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Ichiro Muto*

Abstract
In this study, we investigate how central bank transparency about views on future productivity growth influences social welfare. To this end, we use a New Keynesian framework in which both the central bank and private agents are engaged in filtering problems regarding the persistence of productivity growth. Since the central bank and private agents do not know the true value of the signal-to-noise ratio, the gain parameters used in the filtering problems can be heterogeneous. If the central bank is not transparent, private agents must conjecture the central bank's estimate of the efficient level of the real interest rate. Under this setup, we show that central bank transparency does not necessarily improve social welfare. It can potentially yield a welfare loss, depending on (i) the gain parameters used by the central bank and private agents and (ii) private agents' conjecture on the gain parameter used by the central bank. If the central bank is uncertain about the combination of these gain parameters, it is sensible for the central bank to respond strongly to the variations of the inflation rate, because the misperceptions about these parameters become the source of demand shock.

Keywords: New Keynesian Model; Monetary Policy; Transparency; Productivity Growth; Learning

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1 Introduction

In recent years, the implications of central bank transparency have been actively investigated in monetary economics. According to Geraats [2002], central bank transparency is defined as “the absence of asymmetric information between monetary policymakers and other economic agents” (p. F533). If we apply this definition to the context of monetary policy, the large degree of transparency indicates the situation in which a central bank provides private agents with ample information regarding monetary policymaking, such as a policy objective, policy strategy, economic perspective, and so on. If this information has some degree of impact on private agents’ activity, especially on their expectation formation, then central bank transparency potentially influences economic dynamics, and ultimately, social welfare.

Among the many aspects of central bank transparency, this study focuses on “economic transparency” in the terminology of Geraats [2002]. Economic transparency concerns the economic information that is used for monetary policy, including economic data, policy models, and central bank forecasts (Geraats [2002], p. F540). In our view, economic transparency is distinct from other kinds of transparency in that it does not deal with the behavior of the central bank itself. Rather, it concerns the central bank’s views on economic conditions or economic structures, which are mainly determined by the activities of private agents. In this sense, economic transparency is more indirectly related to a central bank’s monetary policymaking than other kinds of transparency, such as political, procedural, policy, and operational transparency, which are mostly related to the behavior of the central bank itself.

In the case of economic transparency, it will be arguable whether a central bank should seek to be perfectly transparent, because the central bank usually faces considerable uncertainty as to economic conditions or economic structures. If we take account of this kind of uncertainty, it is not so straightforward a task to evaluate the value of central bank transparency because the information provided by the central bank to private agents might be inaccurate, and such inaccurate information might cause economic fluctuations.

The problem of uncertainty becomes particularly serious with respect to the trend growth of aggregate productivity. There is no doubt that trend productivity growth is the key variable for monetary policymaking, because theoretically it is the crucial determinant of the potential growth of GDP and the equilibrium level of the real interest rate. However, it is widely recognized that it is quite difficult to obtain an accurate estimate of the trend growth of aggregate productivity, especially in real time. Concerning this issue, Bernanke [2005] remarks that “notably, imperfect data and the difficulties of distinguishing permanent from temporary changes will make

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1 See Geraats [2002] and Cruijssen and Eijffinger [2007] for a survey of the literature on central bank transparency.
changes in secular productivity growth exceptionally difficult to identify in real time, both for the private sector and for the Federal Reserve. The need to discern the underlying economic forces and to react appropriately in an environment of incomplete information makes monetary policy an exceptionally challenging endeavor."

Once we take account of the large uncertainty, the issue of whether a central bank should be greatly transparent, even if it is quite uncertain about the views on future productivity growth, will deserve the attention of monetary policymakers. In particular, the issue is complicated because, as is noted by Bernanke, not only the central bank but also private agents face uncertainty regarding the persistence of productivity growth. In such a case, the desirability of central bank transparency is likely to depend on private agents’ forecast about the future productivity growth. Therefore, an analysis of central bank transparency in this respect should clarify how the value of such transparency depends on the forecasting mechanisms for future productivity growth used by the central bank and private agents.

Furthermore, if economic dynamics depend on central bank transparency and the forecasting mechanisms used by the central bank and private agents, an optimal policy response will not be independent of these aspects. Therefore, it is also important to study how optimal monetary policy depends on central bank transparency and the forecasting mechanisms used by the central bank and private agents. In considering this issue, it is particularly important to analyze what kind of monetary policy robustly performs well against a wide variety of private agents’ forecasting mechanisms because, in practice, a central bank faces great uncertainty regarding the forecasting mechanism used by private agents.

Based on the above argument, we investigate how central bank transparency about the views on future productivity growth influences social welfare. To this end, we introduce a simple version of a New Keynesian model, which is very close to the model of Galí, Lopez-Salido, and Valles [2003] or Ireland [2004]. Since we judge that central bank transparency mainly influences economic dynamics through the process of private agents’ expectation formation, the forward-looking nature of the New Keynesian model is suitable for carrying out our analysis. In addition, we consider that the simplicity of our version of the New Keynesian model is favorable, since we can explicitly calculate the analytical solution and evaluate the impact of central bank transparency in terms of social welfare, not in terms of some ad hoc central bank’s loss function.

In this study, we assume that the central bank and private agents cannot fully identify the transitory and persistent components of productivity growth and that they are engaged in filtering problems regarding the persistence of productivity growth. This setup has already been introduced in some previous studies, such as Tambalotti [2003], Edge, Laubach, and Williams [2005, 2007], and Gilchrist and Saito [2007]. These studies have shown that private agents’ gradual recognition of
the persistence of productivity growth can replicate the persistent movements of major macroeconomic variables, which are usually found in vector autoregression (VAR) analysis. Therefore, these studies imply that the inclusion of a filtering mechanism is beneficial in yielding a realistic impulse response to productivity shocks. However, none of the studies analyze the influence of central bank transparency.

The contribution of our study is that we investigate the influence of central bank transparency in an environment in which both central bank and private agents are filtering with respect to the persistence of productivity growth. In carrying out the analysis, we introduce heterogeneity in the forecasting mechanisms used by the central bank and private agents. Figure 1 presents forecasts on real output growth made by the Federal Reserve Board (FRB) and economists in the private sector. This figure shows that the FRB and economists in the private sector do not necessarily share the same forecasts for future output growth in each period. The possibility of heterogeneous forecasts is essential in examining the issue of transparency, because it provides private agents with necessity to conjecture the central bank’s forecast on future productivity growth and gives rise to the possibility that central bank transparency has some impact on private agents’ expectations concerning future monetary policy.

In this analysis, we assume that the heterogeneous forecasts arise because the central bank and private agents use different forecasting rules. More concretely, they use different gain parameters in the filtering problem. The reason why they do this is explained by the uncertainty about the variances of transitory and persistent productivity shocks. The uncertainty on this respect is highly plausible, because some empirical studies (Stock and Watson [1998] and Roberts [2001]) show that the uncertainty regarding these shock variances is large in the U.S. economy, and the recent analysis of Justiniano and Primiceri [2006] further shows that there have been large structural changes in shock variances in the U.S. economy, which explains the decline of the volatility of U.S. major macroeconomic variables. In this study, we assume that whereas the central bank and private agents use the same information set concerning current productivity growth, they can use different gain parameters since they can differently assess the possibility of structural change in shock variances. The heterogeneity in gain parameters yields the heterogeneous forecasts for future

\footnote{Although output growth does not directly correspond to productivity growth, it is fair to judge that at least some portion of the different forecasts on output growth comes from the heterogeneity in the views on future productivity growth.}

\footnote{In the context of adaptive learning, Honkapohja and Mitra [2005] examine the E-stability condition in an environment where central bank and private agents use heterogeneous constant gain parameters to estimate their subjective reduced-form model. As is well explained in Evans and Honkapohja [2001], constant gain is used when agents take account of the possibility of structural change. Although our problem is filtering (not learning), our usage of heterogeneous gain parameters could be explained by the central bank’s and private agents’ awareness of the possible future structural change in the variances of productivity shocks.}
productivity growth, which become the disturbance to economic fluctuations.4

Our study is distinct from those that analyze central bank transparency in an environment where a central bank or private agents have private information on current economic conditions (Amato and Shin [2003], Morris and Shin [2005], Hellwig [2005], Walsh [2007], and Lorenzoni [2007]) because we do not explicitly introduce any private information to both agents. As a result, our study does not introduce any strategic interaction between the central bank and private agents in the formation of their expectations for future productivity growth. Therefore, the situation we analyze can be seen as a simplified benchmark. However, we still consider that the existence of private information is an empirical problem because, as noted by Kohn [2005], most of the data used by central bank in forecasting aggregate economic variables is also available to private agents.5

We define a central bank as transparent (or a central bank as adopting a transparent regime) if the central bank announces its forecast on future productivity growth, and we also define a central bank as opaque (or a central bank as adopting an opaque regime) if the central bank does not announce the forecast. Private agents have to conjecture the central bank’s estimate on future productivity growth to form their expectations for future output and the inflation rate, since the central bank’s forecast on future productivity growth corresponds to the central bank’s estimate on the efficient level of the real interest rate, which influences the future interest rate. If the central bank is transparent, private agents’ conjecture on central bank’s estimate on the efficient level of the real interest rate is just equal to the true value of the central bank’s estimate. However, if the central bank is opaque, private agents’ conjecture does not necessarily coincide with the true value of the central bank’s estimate. In an opaque regime, private agents estimate the central bank’s forecast on future productivity growth by using private agents’ conjecture about the gain parameter used by the central bank, which is not necessarily the same as the central bank’s true gain parameter.

