## IMES DISCUSSION PAPER SERIES

The Effectiveness of Forecasting Methods Using Multiple Information Variables

Tomiyuki Kitamura and Ryoji Koike

Discussion Paper No. 2002-E-20

## IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN
C.P.O BOX 203 TOKYO

100-8630 JAPAN

You can download this and other papers at the IMES Web site:
http://www.imes.boj.or.jp

NOTE: IMES Discussion Paper Series is circulated in order to stimulate discussion and comments. Views expressed in Discussion Paper Series are those of authors and do not necessarily reflect those of the Bank of Japan or the Institute for Monetary and Economic Studies.

# The Effectiveness of Forecasting Methods Using Multiple Information Variables 

Tomiyuki Kitamura* and Ryoji Koike**


#### Abstract

This paper examines the effectiveness of forecasting methods using multiple information variables in forecasting the rate of changes in CPI and real GDP in Japan, and also investigates the background of forecast performance improvement and its limitations. We first examine the performance of forecasts that use individual information variables as well as forecasts that use multiple information variables. The results show that no single variable improves forecasts in all periods for either CPI or GDP, but combining the information from individual forecasts can lead to a stable forecast performance. Next, in order to explore the backdrop to these improvements in forecast performance, we decompose and analyze the forecast error of forecast combinations using a simple mean. We then discover that the irregular movements of forecast errors generally cancel each other out, which in turn leads to a reduction in errors. At the same time, the effect of reducing forecast errors rapidly diminishes with the addition of variables, and we verify that forecast performance stops improving after two to four variables are added. For this reason, it is necessary to consider both the performance of original forecast series that comprise the combination, and the combination of variables that best reduces the correlation among forecast error series in order to obtain the optimal combination of series.


Keywords: Information variable, multivariate forecast, out-of-sample forecast, forecast combination

JEL classification: C30, C53, E37

[^0]The authors are grateful to Masao Ogaki (Ohio State University), Yukinobu Kitamura (Hitotsubashi University) and the staff of Policy Planning Office, Research and Statistics Department and Institute for Monetary and Economic Studies of the Bank of Japan for their helpful comments. The views expressed in this paper are those of the authors and do not represent those of the Bank of Japan or the Institute for Monetary and Economic Studies.

## Table of Contents

I. Introduction ..... 1
II. Bivariate Forecasting and Its Results ..... 4
A. Forecasting Model ..... 5
B. Data ..... 6
C. Forecast Results ..... 7
III. Multivariate Forecasting and Its Results ..... 8
A. Multivariate Forecasting Methods ..... 9
B. Performance of Multivariate Forecasting ..... 10
IV. Performance Improvement Mechanisms Involved in Forecast Combination and Their Limitations ..... 14
A. Preliminary Considerations ..... 15
B. The Actual Importance of Each Factor ..... 18
C. Optimal Forecast Combination: Stepwise Combination Selection ..... 19
V. Conclusions ..... 24
References ..... 25

## I. Introduction

In recent years, both inside and outside of Japan, much attention has been given to multivariate forecasting methods for inflation and the growth rate of output. In this paper, we first evaluate the effectiveness of these methods in Japanese data, and then attempt to highlight mechanisms that improve forecasting performance and also consider the limitations of these mechanisms.

In formulating monetary policy, it is crucially important to grasp the current state of the economy and provide an economic outlook. For this purpose, it is necessary to forecast inflation and output growth rates for a certain period ahead, for example, six months, one year and two years. Thus, there has been a variety of research conducted on such economic forecasting. ${ }^{1}$

One typical method of forecasting is to use an individual indicator as an information variable based on economic theory. ${ }^{2}$ For example, real economic variables such as the unemployment rate are likely to contain some information on future inflation, as the Phillips Curve can be derived under assumptions such as rigid nominal prices. Another example is that asset prices such as share prices can be considered to contain some information on the future course of the economy, because an asset price is theoretically equal to the present discounted value of future income generated by that asset. ${ }^{3}$ For this reason, there has been much research conducted from the viewpoint of whether useful information for forecasting can be extracted from each individual variable such as money balance, long-term interest rates, share prices, commodity prices and the unemployment rate. ${ }^{4}$

[^1]However, as a comprehensive survey undertaken by Stock and Watson [2001] shows, a variable with satisfactory effectiveness in forecasting across periods and countries is yet to be found. More specifically, there are virtually no cases in which the theoretical relationship between variables is sufficiently stable in forecasting. ${ }^{5}$

Based on these results, in recent years there have been many attempts to obtain more accurate forecasts by integrating information from a variety of variables, without relying on a particular information variable that, in theory, appears to contain useful information.

Among the many kinds of multivariate forecasting methods, two basic approaches have gained attention in recent years. The difference between the two lies in whether combining information first and then forecasting, or reverse order. The former is an "index approach," where a small number of indices are first constructed from many information variables, and the resulting indices are used for forecasting. The latter is a "forecast combination approach," where some forecasts are made using individual variables separately at first, and then these forecasts are combined by some means to create a final forecast. ${ }^{6}$

Stock and Watson [1998] adopt the index approach, and they estimate a dynamic factor model for 170 time series data in the United States. ${ }^{7}$ They confirm that the forecast performance of the models to forecast inflation and real industrial production was better than the forecast performance of the AR (autoregressive) model or other

[^2]models using individual variables such as the unemployment rate. Similar studies using the index approach are Marcellino et al. [2000] and Forni et al. [2002] for Europe, and Artis et al. [2002] for the United Kingdom. All of these reported high forecast performance for the dynamic factor model.

On the other hand, an example of research that adopts the forecast combination approach is Stock and Watson [2001]. ${ }^{8}$ They first construct forecasts for production growth and inflation using 38 economic indicators as individual information variables for seven OECD countries. They then make combination forecasts by taking the median, mean, and trimmed mean of different individual forecasts, and finally evaluate the forecast performance. As a result, they report that the combination forecasts always outperformed forecasts using individual variables, even if the performance of individual forecasts to be combined are unstable. ${ }^{9}$

Stock and Watson [1999] examine both of these approaches. In their paper, they adopt principal component analysis ${ }^{10}$ as the method of constructing "indexes" for the index approach. The approach taken for forecast combination uses the simple mean, the median and weighted averages whose weights are calculated based on ridge regression. ${ }^{11}$ According to their results, both of these approaches outperform forecast models using individual variables, and in particular the forecast performance using the first principal component extracted by principal component analysis is shown to be superior.

While all of the preceding studies listed above provide evidence supporting the

[^3]effectiveness of the index or the combination approach, there are still important issues that remain unresolved: Why do these forecasting methods work better? How many variables should we use in these two methods? In fact, forecasting performance does not simply improve as the number of variables included increases; it has been found that results can actually deteriorate if the number of variables is too large (see Stock and Watson [2001]). ${ }^{12}$ It is therefore necessary to show both the mechanism that improves forecasting performance in these methods and its limitations, and to investigate the optimal number of variables to be included.

In this paper we evaluate the effectiveness of these multivariate forecasting methods using Japanese data, based on the framework presented in Stock and Watson [1999], and examine both the mechanism of performance enhancement and the limitation of that mechanism. Our results confirm that these forecasting methods are also effective for Japanese data. We also discover that the improvement and stabilization of performance are primarily created by a canceling out of the irregular movements of forecast errors. At the same time, forecast performance stops improving after the inclusion of two to four variables due to the addition of poorly performing forecast series, because the effect of reducing forecast errors rapidly decreases with the addition of variables.

The remainder of this paper is organized as follows. In section 2 , we construct bivariate forecasts for the rate of change in Japan's CPI and real GDP, by using a variety of financial and economic indicators as information variables within the framework of Stock and Watson [2001]. We then review the performance of these bivariate forecasts compared with the forecasting performance based on AR model. In section 3, we create multivariate forecasts and observe their performance. Section 4 considers the mechanisms involved in forecast performance improvements and their limitations. Section 5 concludes the paper.

## II. Bivariate Forecasting and Its Results

As a preparation for examining the performance improvement mechanism of combined forecasts and its limitations, we first construct bivariate forecasts using various

[^4]individual information variables based on the framework of Stock and Watson [2001]. Specifically, we construct half a year (two-quarter), one year (four-quarter) and two year (eight-quarter) ahead forecasts for the rate of changes in CPI and real GDP.

## A. Forecasting Model

The model here predicts the change in the variable being forecast $(Y)$ up to $h$ periods in the future, using the current and past information of both the variable being forecast and an information variable $(X)$, or the current and lagged value of these two variable at the time of the forecast. ${ }^{13}$

$$
\begin{equation*}
y_{t+h}^{h}=\alpha+\beta(L) y_{t}+\gamma(L) X_{t}+\varepsilon_{t+h}^{h}, \tag{1}
\end{equation*}
$$

where $y_{t}=\ln \left(Y_{t}\right)-\ln \left(Y_{t-1}\right)$ is the logarithmic first difference of $Y_{t}$, i.e. the rate of changes in CPI or real GDP from a quarter earlier; $y_{t+h}^{h}=\ln \left(Y_{t+h}\right)-\ln \left(Y_{t}\right)$ is the rate of changes in CPI or real GDP from $h$-quarter earlier; $X_{t}$ is a candidate leading indicator; $\alpha$ is a constant; and $\beta(L)$ and $\gamma(L)$ are the lag polynomials for $y_{t}$ and $X_{t}$.

The performance of the bivariate forecast model is evaluated using a comparison with an AR forecast imposing the restriction of $\gamma(L)=0$ in equation (1). That is, the bivariate model above produces an out-of-sample forecast and its mean squared forecast error (MSFE), as does the AR model. Then we compare forecast performance using the relative mean squared forecast error (MSFE bivariate forecast $/ \mathrm{MSFE}_{\text {AR forecast }}$ ), where the MSFE of the bivariate model is standardized by the MSFE of the corresponding AR model. ${ }^{14}$ Therefore, the forecast performance of the AR model can be improved by adding information variables if the relative MSFE is below 1 , meaning an increase in performance.

[^5]It should be noted that the addition of information variables does not necessarily improve the out-of-sample forecast performance of a forecast model, while it does improve the in-sample forecast performance. ${ }^{15}$

## B. Data

56 quarterly variables starting in 1970-73 and ending in the first half of 2001 were used as information variable candidates (Chart 1). ${ }^{16}$ We classified these variables into four groups. These are variables regarding real economic activities (index of industrial production, unemployment, TANKAN D.I., etc), price/wage/market related variables (wholesale prices, CRB index, etc.), money related variables (monetary base, M2+CDs, etc) and asset price variables (foreign exchange, interest rates, share prices, land prices, etc).

Most of these data series are then transformed as follows. First, for series that showed significant seasonal variation, we use seasonally adjusted series where data is officially available, and we create a seasonally adjusted series using X12-ARIMA where it is not. ${ }^{17}$ Second, when converting monthly data and daily data to quarterly data, we use the end-of-quarter value or the average for the quarter according to the characteristics of each variable. (However we use both the end-of-quarter value and the average for some series, where it cannot be determined which conversion is better.) Third, in some cases, we used not only the original series but also series transformed by

[^6]logarithm, logarithmic difference or HP filter ( $\lambda=1,600$ ).
As a result of these transformations, 148 series are used for forecasting the CPI rate of change and 147 for forecasting the real GDP rate of change (Chart 2).

## C. Forecast Results

In this section, we examine the forecast performance of the bivariate model presented in the subsection II-A in relation to the benchmark AR model for the rate of change in CPI and real GDP. Here we divide the entire sample period from the first quarter of 1983 to the second quarter of 1999, where out-of-sample forecasting and its evaluation are feasible, into four sub-sample periods: the pre-bubble period (1983-86), the bubble formation period (1987-90), the bubble collapse period (1991-94) and the post-bubble period (1995-99/2Q). We then observe the differences between the forecast performance in these four periods.

## 1. CPI forecast

First, we plot the relative MSFE of the bivariate model in two subsequent samples into scatter graphs by forecast horizon, in order to compare the performance of CPI inflation forecast by sample period and by forecast horizon (Chart 3). The chart shows that the right column of panels has the highest density in the third quadrant, suggesting that the number of series with improved performance increased in the bubble collapse period and the post-bubble period. A closer look at the forecast performance of each information variables offers the observation that the forecast improvement effect of price/wage related variables is relatively high (Chart 4). ${ }^{18} 19$ However, unfortunately no variable is found that improves forecast performance across all forecast horizons and all sample periods. ${ }^{20}$

[^7]
## 2. Real GDP forecast

We also calculate the relative MSFE of the bivariate forecast of real GDP growth for each series as we did for CPI. We graph them by forecast horizon and by sample period (Chart 5). Comparing the rows and columns of panels, it appears that the best performance is achieved for one- and two-year forecasts in the bubble formation period, bubble collapse period and post-bubble period. Moreover, examining the forecast performance of each information variable indicates a high forecast improvement effect in money and asset prices (Chart 6). ${ }^{21}$ However, forecast performance does not improve across all forecast horizons and sample periods, which is similar to CPI forecasts in terms of robustness toward differences between samples. ${ }^{22}$

## 3. Overview: bivariate forecasting

According to the results in the previous subsections, bivariate forecasts are deemed not to constantly improve the performance of AR forecasts in either the case of CPI forecasts or real GDP forecasts. That is, although performance may be improved in one sample period, it is uncertain that information variables will improve forecast performance in another sample period. The evidence shows that the results of Stock and Watson [2001] are consistent with Japanese time series data.

## III. Multivariate Forecasting and Its Results

In the previous section we have shown that in a bivariate forecasting framework using individual information variables, forecast performance cannot always be improved for either CPI inflation or real GDP growth across different forecast horizons or sample periods. However, there is a possibility that forecast performance can be improved by appropriately extracting information useful for forecasting from more variables. In this

[^8]regard, this section first describes multivariate forecasting methods and then compares the performance of the forecasts.

## A. Multivariate Forecasting Methods

We now consider forecasting methods used when not only one information variable, but $n$ series $\left(X^{1}, X^{2}, \ldots, X^{\mathrm{n}}\right)$ are available. There are several variations of actual combination methods, but as we note in section 1, these can be classified into the groups of "index approach" and "forecast combination approach" according to whether the combining of information is conducted before or after the forecast. Following Stock and Watson [1999], this paper adopts principal component analysis for the former and variations of both simple and weighted averages for the latter.

