germany}_{data}$ with $\tau = 0.5$. When $\tau = 0.5$, the distance between the country-specific bank loan stickiness and that of Germany by 50%. The third and fourth columns report the case when $\tau = 0.25$ and $\tau = 0$, respectively. It is shown in the table that the priority ranking of the countries for a central bank changes in each stage of the financial integration. Moreover, the ranking does not converge to that by the financial market share of each country because of the heterogeneity of the steady-state markup on the loan rate. Therefore, the steady-state markup on the loan rate as well as the loan rate stickiness are also very important factors for the ECB in conducting monetary policy.

Second, we consider a scenario in which the levels of steady-state markups on the loan rates of all countries approach that of Luxemburg, which has the lowest steady-state

[23] Using data from the euro area, van Leuvenstijn *et al.* (2008) shows that higher bank competition on the interest rate induces larger pass-through of the policy interest rate to the loan rates.

markup on the bank loan rate as before. Table 6 displays the cumulative impulse response (denoted as CIR in the table) of the policy rate to a shock in the loan rate at various levels of financial integration. The numbers in parentheses are the cumulative impulse response values. The second column indicates the cumulative impulse response and the monetary policy priority ranking of the countries when $\epsilon^j = \left(\epsilon^j_{data} - \epsilon^{Luxemburg}_{data}\right) \times \tau + \epsilon^{Luxemburg}_{data}$ with $\tau = 0.5$. The third and fourth columns report the case when $\tau = 0.25$ and $\tau = 0$, respectively. Regardless of the value of τ, the monetary policy priority does not change. The outcomes, however, show that the ranking does not converge to that by the financial market share of each country because of the heterogeneity of the loan rate stickiness. We confirm that the loan rate stickiness and the steady-state loan rate markups are both important factors for the ECB.

Third, we consider a scenario in which both the levels of the steady-state markups on the loan rates and the loan rate stickinesses of all countries approach those of Germany and Luxemburg, respectively, as before. Table 7 shows that the ranking finally converges to the ranking by the financial market share of each country without the heterogeneity in both the loan rate stickinesses and the steady-state markups on the loan rates.

6 Concluding remarks

We developed a New Keynesian dynamic stochastic general equilibrium model under the imperfect financial integration, *i.e.,* cross-country differences in the loan rate stickiness and the markup from the policy rate to the loan rate, within a single currency area. We then investigated the optimal monetary policy rule under such an environment. We also carried out the welfare analysis, indicating that the central bank needs to take account of the heterogeneities in the bank loan markets, such as the size of the bank loan rate stickiness, the level of the steady-state markup, and the share of the market in the bank loan market.

We calibrated our model using data from the euro area and computed the monetary policy priority ranking of euro countries when country-specific shocks in the bank loan markets are considered. The ordering is not equivalent to the order that is based on the size of the financial markets. As financial integration deepens further, however, it is predicted that the share of the bank lending will become the only relevant measure in implementing the optimal monetary policy.

The analysis in this paper suggests several directions for future research. It would be of interest to examine the role of other types of heterogeneity across the different types of interest rates. In particular, previous studies report the prominent difference between the bank loan rates and market interest rates in terms of their stickinesses. In this paper, we concentrated on the difference within the bank loan rates across countries in the euro area. However there is ample evidence that firms directly finance their funds from the financial market. The extension of our model, in this direction, enabled us to analyze monetary policy in an economy where firms can substitute between borrowing from the banks and issuing corporate bonds. As suggested by Taylor (2008), we can also examine the heterogeneity in interest rates between a risk-free asset, such as government bonds, and a risky asset, such as commercial paper, in our framework. A model that captures this heterogeneity may be important to understand better the transmission mechanism of monetary policy.

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Table 1: Loan interest rate stickiness in the euro area

	Markup	Stickiness α_j
Austria	1.016	-0.21 (-4.13)
Belgium	1.017	-0.22 (-2.38)
Finland	1.014	-0.35 (-3.59)
France	1.015	-0.25 (-2.25)
Germany	1.024	-0.42 (-5.84)
Greece	1.035	-0.26 (-1.58)
Ireland	1.020	-0.28 (-4.34)
Italy	1.026	-0.25 (-3.84)
Luxemburg	1.012	-0.21 (-2.28)
Netherlands	1.019	-0.30 (-2.92)
Portugal	1.024	-0.10 (-2.08)
Spain	1.015	-0.07 (-3.00)

Table 2: Parameter values

Parameters	Values	Explanation
β	0.99	Discount factor
σ	6.25	Elasticity of output with respect to real interest rate
κ	0.032	Elasticity of inflation with respect to output
α	0.66	Probability of price change
φ^M	0.6	Probability of loan interest rate change in type M bank
φ^L	0.5	Probability of loan interest rate change in type L bank
θ	7.66	Substitutability of differentiated consumption goods
ϕ_h	1.33	Elasticity of output to labor input in production function
ϵ_j	7.66	Substitutability of differentiated laborers of j type
ν	0.11	Elasticity of marginal disutility to labor
ω	0.47	Total elasticity of marginal cost with respect to y
ω_p	0.33	Elasticity of marginal cost with respect to y regarding to production
n_j	0.5	Share of the lending volume of type j bank

Table 3: Loan rate stickiness of euro countries

	Stickiness φ^j	Markup ϵ_j	GDP share
Austria	0.49	64.5	0.0309
Belgium	0.48	59.7	0.0362
Finland	0.28	74.7	0.0180
France	0.42	65.6	0.1993
Germany	0.20	42.7	0.2731
Greece	0.40	29.8	0.0317
Ireland	0.37	50.7	0.0181
Italy	0.42	39.2	0.1738
Luxemburg	0.48	83.5	0.0037
Netherlands	0.34	52.8	0.0616
Portugal	0.72	42.1	0.0224
Spain	0.81	68.2	0.1310

Table 4: CIRs of the policy interest rate

Ranking by CIR	Ranking by GDP
ES (-0.91)	DE (Germany)
FR (-0.36)	FR (France)
IT (-0.29)	IT (Italy)
PG (-0.18)	ES (Spain)
DE (-0.15)	NL (Netherlands)
BE (-0.09)	BE (Belgium)
AT (-0.08)	GR (Greece)
NL (-0.07)	AT (Austria)
GR (-0.05)	PG (Portugal)
FI (-0.02)	IE (Ireland)
IE (-0.02)	FI (Finland)
LUX (-0.01)	LUX (Luxemburg)

Table 5: CIRs of the policy interest rate under convergence in loan rate stickiness

50% conv.	75% conv.	100% conv.	
Ranking by CIR	Ranking by CIR	Ranking by CIR	Ranking by GDP
ES (-0.27)	FR (-0.16)	DE (-0.15)	DE (Germany)
FR (-0.21)	ES (-0.15)	FR (-0.12)	FR (France)
IT (-0.17)	DE (-0.14)	IT (-0.09)	IT (Italy)
DE (-0.14)	IT (-0.13)	ES (-0.08)	ES (Spain)
NL (-0.05)	NL (-0.04)	NL (-0.04)	NL (Netherlands)
PG (-0.05)	BE (-0.03)	BE (-0.03)	BE (Belgium)
BE (-0.04)	AT (-0.03)	AT (-0.02)	GR (Greece)
AT (-0.04)	PG (-0.03)	GR (-0.02)	AT (Austria)
GR (-0.03)	GR (-0.02)	PG (-0.01)	PG (Portugal)
IE (-0.02)	IE (-0.01)	FI (-0.01)	IE (Ireland)
FI (-0.01)	FI (-0.01)	IE (-0.01)	FI (Finland)
LUX (-0.00)	LUX (-0.00)	LUX (-0.00)	LUX (Luxemburg)

Table 6: CIRs of the policy interest rate under convergence in loan rate markup

50% conv.	75% conv.	100% conv.	
Ranking by CIR	Ranking by CIR	Ranking by CIR	Ranking by GDP
ES (-0.91)	ES (-0.91)	ES (-0.92)	DE (Germany)
FR (-0.36)	FR (-0.36)	FR (-0.36)	FR (France)
IT (-0.31)	IT (-0.31)	IT (-0.32)	IT (Italy)
PG (-0.20)	PG (-0.20)	PG (-0.20)	ES (Spain)
DE (-0.16)	DE (-0.16)	DE (-0.16)	NL (Netherlands)
BE (-0.09)	BE (-0.09)	BE (-0.09)	BE (Belgium)
AT (-0.08)	AT (-0.08)	AT (-0.08)	GR (Greece)
NL (-0.07)	NL (-0.07)	NL (-0.08)	AT (Austria)
GR (-0.05)	GR (-0.05)	GR (-0.05)	PG (Portugal)
IE (-0.03)	IE (-0.03)	IE (-0.03)	IE (Ireland)
FI (-0.02)	FI (-0.02)	FI (-0.02)	FI (Finland)
LUX (-0.01)	LUX (-0.01)	LUX (-0.01)	LUX (Luxemburg)

Table 7: CIRs of the policy interest rate under convergence in both loan rate stickiness and loan rate markup