We evaluate the welfare gains (or possibly the losses) from the central bank transparency. In doing so, we simply examine how welfare losses differ between the transparent regime and the opaque regime. To restrict our attention to the pure impact

4Bullard and Eusepi [2005] investigate the economic dynamics of the New Keynesian model under a mechanism in which both the central bank and private agents are learning the structural parameters, including the process of productivity growth. Although their analysis is close to ours, they do not introduce the heterogeneity in learning mechanisms between the central bank and private agents. In addition, they do not investigate the influence of central bank economic transparency.

5Kohn [2005] remarks that “in the United States, we have some indirect evidence that crowding out of private views has not increased even as the Federal Reserve has become more talkative. Market interest rates have continued to respond substantially to surprises in economic data.” He also states, “that markets continue to react strongly to incoming data is not surprising. Predicting interest rates far enough into the future is not just about what others—including the central bank—think; over time those rates should be tied to objective factors—for example, the forces of productivity and thrift. Differing views about these factors give scope for opportunities to profit from independent research and betting against the crowd.”
of the central bank’s information provision to private agents, we exclude the possibility that the central bank changes the regime (transparent or opaque) period by period, because this possibility inevitably raises the problem of credibility. For the same reason, we rule out the possibility that central bank announces a forecast of productivity growth that differs from its true forecast.

Our results show that central bank transparency about the views on future productivity growth does not necessarily improve social welfare. It can potentially yield a welfare loss, depending on (i) gain parameters used by the central bank and private agents and (ii) private agents’ conjecture about the gain parameter used by the central bank. If the gain parameters used by the central bank and private agents are homogeneous, then central bank transparency always improves social welfare. However, if these gain parameters are heterogeneous, central bank transparency can be either welfare-improving or welfare-reducing. In the latter case, the value of central bank transparency crucially depends on private agents’ conjecture on the gain parameter used by the central bank. Our study shows that if the central bank is uncertain about the combination of the gain parameters (including private agents’ conjecture), it is sensible for the central bank to respond strongly to the variations of the inflation rate, because the misperceptions on these parameters become the source of demand shock.

The rest of this paper is organized as follows. In Section 2, we present our model, including the economic structure, the process of productivity growth, and the mechanisms of forecasting future productivity growth used by the central bank and private agents. In Section 3, we investigate the influence of central bank transparency on economic dynamics and social welfare. In particular, we clarify under what conditions central bank transparency is welfare-improving or welfare-reducing. In Section 4, we investigate how the desirable monetary policy actions depend on central bank transparency and the forecasting mechanisms used by the central bank and private agents. Specifically, we investigate the optimal response to inflation rate in the central bank’s simple monetary policy rule. In doing so, we also examine the influence of private agents’ learning mechanism regarding the gain parameter used by the central bank. In Section 5, we summarize the results of this study and present some possible extensions of it.

2 Model

We use a simple version of a New Keynesian model in which all the existing goods are consumption goods and there are no frictions other than price stickiness and markup fluctuations. This version is quite similar to the model of Galí, Lopez-Salido, and Valles [2003] or Ireland [2004]. However, in contrast to their models, ours introduces a filtering problem in which the central bank and private agents estimate the persistence
of productivity growth. In addition, our model describes a situation in which private agents conjecture the central bank’s views on future productivity growth when the central bank is opaque in this respect.

2.1 Household

The representative household maximizes the following inter-temporal utility function:

$$E_t \sum_{k=0}^{\infty} \beta^{k} \left( \ln Y_{t+k} - \frac{1}{\eta} N_{t+k}^{\eta} \right),$$

where $Y_t$ is aggregate consumption (equal to aggregate output), $N_t$ is labor supply, $\beta$ is the discount factor, and $\eta$ is the parameter related to labor supply elasticity.

Utility maximization yields the following first-order conditions:

$$\ln Y_t = E_t \ln Y_{t+1} - (i_t - E_t \pi_{t+1} - \rho),$$

$$W_t / P_t = Y_t N_t^{\eta-1},$$

where $i_t$ is the nominal interest rate, $\pi_t$ is the inflation rate, $\rho$ is the discount rate (calculated as $\rho = -\ln \beta$), $W_t$ is the nominal wage rate, and $P_t$ is the price level.

$Y_t^*$ and $r_t^*$ denote the output and the real interest rate that should be realized in an environment in which both price stickiness and the distortion due to the time-varying markup are absent. Following Galí [2006], we call this environment an "efficient steady state." Similarly, we call $Y_t^*$ "efficient output" and $r_t^*$ the "efficient interest rate." These differ from the popular concept of "natural output" and the "natural interest rate," which will be realized in an environment in which only price stickiness is absent. This distinction has quite an important implication for welfare analysis (see Galí [2006]).

Since the Euler equation (2) must hold even in the efficient steady state, $Y_t^*$ and $r_t^*$ satisfy the following relationship:

$$\ln Y_t^* = E_t \ln Y_{t+1}^* - (r_t^* - \rho).$$

We define $x_t$ as the output gap $(x_t \equiv \ln Y_t - \ln Y_t^*)$. Then, (2) and (4) yield the following dynamic IS equation:

$$x_t = E_t x_{t+1} - (i_t - E_t \pi_{t+1} - r_t^*).$$

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6 In our model, the desired markup varies around a steady-state level. In an efficient steady state, the markup is fixed at the steady-state level, though it is not equal to unity. Therefore, the distortion due to the steady-state level of the desired markup remains even in the efficient steady state.

7 $x_t$ is the welfare-relevant output gap, in the terminology of Galí [2006].
2.2 Firm

The representative firm’s production function is given by

\[ Y_t = A_tN_t, \]  

(6)

where \( A_t \) is the level of aggregate productivity. The nominal marginal cost \((MC_t)\) is calculated as follows:

\[ MC_t = W_tN_t/Y_t, \]  

(7)

where \( W_t \) is the nominal wage rate, which is assumed to be given for each firm.

We define \( \psi_t \) as the desired markup, which should be realized under a flexible price economy. Under the conventional aggregator \((Y_t \equiv (\int_0^1 Y_t(i) \frac{\theta_{t-1}^{i-1}}{\theta_{t-1}} di) \frac{\theta_t}{\theta_{t-1}})\), \( \psi_t \) is determined by the elasticity of substitution among individual goods \((\theta_t)\) as follows:

\[ \psi_t = \frac{\theta_t}{\theta_t - 1}, \]  

(8)

where \( \theta_t \) moves around the steady-state value \((\theta)\) in each period.

If we apply Calvo’s [1983] and Yun’s [1996] specification of sticky prices in which each period a measure of \( 1 - \alpha \) firms can reset prices, firms’ profit maximization yields the New Keynesian Phillips curve (NKPC) with respect to real marginal cost:

\[ \pi_t = \beta E_t T_{t+1} + \left(1 - \alpha\right)\left(1 - \beta \alpha / \alpha\right) \left(\ln RMC_t + \ln \psi_t\right), \]  

(9)

where \( RMC_t \) represents real marginal cost \((RMC_t = MC_t/P_t)\).