## 1. Forecasting using the principal component

We use principal component analysis to follow the index approach, which produces an index that embodies information commonly contained in many information variables and uses this to perform forecasting. ${ }^{23}$ Specifically, we extract the first principal component common to each information variable $X^{t}$ at a certain point $t$. Let $D^{(t)}$ denote the extracted first principal component, which can be considered as an "index" that embodies common information contained in $n$ variables and removes miscellaneous and idiosyncratic noise. Using this $D^{(t)}$ as a new information variable in the bivariate forecast framework described in the previous section, we produce a forecast of $y_{t+h}^{h}$ using the principal component (fctr). ${ }^{2425}$

## 2. Simple and weighted averages of bivariate forecasts

For the forecast combination approach that combines a number of individual forecast series with some form of weights, there are several variations according to the method used to produce the weights.

First, relatively simple methods involve taking the mean (mean), median (median)

[^9]and trimmed mean of $n$ bivariate forecasts at the same point in time. Three types of trimmed mean are produced. These are the mean with the maximum and minimum excluded (tr.mean), the mean with two series of both the maximum and minimum excluded ( $T M_{-}-2$ ), and the mean with the uppermost and lowermost $15 \%$ excluded (TM_-15).

Another method is to allocate a greater weight to forecasts that are judged to perform well based on past data. Specifically, we produce a forecast combination (ridge), where the variable weighting is estimated by ridge regression ${ }^{26}$ from the following framework.

Let us denote the forecast value at $t$ using information variable $X^{t}$ as $f_{t}^{i}=\hat{y}_{t+h}^{h, i}$; the weight of the forecast made using variable $i$ at time $t$ as $w_{t:}^{i} \cdot f_{t}=\left(f_{t}^{1}, f_{t}^{2}, \ldots, f_{t}^{\prime \prime}\right) ; w_{t}=\left(w_{t}^{1}\right.$, $\left.w_{t}^{2}, \ldots, w_{t}^{\prime \prime}\right)^{\prime}$; and $c=k \times \operatorname{TR}\left(n^{-1} \sum_{s=1}^{t} f_{s} f_{s}^{\prime}\right)$ under a certain parameter $k$, where $\operatorname{TR}(\cdot)$ is the sum of diagonal elements in the matrix. ${ }^{27}$ Then variable weight forecast combination using ridge regression is defined as follows.

$$
\begin{equation*}
y_{t+h}^{h, r i d g e}=\sum_{i=1}^{n} w_{t}^{i} f_{t}^{i}, w_{t}=\left(c I_{n}+\sum_{s=1}^{t} f_{s} f_{s}^{\prime}\right)^{-1}\left(\sum_{s=1}^{t} f_{s} y_{s+h}^{h}+\frac{c}{n} \mathbf{i}\right), \tag{2}
\end{equation*}
$$

where $\mathbf{i}$ is the $n$-dimensional column vector of ones. We calculate this below for $k=0.25,0.5,1,10,100,500 .{ }^{28}$

## B. Performance of Multivariate Forecasting

Next, we compare the performance of the multivariate forecasting methods illustrated in the previous subsections using Japanese data. We conduct multivariate forecasting based on four groups of series (See Chart 7 for a list of the series used): ${ }^{29}$ real economic

[^10]activities (react), prices/wage/market prices (pr_wa), money (money), and asset prices (asset). In addition, multivariate forecasts using all selected variables included in these 4 groups (licatall) ${ }^{30}$ and those using all series (liall) are constructed (we refer to the former as "partial forecast combination" below). By doing this, it is possible to compare not only the differences between the types of information variable but the degree of forecast improvement when increasing the number of information variables without considering the characteristics of the variables.

## 1. CPI forecast

First, looking at the results for CPI forecast for different groups of information variables, we find that the forecasts using the price/wage variables ( $p r_{-} w a$ ) and money variables (money) generally perform well. The forecast performance improved in all sample periods and also in all forecast horizons for the mean, median, trimmed mean and some for ridge regression (ridge $k=100$, 500) using price/wage variables (pr_wa). Furthermore, except the bubble expansion period of 1987-90, there was also stable improvement in the forecasts using money variables (money). Additionally, improvements in forecast performance were conspicuous for 1991-94 and 1995-99 when looking at the results by period, and for four- and eight-quarter ahead forecasts when looking at the results by forecast horizon. ${ }^{31}$
(1) Of all of the bivariate forecast series using individual information variables, forecast series that had markedly inferior performance (if the relative MSFE was over two in any of the periods) in one of the forecast periods of 1987-90, 1991-94, 1995-99 are excluded.
(2) Series that improved AR forecasts (relative MSFE averaging less than one) are subject to combination. Series with relatively good performance (relative MSFE averaged less than 1.5) are also combined where possible. When there are multiple processed series (end of quarter value and average for quarter, logarithmic difference and level, etc) using the same variable, we select the series with the best performance.
(3) This selection is performed for each type of variable (real economic activity, prices/wage, money, asset prices). When doing so, if the number of candidates included in a certain type was less than six, we add series that showed large improvements in other periods or forecast horizons as candidates for the type even if the relative MSFE for that particular period or forecast horizon exceeded two. As a result, the bivariate forecasts of 6-14 series for each type are subject to combination.
${ }^{30}$ As a result, for this multivariate forecasts, 31 of 148 series are used for CPI forecasting and 32 of 147 for real GDP growth forecasting.
${ }^{31}$ For example, forecast combinations of prices/wage (pr_wa mean) showed improvement (-0.280) over all bivariate forecasts for $h=2$, in addition to the improvement over bivariate forecasts using nominal GDP/nominal wages being particularly large for $h=2,4$ in the period of 1995-99.

However, forecast performance does not necessarily improve by merely increasing the number of information variables used. In fact, the performance of forecasts using more than 140 series (liall) was lower than forecasts using approximately 30 selected variables (licatall) in almost all sample periods. Moreover, in many sample periods, the partial forecast combination (licatall) only gives performance lower than that of the forecast by variable type for prices/wage ( $p r_{-} w a$ ).

Meanwhile, by looking at the differences in performance across various combination methods, improvement can be seen to vary depending on the sample period and no particular method can be regarded as the most preferable. Let us take price/wage (pr_wa) that has the most sample periods with improvements on AR forecasts as an example, and compare the forecast combinations using the mean, median, trimmed mean and ridge regression (ridge $k=100,500$ ). Then, the combination method (the case in bold in Chart 8) that improves the relative MSFE the most varies depending on each sample period and the forecast horizon, and we find that no method can be singled out as showing performance superior to the others. At the same time, although ridge regression forecasts show superior performance for the two consecutive periods of the bubble-collapse period and the post-bubble period, it greatly deteriorates in comparison to the AR forecast for the preceding bubble-formation period, and thus is unstable with regard to changes in the sample period. ${ }^{32}$

## 2. Real GDP forecast

Next, looking at the performance of multivariate forecasts of real GDP (Chart 9), it can be noticed that none of the forecasts for different groups of indicators (react, pr_wa, money, asset) shows a relative MSFE of one or less for all sample periods and all forecast horizons. At the same time, forecasts using money variables show improvements in all three sample periods after 1987 for most forecast horizons, and the improvements are sizeable. ${ }^{33}$ Moreover, forecasts using asset price variables (asset mean) also show improvements throughout the three periods after 1987 for most

[^11]forecast horizons, and the improvements on forecast performance were more stable and larger than for bivariate forecasts.

Meanwhile, similar to CPI, we also confirm that the larger number of information variables used in forecasting does not necessarily lead to improvements in forecast performance. That is, the improvement for partial forecast combination is greater than for forecasts using all individual forecasts, in particular displaying improvements in all sampling periods for $h=8$, and the improvement for forecasts using money variables and those using asset value variables were generally greater for 1987-99.

Looking at the differences in performance by type of forecasting method, principal component (fctr), ridge regression (ridge) and simple mean (mean) consistently tend to show superior performance to other methods in the case of GDP forecasting, compared with the case of CPI forecasting. For example, in four- and eight-quarter ahead principal component (fctr) forecasts and two-quarter ahead ridge regression (ridge $k=0.25$ ) forecasts, the relative MSFE is smaller than those of other forecast methods for the three consecutive sample periods from 1987 to 1999. Also, the simple mean shows improvements on the AR forecast in four-quarter ahead forecasts using money variables (money) and eight-quarter ahead forecasts using prices/wage variables (pr_wa) for all four sub-sample periods, in addition to boasting the best performance in three of the four sample periods. However, as these multivariate forecasting methods are unstable with regard to changes in the forecast horizon and the variables used in combination (or principal component extraction), we are unable to choose a particular method as being the most preferable, as was also the case for CPI forecasting.

## 3. Overview: multivariate forecasting

We have shown that multivariate forecasting improves forecast performance more stably than bivariate forecasting for both CPI and GDP. However, the performance order of the various multivariate forecasting methods (simple mean, trimmed mean, ridge regression, principal component) varies depending on the type of variable used and the forecast horizon, and cannot be determined outright. ${ }^{34}$ It is also identified that

[^12]forecast performance does not necessarily improve by merely increasing the number of information variables.

## IV. Performance Improvement Mechanisms Involved in Forecast Combination and Their Limitations

In the previous section, we considered the performance of forecasts using many variables and confirmed that on the whole, these showed better performance than bivariate forecasts. When the sample period was divided, it was also observed that the performance of forecasts using many variables was stable across the sample periods.

These results are consistent with the intuition that forecast performance will improve as more information is reflected according to the increasing number of information variables. However, as we observed in the previous section, an increase in the number of series to be combined (or variables used in principal component analysis) does not always lead to improvements in forecast performance.

In this section, we investigate what mechanisms are working behind the forecast improvements resulting from the use of multiple information variables. For this investigation, it might be desirable to focus on the forecasting method that shows the best forecast performance among several methods. However, as noted in the previous section, it is difficult to pin down the most preferable method of combing information among simple mean, weighted mean or principal component analysis. For this reason, we will focus on forecast combination using the simple mean, as it is the most convenient to conduct analysis. ${ }^{35}$

[^13]
## A. Preliminary Considerations

We first show the way to decompose MSFE into several components, and then compare the MSFEs of simple mean forecast combination with those of original forecasts to be combined by component. By doing this we show what mechanisms produce higher performance and stability for simple mean forecast combinations.

## 1. Decomposition of MSFEs

The MSFE of forecasts using individual variables is comprised of components, one generated by forecast bias and the other by fluctuations in forecast errors. Letting $e_{i t}$ denote the (out-of-sample) forecast error of original forecast model using the individual variable $i(i=1, \ldots, n)$ in period $t$, the mean and the variance of the forecast error for the model are $\mu_{i}=1 / T \sum_{i=1}^{T} e_{i t}$ and $\sigma_{i}^{2}=1 / T \sum_{t=1}^{T}\left(e_{i t}-\mu_{i}\right)^{2}$, respectively (the former can be called the forecast bias). ${ }^{36}$ Then the MSFE for the model is written as:

$$
\begin{equation*}
\operatorname{MSFE}_{i}=\frac{1}{T} \sum_{t=1}^{T} e_{i t}^{2}=\mu_{i}^{2}+\sigma_{i}^{2} \tag{3}
\end{equation*}
$$

The MSFE of forecast combinations using a simple mean is also comprised of a component generated by forecast bias and a component generated by fluctuations in forecast errors. However, the latter can be further divided into two components: one generated by variability in forecast errors for each original forecast and the other involving the canceling out of movements between the forecast errors of the original forecasts. That is, noting that the forecast error in period $t$ is $1 / n \sum_{i=1}^{n} e_{i t},{ }^{37}$ we can write the MSFE for forecast combination using a simple mean, $M S F E_{\text {com }, n}$, as:

$$
\begin{align*}
\operatorname{MSFE}_{\text {com }, n} & =\frac{1}{T} \sum_{t=1}^{T}\left(\frac{1}{n} \sum_{i=1}^{n} e_{i t}\right)^{2} \\
& =\left(\frac{1}{n} \sum_{i=1}^{n} \mu_{i}\right)^{2}+\left(\frac{1}{n} \sum_{i=1}^{n} \sigma_{i}\right)^{2}-\sum_{i=1}^{n} \sum_{j=1}^{n}\left\{\frac{\sigma_{i}}{n} \frac{\sigma_{j}}{n}\left(1-\rho_{i j}\right)\right\}, \tag{4}
\end{align*}
$$

where $\rho_{i j}=\left(1 / \sigma_{i} \sigma_{j} T\right) \sum_{t=1}^{T}\left(e_{i t}-\mu_{i}\right)\left(e_{j t}-\mu_{j}\right) \quad i, j=1, \ldots, n$ is the correlation coefficient between $e_{i t}$ and $e_{j t}$.

[^14]As the forecast bias of forecast combination using a simple mean is $1 / n \sum_{i=1}^{n} \mu_{i}$, the first term on the right-hand side of the equation represents the component generated by forecast bias. At the same time, the second and third terms on the right-hand side of the equation are the parts produced by changes in forecast error and correspond to the second term in the right-hand side of Equation (3). Of these, the third term is the sum of the fluctuations in the original forecast error series multiplied by one minus the correlation coefficient, and thus it becomes zero if the correlation coefficients between all of the original series are one. Since the value obtained by subtracting the correlation coefficient from one can be interpreted as the extent that movement of forecast error series differs, the third term represents the part of the MSFE that is cancelled out by combining original forecasts that move differently. In contrast to this, the second term is the fluctuation in forecast error even when the third term is zero, that is, when all original forecast errors are moving together. Therefore, this term represents the component generated by fluctuations in forecast errors of the original forecast series.

## 2. Comparison of the MSFE of forecast combination and the average MSFE of original forecasts

Next, using the MSFE equations obtained above, we compare the MSFE of simple mean forecast combination with the average MSFE of the original forecasts that are to be combined.

Averaging both sides of Equation (3) for $i$ and taking the difference between it and Equation (4) yields:

$$
\begin{align*}
\operatorname{MSFE}_{c o m, n}-\frac{1}{n} \sum_{i=1}^{n} \text { MSFE }_{i} & =-\left[\frac{1}{n} \sum_{i=1}^{n} \mu_{i}{ }^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} \mu_{i}\right)^{2}\right] \\
& -\left[\frac{1}{n} \sum_{i=1}^{n} \sigma_{i}^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} \sigma_{i}\right)^{2}\right]-\sum_{i=1}^{n} \sum_{j=1}^{n}\left\{\frac{\sigma_{i}}{n} \frac{\sigma_{j}}{n}\left(1-\rho_{i j}\right)\right\} . \tag{5}
\end{align*}
$$

All of the terms on the right-hand side of Equation (5) are always negative. As the correlation coefficient is always less than 1 , it is clear that the third term is always negative. The first and second terms can be rewritten as:

$$
\begin{align*}
& {\left[\frac{1}{n} \sum_{i=1}^{n} \mu_{i}^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} \mu_{i}\right)^{2}\right]=\frac{1}{n} \sum_{i=1}^{n}\left(\mu_{i}-\frac{1}{n} \sum_{j=1}^{n} \mu_{j}\right)^{2},}  \tag{6}\\
& {\left[\frac{1}{n} \sum_{i=1}^{n} \sigma_{i}^{2}-\left(\frac{1}{n} \sum_{i=1}^{n} \sigma_{i}\right)^{2}\right]=\frac{1}{n} \sum_{i=1}^{n}\left(\sigma_{i}-\frac{1}{n} \sum_{j=1}^{n} \sigma_{j}\right)^{2} .} \tag{7}
\end{align*}
$$

These are both always positive, making them both negative in Equation (5).