50% conv.	75% conv.	100% conv.	
Ranking by CIR	Ranking by CIR	Ranking by CIR	Ranking by GDP
ES (-0.27)	FR (-0.16)	DE (-0.16)	DE (Germany)
FR (-0.20)	DE (-0.15)	FR (-0.12)	FR (France)
IT (-0.18)	ES (-0.15)	IT (-0.10)	IT (Italy)
DE (-0.15)	IT (-0.13)	ES (-0.08)	ES (Spain)
NL (-0.05)	NL (-0.04)	NL (-0.04)	NL (Netherlands)
PG (-0.05)	BE (-0.03)	BE (-0.02)	BE (Belgium)
BE (-0.04)	AT (-0.03)	GR (-0.02)	GR (Greece)
AT (-0.04)	PG (-0.03)	AT (-0.02)	AT (Austria)
GR (-0.03)	GR (-0.02)	PG (-0.01)	PG (Portugal)
IE (-0.02)	IE (-0.01)	IE (-0.01)	IE (Ireland)
FI (-0.01)	FI (-0.01)	FI (-0.01)	FI (Finland)
LUX (-0.00)	LUX (-0.00)	LUX (-0.00)	LUX (Luxemburg)

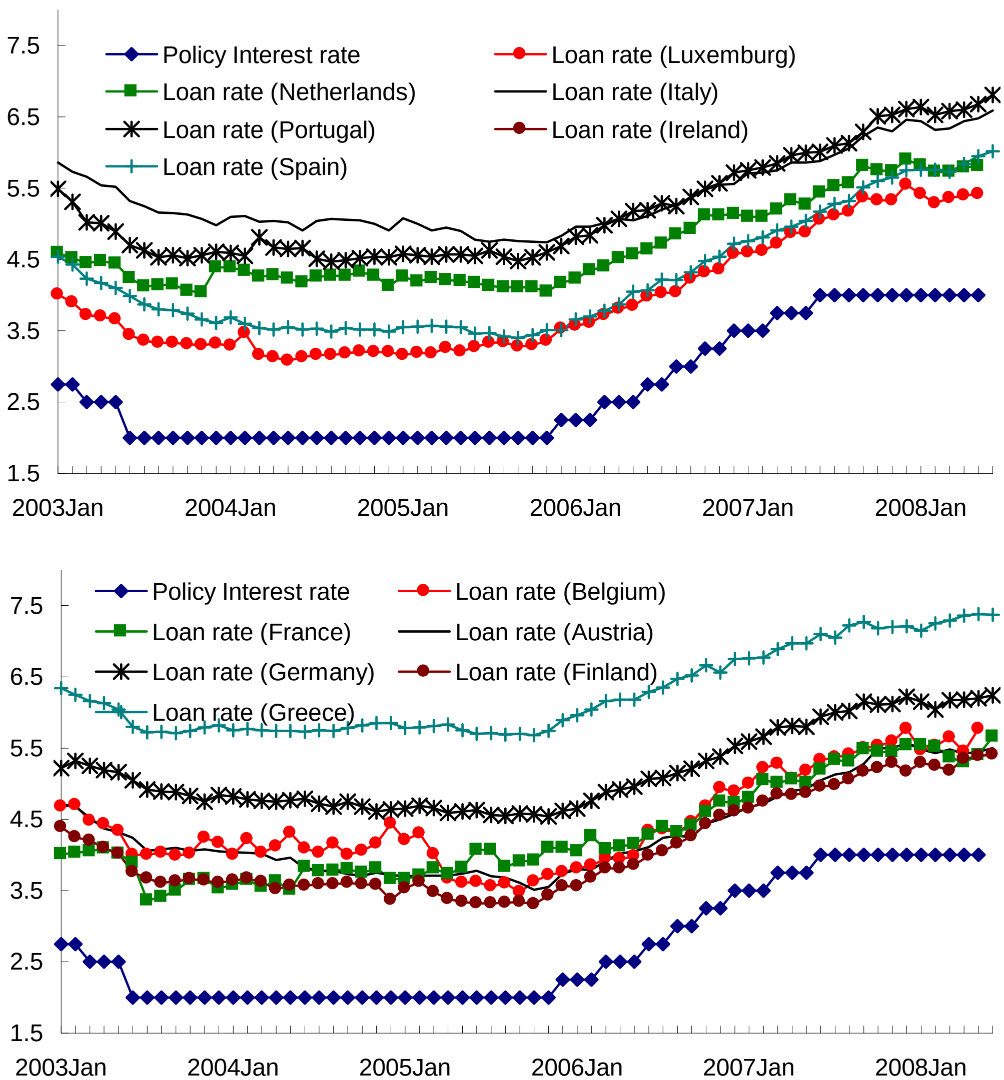
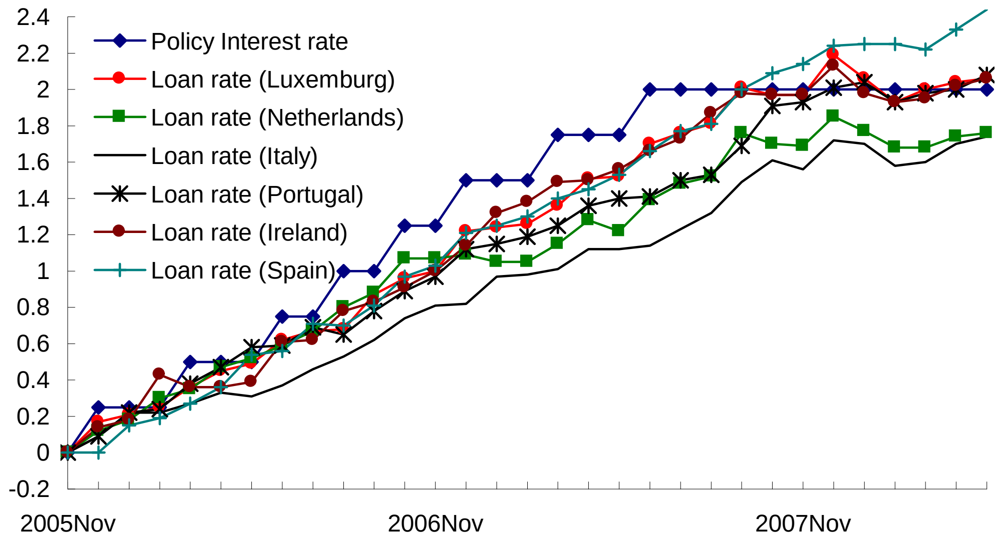


Figure 1: Time path of cross-country bank loan rates.



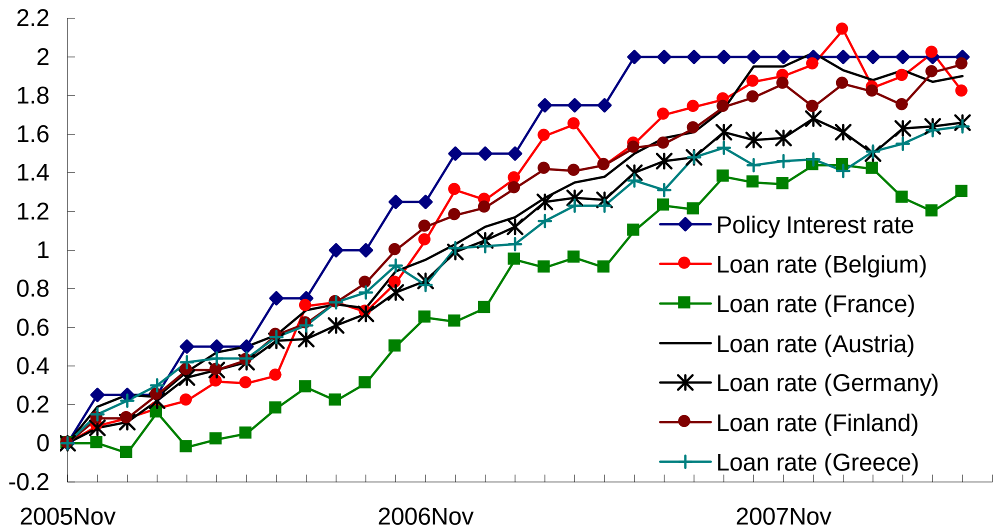


Figure 2: Changes of bank loan rates from November 2005.