To rewrite NKPC in terms of the output gap, we provide the firm’s optimality condition in the efficient steady state as follows:

\[ P_t = \psi MC_t, \]  

(10)

where \( \psi \) is the steady-state value of the desired markup \((\psi = \frac{\theta}{\theta - 1})\). Then, from (3), (6), (7), and (10), we can express efficient output \( Y^*_t \) as follows:\(^8\)

\[ Y^*_t = \psi^{-1/\eta} A_t. \]  

(11)

From (3), (6), and (7), we calculate the actual output as follows:

\[ Y_t = RMC_t^{1/\eta} A_t. \]  

(12)

Finally, from (9), (11), and (12), we derive NKPC in terms of the output gap as

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\(^8\) As we have already explained, efficient output corresponds to the output that should be realized in the absence of price stickiness and time-varying components of the markup \((\psi_t - \psi)\). So the distortion that arises from the steady-state markup \((\psi)\) remains even in the efficient steady state. This is the reason why efficient output depends on \( \psi \).
follows:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \zeta_t, \quad (13) \]

where \( \kappa \) is the slope of NKPC \( (\kappa \equiv \eta(1-\alpha)(1-\beta\alpha)/\alpha) \) and \( \zeta_t \) is the cost-push shock, which is defined as follows:

\[ \zeta_t \equiv (1-\alpha)(1-\beta\alpha)/\alpha (\ln \psi_t - \ln \psi), \quad \zeta_t \sim i.i.d. N(0, \sigma_\zeta). \]

### 2.3 Monetary Policy

The central bank introduces the following simple monetary policy rule:

\[ i_t = r_t^{*C} + \gamma \pi_t + \xi_t, \quad \xi_t \sim i.i.d. N(0, \sigma_\xi), \quad (14) \]

where \( r_t^{*C} \) is the central bank’s estimate of the efficient interest rate, \( \gamma \) is the responsiveness to the inflation rate, and \( \xi_t \) is the monetary policy shock. Since we assume that the central bank cannot directly observe the efficient interest rate, the rule introduces the central bank’s estimate on the efficient interest rate \( (r_t^{*C}) \), not the true value of \( r_t^* \).

Throughout this study, we assume that private agents know the functional form of (14), including the value of \( \gamma \). However, private agents cannot directly observe the value of \( r_t^{*C} \) and \( \xi_t \) unless the central bank announces these values. In that case, they must conjecture the values of \( r_t^{*C} \) and \( \xi_t \). \( r_t^{*P[C]} \) and \( \xi_t^{P[C]} \) denote these conjectures. Private agents form expectations \( (E_t x_{t+1} \text{ and } E_t \pi_{t+1}) \) by using the following monetary policy rule:

\[ i_t = r_t^{*P[C]} + \gamma \pi_t + \xi_t^{P[C]}, \quad (15) \]

### 2.4 Social Welfare

In the simulations of later sections, we evaluate the value of central bank transparency in terms of social welfare. Woodford [2003] shows that in the simple version of the New Keynesian framework, including our model, welfare loss \( (L) \) can be represented as follows:

\[ L = E_t \sum_{k=0}^\infty \beta^k (\kappa x_{t+k}^2 + \theta \pi_{t+k}^2). \quad (16) \]

### 2.5 Process of Productivity Growth

In modeling the process of productivity growth, we follow previous studies, such as Tambalotti [2003], Edge, Laubach, and Williams [2005, 2007], and Gilchrist and

\[ \text{We assume that private agents regard the process of } \xi_t^{P[C]} \text{ as i.i.d.} \]

\[ \text{In deriving this social welfare function, we assume the existence of an output subsidy that offsets the distortion due to the presence of the desired markup in the steady state (\( \psi \)).} \]
Saito [2007]. In these studies, productivity growth is determined as the combination of transitory and persistent components as follows:

\[ z_t \equiv \ln A_t - \ln A_{t-1} = \bar{z} + \varepsilon_t + \mu_t, \quad \varepsilon_t \sim i.i.d.N(0, \sigma_\varepsilon), \]

(17)

\[ \mu_t = \phi \mu_{t-1} + \nu_t, \quad \nu_t \sim i.i.d.N(0, \sigma_\nu), \]

(18)

where \( z_t \) is productivity growth, \( \bar{z} \) is the long-run equilibrium productivity growth, \( \varepsilon_t \) is the transitory productivity shock, \( \mu_t \) is the persistent productivity shock, \( \phi \) is the persistence of \( \mu_t \) \((0 < \phi < 1)\), and \( \nu_t \) is the innovation to \( \mu_t \).

We assume that the central bank and private agents cannot fully identify the values of \( \varepsilon_t \) and \( \mu_t \), though they can observe the values of \( z_t, \bar{z}, \text{and} \phi \). So they are engaged in the filtering problem to estimate the persistence of the productivity shock. In the following argument, \( \mu^C_t \) and \( \mu^P_t \) denote the subjective estimates about the persistent productivity shocks estimated by the central bank and private agents, respectively.

### 2.6 Efficient Interest Rate

The efficient interest rate is the key variable in this study because, under monetary policy rule (14), central bank transparency about the views on future productivity growth influences economic dynamics through private agents’ conjecture on the central bank’s estimate on the efficient interest rate.

From (4) and (11), the true value of the efficient interest rate is calculated as follows:

\[ r^*_t = \rho + E_t \ln A_{t+1} - \ln A_t. \]

(19)

Notice that \( r^*_t \) is determined by private agents’ forecast on future productivity growth \((E_t \ln A_{t+1} - \ln A_t)\). Therefore, \( r^*_t \) depends on private agents’ information set available at time \( t \). Since we assume that private agents have their subjective estimate on persistent productivity shock \((\mu^P_t)\), \( r^*_t \) is calculated as follows:

\[ r^*_t = \rho + \bar{z} + \phi \mu^P_t. \]

(20)

Thus, \( r^*_t \) depends on private agents’ estimate on the persistent productivity shock \((\mu^P_t)\), rather than on the true value of the persistent productivity shock \((\mu_t)\).

The central bank knows that the efficient interest rate is determined by (19). However, the central bank cannot directly observe private agents’ forecast on future productivity growth \((\mu^P_t)\), because the central bank cannot directly observe private agents’ expectations. For this reason, the estimated efficient interest rate \((r^*_t)^C\), which is included in monetary policy rule (14), depends on the central bank’s subjective
estimate on the persistent productivity shock \((\mu^C_t)\):

\[ rt^C_t = \rho + \varepsilon + \phi \mu^C_t. \quad (21) \]

Private agents need to conjecture the value of \(rt^C_t\) to form the expectations for future output and the inflation rate because, under (14), (17), (18), and (21), the future interest rate depends on the central bank’s current estimate on the efficient interest rate. We define \(\mu^P_t\) as private agents’ conjecture about the central bank’s estimate on the persistent shock \((\mu^C_t)\). Then, \(rt^P_t\) is calculated as follows:

\[ rt^P_t = \rho + \varepsilon + \phi \mu^P_t. \quad (22) \]

### 2.7 Filtering Problem

As already explained, both private agents and the central bank cannot directly observe each component of productivity shock \((\varepsilon_t \text{ and } \mu_t)\). So they estimate the persistence of productivity growth through filtering problems.

Notice that (17) and (18) constitute a state-space model. Therefore, if the central bank and private agents know the true value of the signal-to-noise ratio, which is defined as the relative size in the variances of persistent and transitory productivity shocks \((\sigma^2_\epsilon/\sigma^2_\mu)\), they can obtain the optimal estimate on \(\mu_t\) by using the optimal Kalman filter algorithm. However, since we assume that the central bank and private agents do not know the true values of shock variances \((\sigma^2_\epsilon \text{ and } \sigma^2_\mu)\), they cannot compute the optimal gain parameter for the filtering problem. Therefore, we assume that the central bank and private agents use their subjective gain parameters \((\lambda^C \text{ and } \lambda^P)\) to obtain their estimates on the persistent productivity shock \((\mu^C_t \text{ and } \mu^P_t)\). The algorithms are given by

\[ \mu^C_t = \phi \mu^C_{t-1} + \lambda^C [(z_t - \bar{z}) - \phi \mu^C_{t-1}], \quad (23) \]

\[ \mu^P_t = \phi \mu^P_{t-1} + \lambda^P [(z_t - \bar{z}) - \phi \mu^P_{t-1}], \quad (24) \]

Here \(\lambda^C\) and \(\lambda^P\) are constant values.\(^{11}\) These are not necessarily equal to the value of the optimal Kalman gain, because the central bank and private agents face large uncertainty on shock variances \((\sigma_\epsilon \text{ and } \sigma_\mu)\).\(^{12}\) In addition, these gain parameters can be heterogeneous, because the central bank and private agents can differentially assess the possibility of structural changes in shock variances. As is evident in (23) and (24), neither the central bank nor private agents have private information on current

\(^{11}\) Edge, Laubach, and Williams [2007] show that the Kalman filter with constant gain can replicate the public and private forecasts on long-run labor productivity growth reported in the survey data.