Therefore, the following holds. ${ }^{38}$

$$
\begin{equation*}
M S F E_{\text {com }, n} \leq \frac{1}{n} \sum_{i=1}^{n} M S F E_{i} . \tag{8}
\end{equation*}
$$

This means that the performance of forecast combination using a simple mean is always better than the mean performance of the original forecasts. ${ }^{3940}$ The degree of this improvement in performance is determined by the first through third terms of Equation (5).

## 3. The MSFE improvement mechanism resulting from combination

Why is the performance of forecast combination better than the average performance of the original forecasts? That is, what kind of mechanism reduces the MSFE as shown by the first through third terms in Equation (5)?

As aforementioned, the third term represents the "cancellation effect of forecast error variations" resulting from the combination of original forecasts with differing movements. This effect is larger when the correlation between original forecasts is smaller, i.e. when the movements of the original forecasts differ more.

The first and second terms each represent the "leveling effect on forecast bias" and the "leveling effect on forecast error fluctuations" brought about by the averaging of multiple original forecasts. This can be understood from the fact that the first term in the right-hand side of Equation (4) is the square of the average forecast bias of the original forecasts and that the second term is the square of the average forecast error standard deviation of the original forecasts. As we can see in Equations (6) and (7), these terms are lower than the average values for the corresponding terms in the MSFE

[^15]of the original forecasts. That is, combining forecasts using a simple mean reduces both the forecast bias and the extent of fluctuation in forecast errors, and thus makes the MSFE smaller.

It can be expected that this leveling effect is greater for the forecast bias than for forecast error fluctuations for the following reason. As shown in Equations (6) and (7), the reduction in the mean squared forecast error that is brought about by leveling increases as the variances of the forecast bias and standard deviations in forecast error fluctuations increase. While the forecast bias $\mu_{i}$ can be positive or negative, and the forecast error standard deviation $\sigma_{i}$ can only be positive, it is expected that forecast bias will have greater variance. Therefore, it is probable that the leveling effect is larger for forecast bias than for forecast error fluctuations.

In summary, it has been shown in this subsection that performance is improved through (a) the leveling effect on forecast bias, (b) the leveling effect on forecast errors, and (c) the cancellation effect between the forecast error behavior of each original forecast. Due to these effects, forecast combination always has better performance than the average performance of original forecasts. The leveling effect is expected to have a greater effect on forecast bias than on fluctuations.

## B. The Actual Importance of Each Factor

The next issues to be considered are to what degree the MSFE reduction effect of forecast combination works in practice and which of the three effects is actually the most dominant. In order to consider these, we decompose the relative MSFE reduction effect of forecast combination that is using a simple mean into three components, using the equation obtained by dividing both sides of Equation (5) by the MSFE of the AR forecast. Results are shown in Chart 10.

Looking at the results for the entire period (the rightmost column of Chart 10), it can be seen that the reduction effect of combination on the relative MSFE is large. Although there are differences in the forecast combination relative MSFE depending on the forecast horizon and the type of variable, it is 0.1-0.5 lower than the average relative MSFE for the original forecasts. Also, in many cases where the average relative MSFE of the original forecasts exceeds one, the relative MSFE of the forecast combination is below one. For example, in four-quarter ahead CPI forecasts, the average relative MSFEs of the original forecasts is over one for partial forecast combination, real
activity variables and price related variables; but the relative MSFE of forecast combination is less than one in all these cases.

Moreover, looking at a breakdown of the MSFE reduction effect, it is found that most of this MSFE reduction effect is brought about by the third term in Equation (5). That is, most of the MSFE reduction effect for forecast combination using a simple mean is brought about by forecast error fluctuations in each of the original forecast series canceling each other out. The effect of both the first and second terms are considerably smaller than that of the third term, but the first term that represents the leveling effect on forecast bias is the larger of the two. This is compatible with the predictions made in the previous subsection.

Next, looking at the results by sample period, (first to third columns in Chart 10), it can be seen that the difference between the relative MSFE of forecast combination and the average relative MSFE of the original forecasts fluctuates significantly. That is, the MSFE reduction effect due to combination varies depending on the sample period. For example, a large MSFE reduction effect occurred in the 1987-90 and 1995-99 periods for CPI forecasts, but was slightly smaller for 1991-94. Furthermore, the MSFE reduction effect was larger in the 1991-94 and 1995-99 periods for CPI forecasts, but was slightly smaller for 1991-94.

Moreover, the results of the decomposition of the MSFE reduction effects by sample period show that although the effect of the third term was large, the first term also produced a large improvement effect. The effect of the second term, which was almost zero over the entire sample period, also appears in places.

Why the effects of the first and second terms, which do not appear in the results for the entire period, appear in the results for sub-sample periods can be explained as follows. When looking at a relatively long period, the forecast bias of the original forecasts and forecast error fluctuations are similar for each of the original forecasts. For this reason, the leveling effect of the first and second terms is smaller over long sample periods. However, as the dispersion of forecast error fluctuations of the original forecasts is larger over short periods, the leveling effect of the first term and second term increases.

## C. Optimal Forecast Combination: Stepwise Combination Selection

Next, we examine how many bivariate forecasts should be combined in simple mean
forecast combination, i.e. the optimal number of variables to be combined. The results thus far show that improvements do not necessarily increase as the number of variables to be combined increases. In order to discover the limitations to this mechanism that brings about improvements in forecast performance, we consider what kind of series comprise the "optimal forecast combination (using a simple mean)" and the extent of the performance shown by this.

Specifically, we search for the optimal group for combination by adding variables one by one in a stepwise manner, leaving only the group with the best performance. ${ }^{41}$ Additionally, the candidates for combination are variables in partial forecast combination ( 31 series for CPI forecasts and 32 series for real GDP forecasts).

## 1. Stepwise optimization procedure

The procedure of stepwise optimization to find the best grouping forecast combination is as follows:
a) Step 1: One variable is chosen from $K$ variables ( $K=31$ for CPI forecasts, or $K=32$ for real GDP forecasts) as the "initial variable," and another variable is added, producing a forecast combination of two variables. Since the number of variables for combination is $K, K-1$ types of forecast combination of 2 variables can be made, and the one with the lowest MSFE is saved.
b) Step 2: Another variable is added to make a forecast combination of three variables. There are $K-2$ configurations and the one with the smallest MSFE is saved. We continue this process for combining four variables, five variables and so on until all candidate series have been added.
c) Step 3: Given a certain initial variable, correspondences among (i) the initial variable, (ii) the number of series included in the combination, and (iii) MSFE are produced in the process of adding each variable. Of these, we describe the

[^16]grouping with the smallest MSFE as the "optimum given initial variable."
d) Step 4: There are as many optimal groupings given initial variable as there are initial variables, and the grouping with the minimum MSFE is the "stepwise optimum." This will be called the "optimal combination grouping" below.

## 2. Optimal combination grouping and forecast performance

According to the stepwise optimization procedure explained in the previous subsection, the optimal combination grouping for all forecast horizons and sample periods are shown in Chart 11 for each variable. In the upper part in Chart 11, the horizontal axis is made up of the initial variables arranged in the order of forecast performance across sample periods. The dotted line shows the relative MSFE of the optimal combination for each given initial variable and the points ( $\rangle$ or $\boldsymbol{\square}$ ) show the number of series. The solid points ( $\mathbf{\square}$ ) describe the number of optimal combination series for initial variables that reach the stepwise optimum. The lower part of Chart 11 shows the composition of the optimal grouping of variables.

Looking at the entire sample period (1987-99), the optimal combination grouping (the $\square$ points in the upper part in Chart 11) is two to four original forecast series for all forecast horizons and sampling periods of both CPI and real GDP forecasts. Also, the original forecasts to be combined are ones with high performance in many cases, but the combinations are not necessarily made up of forecasts with superior performance, in fact many of the original forecasts with mid-range performance were included. In CPI forecasts, nominal GDP ( $n g d p$ ) is included in all three sample periods, but we see no common indicator in all sample periods for real GDP forecasts. A similar tendency can be seen in the results when dividing into several sample periods (Chart 12). ${ }^{42}$

What degree of performance does this optimal grouping show? Looking at the entire sample period (Chart 13), the optimal forecast combination shows an improvement due to not only combination but reduction in the average relative MSFE of the original forecasts, resulting in a relative MSFE of 0.2-0.6, an improvement of 0.40.8 over AR forecast. Of this improvement over AR forecasting, the effect of combination is around $0.1-0.5$ points, which is similar to the results for combination by

[^17]variable type and combination forecasts using partial forecast combination. Most of this reduction effect is brought about by the canceling out of forecast errors in the original forecasts as is shown on the right-hand side of Equation (5).

Observing the results by sample period (Chart 14), we see that the improvement effect in the third term of Equation (5) is high, just like that for the entire period, but its size varies depending on the period. For example, while the relative MSFE for CPI forecasting between 1991-94 is smaller than that of others, the improvement effect is also smaller than for others. Also, we see a certain amount of leveling effect of the first term, which represents forecast bias, to be functioning in GDP forecasts for 1991-94.

## 3. Stepwise variable addition and factors in MSFE improvement

Next, we consider why improvements in forecast performance cease to occur after the number of series to be combined reaches around five.

Decomposing the factors of changes in the relative MSFE when series to be combined are added in a stepwise manner (Chart 15) shows that, when the forecast combination improvement is positive until two to four series are added, the forecast improvement is attributable primarily to the third term in Equation (5). However, this effect lasts for at most up to four variables in the grouping, and diminishes when five or more variables are added. ${ }^{43}$

Looking at sample periods separately (Chart 16), the improvement due to the third term is dominant in most cases as is the case when observing the entire sample period, although there are some periods in which the improvement due to the first term is also large. These results show that the maximum number of series in a forecast combination is two to four for all forecast horizons and sample periods of both CPI and real GDP.

In summary, as variables being added in a stepwise manner, on the one hand the forecast improvement effect of combination becomes smaller, and on the other it becomes harder to find a information variable that have a low correlation with forecast errors. ${ }^{44}$ Consequently, the effect of raising the average relative MSFE becomes larger

[^18]due to addition of original forecasts with poor performance. For this reason, forecast performance improvements due to combination stop after four variables are added in practice.

## 4. Optimization and comparison by sample period

Finally, we briefly check the robustness of the optimal combination grouping throughout the sample period. Using the optimal grouping for the period, Chart 17 computes its relative MSFE for each sub-sample period, and compares them with those for the optimal grouping for each sub-sample period. In this chart, we refer to the optimum for each sample period being described as the best, and the optimal grouping for the entire period as the second best, intending to let the latter approximate the former. ${ }^{45}$

The upper section of Chart 17 shows that the divergence of relative MSFEs between the best and the second best is 0.2 points for CPI and 0.3 points for GDP at the maximum. When relative MSFEs expand, the best and second best groupings contain few variables in common. For example, when a divergence of 0.2 occurs in the fourquarter ahead CPI forecast in the 1995-99 sample period, the best and second best groupings have more variables that are not in common. Similarly, the eight-quarter ahead GDP forecast for the same sample period has only one variable in common. ${ }^{46}$

We then decompose the improvements for the best and the second best groupings, and compare the improvements in forecast performance for each of the groupings (lower section of Chart 17). The result shows that differences between the two are attributable to not the improvement effect caused by combination, but higher average relative MSFE of the original forecasts. That is, while a variable in the second best grouping shows inferior performance to a variable in the best, the improvement effect of the second best is virtually as large as that of the best grouping. Consequently, the forecast combination of the second best grouping as well as that of the best grouping

[^19]has a relative MSFE of less than one, and outperform the AR forecast.
From the above results we find that the divergence of the forecast performance between the optimal grouping for the entire period and for each period is approximately 0.2-0.3 points in relative MSFE, and that the cause of this divergence is not a lesser improvement effect of forecast combination but the lowered average performance of bivariate forecasts. Even so, as we have seen before, the relative MSFE of the optimal grouping for the entire period is approximately $0.2-0.6$, substantially lower than one in all cases. Therefore, the second best grouping (best over the entire period) can be considered robust in that it is useful for improving AR forecasting.

## V. Conclusions

In this paper, we have examined how we can improve forecast performance of CPI and real GDP changes, by combining forecasts using individual indicators or extracting common elements from this variety of financial and economic indicators. We have further considered what is occurring as a background mechanism to this improvement.

We summarize the results of our analysis as follows: the predictive power of information variables varies depending on the sample period and the forecast horizon, but more stable performance can be obtained by combining forecasts or using the principal component of variables. At the same time, this improvement on forecast performance due to combination ceases to occur after the inclusion of two to four variables, since each variables added shows increasingly poorer performance, and the forecast error is rapidly attenuated by the further addition of variables. For this reason, in order to improve the performance of forecast combination, it is important to find indicators with high forecastability as well as with lower correlation with forecasts of other information variables.

## References

Artis, Michael J, Anindya Banerjee and Massimiliano Marcellino, "Factor Forecasts for the UK," CEPR Working Paper No.3119, 2002.
Bates, J.M. and C.W.J. Granger, "The Combination of Forecasts," Operations Research Quarterly, Vol.20, 1969, pp.319-25.

Bernanke, Ben and Jean Boivin, "Monetary Policy in a Data-Rich Environment," NBER Working Paper No.8379, 2001.

Ban, Kanemi, and Makoto Saito, "Makuro Keiryou Moderu ni yoru Infure-ritsu Yosoku Gosa no Bunseki" (Analysis of Inflation Rate Forecast Errors Using a Macroeconometric Model), Bank of Japan Research and Statistics Department Working Paper, No. 01-12, Bank of Japan, 2001 (in Japanese).
Cecchetti, Stephen G., Rita S. Chu and Charles Steindel, "The Unreliability of Inflation Indicators," Current Issues in Economics and Finance 6(4), Federal Reserve Bank of New York, 2000.

Clements, M.P. and David F. Hendry, Forecasting Economic Time Series, Cambridge University Press, 1996.