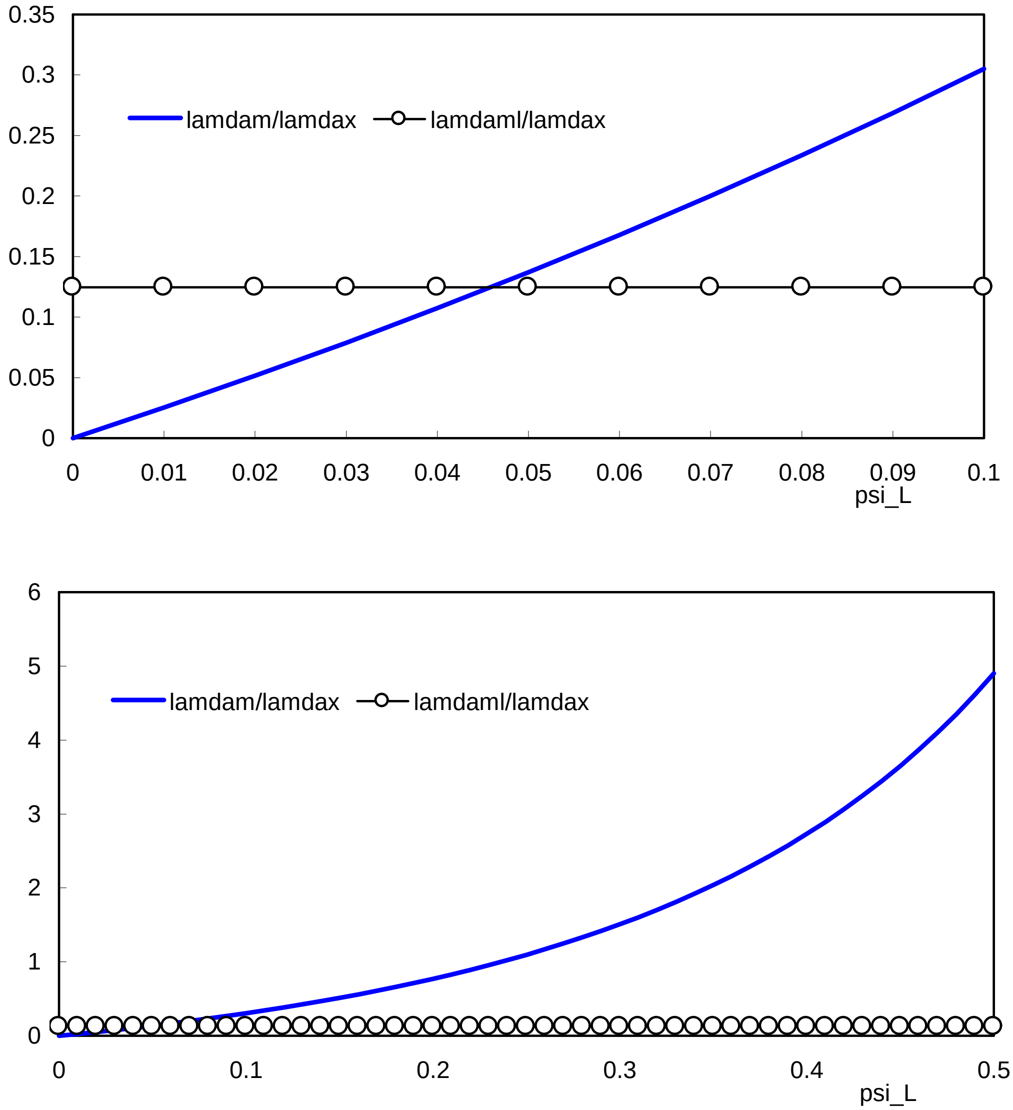


Figure 3: Changing degree of stickiness φ^L.

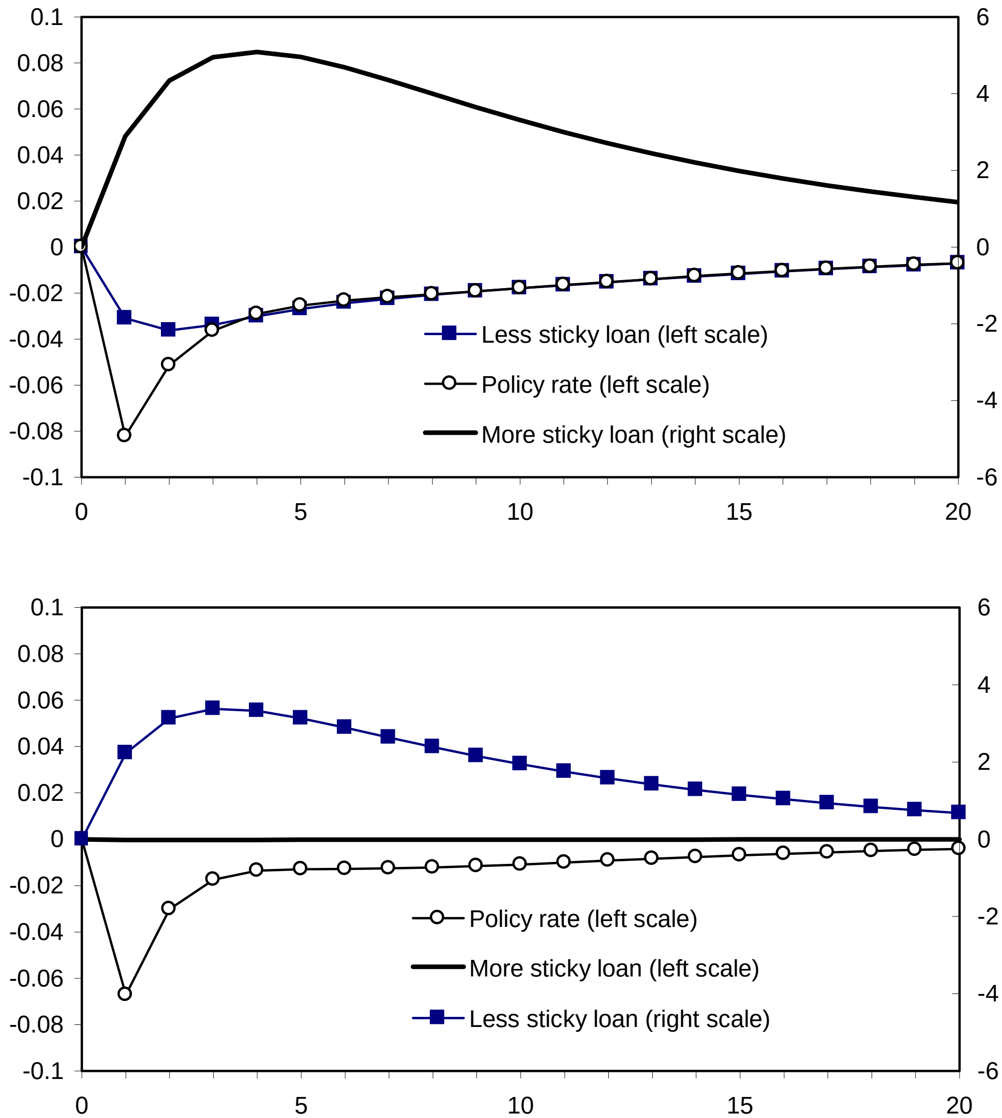


Figure 4: Impulse response of loan rates to a shock in type L bank (upper panel) and type M bank (lower panel) under heterogeneous loan rate stickiness.