\(^{12}\) The optimal Kalman gain is given by

\[ \lambda^* = 1 - 2[1 + \sigma^2_\epsilon/\sigma^2_\mu + \phi^2 + ((1 - \phi^2)^2 + (\sigma^2_\epsilon/\sigma^2_\mu)^2 + 2(1 + \phi^2)(\sigma^2_\epsilon/\sigma^2_\mu))^{1/2}]^{-1}. \]
productivity growth \((z_t)\). Therefore, this framework does not raise the possibility of strategic interactions between the central bank and private agents in estimating the values of \(\mu^C_t\) and \(\mu^P_t\).

Next, we specify the process through which private agents form their conjecture on the central bank’s estimate on the persistent productivity shock \((\mu^C_t)\). In this respect, we assume that private agents know the central bank’s filtering algorithm \((23)\), though they do not know the value of \(\lambda^C\). In other words, private agents know that the central bank uses the same information set as theirs regarding current productivity growth \((z_t)\) and that the only difference from private agents is in the value of the gain parameter. Under this assumption, private agents estimate the value of \(\mu^C_t\) \((\mu^P_t)^{C}\) denotes the estimate) from the following algorithm:

\[
\mu^P_t = \phi \mu^P_{t-1} + \lambda^P [(z_t - \bar{z}) - \phi \mu^P_{t-1}],
\]

where \(\lambda^P\) is private agents’ conjecture on the central bank’s gain parameter \((\lambda^C)\).

In most of the simulations, we assume that \(\lambda^P\) is constant. However, in Section 4.3, we introduce a mechanism through which private agents gradually learn the value of \(\lambda^C\) by observing the central bank’s policy actions.

### 2.8 Reduced-Form Solution

In this subsection, we derive the reduced-form solution of our model. This solution is useful to obtain an intuitive understanding of simulation results in the later sections.

The key issue in deriving a reduced-form solution is how we specify the process of private agents’ expectation formation. In this respect, we assume that private agents possess knowledge about the structure of the economy. That is, private agents know the functional forms and the parameters of structural equations. This is the same assumption as in standard rational expectations. The only difference is that in our study private agents substitute their conjecture about the efficient interest rate into the monetary policy rule if the central bank adopts an opaque regime. In other words, private agents form expectations for the future output gap and inflation rate by using their subjective monetary policy rule \((15)\), not the true monetary policy rule \((14)\). Then the model for determining private agents’ expectations consists of \((5)\), \((13)\), \((15)\), \((20)\), and \((22)\). By substituting \((15)\), \((20)\), and \((22)\) into \((5)\), we obtain the following expression of the dynamic IS equation:

\[
x_t = E_t x_{t+1} - (\gamma \pi_t - E_t \pi_{t+1}) - \phi (\mu^P_t - \mu^P_t) - \xi_t^P.
\]

Now the model for determining private agents’ expectation is reduced to \((13)\) and \((26)\). To calculate the expectations, we apply the undetermined coefficient method.
The simplest solution form of this model is presented as follows:

\[ x_t = a_1(\mu_t^{P[C]} - \mu_t^P) + a_2\xi_t^{P[C]} + a_3\zeta_t, \]  

(27)

\[ \pi_t = b_1(\mu_t^{P[C]} - \mu_t^P) + b_2\xi_t^{P[C]} + b_3\zeta_t. \]  

(28)

Based on (27) and (28), the expectations are calculated as follows:

\[ E_t x_{t+1} = a_1 \phi(\mu_t^{P[C]} - \mu_t^P), \]  

(29)

\[ E_t \pi_{t+1} = b_1 \phi(\mu_t^{P[C]} - \mu_t^P). \]  

(30)

Then, by substituting (27), (28), (29), and (30) into (13) and (26), the coefficients are computed as follows:

\[ a_1 = \frac{-(1-\beta\phi)\phi}{(1-\beta\phi)(1-\phi)+\gamma}, \quad a_2 = \frac{-1}{1+\kappa\gamma}, \quad a_3 = \frac{-\gamma}{1+\kappa\gamma}, \]

\[ b_1 = \frac{-\kappa\phi}{(1-\beta\phi)(1-\phi)+\gamma}, \quad b_2 = \frac{-\kappa}{1+\kappa}, \quad b_3 = \frac{1}{1+\kappa}. \]

The intuition for the determination of expectations (29) and (30) is as follows. As in (20), the efficient interest rate is determined by private agents’ estimate on the persistent productivity shock (\(\mu_t^P\)). Suppose that at period \(t\) private agents raise \(\mu_t^P\) by 1 percentage point. In addition, suppose that private agents raise the conjecture about the central bank’s estimate on the persistent productivity shock (\(\mu_t^{P[C]}\)) by 0.6 percentage point. Then, private agents consider that the remaining 0.4 percentage point is not offset by monetary policy at period \(t\). Since private agents regard that \(\mu_t^P\) and \(\mu_t^{P[C]}\) are determined in (24) and (25), they consider that the difference between \(\mu_t^P\) and \(\mu_t^{P[C]}\) multiplied by the persistent parameter \(\phi\) (0.4 \times \phi\% in this numerical example) remains at period \(t + 1\). Then, private agents expect that the output gap and inflation rate at period \(t + 1\) will not be neutralized by monetary policy at period \(t + 1\). Therefore, their expectations for the output gap and inflation rate deviate from zero.

Once private agents’ expectations are calculated as (29) and (30), we derive the solutions of the actual output gap and inflation rate by substituting the expectations into the model that includes the central bank’s true monetary policy rule (14). The model consists of (5), (13), (14), (20), and (21). By substituting (14), (20), and (21) into (5), we obtain the following expression of the dynamic IS equation:

\[ x_t = E_t x_{t+1} - (\gamma\pi_t - E_t \pi_{t+1}) - \phi(\mu_t^C - \mu_t^P) - \xi_t. \]  

(31)

The model for determining the output gap and inflation rate is reduced to (13) and

\[ \text{This solution is called the minimal-state-variable (MSV) solution. We introduce the MSV solution to restrict our attention to bubble-free solutions. See McCallum[1983, 1999] for the details of the MSV solution.} \]
By substituting (29) and (30) into (13) and (31), we obtain the reduced-form solution of our model as follows:

\[ x_t = c_1 (\mu_t^{P|C} - \mu_t^C) + c_2 (\mu_t^C - \mu_t^P) + c_3 \xi_t + c_4 \xi_t, \tag{32} \]

\[ \pi_t = d_1 (\mu_t^{P|C} - \mu_t^C) + d_2 (\mu_t^C - \mu_t^P) + d_3 \xi_t + d_4 \xi_t, \tag{33} \]

where the coefficients are given by

\[ c_1 = \frac{-\phi^2 [1 + \kappa - \beta (\kappa \gamma + \phi)]}{(1 + \kappa \gamma)[(1 - \beta \phi)(1 - \phi) + (\gamma - \phi)\kappa]}, \quad c_2 = \frac{-\phi (1 - \beta \phi)}{(1 - \beta \phi)(1 - \phi) + (\gamma - \phi)\kappa}, \quad c_3 = \frac{-\gamma}{1 + \kappa \gamma}, \quad c_4 = \frac{-1}{1 + \kappa \gamma}, \]

\[ d_1 = \frac{-\kappa \phi^2 [1 + \kappa - \beta (1 - \phi)]}{(1 + \kappa \gamma)[(1 - \beta \phi)(1 - \phi) + (\gamma - \phi)\kappa]}, \quad d_2 = \frac{-\kappa \phi}{(1 - \beta \phi)(1 - \phi) + (\gamma - \phi)\kappa}, \quad d_3 = \frac{1}{1 + \kappa \gamma}, \quad \text{and} \quad d_4 = \frac{-\kappa}{1 + \kappa \gamma}. \]

(32) and (33) indicate that the output gap and inflation rate are determined by four components: (i) the difference between \( \mu_t^{P|C} \) and \( \mu_t^C \); (ii) the difference between \( \mu_t^C \) and \( \mu_t^P \); (iii) the cost-push shock; and (iv) the monetary policy shock. Of these, the first two components are quite important in this study.

The first component \( (\mu_t^{P|C} - \mu_t^C) \) represents private agents’ misperception regarding the central bank’s estimate on the persistent productivity shock \( \mu_t^C \). If the central bank adopts a transparent regime, the first terms of (32) and (33) vanish, because private agents correctly recognize the value of \( \mu_t^C \). However, if the central bank adopts an opaque regime, the first terms of (32) and (33) are not necessarily zero. Therefore, economic dynamics can differ under the transparent regime and the opaque regime.