Forni, Mario, Marc Hallin, Marco Lippi and Lucrezia Reichlin, "Do Financial Variables Help Forecasting Inflation and Real Activity in the Euro Area?" CEPR Working Paper No.3146, 2002.

Fukuda, Shin'ichi, and Masayuki Keida, "Infure Yosoku ni kansuru Jisshou Kenkyuu no Tenbou - Firippusu Kyokusen no Nihon ni okeru Yosokuryoku wo Chuushin ni" (Outlook of Empirical Analysis for Inflation Forecasting - A focus on the Forecast Performance of the Phillips Curve in Japan), Bank of Japan Research and Statistics Department Working Paper, No.01-21, Bank of Japan, 2001(in Japanese).
Hirata, Hideaki and Kazuo Ueda, "The Yield Spread as a Predictor of Japanese Recessions," Bank of Japan Research and Statistics Department Working Paper, No. 98-3, Bank of Japan, 1998.

Honda, Yuuzou, and Mikihiro Matsuoka, "Keiki Shihyou he no Ikutsuka no Toukeigakuteki Sekkin - Senkou Shihyou wo Chuushin to shite" (Some Statistical Approaches to Business Cycles - Focusing on Leading Indicators), Financial Review, Ministry of Finance Policy Research Institute, 2001 (in Japanese).

Kasuya, Munehisa, and Kazuhiko Shinki, "Bukka Hendou no Tenkanten Yosoku ni tsuite" (On the Forecasting of Turning Points in Price Changes), Bank of Japan

Research and Statistics Department Working Paper, No. 01-20, Bank of Japan, 2001(in Japanese).
Kato, Kengo, "The Information Content of Financial and Economic variables: Empirical Tests of Information Variables in Japan," Monetary and Economic Studies, 9(1), Institute for Monetary and Economic Studies, Bank of Japan, 1991, pp. 61-86
Kitagawa, Genshiro, and Yoshinori Kawasaki, "Jikeiretsu Moderu ni yoru Infure-ritsu Yosoku Gosa no Bunseki" (Analysis of Inflation Rate Forecast Errors Using a Time Series Model), Bank of Japan Research and Statistics Department Working Paper, No. 01-13, Bank of Japan, 2001 (in Japanese).
Marcellino, Massimiliano, "Forecast Pooling for Short Time Series of Macroeconomic Variables," CEPR Working Paper No.3313, 2002.
Marcellino, Massimiliano, James H. Stock and Mark W. Watson, "Macroeconomic Forecasting in the Euro Area: Country Specific versus Area-Wide Information," mimeo, 2000.
Mio, Hitoshi, "The Phillips Curve and Underlying Inflation", Monetary and Economic Studies, 19(2), Institute for Monetary and Economic Studies, Bank of Japan, 2001, pp.85-108
Okina, Kunio, and Shigenori Shiratsuka, "Asset Price Bubble, Price Stability, and Monetary Policy: Japan's Experiences," Monetary and Economic Studies, 20(3), Institute for Monetary and Economic Studies, Bank of Japan, 2002, pp. 35-76
Oyama, Shinsuke, "Forecast Combination ni yoru Jisshitsu GDP no Yosoku" (Real GDP Forecasting Using Forecast Combination), Bank of Japan Research and Statistics Department Working Paper, No. 01-3, Bank of Japan, 2001
Stock, James H. and Mark W. Watson, "Evidence on Structural Instability in Macroeconomic Time Series Relations," Journal of Business and Economic Statistics, 14, 1996, pp.11-30.
——, "Diffusion Index," NBER Working Paper No. 6702, 1998.
__, "Forecasting Inflation," Journal of Monetary Economics, 44, 1999, pp. 293-335.
__, "Forecasting Output and Inflation: The Role of Asset Prices," NBER Working Paper No. 8180, 2001.
Watson, Mark W., "Macroeconomic Forecasting Using Many Predictor," mimeo, 2000.

Chart 1: Variables used in testing forecast ability

| Variable | Code | Seasonal Adjustment | Nominal/real |  | Average/ end- |  | Processed calculation |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Nominal | Real | Ave $a v$ | End <br> ed | Level <br> lev | $\begin{gathered} \mathrm{Log} \\ \mathrm{ln} \end{gathered}$ | $\begin{aligned} & \hline \text { Log. } \\ & \text { ln } 1 d \end{aligned}$ | $\begin{gathered} \hline \text { GAP } \\ \text { gap } \end{gathered}$ |
| Real GDP | $r g d p$ | SA | - | 0 | O | - | - | - | O | 0 |
| Index of industrial production | ip | SA | - | O | 0 | - | - | - | O | 0 |
| Index of Tertiary Industries Activities | sanji | SA | - | 0 | 0 | - | - | - | O | 0 |
| Industrial capacity utilization index | сари | SA | - | 0 | 0 | - | - | - | O | 0 |
| Business conditions Diffusion Index of TANKAN (Short-term economic survey of principal enterprises) | tnkzen | SA | - | O | 0 | - | O | - | - | 0 |
| ** (manufacturing Industry) | tnksei | SA | - | 0 | 0 | - | 0 | - | - | 0 |
| ** (nonmanufacturing Industry) | tnkhi | SA | - | 0 | 0 | - | 0 | - | - | 0 |
| Unemployment rate | ипетр | SA | - | 0 | 0 | - | 0 | - | - | 0 |
| Ratio of job offers to applicant | Kyujin | SA | - | 0 | 0 | - | 0 | - | - | 0 |
| Machinery orders (private demand) | kijmi | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Machinery orders (manufacturing Industry) | kijse | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Machinery orders (nonmanufacturing Industry) | kijhi | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Construction orders | kenjal | SA | - | O | 0 | - | - | - | O | 0 |
| Construction orders (private) | kenjimi | SA | - | O | 0 | - | - | - | O | 0 |
| Construction orders (nonmanufacturing) | kenjhi | SA | - | 0 | 0 | - | - | - | O | 0 |
| Value of public works contracted | ukeall | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Value of public works contracted (Central gov't) | ukekun | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Value of public works contracted (local gov't) | ukechi | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Housing starts | juckko | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Floor area of housing starts | juckme | SA | - | 0 | 0 | - | - | - | O | 0 |
| Floor area of construction starts | ckhime | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Number of new car registrations | car | SA | - | 0 | 0 | - | - | - | O | 0 |
| Sales of large-scale retail stores | kouri | SA | - | 0 | 0 | - | - | - | O | 0 |
| Sales of department store | hyaka | SA | - | 0 | 0 | - | - | - | O | 0 |
| Total exports (custom clearance) | expt | SA | - | 0 | 0 | - | - | - | 0 | 0 |
| Total imports (custom clearance) | impt | SA | $\therefore$ | O | 0 | - |  | - | O | O |
| Nominal GDP | $n \mathrm{n}$ dp | SA | 0 |  | 0 |  |  |  | O |  |
| GDP deflator | $p g d p$ | SA | 0 | - | 0 | - | - | - | 0 | - |
| CPI | cpi | SA | 0 | - | 0 | - | - | - | 0 | - |
| Domestic WPI aggregate average | wpi | NSA | 0 | - | 0 | - | - | - | 0 | - |
| Domestic WPI intermediate goods | wpiin | NSA | 0 | - | 0 | 0 | - | - | 0 | - |
| Import price index (total average) | ipiav | NSA | 0 | - | 0 | - | - | - | 0 | - |
| Import price index (raw materials) | ipiso | NSA | 0 | - | 0 | 0 | - | - | 0 | - |
| Wage index | wage | SA | 0 | 0 | 0 | - | - | - | 0 | - |
| Crude oil | oil | NSA | 0 | 0 | - | 0 | - | - | O | - |
| Domestic commodity price index | commed | NSA | 0 | 0 | - | 0 | - | - | O | - |
| Reuters index | reu | NSA | 0 | 0 | - | 0 | - | - | 0 | - |
| CRB index | crb | NSA | 0 | 0 | - | 0 | - | - | 0 | - |
| Gold | gld | NSA | 0 | 0 | $\therefore$ | O | - | - | 0 | - |
| Monetary base | mon0 | SA | 0 | 0 | 0 | - |  | - | 0 |  |
| M1 | mon1 | SA | 0 | 0 | 0 | - | - | - | 0 | - |
| M2+CD | mon2 | SA | 0 | 0 | 0 | - | - | - | 0 | - |
| Broadly-defined credit aggregate | mon4 | SA | 0 | 0 | - | 0 | - | - | 0 | - |
| Bank lending | lended | SA | 0 | 0 | - | 0 | - | - | 0 | - |
| Credit multipler | $m l p$ | SA | 0 | - | 0 | - | 0 | - | - | 0 |
| Circuit velocity of money [M2+CD] | velo | SA | 0 | - | 0 | - | - | - | 0 | 0 |
| Bank notes in circulation | note | SA | 0 | 0 | 0 |  |  | - | O | O |
| Long-term government bonds | $i g b$ | NSA | 0 | O- | 0 | 0 | 0 | - |  |  |
| Interest rate spread | sprd | NSA | 0 | - | - | 0 | 0 | - | - | - |
| Loan contract rates | alnd | NSA | 0 | 0 | 0 | - | 0 | - | - | - |
| Yen/dollar rate | rate | NSA | 0 | 0 | 0 | 0 | - | - | 0 | 0 |
| Effective exchange rate | efrat | NSA | 0 | 0 | 0 | 0 | - | - | 0 | 0 |
| Nikkei average | nik | NSA | 0 | O | 0 | 0 | - | - | 0 | - |
| TOPIX | $t p x$ | NSA | 0 | 0 | 0 | 0 | - | - | O | - |
| TSE amount traded | tosho | NSA | - | 0 | 0 | - | - | 0 | O | - |
| Land prices for commercial areas in 6 major cities | land | NSA | 0 | 0 | - | 0 | - | - | 0 | - |

Notes: 1. "SA" is seasonally adjusted and "NSA" is not seasonally adjusted.
2. "level", "log", "log diff" and "gap" are the processed values of the original values, the logarithmic values, logarithmic difference and HP filter.

Chart 2: Codes, variables and conversion methods used in each series.