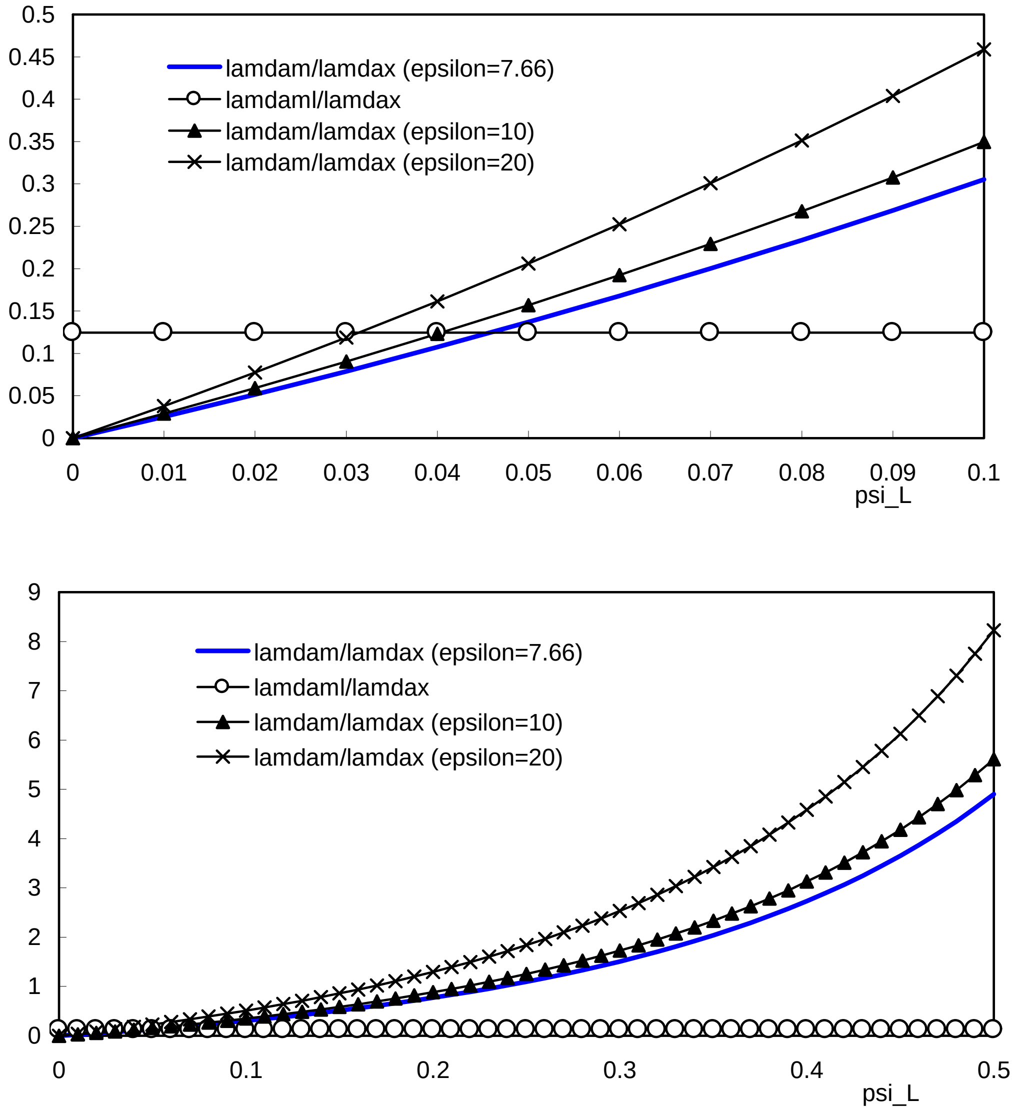


Figure 5: Changing degree of labor difference ϵ_L.

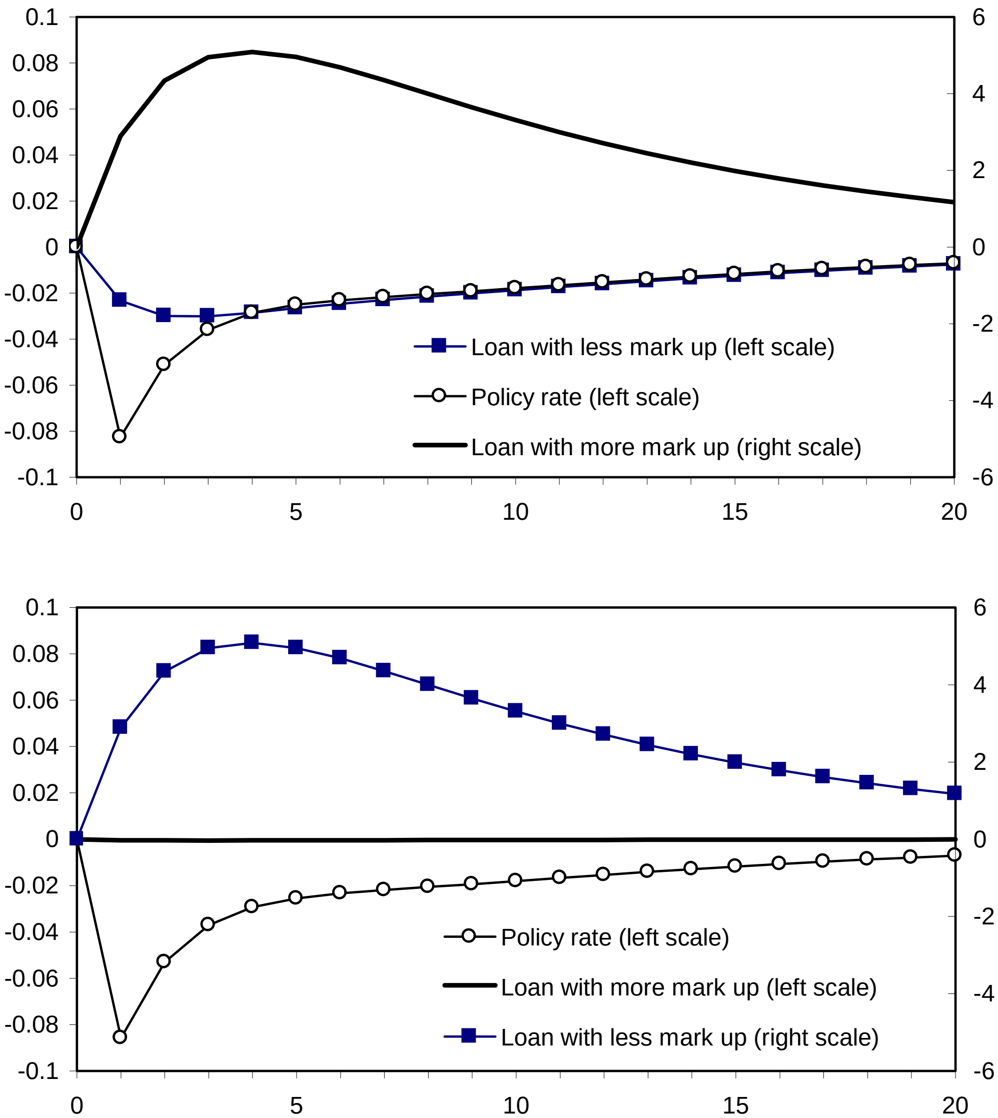


Figure 6: Impulse response of loan rates to a shock in type L bank (upper panel) and type M bank (lower panel) under heteregeneous mark up.

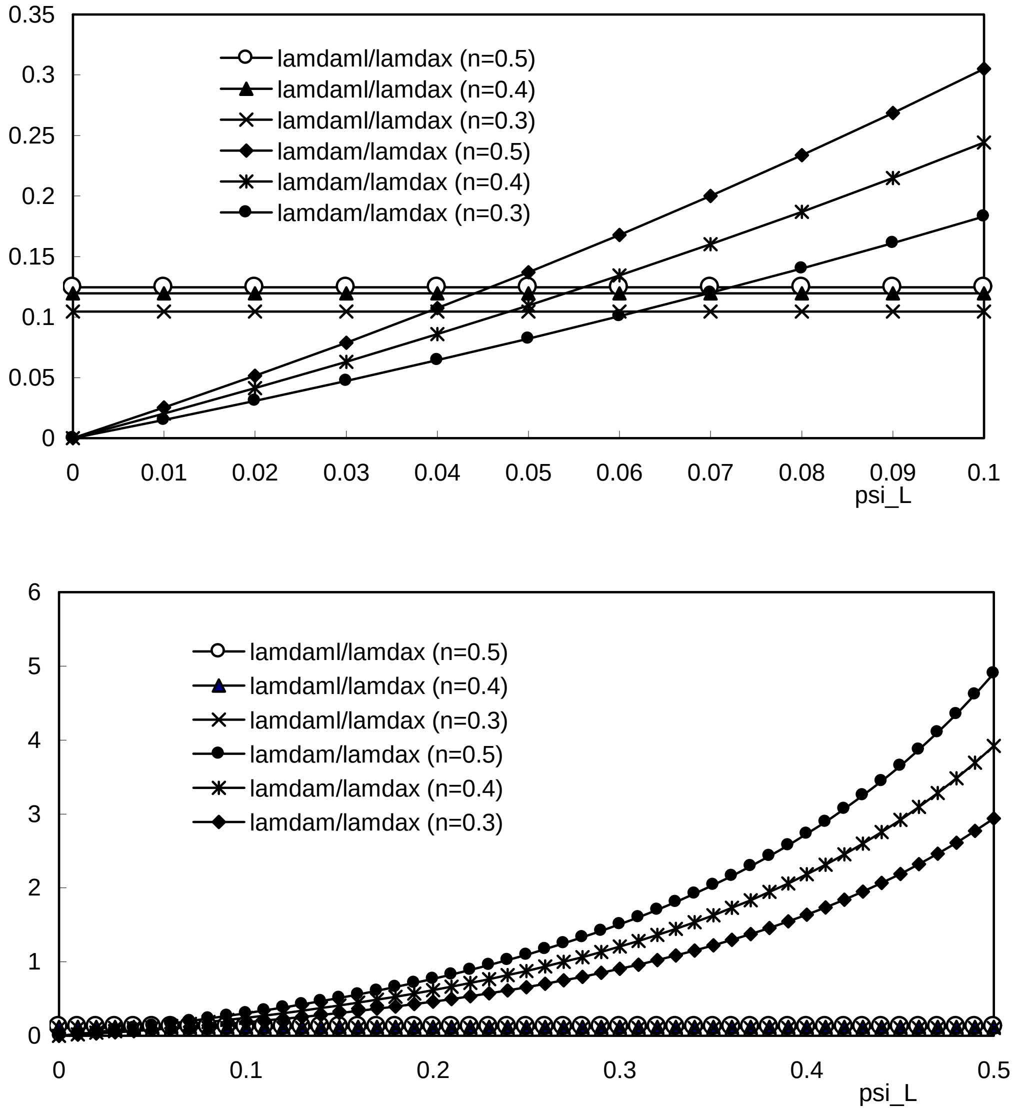


Figure 7: Changing market share n_L.