The second component \( (\mu_t^C - \mu_t^P) \) represents the heterogeneity between the central bank and private agents regarding the estimates on the persistent productivity shock. The difference between \( \mu_t^C \) and \( \mu_t^P \) influences economic dynamics, because it corresponds to the central bank’s misperception about the efficient interest rate. Since \( \mu_t^C \) and \( \mu_t^P \) are determined respectively by the central bank and private agents, the difference between \( \mu_t^C \) and \( \mu_t^P \) does not vanish even if the central bank announces the value of \( \mu_t^C \). Thus, the second terms of (32) and (33) express the direct impact of the central bank’s misperception about the efficient interest rate on the current output and inflation rate.\(^{14}\)

In the next section, we examine the economic dynamics and the influence of central bank transparency. In doing so, we pay particular attention to the first two components of (32) and (33).

\(^{14}\)Orphanides and Williams [2002, 2005] investigate the direct impact of the central bank’s misperception of the natural interest rate.
3 Economic Dynamics

3.1 Parameter Setting

In setting parameters, we refer to previous studies. The discount factor is $\beta = 0.99$, as in many studies. As for the slope of NKPC, we set $\kappa = 0.10$, following Ireland [2007]. The elasticity of substitution between each individual good is $\theta = 3.778$, following Christiano, Eichenbaum, and Evans [2005]. The policy responsiveness to the inflation rate is $\gamma = 1.5$. The parameters for the process of productivity growth are $\phi = 0.95, \sigma_{\epsilon} = 0.01, \text{and } \sigma_{\nu} = 0.001$, following Gilchrist and Saito [2007]. The standard error on the monetary policy shock is $\sigma_\xi = 0.000975$, which is estimated in Christiano, Eichenbaum, and Evans [1999]. The standard error on the cost-push shock is $\sigma_\xi = 0.0007$, following the estimation result of Ireland [2007].

3.2 Impulse Response in a Transparent Regime

Here, we examine the impulse response to transitory and persistent productivity shocks when the central bank adopts transparent regime. In this regime, the central bank announces the estimate on the persistent productivity shock ($\mu_{Ct}$). This announcement virtually implies the central bank’s disclosure about the estimate on the efficient interest rate ($r_{t^*C}$), because there is a one-to-one correspondence between $\mu_{Ct}$ and $r_{t^*C}$, as in (21).

In a transparent regime, private agents do not have any misperception about the central bank’s estimate on the persistent productivity shock ($\mu_{Ct}$). Therefore, $\mu_{P[C]}$ is always equal to $\mu_{Ct}$. However, $\mu_{P}$ can differ from $\mu_{Ct}$, because $\mu_{Ct}$ and $\mu_{P}$ are determined by the gain parameters respectively set by the central bank and private agents ($\lambda^C$ and $\lambda^P$). To examine the influence of heterogeneity between $\lambda^C$ and $\lambda^P$, we compare the impulse responses in two cases: the case of a homogeneous gain ($\lambda^C = \lambda^P = 0.05$) and the case of a heterogeneous gain ($\lambda^C = 0.05$ and $\lambda^P = 0.10$).

Figure 2 shows the impulse response to one standard deviation of a transitory productivity shock ($\epsilon_t$) in a transparent regime. In response to the transitory productivity shock, $\mu_{Ct}$ and $\mu_{Pt}$ immediately increase, and then gradually decrease to zero. This represents private agents’ gradual recognition of the persistence of the productivity shock, which is shown by some previous studies (Tambalotti [2003], Edge, Laubach, and Williams [2005, 2007], and Gilchrist and Saito [2007]) as the key mechanism to replicate the persistent movements of major macroeconomic variables.

In the case of the homogeneous gain (the solid line), the movements of $\mu_{Ct}$ and $\mu_{Pt}$ are exactly the same. Then, the output gap and the inflation rate are always zero. This means that in the case of the homogeneous gain, the central bank perfectly

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15 The data frequency is quarterly.
16 This value corresponds to the case of unconditional price indexation in Christiano, Eichenbaum, and Evans [2005].
stabilizes the output gap and inflation rate by completely offsetting the variations in
the efficient interest rate. However, in the case of the heterogeneous gain (the dashed
line), the output gap and inflation rate are never neutralized. In this case, the initial
rise of $\mu_C^t$ is less than that of $\mu_P^t$, which means that the central bank underestimates
the rise of the efficient interest rate. Since this implies that the tightening of monetary
policy is insufficient, the output gap and inflation rate are pushed upward for some
sustained periods.

Figure 3 indicates the impulse response to one standard deviation of a persistent
productivity shock ($\nu_t$). Now the responses of $\mu_C^t$ and $\mu_P^t$ are hump-shaped, because
the shock itself is sustained in the case of persistent productivity shock. However,
the contrast between the cases of a homogeneous gain and a heterogeneous gain is
essentially the same as in Figure 2. That is, the output gap and inflation rate are
always zero in the case of a homogeneous gain, though they go upward in the case of
a heterogeneous gain.

Therefore, the results in this subsection indicate that, in a transparent regime,
the central bank perfectly offsets the movements of the efficient interest rate if the
gains are homogeneous. However, if the gains are heterogeneous, the central bank
does not perfectly offset the movements of the efficient interest rate. Consequently,
the output gap and inflation rate are not neutralized to productivity shocks even if
the central bank is transparent.

3.3 Impulse Response in an Opaque Regime

Next, we examine the impulse response to productivity shocks when the central bank
adopts an opaque regime. In an opaque regime, the impulse response depends not
only on the gain parameters of private agents and the central bank themselves ($\lambda^C$ and
$\lambda^P$), but also on private agents’ conjecture about the central bank’s gain parameter
($\lambda^{P[C]}$).

Figure 4 shows the impulse response to a transitory productivity shock in the case
of a homogeneous gain ($\lambda^C = \lambda^P = 0.05$). Evidently, the impulse response depends
on the value of $\lambda^{P[C]}$. Note that the case of $\lambda^{P[C]} = 0.05$ (the solid line) corresponds
to a transparent regime, because private agents do not have any misperception about
$\lambda^C$. Then, the output gap and inflation rate are always zero. This replicates the
result in the previous subsection.

In contrast to this result, when $\lambda^{P[C]}$ is not equal to 0.05, the output gap and
inflation rate are never neutralized even if $\lambda^C$ and $\lambda^P$ are homogeneous. If $\lambda^{P[C]} =
0.00$ (the dashed line), $\mu^{P[C]}_t$ does not respond to a transitory shock. Then, the output
gap and inflation rate are pushed upward. Note that in this case, the central bank
perfectly offsets the movement of the efficient interest rate, because the rise of $\mu_C^t$ is
exactly the same as the rise of $\mu_P^t$. Nevertheless, the output gap and inflation rate
are not neutralized.\textsuperscript{17} This result is explained as follows. If $\lambda^{P[C]} = 0.00$ (or 0.10), private agents consider that the central bank underestimates (or overestimates) the rise of the efficient interest rate. Since the efficient interest rate is determined by the persistent productivity shock, private agents expect that the central bank’s misperception about the efficient interest rate remains at the next period. Based on this reasoning, private agents raise (or lower) the expectations for the output gap ($E_t x_{t+1}$) and inflation rate ($E_t \pi_{t+1}$). This process is shown in (29) and (30). Then, the increases (or decreases) of $E_t x_{t+1}$ and $E_t \pi_{t+1}$ raise (or lower) the current output gap and inflation rate through the dynamic IS equation (5) and NKPC (13). This is the basic mechanism working in Figure 4.

Next, we examine the impulse response in the case of a heterogeneous gain. Figure 5 shows the impulse response to a transitory productivity shock when the central bank’s gain parameter is smaller than private agents’ gain parameter ($\lambda^C = 0.05, \lambda^P = 0.10$). The solid line is the case of $\lambda^{P[C]} = 0.05$, which corresponds to the transparent regime (the same as the dashed line in Figure 2). Since $\lambda^C$ and $\lambda^P$ differ, monetary policy does not offset the movements of the efficient interest rate. Therefore, the output gap and inflation rate are not neutralized even if the central bank adopts a transparent regime.

In the case of a heterogeneous gain, we find that the variations of the output gap and inflation rate in an opaque regime can become either smaller or larger than in the transparent regime, depending on the value of $\lambda^{P[C]}$. If $\lambda^{P[C]} = 0.00$ (the dashed line), the responses of the output gap and inflation rate are larger in an opaque regime than in the transparent regime. However, if $\lambda^{P[C]} = 0.10$ (the dotted line), the result is overturned. In this case, the responses are smaller in an opaque regime than in a transparent regime. This result implies that the welfare loss becomes smaller in an opaque regime than in a transparent regime. This result might be surprising, because the central bank transparency is widely recognized as welfare-improving.