| code of series |  | name of variable(process) | code of series |  | name of variable(process) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| rgdp | $\ln 1 d$ | real GDP ( $\log$ diff.) | crbed | $\ln 1 \mathrm{~d}$ | CRB index (log diff.) |
| $r g d p$ | gap | real GDP (gap) | rcrbed | $\ln 1 d$ | CRB index (log diff.) |
| ip | $\ln 1 \mathrm{~d}$ | Index of industrial production (log diff.) | lccrbed | lnld | CRB index (log diff.) |
| ip | gap | Index of industrial production (gap) | rlccrbed | $\ln 1 d$ | CRB index (log diff.) |
| sanji | $\ln 1 d$ | Index of Tertiary Industries Activities (log diff.) | glded | $\ln 1 \mathrm{~d}$ | Gold (log diff.) |
| sanji | gap | Index of Tertiary Industries Activities (gap) | rglded | $\ln 1 \mathrm{~d}$ | Gold (log diff.) |
| сари | $\ln 1 \mathrm{~d}$ | Industrial capacity utilization index ( $\log$ diff.) | lcglded | $\ln 1 \mathrm{~d}$ | Gold (log diff.) |
| сари | gap | Industrial capacity utilization index (gap) | rlcglded | lnld | Gold (log diff.) |
| tnksei | lev | Business conditions Diffusion Index of TANKAN (level) | mon0 | $\ln 1 \mathrm{~d}$ | Monetary base (log diff.) |
| tnkhi | lev | ** (manufacturing Industry) (level) | mon0 | gap | Monetary base (gap) |
| tnkzen | lev | ** (nonmanufacturing Industry) (level) | rmon0 | $\ln 1 \mathrm{~d}$ | Monetary base (log diff.) |
| unemp | lev | Unemployment rate (level) | rmon0 | gap | Monetary base (gap) |
| unemp | gap | Unemployment rate (gap) | monl | $\ln 1 d$ | M1 (log diff.) |
| kyujin | lev | Ratio of job offers to applicant (level) | monl | gap | M1 (gap) |
| kyujin | gap | Ratio of job offers to applicant (gap) | rmon1 | $\ln 1 d^{\prime}$ | M1 (log diff.) |
| kijmi | $\ln 1 \mathrm{~d}$ | Machinery orders (private demand) (log diff.) | rmon 1 | gap | M1 (gap) |
| kijmi | gap | Machinery orders (private demand) (gap) | mon2 | lnld | M2+CD (log diff.) |
| kijse | $\ln 1 \mathrm{~d}$ | Machinery orders (manufacturing Industry) (log diff.) | mon2 | gap | M2+CD (gap) |
| kijse | gap | Machinery orders (manufacturing Industry) (gap) | rmon2 | $\ln 1 d$ | M2+CD (log diff.) |
| kijhi | $\ln 1 \mathrm{~d}$ | Machinery orders (nonmanufacturing Industry) (log diff.) | rmon2 | gap | M2+CD (gap) |
| kijhi | gap | Machinery orders (nonmanufacturing Industry) (gap) | mon4 | $\ln 1 d$ | Broadly-defined credit aggregate (log diff.) |
| kenjal | $\ln 1 d$ | Construction orders (log diff.) | mon4 | gap | Broadly-defined credit aggregate (gap) |
| kenjal | gap | Construction orders (gap) | rmon4 | $\ln 1 d$ | Broadly-defined credit aggregate (log diff.) |
| kenjmi | $\ln 1 \mathrm{~d}$ | Construction orders (private) (log diff.) | rmon4 | gap | Broadly-defined credit aggregate (gap) |
| kenjmi | gap | Construction orders (private) (gap) | lended | lnld | Bank lending (log diff.) |
| kenjhi | $\ln 1 \mathrm{~d}$ | Construction orders (nonmanufacturing) (log diff.) | lended | gap | Bank lending (gap) |
| kenjhi | gap | Construction orders (nonmanufacturing) (gap) | rlended | lnld | Bank lending (log diff.) |
| ukeall | $\ln 1 d^{1}$ | Value of public works contracted (log diff.) | rlended | gap | Bank lending (gap) |
| ukeall | gap | Value of public works contracted (gap) | $m \mathrm{lp}$ | lev | Credit multipler (level) |
| ukekun | $\ln 1 \mathrm{~d}$ | Value of public works contracted (Central gov't) (log diff.) | $m l p$ | gap | Credit multipler (gap) |
| ukekun | gap | Value of public works contracted (Central gov't) (gap) | velo | $\ln 1 \mathrm{~d}$ | Circuit velocity of money [M2+CD] (log diff.) |
| ukechi | ln1d | Value of public works contracted (local gov't) (log diff.) | velo | gap | Circuit velocity of money [M2+CD] (gap) |
| ukechi | gap | Value of public works contracted (local gov't) (gap) | note | $\ln 1 \mathrm{~d}$ | Bank notes in circulation (log diff.) |
| juckko | $\ln 1 \mathrm{~d}$ | Housing starts (log diff.) | note | gap | Bank notes in circulation (gap) |
| juckko | gap | Housing starts (gap) | rnote | $\ln 1 d^{1}$ | Bank notes in circulation (log diff.) |
| juckme | $\ln 1 \mathrm{~d}$ | Floor area of housing starts (log diff.) | rnote | gap | Bank notes in circulation (gap) |
| juckme | gap | Floor area of housing starts (gap) | jgbed | lev | Long-term government bonds (level) |
| ckhime | $\ln 1 \mathrm{~d}$ | Floor area of construction starts (log diff.) | righed | lev | Long-term government bonds (level) |
| ckhime | gap | Floor area of construction starts (gap) | sprded | lev | Interest rate spread (level) |
| car | $\ln 1 \mathrm{~d}$ | Number of new car registrations (log diff.) | alndav | lev | Loan contract rates (level) |
| car | gap | Number of new car registrations (gap) | ralndav | lev | Loan contract rates (level) |
| kouri | ln1d | Sales of large-scale retail stores (log diff.) | rateav | $\ln 1 d$ | Yen/dollar rate (log diff.) |
| kouri | gap | Sales of large-scale retail stores (gap) | rateav | gap | Yen/dollar rate (gap) |
| hyaka | $\ln 1 d$ | Sales of department store (log diff.) | rateed | $\ln 1 \mathrm{~d}$ | Yen/dollar rate (log diff.) |
| hyaka | gap | Sales of department store (gap) | rateed | gap | Yen/dollar rate (gap) |
| lcexpt | $\ln 1 \mathrm{~d}$ | Total exports (custom clearance) (log diff.) | ratest | lev | Yen/dollar rate (level) |
| lcexpt | gap | Total exports (custom clearance) (gap) | ratesk | lev | Yen/dollar rate (level) |
| lcimpt | $\ln 1 d$ | Total imports (custom clearance) (log diff.) | ratekr | lev | Yen/dollar rate (level) |
| lcimpt | gap | Total imports (custom clearance) (gap) | rrateav | lnld | real exchange rate (log diff.) |
| expt | $\ln 1 d^{1}$ | Total exports (custom clearance) (log diff.) | rrateav | gap | real exchange rate (gap) |
| expt | gap | Total exports (custom clearance) (gap) | rrateed | $\ln 1 d$ | real exchange rate (log diff.) |
| impt | $\ln 1 \mathrm{~d}$ | Total imports (custom clearance) (log diff.) | rrateed | gap | real exchange rate (gap) |
| impt | gap | Total imports (custom clearance) (gap) | efratav | lnld | Nominal effective exchange rate (log diff.) |
| $n g d p$ | $\ln 1 d$ | Nominal GDP (log diff.) | efratav | gap | Nominal effective exchange rate (gap) |
| pgdp | $\ln 1 d$ | GDP deflator (log diff.) | efrated | $\ln 1 \mathrm{~d}$ | Nominal effective exchange rate (log diff.) |
| cpi | $\ln 1 d$ | CPI (log diff.) | efrated | gap | Nominal effective exchange rate (gap) |
| wpi | $\ln 1 d$ | Domestic WPI aggregate average (log diff.) | refrated | $\ln 1 \mathrm{~d}$ | Real Effective exchange rate (log diff.) |
| wpiinav | $\ln 1 d$ | Domestic WPI intermediate goods (log diff.) | refrated | gap | Real Effective exchange rate (gap) |
| wpiined | $\ln 1 d$ | Domestic WPI intermediate goods (log diff.) | refratav | $\ln 1 \mathrm{~d}$ | Real Effective exchange rate (log diff.) |
| ipiav | $\ln 1 d$ | Import price index (total average) (log diff.) | refratav | gap | Real Effective exchange rate (gap) |
| ipisoav | $\ln 1 d$ | Import price index (raw materials) (log diff.) | nikav | $\ln 1 \mathrm{~d}$ | Nikkei average (log diff.) |
| ipisoed | $\ln 1 d$ | Import price index (raw materials) (log diff.) | niked | $\ln 1 d^{1}$ | Nikkei average (log diff.) |
| wage | $\ln 1 d$ | Wage index (log diff.) | nikst | lev | Nikkei average (level) |
| rwage | $\ln 1 d$ | Wage index (log diff.) | niksk | lev | Nikkei average (level) |
| oil | $\ln 1 d$ | Crude oil ( $\log$ diff.) | nikkr | lev | Nikkei average (level) |
| roil | $\ln 1 d$ | Crude oil (log diff.) | rnikav | $\ln 1 \mathrm{~d}$ | Nikkei average (log diff.) |
| lcoil | $\ln 1 \mathrm{~d}$ | Crude oil (log diff.) | rniked | lnld | Nikkei average (log diff.) |
| rlcoil | $\ln 1 d$ | Crude oil (log diff.) | tpxav | $\ln 1 \mathrm{~d}$ | TOPIX (log diff.) |
| commed | $\ln 1 d$ | Domestic commodity index (log diff.) | tpxed | $\ln 1 \mathrm{~d}$ | TOPIX (log diff.) |
| rcommed | $\ln 1 d$ | Domestic commodity index (log diff.) | rtpxav | $\ln 1 d$ | TOPIX (log diff.) |
| reued | $\ln 1 \mathrm{~d}$ | Reuters index (log diff.) | rtpxed | $\ln 1 \mathrm{~d}$ | TOPIX (log diff.) |
| rreued | $\ln 1 d$ | Reuters index (log diff.) | tosho | ln | TSE amount traded (log) |
| lcreued | $\ln 12$ | Reuters index (log diff.) | tosho | $\ln 1 \mathrm{~d}$ | TSE amount traded (log diff.) |
| rlcreued | $\ln 1 d$ | Reuters index (log diff.) | land | $\ln 1 \mathrm{~d}$ | Land prices for commercial areas in 6 major cities (log diff.) |
|  |  |  | rland | $\ln 1 d$ | Land prices for commercial areas in 6 major cities (log diff.) |

Notes: The start of the data is 1970, except the following series: 1973 for the index of tertiary industries activities, 1971 for construction orders (kenjal, kenjmi, kenjhi), 1973 for the value of public works contracted (ukeall, ukekun, ukechi), 1971 for long-term government bonds (rjgb), 1973 for the yen/dollar rate and related indicators (rateav, rateed, ratest, ratesk, ratekr, rrateav, rrateed, efratav, efrated, refrated, refratav), and 1972 for some of the Nikkei average (nikst, niksk, nikkr).

## Chart 3: Performance of CPI forecasts



Notes: Each group of graphs of (horizontal axis and vertical axis) show the relative MSFE for (1983-86, 1987-90), (1987-90, 1991-94), and (1991-94, 1995-99/2Q).
If the relative MSFE is one or less, the performance is an improvement on the AR model, meaning that the information variables in the third quadrant of each graph show improved performance over the AR model for two consecutive sample periods. Additionally, a comparison of the graphs along the horizontal direction confirms how forecast performance changes between sample periods while the forecast horizon, $h$ in Equation (1), is constant. Also a comparison of the graphs along the vertical direction confirms how forecast performance changes between forecast horizons while the sample period is kept constant.

Chart 4: Performance of CPI forecasts

| $h=2$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Indicator | Trans. | 83-86 | 87-90 | 91-94 | 95-99 |
| AR | RMSFE | 1.38 | 0.91 | 0.72 | 0.55 |
|  |  |  |  |  |  |
| rgdp | $\ln 18$ | 2.03 | 1.63 | 0.68 | . 36 |
| rgdp | gap | 0.91 | 1.31 | 0.92 | 1.74 |
| ip | $\ln 1 d$ | 0.73 | 1.77 | 1.07 | 1.60 |
| ip | gap | 0.58 | 1.36 | 0.98 | 2.11 |
| sanji | $\ln 1 d^{\text {d }}$ | 1.49 | 2.51 | 0.92 | 0.97 |
| sanji | gap | 1.61 | 1.77 | 0.95 | 1.04 |
| сари | $\ln 1 d$ | 0.80 | 1.79 | 1.01 | 1.10 |
| сари | gap | 0.93 | 1.18 | 1.00 | 2.38 |
| tnksei | lev | 1.08 | 1.69 | 0.81 | 2.01 |
| tnkhi | lev | 0.70 | 1.00 | 0.72 | 1.06 |
| tnkzen | lev | 0.94 | 2.03 | 0.85 | 2.12 |
| unemp | lev | 4.16 | 1.38 | 1.25 | 3.59 |
| unemp | gap | 1.11 | 1.68 | 0.87 | 2.71 |
| kyujin | lev | 0.54 | 8.09 | 1.12 | 0.60 |
| kyujin | gap | 0.75 | 3.33 | 0.99 | 3.14 |
| kijmi | $\ln 18$ | 1.06 | 1.23 | 1.01 | 1.56 |
| kijmi | gap | 0.84 | 1.61 | 0.94 | 2.15 |
| kijse | $\ln 1 \mathrm{~d}$ | 0.72 | 1.35 | 0.97 | 1.32 |
| kijse | gap | 0.76 | 2.03 | 0.91 | 1.64 |
| kijhi | $\ln 1 \mathrm{~d}$ | 1.95 | 1.14 | 0.92 | 1.35 |
| kijhi | gap | 1.61 | 0.99 | 1.17 | 2.12 |
| kenjal | $\ln 1 d^{\prime}$ | 1.09 | 1.01 | 1.01 | 1.04 |
| kenjal | gap | 0.95 | 1.01 | 0.95 | 1.06 |
| kenjmi | $\ln 1 d^{\text {d }}$ | 0.78 | 1.95 | 1.04 | 1.14 |
| kenjmi | gap | 0.88 | 1.25 | 1.22 | 1.16 |
| kenjhi | $\ln 1 d^{1}$ | 0.87 | 1.54 | 1.09 | 1.16 |
| kenjhi | gap | 0.80 | 1.82 | 1.09 | 1.21 |
| ukeall | $\ln 18$ | 1.00 | 1.14 | 1.28 | 1.04 |
| ukeall | gap | 1.23 | 1.16 | 1.32 | 1.21 |
| ukekun | $\ln 1 d^{\text {d }}$ | 1.02 | 1.08 | 1.29 | 1.01 |
| ukekun | gap | 1.29 | 1.08 | 1.58 | 1.15 |
| ukechi | $\ln 1 \mathrm{~d}$ | 0.96 | 1.15 | 1.04 | 1.04 |
| ukechi | gap | 1.10 | 1.16 | 1.08 | 1.23 |
| juckko | $\ln 1 \mathrm{~d}$ | 1.24 | 0.97 | 1.11 | 1.41 |
| juckko | gap | 1.34 | 0.65 | 1.26 | 2.24 |
| juckme | $\ln 1 \mathrm{~d}$ | 1.54 | 0.96 | 0.94 | 1.69 |
| juckme | gap | 1.33 | 0.55 | 1.26 | 2.90 |
| ckhime | $\ln 1 \mathrm{~d}$ | 1.17 | 1.16 | 1.07 | 1.33 |
| ckhime | gap | 0.74 | 0.99 | 1.02 | 1.47 |
| car | $\ln 1 d^{1}$ | 0.98 | 1.01 | 0.99 | 0.65 |
| car | gap | 0.60 | 1.88 | 1.27 | 1.18 |
| kouri | $\ln 1 \mathrm{~d}$ | 0.92 | 1.10 | 0.87 | 1.23 |
| kouri | gap | 1.67 | 2.39 | 0.95 | 1.33 |
| hyaka | $\ln 1 \mathrm{~d}$ | 1.34 | 1.54 | 1.09 | 1.48 |
| hyaka | gap | 1.05 | 1.66 | 0.85 | 1.95 |
| lcexpt | $\ln 1 d^{\text {d }}$ | 0.67 | 0.77 | 1.46 | 1.62 |
| lcexpt | gap | 0.64 | 2.11 | 1.17 | 3.01 |
| lcimpt | $\ln 1 \mathrm{~d}$ | 0.95 | 2.47 | 1.09 | 0.95 |
| lcimpt | gap | 0.75 | 4.09 | 0.86 | 2.85 |
| expt | $\ln 1 d$ | 0.88 | 1.89 | 0.95 | 1.75 |
| expt | gap | 0.89 | 0.88 | 1.15 | 2.30 |
| impt | $\ln 1 \mathrm{~d}$ | 1.10 | 3.96 | 0.95 | 1.38 |
| impt | gap | 1.26 | 3.20 | 0.91 | 1.87 |
| $n \mathrm{n}$ dp | $\ln 1 d$ | 0.87 | 0.93 | 0.56 | . 04 |
| $p g d p$ | $\ln 1 d$ | 1.32 | 1.12 | 0.74 |  |
| wpi | $\ln 1 \mathrm{~d}$ | 0.44 | 2.28 | 0.82 | 1.45 |
| wpiinav | $\ln 1 d$ | 0.54 | 2.01 | 1.20 | 1.57 |
| wpiined | $\ln 18$ | 0.30 | 1.94 | 1.15 | 1.18 |
| ipiav | lnld | 0.82 | 1.45 | 1.02 | 1.04 |
| ipisoav | $\ln 1 \mathrm{~d}$ | 1.30 | 1.73 | 1.02 | 1.03 |
| ipisoed | ln1d | 1.37 | 1.73 | 1.08 | 1.03 |
| wage | $\ln 1 \mathrm{~d}$ | 1.01 | 0.80 | 0.70 | 1.77 |
| rwage | $\ln 18$ | 1.13 | 0.89 | 0.67 | 1.11 |
| oil | $\ln 1 \mathrm{~d}$ | 1.32 | 2.64 | 1.43 | 1.02 |
| roil | $\ln 1 \mathrm{~d}$ | 1.39 | 2.71 | 1.43 | 1.02 |
| lcoil | $\ln 1 \mathrm{~d}$ | 1.16 | 1.76 | 1.36 | 1.12 |
| rlcoil | $\ln 1 \mathrm{~d}$ | 1.18 | 1.70 | 1.36 | 1.12 |
| commed | $\ln 1 \mathrm{~d}$ | 0.77 | 3.74 | 0.92 | 1.02 |
| rcommed | $\ln 1 \mathrm{~d}$ | 0.77 | 3.69 | 0.93 | 1.03 |
| reued | $\ln 1 \mathrm{~d}$ | 1.64 | 1.49 | 0.96 | 1.21 |
| rreued | $\ln 1 \mathrm{~d}$ | 1.93 | 1.32 | 0.95 | 1.22 |
| lcreued | $\ln 1 \mathrm{~d}$ | 0.99 | 1.97 | 1.31 | 0.89 |
| rlcreued | $\ln 1 \mathrm{~d}$ | 0.98 | 2.02 | 1.29 | 0.89 |
| crbed | $\ln 1 \mathrm{~d}$ | 1.87 | 2.19 | 1.02 | 1.44 |
| rcrbed | $\ln 1 d$ | 2.48 | 2.13 | 1.03 | 1.20 |
| lccrbed | $\ln 1 \mathrm{~d}$ | 1.22 | 1.70 | 1.41 | 1.67 |
| rlccrbed | $\ln 1 \mathrm{~d}$ | 1.26 | 1.68 | 1.41 | 1.69 |
| glded | $\ln 1 \mathrm{~d}$ | 1.78 | 0.99 | 0.94 | 1.80 |
| rglded | $\ln 1 \mathrm{~d}$ | 1.81 | 0.99 | 0.92 | 1.78 |
| lcglded | $\ln 1 \mathrm{~d}$ | 1.52 | 1.24 | 1.18 | 1.05 |
| rlcglded | lnld | 1.54 | 1.25 | 1.20 | 1.06 |
| mon0 | $\ln 1 \mathrm{~d}$ | 0.94 | 3.23 | . 00 | . 30 |
| mon0 | gap | 3.32 | 0.94 | 1.74 | 1.57 |
| rmon0 | $\ln 1 \mathrm{~d}$ | 1.06 | 2.56 | 0.99 | 1.40 |
| rmon0 | gap | 1.19 | 1.65 | 2.80 | 1.32 |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| 83-86 | 87-90 | 91-94 | 95-99 |
| 1.66 | 0.76 | 0.69 | 0.59 |
| 2.02 | 2.56 | 0.48 | 0.79 |
| 1.15 | 1.78 | 0.63 | 1.23 |
| 0.58 | 2.27 | 0.92 | 1.38 |
| 0.59 | 1.52 | 0.77 | 2.38 |
| 1.12 | 12.71 | 0.93 | 0.95 |
| 1.52 | 4.73 | 1.07 | 1.12 |
| 0.71 | 2.22 | 1.04 | 1.51 |
| 1.12 | 1.29 | 0.96 | 2.42 |
| 1.10 | 2.93 | 0.31 | 1.37 |
| 0.59 | 1.91 | 0.30 | 0.71 |
| 0.95 | 3.52 | 0.34 | 1.24 |
| 4.49 | 2.14 | 1.41 | 4.70 |
| 1.40 | 2.15 | 0.45 | 2.89 |
| 0.44 | 28.76 | 0.82 | 0.18 |
| 0.39 | 4.97 | 0.46 | 3.38 |
| 1.07 | 1.44 | 0.88 | 1.52 |
| 0.99 | 2.05 | 0.83 | 2.55 |
| 0.89 | 1.30 | 0.87 | 1.94 |
| 0.78 | 2.58 | 0.93 | 2.38 |
| 1.60 | 1.13 | 1.02 | 0.97 |
| 2.60 | 1.08 | 1.53 | 2.35 |
| 0.97 | 1.09 | 0.92 | 0.63 |
| 0.94 | 1.07 | 0.78 | 1.04 |
| 0.90 | 1.33 | 0.98 | 0.57 |
| 0.73 | 1.42 | 1.09 | 1.26 |
| 0.78 | 1.37 | 0.88 | 0.78 |
| 0.64 | 1.28 | 1.58 | 1.26 |
| 1.00 | 1.11 | 1.21 | 1.17 |
| 0.96 | 1.70 | 1.33 | 1.91 |
| 1.00 | 1.11 | 1.18 | 1.01 |
| 1.67 | 1.08 | 1.63 | 1.41 |
| 0.99 | 1.06 | 1.06 | 1.12 |
| 0.83 | 1.87 | 1.18 | 1.50 |
| 1.08 | 1.06 | 1.06 | 1.76 |
| 1.21 | 0.72 | 1.75 | 2.90 |
| 1.27 | 1.12 | 0.81 | 2.00 |
| 1.14 | 0.73 | 1.81 | 3.63 |
| 1.03 | 1.21 | 1.40 | 1.12 |
| 0.61 | 0.91 | 0.77 | 2.43 |
| 1.37 | 1.18 | 0.68 | 0.65 |
| 0.41 | 5.04 | 1.25 | 0.91 |
| 0.78 | 1.09 | 1.42 | 0.52 |
| 1.80 | 6.92 | 0.49 | 0.98 |
| 1.21 | 1.55 | 0.85 | 1.55 |
| 1.35 | 3.84 | 0.57 | 2.35 |
| 0.66 | 1.95 | 1.27 | 1.60 |
| 0.69 | 5.84 | 1.28 | 3.46 |
| 0.88 | 2.49 | 1.43 | 1.12 |
| 0.86 | 6.59 | 1.01 | 2.86 |
| 1.00 | 0.99 | 0.98 | 1.51 |
| 0.74 | 0.64 | 1.31 | 2.34 |
| 1.08 | 7.55 | 0.97 | 1.43 |
| 1.33 | 5.36 | 0.75 | 2.07 |
| 0.94 | 0.79 | 0.38 | 0.73 |
| 1.35 | 1.74 | 0.65 | 1.61 |
| 0.42 | 2.58 | 1.16 | 1.76 |
| 0.40 | 2.41 | 1.01 | 2.04 |
| 0.26 | 2.07 | 1.04 | 2.02 |
| 0.89 | 1.50 | 1.21 | 0.97 |
| 1.40 | 2.07 | 1.20 | 1.03 |
| 1.59 | 4.41 | 1.19 | 0.94 |
| 1.03 | 0.40 | 0.61 | 1.85 |
| 1.08 | 1.49 | 0.83 | 1.07 |
| 0.90 | 4.31 | 1.22 | 1.01 |
| 1.00 | 4.50 | 1.23 | 1.02 |
| 1.64 | 2.12 | 1.26 | 1.00 |
| 1.61 | 2.06 | 1.26 | 1.00 |
| 0.79 | 3.54 | 1.01 | 1.11 |
| 0.80 | 3.49 | 1.02 | 1.14 |
| 1.69 | 2.93 | 0.98 | 1.33 |
| 1.97 | 2.60 | 0.96 | 1.42 |
| 1.24 | 2.57 | 1.36 | 0.87 |
| 1.19 | 2.58 | 1.35 | 0.86 |
| 2.35 | 2.72 | 1.20 | 0.96 |
| 3.07 | 2.48 | 1.29 | 1.17 |
| 1.39 | 1.70 | 1.49 | 1.22 |
| 1.37 | 1.72 | 1.50 | 1.25 |
| 1.14 | 1.58 | 0.97 | 1.97 |
| 1.22 | 1.56 | 1.01 | 1.97 |
| 0.77 | 1.58 | 1.16 | 0.96 |
| 0.78 | 1.52 | 1.12 | 0.95 |
| 1.11 | 7.47 | 0.91 | 1.36 |
| 3.45 | 2.70 | 1.43 | 1.54 |
| 1.09 | 6.43 | 0.92 | 1.33 |
| 1.19 | 5.32 | 3.13 | 1.31 |