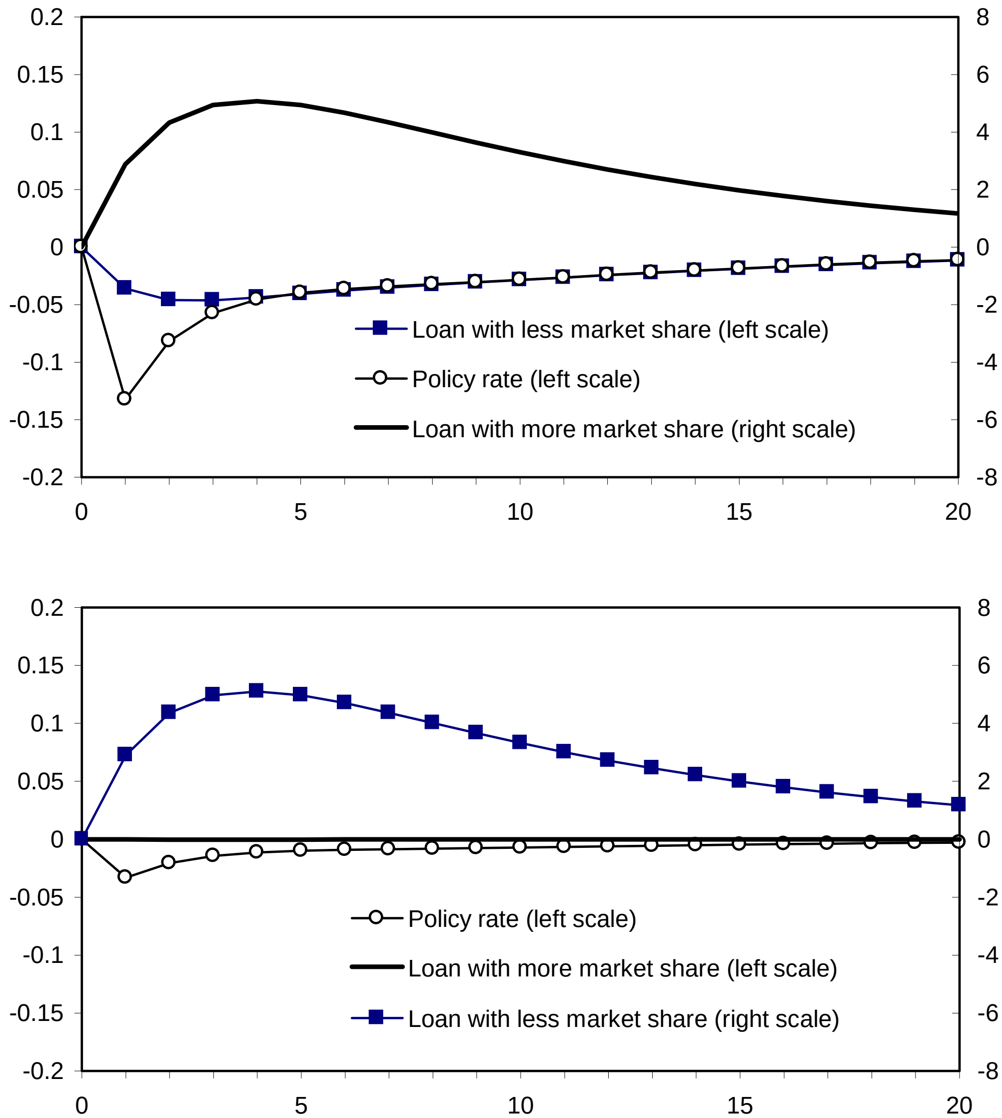


Figure 8: Impulse response of loan rates to a shock in type L bank (upper panel) and type M bank (lower panel) under different market share.

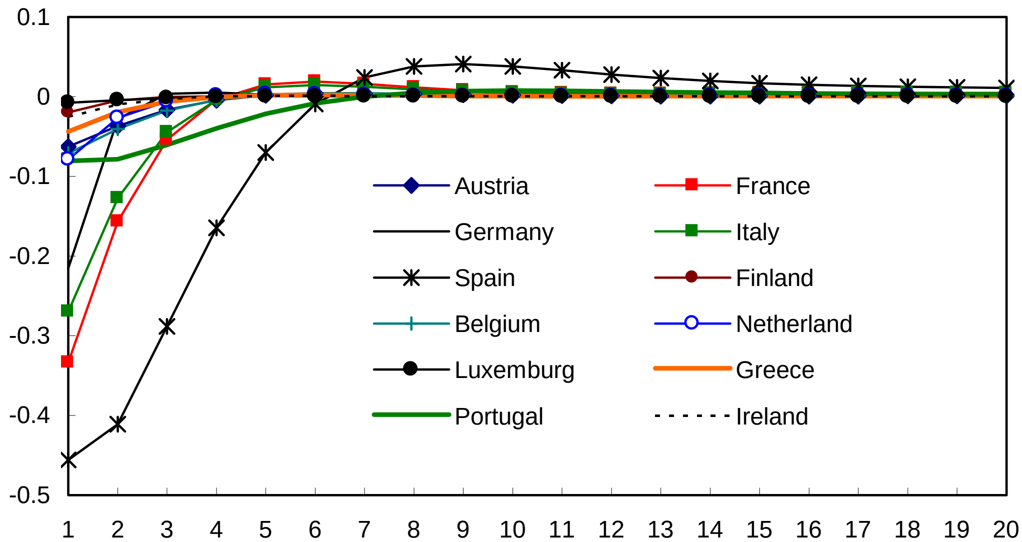


Figure 9: Impulse response of the policy rates to country-specific credit shock.

Appendix (not for publication)

A Derivation of loss function

In this subsection, we derive a second-order approximation to the welfare function, following Woodford (2003). Except for x_t and π_t, we denote the log-linearized version of variable m_t by $\widehat{m}_t = \ln(m_t/\overline{m})$, where $\overline{m}$ is the steady-state value of m_t.

As market clearing conditions $Y_t = C_t$ and $y_t(f) = c_t(f)$ for any f hold, the welfare criterion of the consumer is given by:

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t UT_t \right\},$$

where:

$$UT_t = U(C_t) - \int_0^{n_M} V(l_{M,t}(h_M))dh_M - \int_0^{n_L} V(l_{L,t}(h_L))dh_L, \tag{26}$$

and:

$$Y_t \equiv \left[\int_0^1 y_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}.$$

Log-linearizing the first term of the equation (26) yields:

$$\begin{aligned}
UT(Y_t;\nu_t) &= \overline{U} + U_c\widetilde{Y}_t + U_\nu \nu_t + \frac{1}{2}U_{cc}\widetilde{Y}_t^2 + \nu_t U_{c\nu}\widetilde{Y}_t + \frac{1}{2}\nu_t' U_{\nu\nu}\nu_t + Order(\| \xi \|^3) \\
&= \overline{U} + \overline{Y}U_c(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2) + U_\nu\nu_t + \frac{1}{2}U_{cc}\overline{Y}^2\widehat{Y}_t^2 + \overline{Y}U_{c\nu}\nu_t\widehat{Y}_t + \frac{1}{2}\nu_t'U_{\nu\nu}\nu_t + Order(\| \xi \|^3) \\
&= \overline{Y}U_c\widehat{Y}_t + \frac{1}{2}\left[\overline{Y}U_c + \overline{Y}^2U_{cc}\right]\widehat{Y}_t^2 - \overline{Y}^2U_{cc}g_t\widehat{Y}_t + t.i.p + Order(\| \xi \|^3) \\
&= \overline{Y}U_c\left[\widehat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\widehat{Y}_t^2 + \sigma^{-1}g_t\widehat{Y}_t\right] + t.i.p + Order(\| \xi \|^3),
\end{aligned} \tag{27}$$

where $\overline{U} \equiv U(\overline{Y};0)$, $\widetilde{Y}_t \equiv Y_t - \overline{Y}$, $\sigma^{-1} \equiv -\frac{\overline{Y}U_{cc}}{U_c} > 0$, and $g_t \equiv -\frac{U_{c\nu}\nu_t}{\overline{Y}U_{cc}}$. $t.i.p$ stands for the terms that are independent from monetary policy and $Order(\| \xi \|^3)$ are the terms higher than the second order. To replace $\widetilde{Y}_t$ by $\widehat{Y}_t \equiv \ln(Y_t/\overline{Y})$ in the second line, we use the Taylor series expansion on $Y_t/\overline{Y}$:

$$Y_t/\overline{Y} = 1 + \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + Order(\parallel \xi \parallel^3).$$