However, this does not mean that the central bank transparency is always welfare-reducing when $\lambda^{P[C]}$ is larger than $\lambda^C$. Notice that, if $\lambda^{P[C]}$ takes a still larger value, such as $\lambda^{P[C]} = 0.25$, then the drop in the inflation rate becomes quite large. Under our parameter setting, the welfare loss becomes larger in an opaque regime than in a transparent regime. Therefore, the result shows that central bank transparency improves social welfare when $\lambda^{P[C]}$ is far greater than $\lambda^C$. This implies that whether central bank transparency improves social welfare or not depends on the direction and the magnitude of private agents’ misperception about $\lambda^C$.\textsuperscript{18}

In sum, the impulse response in an opaque regime depends on the value of $\lambda^{P[C]}$.\textsuperscript{17}

\textsuperscript{17}These results can be also confirmed in the case of a persistent productivity shock.

\textsuperscript{18}We have confirmed that essentially the same result can be obtained in the case of a persistent productivity shock.
When \( \lambda^{P[C]} \) differs from \( \lambda^C \), the central bank cannot perfectly stabilize the output gap and inflation rate even if the central bank completely offsets the variations of the efficient interest rate. In addition, if \( \lambda^C \) and \( \lambda^P \) are heterogeneous, central bank transparency can either improve or worsen social welfare, depending on the value of \( \lambda^{P[C]} \). In the next subsection, we examine exactly how the influence of central bank transparency depends on the combinations of \( \lambda^C \), \( \lambda^P \), and \( \lambda^{P[C]} \).

### 3.4 Gains and Losses from Transparency

The previous subsection has shown that welfare loss in an opaque regime can be smaller than in a transparent regime, depending on the value of \( \lambda^{P[C]} \). This result could be considered striking, because central bank transparency is widely recognized as welfare-improving.

To understand the reason for this result, it is useful to look at the reduced-form solutions (32) and (33). If we ignore the cost-push shock and monetary policy shock, the output gap and inflation rate are determined by the difference between \( \mu^C_t \) and \( \mu^P_t \) and the difference between \( \mu^P_t \) and \( \mu^C_t \). Suppose that the movements of \( \mu^C_t \) and \( \mu^P_t \) are exactly the same. Then, the second terms of (32) and (33) become zero. This is the situation in which the central bank perfectly offsets the variations of the efficient interest rate. Then, the welfare loss is minimized when the first terms of (32) and (33) are zero, which is attained in the absence of private agents’ misperception of \( \mu^C_t \) (\( \mu^{P[C]}_t = \mu^C_t \)). Therefore, in the case of a homogeneous gain, central bank transparency is always desirable, because private agents’ misperception of \( \mu^C_t \) is merely a source of disturbance to the economy.

However, in the case of a heterogeneous gain, private agents’ misperception of \( \mu^C_t \) is not necessarily harmful to the economy. Suppose that \( \mu^C_t \) is much smaller than \( \mu^P_t \), which means that the central bank largely underestimates the level of the efficient interest rate. Then, the second terms of (32) and (33) take large positive values, which means that the output gap and inflation rate are pressured upward by the central bank’s unintentional monetary easing. In this environment, private agents’ misperception of \( \mu^C_t \) might mitigate the impact of monetary easing. That is, if \( \mu^{P[C]}_t \) is larger than \( \mu^C_t \) (but smaller than \( \mu^P_t \)), then the first terms of (32) and (33) become negative and they offset the positive impacts of the second terms of (32) and (33).

Intuitively, this occurs because private agents underestimate the strength of current monetary easing and, for that reason, the expectations for the future output gap and inflation rate are sustained at a lower level than that under the transparent regime.

So far, we have explained the influence of central bank transparency by regarding \( \mu^P_t \), \( \mu^C_t \), and \( \mu^{P[C]}_t \) as given. However, since \( \mu^P_t \), \( \mu^C_t \), and \( \mu^{P[C]}_t \) depend on the gain parameters (\( \lambda^P \), \( \lambda^C \), and \( \lambda^{P[C]} \)), we can clarify how the welfare loss depends on these gain parameters. For this purpose, we carry out stochastic simulations, in which we introduce one standard deviation of all the stochastic shocks (the transitory and
persistent productivity shocks, cost-push shock, and monetary policy shock). Then we simply compare the social welfare loss in a transparent regime and in an opaque regime. For this comparison, we calculate the “welfare gain from transparency”, which is defined as the welfare loss in an opaque regime minus the welfare loss in a transparent regime. If the welfare gain from transparency is negative, central bank transparency is welfare-reducing.

The upper panel of Figure 6 shows the welfare gain from transparency in the case of a homogeneous gain ($\lambda^C = \lambda^P = 0.05$). In this case, the welfare gain is minimized when $\lambda^{P|C}$ is equal to $\lambda^C$. If $\lambda^{P|C}$ takes a different value from $\lambda^C$, the welfare gain from transparency becomes strictly positive. Therefore, central bank transparency always improves social welfare in the case of a homogeneous gain.

However, in the case of a heterogeneous gain, transparency can either improve or worsen social welfare. The middle panel of Figure 6 corresponds to the case where the central bank’s gain parameter is smaller than private agents’ gain parameter ($\lambda^C = 0.05$ and $\lambda^P = 0.10$). Now the welfare gain from transparency can be either positive or negative, depending on the value of $\lambda^{P|C}$. If $\lambda^{P|C}$ is smaller than $\lambda^C$, the welfare gain from transparency is positive. This is because, in this case, private agents overestimate the magnitude of heterogeneity between the central bank and private agents ($|\lambda^C - \lambda^P|$. In this situation, central bank transparency contributes to reduce private agents’ overestimation of heterogeneity. However, if $\lambda^{P|C}$ is larger than $\lambda^C$ and smaller than the critical value ($\lambda^C$), private agents’ misperception offsets the distortion due to the heterogeneity between $\lambda^C$ and $\lambda^P$. Then, the central bank’s disclosure about the value of $\lambda^C$ removes this offsetting effect of private agents’ misperception. This is the reason why central bank transparency is undesirable in this case. However, if $\lambda^{P|C}$ is a still larger value (such as $\lambda^{P|C} = 0.25$), then the welfare gain from transparency again becomes positive. This is because the central bank’s disclosure about $\lambda^C$ removes the distortion due to private agents’ large misperception of heterogeneity in gain parameters ($\lambda^P - \lambda^{P|C} < 0$), which is in completely opposite direction to the actual heterogeneity ($\lambda^P - \lambda^C > 0$). This result implies that if private agents’ misperception of $\lambda^C$ is quite large, central bank transparency is desirable regardless of the sign of misperception ($\lambda^{P|C} - \lambda^C < 0$ or $> 0$). In other words, transparency can be welfare-reducing only if private agents’ misperception of $\lambda^C$ is not too large.

In sum, Figure 6 shows that the desirability of central bank transparency depends on the combinations of the values of $\lambda^C$, $\lambda^P$, and $\lambda^{P|C}$. In the case of a homogeneous gain ($\lambda^C = \lambda^P$), transparency improves social welfare, regardless of the value of $\lambda^{P|C}$. However, in the case of a heterogeneous gain ($\lambda^C \neq \lambda^P$), transparency can be either welfare-improving or welfare-reducing, depending on the gain parameters used by the central bank and private agents ($\lambda^C$ and $\lambda^P$) and private agents’ conjecture on the gain parameter used by the central bank ($\lambda^{P|C}$).
By the way, which case corresponds to the situation monetary policymakers usually face? In our view, a homogeneous gain could be regarded as a relatively special case, which is only achieved in the long run, because we have large uncertainty and the possible structural changes on the variances of transitory and persistent productivity shocks \( (\sigma^2_\xi \text{ and } \sigma^2_\nu) \). On that ground, it could be considered that a central bank typically faces uncertainty as to the desirability of central bank transparency about the views on future productivity growth.

4 Implications for Monetary Policy Actions

In the previous section, we examined how central bank transparency influences social welfare. However, we have not examined how the desirable monetary policy action depends on central bank transparency or the forecasting mechanisms used by the central bank and private agents. Therefore, in this section, we investigate this issue. To do so, we specifically examine the optimal policy response to the inflation rate (i.e., the optimal value of \( \gamma \) in monetary policy rule (14)) under a transparent regime and under an opaque regime, respectively.

4.1 Optimal Response to Inflation in a Transparent Regime

To investigate the optimal policy response to the inflation rate, we first calculate the optimal value of \( \gamma \) in a case where productivity shocks are absent. This virtually corresponds to the case of a homogeneous gain in a transparent regime, because productivity shocks become irrelevant to the economic dynamics in that case. We regard this case as the benchmark in this section.