| $h=8$ |  |  |  |
| :---: | :---: | :---: | :---: |
| 83-86 | 87-90 | 91-94 | 95-99 |
| 2.98 | 0.62 | 0.88 | 0.77 |
|  |  |  |  |
| 0.55 | 4.10 | 0.52 | 0.29 |
| 0.59 | 2.57 | 0.42 | 1.00 |
| 0.71 | 2.50 | 0.89 | 0.93 |
| 0.69 | 2.48 | 0.84 | 2.03 |
| 2.58 | 97.55 | 0.99 | 0.92 |
| 2.68 | 29.89 | 1.28 | 1.00 |
| 0.87 | 2.57 | 1.04 | 1.33 |
| 1.44 | 1.86 | 0.90 | 2.05 |
| 0.94 | 5.33 | 0.38 | 0.83 |
| 0.82 | 20.39 | 0.24 | 0.42 |
| 0.89 | 8.12 | 0.41 | 0.65 |
| 1.12 | 3.01 | 2.40 | 1.62 |
| 1.72 | 1.31 | 0.45 | 1.63 |
| 0.51 | 92.43 | 0.68 | 0.41 |
| 0.36 | 6.44 | 0.31 | 2.50 |
| 1.08 | 1.23 | 0.85 | 0.83 |
| 1.16 | 1.67 | 0.70 | 2.01 |
| 0.95 | 2.38 | 0.87 | 1 |
| 1.01 | 2.81 | 0.66 | 2.20 |
| 1.37 | 1.41 | 0.97 | 0.72 |
| 1.93 | 1.21 | 1.55 | 1.71 |
| 1.27 | 1.03 | 0.91 | 0.43 |
| 1.44 | 1.08 | 0.79 | 1.14 |
| 1.18 | 0.89 | 0.88 | 0.55 |
| 1.07 | 0.99 | 1.06 | 1.48 |
| 1.15 | 1.20 | 0.92 | 0.56 |
| 0.95 | 0.99 | 1.73 | 1.32 |
| 0.99 | 1.87 | 1.06 | 1.00 |
| 2.07 | 6.75 | 1.10 | 1.02 |
| 1.00 | 1.13 | 1.07 | 0.99 |
| 3.00 | 6.16 | 1.23 | 1.20 |
| 1.01 | 1.79 | 1.01 | 1.01 |
| 1.54 | 5.94 | 1.03 | 0.94 |
| 0.68 | 1.39 | 1.21 | 2.32 |
| 1.06 | 0.95 | 1.93 | 3.53 |
| 0.49 | 1.85 | 0.93 | 2.60 |
| 0.85 | 1.54 | 1.31 | 4.63 |
| 0.64 | 0.82 | 0.54 | 0.92 |
| 0.72 | 0.96 | 0.63 | 2.21 |
| 0.80 | 8.89 | 0.44 | 0.74 |
| 0.46 | 13.18 | 0.76 | 1.45 |
| 0.47 | 4.05 | 0.34 | 0.45 |
| 1.78 | 27.46 | 0.59 | 1.76 |
| 0.53 | 3.81 | 0.32 | 1.11 |
| 1.68 | 9.76 | 0.38 | 2.68 |
| 0.92 | 5.16 | 1.03 | 2.54 |
| 1.01 | 15.65 | 1.06 | 4.78 |
| 1.06 | 2.58 | 1.28 | 2.12 |
| 1.08 | 15.46 | 0.93 | 3.05 |
| 1.00 | 0.97 | 1.05 | 1.10 |
| 1.13 | 0.67 | 0.98 | 2.08 |
| 0.98 | 7.90 | 1.27 | 0.75 |
| 1.17 | 11.11 | 0.98 | 1.23 |
| 0.48 | 1.04 | 0.43 | 0.19 |
| 1.11 | 0.95 | 0.89 | 1.12 |
| 1.03 | 4.20 | 0.97 | 1.53 |
| 0.95 | 3.66 | 0.99 | 1.95 |
| 0.66 | 2.96 | 1.02 | 1.90 |
| 0.90 | 1.73 | 1.27 | 1.88 |
| 0.97 | 4.85 | 1.28 | 1.51 |
| 1.04 | 1.97 | 1.40 | 1.28 |
| 0.71 | 0.41 | 0.29 | 1.12 |
| 0.67 | 1.87 | 0.99 | 0.97 |
| 0.86 | 3.02 | 1.04 | 1.00 |
| 0.83 | 3.17 | 1.04 | 1.00 |
| 0.92 | 2.20 | 1.01 | 1.07 |
| 0.91 | 2.22 | 1.01 | 1.07 |
| 0.57 | 2.61 | 0.72 | 1.52 |
| 0.57 | 2.68 | 0.77 | 1.52 |
| 0.97 | 4.52 | 1.03 | 1.11 |
| 1.06 | 3.89 | 1.02 | 1.12 |
| 0.92 | 1.45 | 1.44 | 1.55 |
| 0.92 | 1.65 | 1.43 | 1.55 |
| 1.26 | 2.57 | 1.21 | 1.00 |
| 1.52 | 2.44 | 1.24 | 1.04 |
| 1.06 | 2.53 | 1.46 | 1.89 |
| 1.00 | 2.75 | 1.28 | 1.84 |
| 0.74 | 3.26 | 1.01 | 1.94 |
| 0.81 | 3.14 | 1.00 | 2.00 |
| 0.89 | 2.31 | 1.11 | 0.97 |
| 0.89 | 2.35 | 1.12 | 0.99 |
| 0.63 | 14.04 | 0.47 | 0.68 |
| 1.85 | 15.19 | 0.68 | 1.57 |
| 0.63 | 11.91 | 0.86 | 0.57 |
| 0.54 | 10.72 | 1.79 | 2.10 |