Next, we log-linearize the second and the third components of the right-hand side of equation (26) in a similar way:

$$\begin{aligned}\frac{1}{n_j}\int_0^{n_j} V(l_{j,t}(h_j);\nu_t)dh_j &= V_l\overline{\overline{L}}(E_{h_j}\widehat{l}_{j,t}(h_j) + \frac{1}{2}E_{h_j}(\widehat{l}_{j,t}(h_j))^2) + \frac{1}{2}V_{ll}\overline{L}^2E_{h_j}(\widehat{l}_{j,t}(h_j))^2 + V_{l\nu}\overline{L}\nu_tE_{h,j}\widehat{l}_{j,t}(h_j) \\ +t.i.p + Order(\parallel \xi \parallel^3) \\ &= \overline{\overline{L}}V_l\left[\widehat{L}_{j,t} + \frac{1}{2}(1+\nu)\widehat{L}_{j,t}^2 - \nu\widetilde{\nu}_t\widehat{L}_{j,t} + \frac{1}{2}(\nu + \frac{1}{\epsilon_j})var_{h_j}\widehat{l}_{j,t}(h_j)\right] \\ +t.i.p + Order(\parallel \xi \parallel^3),\end{aligned} \tag{28}$$

where $\widetilde{\nu}_t \equiv -\frac{V_{l\nu}\nu_t}{\overline{\overline{L}}V_{ll}}$, $\nu \equiv \frac{\overline{\overline{L}}V_{ll}}{V_l}$, $\phi_h \equiv \frac{\overline{Y}}{\overline{\overline{L}}f_L}$, $\omega_p \equiv \frac{f_Lf_{LL}}{(f_L)^2}$, $q_t \equiv (1+\omega^{-1})a_t + \omega^{-1}\nu\widetilde{\nu}_t$, $a_t \equiv \ln A_t$. $var_{h_j}\widehat{l}_{j,t}(h_j)$ is the cross-sectional variance of the labor type j worker and $var_f\widehat{p}_t(f)$ is the cross-sectional variance of differentiated good prices $\widehat{p}_t(f)$. We define a subaggregator for $L_{j,t}(h_j)$ as follows:

$$L_{j,t} \equiv \left[\left(\frac{1}{n_j}\right)^{\frac{1}{\epsilon_j}}\int_0^{n_j} l_{j,t}(h_j)^{\frac{\epsilon_j-1}{\epsilon_j}}dh_j\right]^{\frac{\epsilon_j}{\epsilon_j-1}},$$

and use its second-order approximation $\widehat{L}_{j,t} = E_{h_j,j}\widehat{l}_{j,t}(h_j) + \frac{1}{2}\frac{\epsilon_j-1}{\epsilon_j}var_{h_j}\widehat{l}_{j,t}(h_j) + Order(\parallel \xi \parallel^3)$ to obtain the second equality above. Note that we assume the wage tax as $\frac{1}{\tau_j} = \left(\frac{\epsilon_j}{\epsilon_j-1}\right)^2$ to realize $\overline{l_L}(h_M) = \overline{l_L}(h_L) = \overline{\overline{L}}$. This comes from the optimization by the social planner in the steady state.[24] Combining equation (28) for $j = L, M$:

$$
\begin{aligned}
& \int_0^{n_M} V(l_{M,t}(h_M);\nu_t)dh_M + \int_0^{n_L} V(l_{L,t}(h_L);\nu_t)dh_L \\
= \ & \overline{\overline{L}}V_l \left[\sum_{j=M,L} \left(n_j \widehat{L}_{j,t} + \frac{n_j}{2}(1+\nu)\widehat{L}_{j,t}^2 - n_j \nu \widetilde{\nu}_t \widehat{L}_{j,t} + \frac{n_j}{2}(\nu + \frac{1}{\epsilon_j}) var_{h_j} \widehat{l}_{j,t}(h_j) \right) \right] \\
& + t.i.p + Order(\ \| \ \xi \|^3) \\
= \ & \overline{\overline{L}}V_l \left[\begin{array}{c} \widehat{\widehat{L}}_t + \frac{1+\nu}{2}\widehat{\widehat{L}}_t^2 - \nu\widetilde{\nu}_t \phi_h^{-1}\widehat{\widehat{L}}_t + n_M n_L \frac{1+\nu}{2}\left(\widehat{L}_{M,t} - \widehat{L}_{L,t}\right)^2 \\ + \sum_{j=M,L} \frac{n_j}{2}(\nu + \frac{1}{\epsilon_j}) var_{h_j}\widehat{l}_{j,t}(h_j) \end{array} \right] \\
& + t.i.p + Order(\ \| \ \xi \|^3),
\end{aligned} \tag{29}
$$

where $\widehat{\widehat{L}}_t = n_M \widehat{L}_{M,t} + n_L \widehat{L}_{L,t}$.

It is notable that the market clearing condition for the labor inputs implies:

$$\widetilde{L}_t = \int_0^1 \widetilde{L}_t(f)df = \int_0^1 f^{-1}(\frac{y_t(f)}{A_t})df.$$

Using the relationship between the cross-sectional variance of differentiated goods and that of prices, $var_f \ln y_t(f) = \theta^2 var_f \ln p_t(f)$, under the Dixit–Stiglitz preferences, the second-order approximation of this equation gives:

$$\widehat{\widehat{L}}_t = \phi_h(\widehat{Y}_t - a_t) + \frac{1}{2}(1 + \omega_p - \phi_h)\phi_h(\widehat{Y}_t - a_t)^2 + \frac{1}{2}(1 + \omega_p\theta)\theta var_f \widehat{p}_t(f) + Order(\| \ \xi \|^3),$$

where $\phi_h \nu$ equals ω_w, the elasticity of the real wage with respect to aggregate output under the flexible-wage labor supply, and $\omega = \omega_p + \omega_w$.

---

[24] The social planner optimizes the following problem:
$max_{C,\bar{l}_M(h_M),\bar{l}_L(h_L)} U(C) - n_M V(\bar{l}_M(h_M)) - n_L V(\bar{l}_L(h_L))$ s.t. $C = \left(\bar{l}_M(h_M)^{n_M}\bar{l}_L(h_L)^{n_L}\right)^{\frac{1}{\phi_h}}$.
Then we have $\frac{1}{\tau_j} = \left(\frac{\epsilon_j}{\epsilon_j - 1}\right)^2$ to satisfy $\overline{l_L}(h_M) = \overline{l_L}(h_L) = \overline{\overline{L}}$.

With these relationships, equation (29) is reduced to:

$$
\begin{aligned}
&\sum_{j=M,L} \int_0^{n_j} V(l_{j,t}(h_j);\nu_t)dh_j \\
&= \phi_h \overline{\overline{L}} V_l \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t + n_M n_L \frac{1+\nu}{2} \left(\widehat{L}_{M,t} - \widehat{L}_{L,t}\right)^2 \\ +\frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ + \sum\limits_{j=M,L} \frac{n_j}{2} \phi_{h_j}^{-1}(\nu + \frac{1}{\epsilon_j}) var_{h_j} \ln l_{t,j}(h_j) \end{array} \right] \\
&+t.i.p + Order(\quad \| \quad \xi \|^3) \\
&= \phi_h \overline{\overline{L}} V_l \left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t \\ +n_M n_L \frac{1+\nu}{2} \left(\frac{1}{1+\nu}\right)^2 \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right)^2 + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ + \sum\limits_{j=M,L} \frac{n_j}{2} \phi_{h_j}^{-1}(\nu + \frac{1}{\epsilon_j}) var_{h_j} \ln l_{j,t}(h_j) \end{array} \right] \\
&+t.i.p + Order(\quad \| \quad \xi \|^3), \qquad (30)
\end{aligned}
$$

where we use the following equality to obtain the third line:

$$
\begin{aligned}
\widehat{L}_{M,t} - \widehat{L}_{L,t} &= \widehat{\Omega}_{L,t} - \widehat{\Omega}_{M,t} \\
&= \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right) + \frac{1}{n_L}\int_0^{n_L} w_t(h_L)dh_L - \frac{1}{n_M}\int_0^{n_M} w_t(h_M)dh_M \\
&= \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right) - \nu\left(\widehat{L}_{L,t} - \widehat{L}_{M,t}\right).
\end{aligned}
$$

We also make use of the log-linear relations from equation (3), equation (11) and the definitions from equation (1), equation (2), and equation (16).