To calculate the optimal value of \( \gamma \) in the benchmark case, we assume that all the values of \( \mu^P_t, \mu^C_t, \text{ and } \mu^{P[C]}_t \) are equal to zero in (32) and (33). Then we substitute (32) and (33) into welfare function (16), and minimize (16) with respect to \( \gamma \). As a result, we obtain the optimal value of \( \gamma \) in the absence of a productivity shock, denoted as \( \gamma^* \), as follows:19

\[
\gamma^* = \theta + (1 + \theta \kappa) \frac{\sigma^2_\xi}{\sigma^2_\zeta}.
\]

Thus, \( \gamma^* \) depends on the relative size in the variances of the monetary policy shock and cost-push shock \( (\sigma^2_\xi / \sigma^2_\zeta) \). If the monetary policy shock is absent (\( \sigma_\xi = 0 \)), then \( \gamma^* \) becomes exactly equal to \( \theta \). However, if the variance of the monetary policy shock is nonzero, then \( \gamma^* \) becomes larger than \( \theta \). Under our parameter setting, \( \gamma^* \) is 4.045, which is slightly larger than \( \theta = 3.778 \).

19When we set \( \gamma = \gamma^* \), the policy rule (14) corresponds to optimal discretionary policy in the absence of a productivity shock.
Figure 7 shows the welfare loss when the central bank adopts transparent regime. Here, we assume that $\lambda^C$ is 0.05. Since the central bank is transparent, $\lambda^{P[C]}$ becomes 0.05. The welfare loss depends on the value of $\lambda^P$. In the case of a homogeneous gain ($\lambda^P = 0.05$), welfare loss is minimized when the central bank chooses $\gamma = \gamma^*$. However, in the cases of a heterogeneous gain ($\lambda^P = 0.00, 0.02, 0.08, \text{and } 0.10$), the loci of the welfare loss are shifted to the upper-right region of Figure 7. Then the optimal value of $\gamma$ for each value of $\lambda^P$ becomes larger than $\gamma^*$, since the central bank can reduce the welfare loss by setting the value of $\gamma$ greater than $\gamma^*$. The optimal value of $\gamma$ is especially large when the difference between $\lambda^C$ and $\lambda^P$ is large, such as the case of $\lambda^P = 0.00$ or 0.10.

We can understand the reason for these results by looking at (31). In (31), the difference between $\mu^C_t$ and $\mu^P_t$ appears as the disturbance to the dynamic IS equation. Therefore, the difference between $\mu^C_t$ and $\mu^P_t$ plays essentially the same role as the monetary policy shock ($\xi_t$), since it constitutes the source of the demand shock. As in (34), the optimal value of $\gamma$ is large when the variance of the demand shock is large. This is the reason why the optimal value of $\gamma$ is large when the heterogeneity between $\lambda^C$ and $\lambda^P$ is prominent.

4.2 Optimal Response to Inflation in an Opaque Regime

In this subsection, we investigate the optimal policy response to the inflation rate when the central bank adopts an opaque regime. In an opaque regime, the welfare loss depends on the value of $\lambda^{P[C]}$, since the economic dynamics depend on $\lambda^{P[C]}$, as shown in Section 3.3.

Figure 8 summarizes the welfare loss in an opaque regime. The upper panel shows the case of a homogeneous gain ($\lambda^C = 0.05$ and $\lambda^P = 0.05$). The case of $\lambda^{P[C]} = 0.05$ corresponds to a transparent regime, in which the optimal value of $\gamma$ is $\gamma^*$. If $\lambda^{P[C]}$ differs from 0.05 (such as $\lambda^{P[C]} = 0.00, 0.10, \text{and } 0.20$), the optimal value of $\gamma$ is larger than $\gamma^*$. The optimal value of $\gamma$ is particularly large when the difference between $\lambda^{P[C]}$ and $\lambda^C$ is large, such as the case of $\lambda^{P[C]} = 0.20$.

The middle and bottom panels of Figure 8 show the case of a heterogeneous gain ($\lambda^C \neq \lambda^P$). In the case of a heterogeneous gain, the welfare loss for the given value of $\gamma$ is not minimized in a transparent regime. This can be confirmed in the middle panel. There, the welfare loss for the given value of $\gamma$ becomes larger in a transparent regime than in the opaque regime of $\lambda^{P[C]} = 0.10$. In addition, the optimal value of $\gamma$ is larger in a transparent regime than in the opaque regime of $\lambda^{P[C]} = 0.10$. Furthermore, the optimal value of $\gamma$ is not necessarily monotonically increasing with the difference between $\lambda^P$ and $\lambda^C$.

The reason for these results can be explained as follows. As we have seen in the previous subsection, the difference between $\mu^C_t$ and $\mu^P_t$ plays the role of the demand shock. However, in contrast to a transparent regime, it is possible that the
expectations for the output gap \((E_t x_{t+1})\) and inflation rate \((E_t \pi_{t+1})\) at least partially offset the impact of heterogeneity between \(\mu^C_t\) and \(\mu^P_t\) in (31) because, as in (29) and (30), these expectations depend on the value of \(\mu^{P[C]}_t\) in an opaque regime. This happens in the case of \(\lambda^{P[C]} = 0.10\) in the middle panel of Figure 8 and also in the case of \(\lambda^{P[C]} = 0.05\) in the bottom panel of Figure 8.

In sum, the social welfare loss depends on the combinations of \(\lambda^C\), \(\lambda^P\), and \(\lambda^{P[C]}\) in an opaque regime. As a result, the optimal value of \(\gamma\) in an opaque regime depends on these gain parameters. A problem here is that the values of \(\lambda^P\) and \(\lambda^{P[C]}\) are not directly observable by the central bank. In this sense, the central bank faces uncertainty about the optimal policy response. However, in any case, the optimal value of \(\gamma\) is at least larger than (or equal to) \(\gamma^*\). In other words, any value less than \(\gamma^*\) cannot be optimal in all the combinations of \(\lambda^C\), \(\lambda^P\), and \(\lambda^{P[C]}\). Therefore, if the central bank is uncertain about the values of \(\lambda^C\), \(\lambda^P\), and \(\lambda^{P[C]}\), then it is sensible for central bank to respond strongly to the variations of the inflation rate.

4.3 Influence of Private Agents’ Learning on \(\lambda^C\)

Until the previous subsection, we have assumed that private agents’ conjecture on the gain parameter used by the central bank (\(\lambda^{P[C]}\)) is time-invariant. However, we can consider the possibility that private agents gradually learn the value of \(\lambda^C\) by observing the central bank’s policy actions. If we introduce such a learning mechanism, the optimal value of \(\gamma\) depends on the speed of learning of the private agents.

As for the mechanism of the private agents’ learning, we introduce a recursive procedure in forming the value of \(\lambda^{P[C]}\). First, we define a variable \(h_t\) as below:

\[
h_t = i_t - \gamma \pi_t - (\rho + \bar{\pi}).
\]

(35)

\(h_t\) represents the residual of policy action, which is calculated as the variation of the nominal interest rate except for the response to the inflation rate (\(\gamma \pi_t\)) and the steady-state value of the real interest rate (\(\rho + \bar{\pi}\)). Since we assume that private agents can observe \(i_t\) and \(\pi_t\), \(h_t\) is computable to private agents at period \(t\).

From (14) and (35), \(h_t\) can be expressed as follows:

\[
h_t = \phi \mu^C_t + \xi_t.
\]

(36)

Thus, \(h_t\) is the amalgam of \(\phi \mu^C_t\) and \(\xi_t\). By substituting (23) into (36), we obtain the following equation:

\[
S_t = \lambda^C X_t + \xi_t.
\]

(37)

where \(S_t\) and \(X_t\) are defined as \(S_t \equiv h_t - \phi^2 \mu^C_{t-1}\) and \(X_t \equiv \phi (z_t - \bar{\pi}) - \phi^2 \mu^C_{t-1}\), respectively. Private agents can estimate \(\lambda^C\) by regressing equation (36) with recursive least squares (RLS), because they know the values of \(S_t\) and \(X_t\) at period \(t\). Suppose
that private agents initially conjecture the value of $\lambda^C$ as $\lambda^P_0$. Then we can apply the following recursive formula to obtain the estimate of $\lambda^P_t$ in each period:

$$
\lambda^P_t = \lambda^P_{t-1} + \omega^P R^{-1}_t X_t (S_t - \lambda^P_{t-1} X_t),
$$

(38)

where $\omega^P$ is the constant gain and $R_t$ is the moment matrix of $X_t$.\textsuperscript{20}

Once we obtain the value of $\lambda^P_t$, the estimate of $\mu^C_{t-1}$, which is denoted as $\mu^P_t$, can be calculated as follows:

$$
\mu^P_t = \phi \mu^P_{t-1} + \lambda^P_t [ (z_t - \bar{z}) - \phi \mu^P_{t-1} ].
$$

(40)

In numerical simulations, we set two alternative values for constant gain $\omega^P$ (0.025 and 0.10). As Figure 9 shows, $\lambda^P_t$ converges to the true value of $\lambda^C$. The speed of convergence is slower when the value of $\omega^P$ is smaller.