(Chart 4: Continued)

| monl | ln1d | 0.81 | 1.05 | 1.19 | 1.03 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| monl | gap | 3.66 | 0.47 | 1.46 | 1.14 |
| rmonl | ln1d | 0.77 | 1.09 | 1.21 | 1.03 |
| rmonl | gap | 1.08 | 1.38 | 1.62 | 1.37 |
| mon2 | $\ln 1 d^{\text {d }}$ | 1.12 | 1.52 | 1.02 | 1.09 |
| mon2 | gap | 2.32 | 1.65 | 1.26 | 1.34 |
| rmon2 | $\ln 1 d^{\text {d }}$ | 1.14 | 1.17 | 0.90 | 0.84 |
| rmon2 | gap | 0.71 | 2.63 | 2.33 | 1.15 |
| mon4 | $\ln 1 d^{\text {d }}$ | 0.59 | 0.80 | 1.50 | 57.19 |
| mon4 | gap | 2.15 | 1.52 | 1.02 | 58.63 |
| rmon4 | $\ln 1 d^{\text {d }}$ | 0.74 | 1.34 | 1.19 | 50.90 |
| rmon4 | gap | 0.42 | 1.26 | 2.01 | 43.98 |
| lended | $\ln 1 d^{\text {d }}$ | 1.54 | 1.48 | 1.01 | 0.92 |
| lended | gap | 2.87 | 2.11 | 1.37 | 1.28 |
| rlended | $\ln 1 d^{\text {d }}$ | 1.35 | 1.68 | 0.97 | 0.83 |
| rlended | gap | 1.15 | 2.43 | 2.76 | 1.08 |
| $m \mathrm{lp}$ | lev | 1.19 | 4.65 | 1.75 | 1.05 |
| $m l p$ | gap | 0.77 | 1.00 | 1.89 | 1.59 |
| velo | $\ln 1 d^{\text {d }}$ | 0.88 | 1.05 | 1.05 | 1.25 |
| velo | gap | 0.89 | 1.72 | 1.29 | 1.22 |
| note | $\ln 1 d^{\text {d }}$ | 0.93 | 1.53 | 1.17 | 1.47 |
| note | gap | 2.61 | 1.29 | 2.01 | 1.58 |
| rnote | $\ln 1 d^{\text {d }}$ | 0.96 | 1.06 | 1.00 | 1.55 |
| rnote | gap | 1.05 | 1.46 | 2.55 | 1.31 |
| ighed | lev | 0.33 | 2.27 | 1.28 | 0.92 |
| rjghed | lev | 0.52 | 1.20 | 1.08 | 0.83 |
| sprded | lev | 0.45 | 1.15 | 1.26 | 1.40 |
| alndav | lev | 1.00 | 2.27 | 1.14 | 0.96 |
| ralndav | lev | 0.96 | 0.95 | 1.24 | 1.55 |
| rateav | $\ln 1 d^{\prime}$ | 1.00 | 1.45 | 1.06 | 2.21 |
| rateav | gap | 1.11 | 1.74 | 1.02 | 2.86 |
| rateed | ln1d | 1.17 | 1.94 | 1.08 | 2.24 |
| rateed | gap | 1.20 | 1.90 | 1.04 | 2.76 |
| ratest | lev | 1.31 | 0.81 | 1.09 | 1.53 |
| ratesk | lev | 1.03 | 1.03 | 1.00 | 0.94 |
| ratekr | lev | 2.37 | 1.01 | 1.04 | 1.20 |
| rrateav | $\ln 1 d^{\text {d }}$ | 1.19 | 1.34 | 1.05 | 2.26 |
| rrateav | gap | 1.01 | 1.79 | 0.96 | 2.93 |
| rrateed | $\ln 1 d^{\text {d }}$ | 0.97 | 1.46 | 1.02 | 2.41 |
| rrateed | gap | 0.94 | 1.81 | 0.95 | 3.11 |
| efratav | $\ln 1 d^{\text {d }}$ | 0.94 | 1.49 | 0.94 | 1.94 |
| efratav | gap | 0.82 | 1.57 | 0.82 | 2.34 |
| efrated | $\ln 1 d^{\text {d }}$ | 1.06 | 1.67 | 0.92 | 1.90 |
| efrated | gap | 0.99 | 2.07 | 0.79 | 2.53 |
| refrated | $\ln 1 d^{\text {d }}$ | 0.94 | 1.41 | 0.99 | 1.81 |
| refrated | gap | 0.93 | 1.75 | 0.85 | 2.17 |
| refratav | $\ln 1 d^{\prime}$ | 1.15 | 1.48 | 0.90 | 1.64 |
| refratav | gap | 1.02 | 1.83 | 0.82 | 2.23 |
| nikav | $\ln 1 d^{\prime}$ | 1.54 | 1.11 | 1.26 | 1.39 |
| niked | $\ln 1 d^{\text {d }}$ | 1.07 | 1.79 | 1.22 | 1.36 |
| nikst | lev | 1.01 | 1.27 | 1.02 | 1.23 |
| niksk | lev | 1.01 | 2.58 | 1.17 | 1.15 |
| nikkr | lev | 1.07 | 1.32 | 1.05 | 1.04 |
| rnikav | $\ln 1 d^{\text {d }}$ | 1.67 | 1.11 | 1.25 | 1.39 |
| rniked | $\ln 1 \mathrm{~d}$ | 1.03 | 1.85 | 1.22 | 1.36 |
| tpxav | $\ln 1 \mathrm{~d}$ | 1.18 | 1.37 | 1.15 | 1.32 |
| tpxed | $\ln 1 \mathrm{~d}$ | 0.82 | 1.71 | 0.97 | 1.36 |
| rtpxav | $\ln 1 \mathrm{~d}$ | 1.23 | 1.38 | 1.15 | 1.32 |
| rtpxed | $\ln 1 d^{\text {d }}$ | 0.82 | 1.77 | 0.97 | 1.36 |
| tosho | ln | 1.54 | 1.03 | 1.00 | 0.94 |
| tosho | $\ln 1 d$ | 1.45 | 0.97 | 1.08 | 0.98 |
| land | $\ln 1 \mathrm{~d}$ | 1.41 | 0.79 | 2.11 | 1.26 |
| rland | ln1d | 1.48 | 0.75 | 2.13 | 1.24 |


|  |  |  |  |
| ---: | ---: | ---: | ---: |
| 0.84 | 3.13 | 1.64 | 1.03 |
| 2.99 | 0.50 | 2.00 | 1.74 |
| 0.78 | 3.74 | 1.54 | 1.02 |
| 0.95 | 9.13 | 2.03 | 2.35 |
| 1.09 | 3.58 | 0.77 | 0.76 |
| 2.21 | 5.41 | 0.93 | 1.49 |
| 1.01 | 3.45 | 0.70 | 0.71 |
| 0.63 | 5.31 | 3.70 | 1.05 |
| 0.59 | 0.92 | 3.60 | 54.57 |
| 2.03 | 5.07 | 0.82 | 92.11 |
| 0.48 | 1.67 | 1.67 | 47.13 |
| 0.42 | 1.52 | 2.78 | 80.89 |
| 1.99 | 1.05 | 0.67 | 0.58 |
| 3.48 | 3.42 | 1.12 | 1.50 |
| 1.83 | 1.19 | 0.60 | 0.53 |
| 0.54 | 8.20 | 5.21 | 1.07 |
| 1.95 | 15.43 | 1.51 | 1.42 |
| 0.75 | 0.67 | 2.13 | 1.81 |
| 0.70 | 1.07 | 1.12 | 1.68 |
| 1.18 | 2.98 | 0.94 | 1.49 |
| 1.00 | 3.96 | 0.95 | 1.42 |
| 3.71 | 2.47 | 1.17 | 1.76 |
| 0.89 | 3.75 | 0.95 | 1.18 |
| 1.11 | 3.72 | 2.59 | 1.39 |
| 0.36 | 1.72 | 1.26 | 0.64 |
| 0.32 | 1.32 | 1.04 | 0.61 |
| 0.30 | 1.26 | 1.83 | 1.42 |
| 1.06 | 4.06 | 0.68 | 0.72 |
| 1.15 | 3.69 | 1.88 | 1.31 |
| 0.61 | 1.13 | 1.00 | 2.02 |
| 1.08 | 4.38 | 1.15 | 3.81 |
| 0.35 | 1.62 | 1.10 | 2.43 |
| 0.67 | 2.77 | 1.13 | 3.90 |
| 1.00 | 0.81 | 1.02 | 1.22 |
| 0.60 | 2.93 | 1.07 | 0.90 |
| 1.36 | 0.96 | 1.13 | 1.12 |
| 0.82 | 0.99 | 0.97 | 2.16 |
| 0.81 | 3.48 | 1.03 | 3.86 |
| 0.61 | 0.99 | 0.97 | 2.60 |
| 0.64 | 3.01 | 1.04 | 4.01 |
| 0.72 | 1.26 | 0.99 | 1.71 |
| 0.88 | 2.76 | 0.53 | 4.12 |
| 0.70 | 1.40 | 1.06 | 1.95 |
| 0.80 | 2.43 | 0.72 | 3.12 |
| 0.67 | 1.17 | 1.03 | 1.49 |
| 0.83 | 1.84 | 0.80 | 3.18 |
| 0.71 | 1.26 | 1.12 | 1.56 |
| 0.75 | 1.74 | 0.92 | 2.95 |
| 1.67 | 0.95 | 1.22 | 0.70 |
| 1.56 | 1.16 | 1.21 | 1.12 |
| 1.14 | 13.98 | 0.99 | 0.70 |
| 0.98 | 2.89 | 2.55 | 1.00 |
| 0.96 | 1.25 | 1.04 | 1.00 |
| 1.63 | 0.95 | 1.22 | 0.81 |
| 1.42 | 1.20 | 1.21 | 1.12 |
| 1.49 | 1.06 | 1.15 | 0.98 |
| 1.26 | 1.36 | 1.15 | 1.16 |
| 1.45 | 1.06 | 1.15 | 0.98 |
| 1.29 | 1.39 | 1.14 | 1.17 |
| 1.05 | 1.15 | 0.46 | 1.38 |
| 1.29 | 1.04 | 1.21 | 1.26 |
| 2.44 | 3.27 | 2.96 | 1.05 |
| 2.17 | 1.01 | 3.18 | 1.02 |
|  |  |  |  |


| 0.39 | 8.95 | 1.13 | 0.96 |
| ---: | ---: | ---: | ---: |
| 1.94 | 5.44 | 1.32 | 2.39 |
| 0.40 | 9.91 | 1.12 | 0.96 |
| 0.68 | 27.32 | 1.71 | 2.43 |
| 0.22 | 9.55 | 0.81 | 0.42 |
| 1.23 | 26.86 | 0.79 | 1.77 |
| 0.22 | 7.51 | 0.75 | 0.36 |
| 0.09 | 10.28 | 4.24 | 1.50 |
| 0.16 | 1.77 | 2.77 | 33.32 |
| 1.38 | 28.56 | 0.96 | 52.17 |
| 0.22 | 3.42 | 1.54 | 33.41 |
| 0.09 | 2.76 | 2.68 | 45.50 |
| 0.99 | 1.68 | 0.48 | 0.16 |
| 2.33 | 5.28 | 2.65 | 1.41 |
| 0.87 | 1.67 | 0.64 | 0.13 |
| 0.17 | 22.08 | 7.48 | 1.03 |
| 1.14 | 44.30 | 2.83 | 0.84 |
| 0.52 | 19.87 | 0.87 | 1.71 |
| 0.42 | 1.72 | 1.09 | 1.14 |
| 0.43 | 6.87 | 0.89 | 1.73 |
| 0.57 | 8.93 | 0.45 | 1.04 |
| 1.79 | 16.50 | 0.50 | 1.90 |
| 0.53 | 6.80 | 0.42 | 0.79 |
| 0.64 | 7.33 | 2.08 | 1.83 |
| 0.87 | 9.92 | 0.93 | 0.22 |
| 0.81 | 2.64 | 0.85 | 0.64 |
| 0.43 | 1.61 | 1.97 | 1.35 |
| 1.30 | 10.81 | 0.87 | 1.72 |
| 1.23 | 8.21 | 2.21 | 1.34 |
| 1.22 | 2.39 | 1.13 | 2.11 |
| 1.96 | 7.53 | 1.20 | 4.37 |
| 0.60 | 3.04 | 1.11 | 2.57 |
| 1.83 | 7.19 | 1.18 | 4.27 |
| 0.87 | 1.14 | 0.92 | 1.51 |
| 1.33 | 2.75 | 1.01 | 0.98 |
| 2.10 | 0.98 | 1.21 | 1.05 |
| 1.00 | 2.73 | 1.06 | 2.05 |
| 1.42 | 6.54 | 1.05 | 4.41 |
| 0.86 | 2.57 | 1.06 | 2.36 |
| 1.48 | 5.75 | 1.00 | 4.41 |
| 0.62 | 1.92 | 0.94 | 2.70 |
| 1.20 | 6.62 | 0.58 | 5.30 |
| 0.53 | 2.01 | 0.98 | 2.76 |
| 1.11 | 5.72 | 0.60 | 4.72 |
| 0.53 | 0.91 | 1.04 | 2.46 |
| 1.41 | 3.71 | 0.70 | 4.09 |
| 0.46 | 0.95 | 1.09 | 2.66 |
| 1.07 | 3.67 | 0.70 | 4.41 |
| 0.98 | 1.12 | 1.23 | 0.95 |
| 1.18 | 1.50 | 1.11 | 1.09 |
| 1.12 | 28.38 | 1.03 | 0.56 |
| 1.01 | 3.74 | 2.94 | 0.91 |
| 0.95 | 1.13 | 1.01 | 0.92 |
| 0.94 | 1.13 | 1.24 | 0.95 |
| 1.17 | 1.54 | 1.12 | 1.09 |
| 1.03 | 1.10 | 1.23 | 1.11 |
| 1.20 | 1.56 | 1.10 | 1.12 |
| 1.05 | 1.11 | 1.72 | 1.12 |
| 1.19 | 1.62 | 1.10 | 1.13 |
| 0.84 | 3.21 | 0.35 | 0.69 |
| 0.97 | 1.19 | 1.15 | 1.33 |
| 1.19 | 56.48 | 2.57 | 0.54 |
| 1.04 | 61.47 | 3.38 | 0.52 |
|  |  |  |  |
|  |  |  |  |

Notes: The value in the first row is the root mean squared forecast error (RMSFE). The second row and onwards are the relative MSFE. The shaded sections are for a relative MSFE of less than one. Note that the forecast data for 1983-85 is missing for series regarding economic activity levels and asset values that start between 1971 and 1973, so 1983-86 uses a mean that does not include the data for this period.

## Chart 5: Performance of real GDP forecasts


$h=4$



$h=8$




Notes: Each group of graphs of (horizontal axis and vertical axis) show the relative MSFE for (1983-86, 1987-90), (1987-90, 1991-94), and (1991-94, 1995-99/2Q).
If the relative MSFE is one or less, the performance is an improvement on the AR model, meaning that the information variables in the third quadrant of each graph show improved performance over the AR model for two consecutive sample periods. Additionally, a comparison of the graphs along the horizontal direction confirms how forecast performance changes between sample periods while the forecast horizon, $h$ in Equation (1), is constant. Also a comparison of the graphs along the vertical direction confirms how forecast performance changes between forecast horizons while the sample period is kept constant. .