Replacing $\phi_h \overline{\overline{L}} V_l$ by $(1-\Phi)\overline{Y}U_c$ yields:

$$
\begin{aligned}
&\int_0^{n_M} V(l_{M,t}(h_M);\nu_t)dh_M + \int_0^{n_L} V(l_{L,t}(h_L);\nu_t)dh_L \\
&= \overline{Y}U_c \left[\begin{array}{c} (1-\Phi)\widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y}_t + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ + \sum\limits_{j=M,L} \frac{n_j}{2} \phi_{h_j}^{-1}(\nu + \frac{1}{\epsilon_j}) var_{h_j} \ln l_{j,t}(h_j) \\ +n_M n_L \frac{1+\nu}{2} \left(\frac{1}{1+\nu}\right)^2 \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right)^2 \end{array} \right] \\
&+t.i.p + Order(\quad \| \quad \xi \|^3). \qquad (31)
\end{aligned}
$$

Here, we use the assumption that distortion of the output level Φ, induced by the firm's price markup through:

$$\prod_{j=M,L}\left\{\left(\frac{1}{n_j}\right)\int_0^{n_j}\left\{\frac{V_l\left[l_{j,T}\left(h_j\right)\right]}{U_C\left(C_t\right)}\frac{\partial\widetilde{L}_{t,T}\left(f\right)}{\partial y_{t,T}\left(f\right)}\right\}^{1-\epsilon_j}dh_j\right\}^{\frac{n_j}{1-\epsilon_j}}, \tag{32}$$

which would exist under flexible prices and no role of monetary policy is of the same order as in Woodford (2003).[25] Thus, in terms of the natural rate of output, we actually assume that the real marginal cost function of firm $Z(\cdot)$ in order to supply a good f is given by:

$$Z(f)_t = Z(y_t(f), Y_t, r_t; \nu_t) = \prod_{j=M,L}\left\{\left(\frac{1}{n_j}\right)\int_0^{n_j}\left[1+r_{j,t}\left(h_j\right)\right]^{1-\epsilon_j}\left\{\frac{V_l\left[l_{j,T}\left(h_j\right)\right]}{U_C\left(C_t,\nu_t\right)}\frac{\partial\widetilde{L}_{t,T}\left(f\right)}{\partial y_{t,T}\left(f\right)}\right\}^{1-\epsilon_j}dh_j\right\}^{\frac{n_j}{1-\epsilon_j}},$$

then the natural rate of output $Y_t^n = Y^n(\xi_t)$ is given by:

$$Z(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta-1}{\theta} = (1-\Phi)\prod_{j=M,L}\left[\frac{\epsilon_j}{\epsilon_j-1}\right]^{n_j}\left[1+\overline{R}_j\right]^{n_j}, \tag{33}$$

where the parameter Φ expresses the distortion of the output level, which is of order one.[26] Then we can combine equation (27) and equation (31) as follows:

[25] We assume that monetary policy has no impact on the level of the natural rate of output.

[26] By assuming a proper proportional tax on sales τ as:

$$Z(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta-1}{\theta}(1-\tau) = (1-\Phi)\prod_{j=M,L}\left[\frac{\epsilon_j}{\epsilon_j-1}\right]^{n_j}\left[1+\overline{R}\right]^{n_j},$$

we can induce $\Phi = 0$ as in Rotemberg and Woodford (1997).

$$
\begin{aligned}
U_t &= \overline{Y}U_c \begin{bmatrix} \Phi\widehat{Y}_t - \frac{1}{2}(\sigma^{-1}+\omega)\widehat{Y}_t^2 + (\sigma^{-1}g_t + \omega q_t)\widehat{Y}_t \\ -\frac{1}{2}\eta_\pi var_f \ln p_t(f) - \sum\limits_{j=M,L} \frac{n_j}{2}\eta_{l_j} var_{h_j} \ln l_{j,t}(h_j) \\ +n_M n_L \frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2 \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right)^2 \end{bmatrix} \\
&+t.i.p + Order(\parallel \xi \parallel^3) \\
&= -\frac{1}{2}\overline{Y}U_c \begin{bmatrix} (\sigma^{-1}+\omega)(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) \\ + \sum\limits_{j=M,L} n_j \eta_{l_j} var_{h_j} \ln l_{j,t}(h_j) \\ +n_M n_L \frac{1+\nu}{2}\left(\frac{1}{1+\nu}\right)^2 \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right)^2 \end{bmatrix} \\
&+t.i.p + Order(\parallel \xi \parallel^3), \qquad (34)
\end{aligned}
$$

where $\eta_\pi \equiv \theta(1+\omega_p\theta)$, $\eta_{l_j} \equiv \phi_{h_j}^{-1}(\nu + \epsilon_j^{-1})$, $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$, and $x^* \equiv \ln(Y^*/\overline{Y})$. Here Y^* is a solution of equation (33) evaluated by $\Phi = 0$, and it is called the efficient level of output (see Woodford, 2003a). In the second line above, we use the log-linearization of equation (33):

$$
\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y}) = \frac{\sigma^{-1}g_t + \omega q_t}{\sigma^{-1}+\omega},
$$

and the relation between the efficient level of output and the natural rate of output implied by equation (32):

$$
\ln(Y_t^n/Y_t^*) = -(\sigma^{-1}+\omega)\Phi + Order(\parallel \xi \parallel).
$$

The latter shows that the percentage difference between Y_t^n and Y_t^* is independent from shocks in the first-order approximation. To evaluate $var_{h_j}\widehat{l}_{j,t}(h_j)$, we use the optimal condition of labor supply and the labor demand function given by equation (5), equation (11). By log-linearizing these equations, we finally have the following relation:

$$
var_{h_j} \ln l_{t,j}(h_j) = \Xi_j var_{h_j} \ln(1 + r_{j,t}(h_j)) + Order(\parallel \xi \parallel^3),
$$

where $\Xi_j \equiv \frac{\epsilon_j^2}{(1+\nu\epsilon_j)^2}$.[27] Then, equation (33) is transformed into:

$$
\begin{aligned}
UT_t &= -\frac{1}{2}\overline{Y}U_c \left[\begin{array}{c} (\sigma^{-1}+\omega)(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) \\ + \sum_{j=M,L} n_j \eta_{r,j} var_{h_j} \ln(1 + r_{j,t}(h_j)) \\ + n_M n_L \frac{1+\nu}{2} \left(\frac{1}{1+\nu}\right)^2 \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right)^2 \end{array} \right] \\
&+ t.i.p + Order(\parallel \xi \parallel^3), \qquad (35)
\end{aligned}
$$

where $\eta_{r,j} \equiv \Xi \eta_{l_j} = \phi_{h_j}^{-1} \frac{\epsilon_j}{(1+\nu\epsilon_j)}$.

What remains is to evaluate $var_f \ln p_t(f)$ and $var_{h_j} \ln(1 + r_{t,j}(h_j))$ in equation (35). Following Woodford (2003a), we define:

$$\overline{P}_t \equiv E_f \ln p_t(f) \text{ and } \triangle_t \equiv var_f \ln p_t(f).$$

Then a change in the aggregate price is described by:

$$
\begin{aligned}
\overline{P}_t - \overline{P}_{t-1} &= E_f \left[\ln p_t(f) - \overline{P}_{t-1}\right] \\
&= \alpha E_f \left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right] + (1-\alpha) E_f \left[\ln p_t^*(f) - \overline{P}_{t-1}\right] \\
&= (1-\alpha) E_f \left[\ln p_t^*(f) - \overline{P}_{t-1}\right]. \qquad (36)
\end{aligned}
$$

For the cross-sectional variance of prices, we have:

[27] We assume $cov_h(\ln w_{t,j}(h_j), \ln(1+r_{j,t}(h_j)))$ is not zero regardless of ambiguity of the policy rule. If we assume $cov_h(\ln w_{t,j}(h_j), \ln(1+r_{j,t}(h_j)))$, $\Xi_j \equiv \epsilon_j^2 (\frac{\epsilon_j^2}{(\nu^{-1}+\epsilon_j)^2} + 1)$. But, this difference does not change the qualitative and quantitative outcomes in this paper.

$$
\begin{aligned}
\triangle_t &= var_f\left[\ln p_t(f) - \overline{P}_{t-1}\right] \\
&= E_f\left\{\left[\ln p_t(f) - \overline{P}_{t-1}\right]^2\right\} - (E_f \ln p_t(f) - \overline{P}_{t-1})^2 \\
&= \alpha E_f\left\{\left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right]^2\right\} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)(var_f(\ln p_t^*(f) - \overline{P}_{t-1}) + \left\{E_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]\right\}^2) - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}(\overline{P}_t - \overline{P}_{t-1}), \qquad (37)
\end{aligned}
$$

where $p_t^*(f)$ is an optimal price set by active firm f at period t. Furthermore, we have the following relation that relates $\overline{P}_t$ to P_t :

$$
\overline{P}_t = \ln P_t + Order(\parallel \xi \parallel^2),
$$

where $Order(\parallel \xi \parallel^2)$ represents terms higher than the first order. Here we make use of the definition of the price aggregator $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$. Then equation (37) can be transformed as follows:

$$
\triangle_t = \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}\pi_t, \qquad (38)
$$

where $\pi_t \equiv \ln \frac{P_t}{P_{t-1}}$. From equation (38), we have:

$$
\triangle_t = \alpha^{t+1}\triangle_{-1} + \sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha}{1-\alpha}\right)\pi_s^2,
$$

and so:

$$
\sum_{t=0}^{\infty}\beta^t\triangle_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p + Order(\parallel \xi \parallel^3). \qquad (39)
$$