Figure 10 shows the welfare loss in the case where private agents update the value of $\lambda^P_t$ by using $\omega^P = 0.025$. The difference between Figure 8 and Figure 10 simply reflects the influence of private agents’ learning on $\lambda^C$. Because of the learning mechanism, each locus of the welfare loss in Figure 10 shifts away from the corresponding locus in Figure 8. In some cases, these shifts are downward. This could be regarded as natural consequences, since private agents’ learning reduces their misperception of the value of $\lambda^C$. However, in other cases (such as the cases of $\lambda^P_0 = 0.10$ in the middle panel and $\lambda^P_0 = 0.20$ in the bottom panel), the shifts are upward. These results suggest that private agents’ learning mechanism does not necessarily reduce the social welfare loss for a given value of $\gamma$. This finding indicates that the optimal value of $\gamma$ does not necessarily approach $\gamma^*$ with the introduction of private agents’ learning mechanism.\textsuperscript{21}

We can understand the reason for this result by looking at Figure 6. In the case of a homogeneous gain, social welfare monotonically decreases while $\lambda^P_t$ approaches $\lambda^C$. However, in the case of a heterogeneous gain, social welfare does not necessarily decrease through the process of learning. For example, in the middle panel, if the initial value of $\lambda^P_t$ is just the same as $\lambda^P$, private agents’ learning process increases the social welfare loss. This is because, as explained in Section 3.4, private agents’ initial misperception of $\lambda^C$ reduces the magnitude of the demand shock in this case.

In this environment, private agents’ learning process magnifies the volatility of the

\textsuperscript{20}See Evans and Honkapohja [2001] for the details of the RLS formula. The use of constant gain implies that private agents consider the possibility that the central bank shifts the gain parameter ($\lambda^C$).

\textsuperscript{21}Notice that the welfare loss in the case that private agents initially guess correctly ($\lambda^P_0 = 0.05$ in the upper panels of Figure 10 and Figure 11) is not the same as the welfare loss in the transparent regime. This is because private agents do not know that the true value of $\lambda^C$ is 0.05 and revise the estimate $\lambda^P_t$ in each period even though they initially guess correctly on $\lambda^C$. 

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demand shock by eliminating the influence of private agents’ misperception that has been favorable to social welfare.

Figure 11 shows the welfare loss in the case where private agents learn the value of $\lambda^C$ by using $\omega^P = 0.10$. In this case, the welfare losses converge for each value of $\lambda_0^{P[C]}$. This result is natural, since the high constant gain implies that private agents quickly learn the value of $\lambda^C$. As a result, the optimal values of $\gamma$ in an opaque regime converge across the alternative values of $\lambda_0^{P[C]}$. In this sense, the central bank faces smaller uncertainty about the optimal policy response when the value of $\omega^P$ is higher.

But, a problem here is that the central bank cannot directly observe the value of $\omega^P$. This means that the central bank faces uncertainty about the speed of convergence of private agents’ learning. Nevertheless, the optimal value of $\gamma$ is still larger at least than $\gamma^*$ for both values of $\omega^P = 0.025$ and 0.10. This suggests that, for any value of $\omega^P$, the central bank should set the value of $\gamma$ larger than $\gamma^*$. Therefore, it is sensible for the central bank to respond strongly to the variations of the inflation rate even if we introduce the influence of private agents’ learning on $\lambda^C$.

5 Conclusions

In this study, we have investigated how central bank transparency about the views on future productivity growth influences social welfare. To this end, we have used a New Keynesian framework in which both the central bank and private agents are engaged in filtering problems about the persistence of productivity growth. Since the central bank and private agents do not know the true value of the signal-to-noise ratio, the gain parameters used in the filtering problems can be heterogeneous. If the central bank is not transparent, private agents must conjecture the central bank’s estimate on the efficient level of the interest rate. Under this setup, we have shown that central bank transparency does not necessarily improve social welfare. It can potentially yield a welfare loss, depending on (i) the gain parameters used by the central bank and private agents and (ii) private agents’ conjecture on the gain parameter used by the central bank. If the gain parameters used by the central bank and private agents are homogeneous, then central bank transparency always improves social welfare. However, if these gain parameters are heterogeneous, central bank transparency can be either welfare-improving or welfare-reducing. Our study has shown that, if the central bank is uncertain about the combination of the gain parameters (including private agents’ conjecture), it is sensible for the central bank to respond strongly to the variations of the inflation rate, because the misperceptions on these parameters become the source of demand shock.

There are some possible extensions of this study. First, we have not incorporated the learning mechanism in forming gain parameters used by the central bank and
private agents ($\lambda^C$ and $\lambda^P$). This is largely due to computational difficulty in learning the theoretical values of shock variances ($\sigma_\varepsilon$ and $\sigma_\nu$), especially in the presence of possible structural change. However, it might be unrealistic to consider that both of the central bank and private agents do not learn these values at all, even in the long run. Therefore, in future research, the learning mechanism for the shock variances should be incorporated. Second, we have excluded the possibility that the central bank does not honestly announce its true view on future productivity growth. If we consider the possibility of central bank’s dishonest information provision, we must investigate a credibility problem. Third, in this study, we have simply compared the economic dynamics and social welfare in a transparent regime and in an opaque regime. Therefore, we do not consider the possibility that the central bank decides whether it should be transparent or opaque in each period. If we take account of this possibility, then we must consider the credibility problem again. Fourth, we have assumed that the central bank chooses between a perfectly transparent regime and a perfectly opaque regime. However, it is possible to extend our analysis to include the possibility that the central bank chooses some intermediate regime. These issues should be explored in future research.
References


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Figure 1: Forecasts for Output Growth in the U.S. Economy

Note: “Forecast by the FRB” denotes the Greenbook projections of the Board of Governors of the Federal Reserve System. “Forecast by economists in the private sector” is the Survey of Professional Forecasters released by the Federal Reserve Bank of Philadelphia. Each series indicates one-year-ahead forecasts made at the beginning of each year (January or February).
Figure 2: Response to a Transitory Productivity Shock in a Transparent Regime
Figure 3: Response to a Persistent Productivity Shock in a Transparent Regime
Figure 4: Response to a Transitory Productivity Shock in an Opaque Regime ($\lambda^C = 0.05$, $\lambda^I = 0.05$)
Figure 5: Response to a Transitory Productivity Shock in an Opaque Regime ($\lambda^C = 0.05, \lambda^P = 0.10$).
1. $\lambda^C = 0.05, \lambda^P = 0.05$

![Graph 1](image1)

2. $\lambda^C = 0.05, \lambda^P = 0.10$

![Graph 2](image2)

3. $\lambda^C = 0.10, \lambda^P = 0.05$

![Graph 3](image3)

Figure 6: Gains and Losses from Transparency
Note: $\lambda^c = \lambda^p = 0.05$ in all cases.

Figure 7: Social Welfare in Transparent Regime
1. $\lambda^C = 0.05$, $\lambda^P = 0.05$

2. $\lambda^C = 0.05$, $\lambda^P = 0.10$

3. $\lambda^C = 0.10$, $\lambda^P = 0.05$

Figure 8: Social Welfare in an Opaque Regime
1. $\omega^P=0.025$

![Graph showing speed of convergence for different periods with $\omega^P=0.025$.]

2. $\omega^P=0.10$

![Graph showing speed of convergence for different periods with $\omega^P=0.10$.]

Figure 9: Speed of Convergence of $\lambda^P[C]$
1. $\lambda^C = 0.05, \lambda^P = 0.05$

2. $\lambda^C = 0.05, \lambda^P = 0.10$

3. $\lambda^C = 0.10, \lambda^P = 0.05$

Figure 10: Social Welfare When Private Agents Learn About $\lambda^C$ ($\omega^P = 0.025$)
1. $\lambda^C = 0.05, \lambda^P = 0.05$

2. $\lambda^C = 0.05, \lambda^P = 0.10$

3. $\lambda^C = 0.10, \lambda^P = 0.05$

Figure 11: Social Welfare When Private Agents Learn About $\lambda^C$ ($\omega^P = 0.10$)