To evaluate $var_{h_j}\ln(1 + r_{j,t}(h_j))$, we define $\overline{R}_{j,t}$ and $\triangle_t^{R_j}$ as:

$$\overline{R}_{j,t} \equiv E_{h_j} \ln(1 + r_{j,t}(h_j)),$$

$$\triangle_t^{R_j} \equiv var_{h_j} \ln(1 + r_{j,t}(h_j)).$$

Then, we can make following relations:

$$\begin{aligned}
\overline{R}_{j,t} - \overline{R}_{j,t-1} &= E_{h_j}\left[\ln(1 + r_{j,t}(h_j)) - \overline{R}_{j,t-1}\right] \\
&= \varphi^j E_{h_j}\left[\ln(1 + r_{j,t-1}(h_j)) - \overline{R}_{j,t-1}\right] + (1-\varphi^j)\left[\ln(1 + r^*_{j,t}) - \overline{R}_{j,t-1}\right] \\
&= (1-\varphi^j)\left[\ln(1 + r^*_{j,t}(h_j)) - \overline{R}_{j,t-1}\right], \qquad (40)
\end{aligned}$$

and:

$$\begin{aligned}
\triangle_t^{R_j} &= var_{h_j}\left[\ln(1 + r_{j,t}(h_j)) - \overline{R}_{j,t-1}\right] \\
&= E_{h_j}\left\{\left[\ln(1 + r_{j,t}(h_j)) - \overline{R}_{j,t-1}\right]^2\right\} - (E_{h_j}\ln(1 + r_{j,t}(h_j)) - \overline{R}_{j,t-1})^2 \\
&= \varphi^j E_{h_j}\left\{\left[\ln(1 + r_{j,t-1}(h_j)) - \overline{R}_{j,t-1}\right]^2\right\} + (1-\varphi^j)\left[\ln(1 + r^*_{j,t}) - \overline{R}_{j,t-1}\right]^2 - (\overline{R}_{j,t} - \overline{R}_{j,t-1})^2 \\
&= \varphi^j \triangle_{t-1}^{R_j} + \frac{\varphi^j}{1-\varphi^j}(\overline{R}_{j,t} - \overline{R}_{j,t-1})^2, \qquad (41)
\end{aligned}$$

where we use equation (40).

A similar discussion applies to the bank loan interest rates. We have:

$$\overline{R}_{j,t} = \ln(1 + R_{j,t}) + Order(\| \xi \|^2), \qquad (42)$$

where we make use of the definition of the aggregate loan rates $1 + R_{j,t} \equiv \int_0^1 \frac{q_{j,t}(h_j)}{Q_t}(1 + r_{j,t}(h_j))dh_j$. For the cross-sectional variance of bank loan interest rates, we have:

$$\triangle_t^{R_j} = \varphi^j \triangle_{t-1}^{R_j} + \frac{\varphi^j}{1-\varphi^j}(\widehat{R}_{j,t} - \widehat{R}_{j,t-1})^2, \qquad (43)$$

where $\widehat{R}_{j,t} \equiv \ln \frac{1+R_{j,t}}{1+\bar{r}}$. From equation (43), it can be written as:

$$\triangle_t^{R_j} = \left(\varphi^j\right)^{t+1} \triangle_{-1}^{R_j} + \sum_{s=0}^{t} \left(\varphi^j\right)^{t-s} \left(\frac{\varphi^j}{1-\varphi^j}\right) (\widehat{R}_{j,s} - \widehat{R}_{j,s-1})^2.$$

Then, we have

$$\sum_{t=0}^{\infty} \beta^t \triangle_t^{R_j} = \frac{\varphi^j}{(1-\varphi^j)(1-\varphi^j\beta)} \sum_{t=0}^{\infty} \beta^t (\widehat{R}_{j,t} - \widehat{R}_{j,t-1})^2 + t.i.p + Order(\| \xi \|^3).$$

Now, equation (35) can be reduced to:

$$\sum_{t=0}^{\infty} \beta^t UT_t \simeq -\Lambda \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 \\ + \sum_{j=M,L} \lambda_j (\widehat{R}_{j,t} - \widehat{R}_{j,t-1})^2 + \lambda_{ML} \left(\widehat{R}_{M,t} - \widehat{R}_{L,t}\right)^2 \end{array} \right), \qquad (44)$$

where $\Lambda \equiv \frac{1}{2}\overline{Y} u_c$, $\lambda_\pi \equiv \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \theta(1+\omega_p\theta)$, $\lambda_x \equiv (\sigma^{-1}+\omega)$, $\lambda_j \equiv n_j \phi_{h_j}^{-1} \frac{\epsilon_j}{(1+\nu\epsilon_j)} \frac{\varphi^j}{(1-\varphi^j)(1-\varphi^j\beta)}$, for $j = L, M$, and $\lambda_{ML} \equiv n_M n_L \frac{1+\nu}{2} \left(\frac{1}{1+\nu}\right)^2$.

B Optimal monetary policy rule

We consider an optimal monetary policy scheme in which a central bank is credibly committed to a policy rule from a *timeless perspective*[28]. In this case, as shown in Woodford (2003), the central bank conducts monetary policy in a forward-looking way by taking account of future economic variables and the monetary policy effects on those variables.

The objective of monetary policy is to minimize the expected value of the loss criterion expressed by (44), under the standard New Keynesian IS curve (10), the augmented Phillips curve (20), the loan rate curve of type M banks, and the loan rate curve of type L banks (23). The optimal monetary policy is described as the solution of the optimization problem,

[28] A detailed explanation of the timeless perspective is in Woodford (2003).

which is represented by the following Lagrangian:

$$\mathcal{L} = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} L_t + 2\Xi_{1t} \left[x_{t+1} - \sigma(\widehat{i}_t - \pi_{t+1} - r_t^n) - x_t \right] \\ +2\Xi_{2t} \left[\kappa x_t + \chi \left(n_M \widehat{R}_{M,t} + n_L \widehat{R}_{L,t} \right) + \beta \pi_{t+1} - \pi_t \right] \\ +2\Xi_{3t} \left[\lambda_1^M \widehat{R}_{M,t+1} + \lambda_2^M \widehat{R}_{M,t-1} + \lambda_3^M \widehat{i}_t - \widehat{R}_{M,t} \right] \\ +2\Xi_{4t} \left[\lambda_1^L \widehat{R}_{L,t+1} + \lambda_2^L \widehat{R}_{L,t-1} + \lambda_3^L \widehat{i}_t - \widehat{R}_{L,t} \right] \end{array} \right\} \right\},$$

where Ξ_1, Ξ_2, Ξ_3, and Ξ_4 are the Lagrange multipliers associated with the IS curve constraint, the Phillips curve constraint, and the loan rate curve constraints, respectively. Taking the first derivatives of the Lagrangian with respect to π_t, x_t, $\widehat{R}_{M,t}$, $\widehat{R}_{L,t}$, and $\widehat{i}_t$ yields the following optimal monetary policy rule:

$$\left[\begin{array}{l} -z_3^{-1} z_4^{-1} (1 - z_5 L)^{-1} (1 - z_6 F)^{-1} \left\{ \begin{array}{c} \lambda_M (\triangle \widehat{R}_{M,t} - \beta \mathrm{E}_t \triangle \widehat{R}_{M,t+1}) + \lambda_{ML} (\widehat{R}_{M,t} - \widehat{R}_{L,t}) \\ -\kappa^{-1} \lambda_x n_M (x_t - x^*) \end{array} \right\} \\ -(z_3^* z_4^*)^{-1} (1 - z_5^* L)^{-1} (1 - z_6^* F)^{-1} \left\{ \begin{array}{c} \lambda_L (\triangle \widehat{R}_{L,t} - \beta \mathrm{E}_t \triangle \widehat{R}_{L,t+1}) - \lambda_{ML} (\widehat{R}_{M,t} - \widehat{R}_{L,t}) \\ -\kappa^{-1} \lambda_x n_L (x_t - x^*) \end{array} \right\} \end{array} \right]$$

$$= \mathrm{E}_t \left[(1 - z_1 L)^{-1} (1 - z_2 L)^{-1} (\kappa \lambda_\pi \pi_t + \lambda_x \triangle x_t) \right], \tag{45}$$

where z_1, z_2, z_3, z_4, z_5, z_6, z_3^*, z_4^*, z_5^*, and z_6^* are parameters that satisfy $z_1 + z_2 = 1 + \beta^{-1} + \kappa\sigma\beta^{-1}$, $z_1 z_2 = \beta^{-1}$ ($z_1 > 1$, $0 < z_2 < 1$), $z_3 = -\frac{\beta\lambda_2^M \sigma}{\lambda_3^L}$, $z_4 z_5 = \frac{1}{z_3}(\frac{\sigma}{\lambda_3^M} - \frac{n_M}{\kappa})$, $z_4 + z_5 = -\frac{1}{z_3}(\frac{n_M}{\beta\kappa} - \frac{\sigma\lambda_1^M}{\beta\lambda_3^M})$, $z_6 = z_4^{-1}$, $z_3^* = -\frac{\beta\lambda_2^L \sigma}{\lambda_3^L}$, $z_4^* z_5^* = \frac{1}{z_3^*}(\frac{\sigma}{\lambda_3^L} - \frac{n_L}{\kappa})$, $z_4^* + z_5^* = -\frac{1}{z_3^*}(\frac{n_L}{\beta\kappa} - \frac{\sigma\lambda_1^L}{\beta\lambda_3^L})$, and $z_6^* = (z_4^*)^{-1}$.