Chart 6: Performance of real GDP forecasts

| $h=2$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Indicator | Trans. | 83-86 | 87-90 | 91-94 | 95-99 |
| AR | RMSFE | 1.06 | 2.39 | 1.81 | 2.65 |
| ip | $\ln 1 d$ | 0.97 | 1.02 | 1.22 | 0.94 |
| $i p$ | gap | 1.00 | 1.09 | 1.32 | 1.06 |
| sanji | $\ln 1 \mathrm{~d}$ | 1.53 | 1.26 | 1.10 | 0.99 |
| sanji | gap | 1.52 | 2.53 | 1.13 | 0.98 |
| сари | $\ln 1 d^{\text {d }}$ | 1.00 | 0.97 | 1.05 | 0.99 |
| сари | gap | 1.09 | 1.06 | 1.26 | 1.13 |
| tnksei | lev | 1.22 | 0.84 | 1.32 | 0.72 |
| tnkhi | lev | 1.01 | 1.06 | 1.75 | 0.91 |
| tnkzen | lev | 1.15 | 0.82 | 1.47 | 0.70 |
| unemp | lev | 1.98 | 0.88 | 1.34 | 1.84 |
| ипетр | gap | 1.02 | 1.07 | 1.25 | 1.29 |
| kyujin | lev | 0.83 | 0.88 | 1.30 | 0.62 |
| kyujin | gap | 1.08 | 0.78 | 1.03 | 1.60 |
| kijmi | $\ln 1 \mathrm{~d}$ | 1.10 | 0.75 | 0.98 | 0.74 |
| kijmi | gap | 1.90 | 0.86 | 1.36 | 1.16 |
| kijse | $\ln 1 \mathrm{~d}$ | 1.27 | 0.94 | 1.05 | 1.07 |
| kijse | gap | 1.58 | 0.88 | 1.22 | 1.13 |
| kijhi | $\ln 1 d^{1}$ | 1.06 | 0.50 | 0.74 | 0.85 |
| kijhi | gap | 2.15 | 0.62 | 0.78 | 1.37 |
| kenjal | $\ln 1 \mathrm{~d}$ | 1.06 | 1.02 | 0.99 | 0.85 |
| kenjal | gap | 1.06 | 1.00 | 0.98 | 0.95 |
| kenjmi | $\ln 1 \mathrm{~d}$ | 1.21 | 1.10 | 1.01 | 1.08 |
| kenjmi | gap | 1.13 | 1.03 | 1.05 | 1.46 |
| kenjhi | $\ln 1 \mathrm{~d}$ | 1.50 | 1.20 | 1.01 | 0.97 |
| kenjhi | gap | 1.36 | 1.13 | 0.89 | 1.40 |
| ukeall | $\ln 1 \mathrm{~d}$ | 1.50 | 1.04 | 1.04 | 1.03 |
| ukeall | gap | 1.46 | 0.75 | 1.86 | 1.05 |
| ukekun | $\ln 1 \mathrm{~d}$ | 1.11 | 1.07 | 1.09 | 0.95 |
| ukekun | gap | 1.32 | 0.74 | 1.74 | 0.95 |
| ukechi | $\ln 1 \mathrm{~d}$ | 2.03 | 1.01 | 0.97 | 1.02 |
| ukechi | gap | 1.65 | 0.78 | 1.56 | 1.07 |
| juckko | $\ln 1 \mathrm{~d}$ | 1.30 | 1.09 | 0.79 | 1.08 |
| juckko | gap | 1.42 | 0.99 | 0.81 | 1.07 |
| juckme | $\ln 1 d^{\prime}$ | 1.17 | 1.12 | 0.97 | 1.09 |
| juckme | gap | 1.40 | 0.87 | 0.93 | 1.03 |
| ckhime | $\ln 1 \mathrm{~d}$ | 1.18 | 1.01 | 1.30 | 1.16 |
| ckhime | gap | 1.06 | 1.01 | 0.98 | 1.32 |
| car | $\ln 1 \mathrm{~d}$ | 1.82 | 1.03 | 0.98 | 0.96 |
| car | gap | 1.27 | 0.94 | 0.98 | 0.94 |
| kouri | $\ln 1 \mathrm{~d}$ | 1.67 | 1.02 | 1.46 | 0.87 |
| kouri | gap | 1.64 | 1.41 | 2.12 | 1.08 |
| hyaka | ln1d | 1.39 | 1.05 | 1.64 | 1.05 |
| hyaka | gap | 1.13 | 1.48 | 1.42 | 1.67 |
| lcexpt | $\ln 1 \mathrm{~d}$ | 2.35 | 1.04 | 1.01 | 0.99 |
| lcexpt | gap | 2.05 | 1.18 | 0.95 | 1.11 |
| lcimpt | $\ln 1 \mathrm{~d}$ | 1.77 | 1.10 | 1.10 | 1.30 |
| lcimpt | gap | 2.07 | 1.32 | 1.07 | 0.80 |
| expt | $\ln 1 \mathrm{~d}$ | 1.20 | 0.94 | 1.05 | 1.07 |
| expt | gap | 1.59 | 1.05 | 1.08 | 1.08 |
| impt | $\ln 1 d^{1}$ | 1.19 | 1.08 | 2.05 | 0.88 |
| impt | gap | 1.56 | 1.01 | 2.00 | 0.63 |
| $n \mathrm{n}$ dp | $\ln 1 \mathrm{~d}$ | 1.13 | 1.39 | 1.52 | 1.11 |
| $p g d p$ | $\ln 1 \mathrm{~d}$ | 1.23 | 1.27 | 1.47 | 1.04 |
| cpi | $\ln 1 \mathrm{~d}$ | 1.23 | 0.97 | 1.47 | 1.19 |
| wpi | $\ln 1 \mathrm{~d}$ | 1.51 | 1.01 | 0.84 | 1.11 |
| wpiinav | lnld | 1.82 | 1.27 | 0.97 | 1.20 |
| wpiined | $\ln 1 \mathrm{~d}$ | 2.05 | 1.20 | 0.96 | 1.17 |
| ipiav | $\ln 1 \mathrm{~d}$ | 3.18 | 1.24 | 1.05 | 1.10 |
| ipisoav | $\ln 1 \mathrm{~d}$ | 4.48 | 1.23 | 1.03 | 1.11 |
| ipisoed | $\ln 1 \mathrm{~d}$ | 4.23 | 1.27 | 1.01 | 1.11 |
| wage | $\ln 1 \mathrm{~d}$ | 1.37 | 1.08 | 1.49 | 1.07 |
| rwage | $\ln 1 d$ | 1.12 | 1.01 | 0.98 | 0.92 |
| oil | $\ln 1 \mathrm{~d}$ | 5.37 | 1.16 | 1.02 | 1.04 |
| roil | $\ln 1 \mathrm{~d}$ | 5.03 | 1.16 | 1.02 | 1.04 |
| lcoil | $\ln 1 d^{1}$ | 5.19 | 1.26 | 1.00 | 1.02 |
| rlcoil | $\ln 1 \mathrm{~d}$ | 4.91 | 1.27 | 1.00 | 1.02 |
| commed | lnld | 1.44 | 1.02 | 1.00 | 1.11 |
| rcommed | ln1d | 1.47 | 0.95 | 1.08 | 1.05 |
| reued | $\ln 1 \mathrm{~d}$ | 1.85 | 1.03 | 1.00 | 0.94 |
| rreued | $\ln 1 \mathrm{~d}$ | 1.90 | 1.02 | 1.00 | 0.94 |
| lcreued | $\ln 1 \mathrm{~d}$ | 1.61 | 1.04 | 1.07 | 1.07 |
| rlcreued | $\ln 1 \mathrm{~d}$ | 1.49 | 1.02 | 1.08 | 1.06 |
| crbed | $\ln 1 \mathrm{~d}$ | 1.19 | 0.98 | 1.07 | 1.14 |
| rcrbed | $\ln 1 d$ | 1.24 | 0.93 | 1.10 | 1.14 |
| lccrbed | $\ln 1 \mathrm{~d}$ | 2.81 | 0.98 | 1.11 | 1.10 |
| rlccrbed | $\ln 1 d$ | 2.81 | 0.99 | 1.12 | 1.10 |
| glded | $\ln 1 \mathrm{~d}$ | 1.29 | 1.01 | 1.04 | 1.09 |
| rglded | $\ln 1 \mathrm{~d}$ | 1.31 | 1.02 | 1.05 | 1.09 |
| lcglded | lnld | 1.69 | 1.10 | 1.13 | 1.13 |
| rlcglded | $\ln 1 d$ | 1.63 | 1.10 | 1.14 | 1.13 |
| mon0 | $\ln 1 \mathrm{~d}$ | 1.23 | 0.80 | 0.58 | 1.62 |
| mon0 | gap | 1.52 | 1.29 | 0.66 | 1.82 |
| rmon0 | $\ln 1 \mathrm{~d}$ | 1.33 | 0.78 | 0.75 | 1.52 |
| rmon0 | gap | 1.20 | 1.29 | 0.70 | 1.51 |
| monl | $\ln 1 \mathrm{~d}$ | 0.90 | 1.23 | 0.93 | 1.03 |
| monl | gap | 1.49 | 1.30 | 1.60 | 1.44 |

$h=4$

| $83-86$ | $87-90$ | $91-94$ | $95-99$ |
| :--- | :--- | :--- | :--- | :--- |
| 0.94 | 204 | 234 |  | | $83-86$ | $87-90$ | $91-94$ | $95-99$ |
| :---: | :---: | :---: | :---: |
| 0.94 | 2.04 | 2.34 | 1.90 |
| 1.02 | 0.91 | 0.78 | 1.03 |
| 1.55 | 1.13 | 0.81 | 1.26 |
| 0.091 | 130 |  |  |

$h=8$

| $83-86$ | $87-90$ | $91-94$ | $95-99$ |
| :---: | :---: | :---: | :---: |
| 0.86 | 172 | 2.61 | 1.66 | | $83-86$ | $87-90$ | $91-94$ | $95-99$ |
| ---: | ---: | ---: | ---: |
| 0.86 | 1.72 | 2.61 | 1.66 |
|  |  |  |  |
| 1.14 | 0.99 | 0.84 | 1.30 |
| 1.26 | 0.96 | 0.94 | 2.70 |
| 0.68 | 1.10 | 1.01 | 0.98 |
| 0.43 | 6.53 | 0.96 | .00 |

(Chart 6: Continued)

| rmonl | ln 1 d | 0.99 | 1.34 | 1.07 | 0.95 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| rmonl | gap | 1.64 | 1.29 | 0.89 | 1.52 |
| mon2 | $\ln 1 d^{\text {d }}$ | 1.19 | 1.15 | 0.43 | 0.87 |
| mon2 | gap | 1.37 | 1.05 | 0.85 | 1.76 |
| rmon2 | $\ln 1 d^{\text {d }}$ | 1.16 | 1.03 | 0.38 | 0.88 |
| rmon2 | gap | 1.03 | 1.00 | 0.66 | 1.35 |
| mon4 | $\ln 1 d^{\prime}$ | 1.01 | 1.30 | 0.94 | 30.45 |
| mon4 | gap | 1.02 | 2.11 | 1.26 | 24.02 |
| rmon4 | $\ln 1 d^{\text {d }}$ | 1.66 | 1.05 | 0.53 | 44.46 |
| rmon4 | gap | 2.76 | 1.11 | 0.68 | 12.25 |
| lended | $\ln 1 d^{\prime}$ | 1.51 | 0.58 | 1.77 | 0.79 |
| lended | gap | 1.92 | 1.12 | 1.37 | 1.93 |
| rlended | $\ln 1 d^{\text {d }}$ | 1.39 | 0.82 | 1.07 | 0.75 |
| rlended | gap | 1.25 | 1.38 | 0.54 | 1.43 |
| mlp | lev | 1.53 | 1.65 | 1.55 | 1.48 |
| $m l p$ | gap | 1.21 | 1.31 | 1.09 | 1.18 |
| velo | $\ln 1 d^{\text {d }}$ | 1.31 | 1.27 | 0.65 | 1.18 |
| velo | gap | 1.49 | 1.26 | 0.90 | 1.28 |
| note | $\ln 1 d^{\text {d }}$ | 1.09 | 0.65 | 0.80 | 1.57 |
| note | gap | 1.51 | 1.11 | 0.90 | 1.83 |
| rnote | $\ln 1 d^{\text {d }}$ | 1.06 | 0.65 | 0.89 | 1.51 |
| rnote | gap | 1.28 | 0.94 | 0.76 | 1.49 |
| ighed | lev | 3.03 | 0.51 | 1.94 | 1.40 |
| rigbed | lev | 0.88 | 1.13 | 1.29 | 1.23 |
| sprded | lev | 1.11 | 1.07 | 1.19 | 1.05 |
| alndav | lev | 1.26 | 0.56 | 2.54 | 1.44 |
| ralndav | lev | 1.16 | 1.26 | 1.04 | 1.31 |
| rateav | $\ln 1 d^{\text {d }}$ | 7.11 | 1.14 | 1.00 | 0.99 |
| rateav | gap | 10.22 | 1.11 | 0.97 | 1.17 |
| rateed | $\ln 1 d^{\prime}$ | 5.64 | 1.01 | 1.02 | 1.07 |
| rateed | gap | 6.32 | 0.99 | 1.19 | 1.09 |
| ratest | lev | 1.31 | 1.25 | 1.06 | 0.91 |
| ratesk | lev | 1.08 | 1.08 | 1.02 | 0.99 |
| ratekr | lev | 1.91 | 1.06 | 0.98 | 1.24 |
| rrateav | $\ln 1 d^{\text {d }}$ | 6.16 | 1.08 | 0.99 | 0.97 |
| rrateav | gap | 7.34 | 0.91 | 1.28 | 1.14 |
| rrateed | $\ln 1 d^{\text {d }}$ | 6.29 | 1.03 | 1.02 | 1.06 |
| rrateed | gap | 7.61 | 0.85 | 1.14 | 1.22 |
| efratav | $\ln 1 d^{\text {d }}$ | 4.08 | 1.19 | 1.01 | 1.06 |
| efratav | gap | 4.73 | 1.13 | 0.88 | 1.09 |
| efrated | $\ln 1 d^{\prime}$ | 4.57 | 1.08 | 1.00 | 1.15 |
| efrated | gap | 4.07 | 1.15 | 1.10 | 1.32 |
| refrated | $\ln 1 d^{\prime}$ | 5.40 | 1.14 | 1.01 | 1.07 |
| refrated | gap | 4.92 | 1.23 | 0.88 | 1.22 |
| refratav | $\ln 1 d^{\text {d }}$ | 4.96 | 1.05 | 1.00 | 1.16 |
| refratav | gap | 5.16 | 1.15 | 1.12 | 1.21 |
| nikav | $\ln 1 \mathrm{~d}$ | 1.41 | 0.98 | 0.90 | 1.12 |
| niked | $\ln 1 d^{\text {d }}$ | 1.20 | 1.23 | 1.00 | 1.08 |
| nikst | lev | 1.26 | 1.84 | 2.04 | 0.95 |
| niksk | lev | 1.37 | 1.13 | 0.94 | 0.96 |
| nikkr | lev | 1.06 | 1.11 | 1.06 | 0.99 |
| rnikav | $\ln 1 d$ | 1.79 | 0.86 | 0.92 | 1.13 |
| rniked | $\ln 1 d^{\text {d }}$ | 1.64 | 1.08 | 1.00 | 1.08 |
| tpxav | ln1d | 1.31 | 1.20 | 0.75 | 1.13 |
| tpxed | $\ln 1 d^{\text {d }}$ | 1.38 | 1.10 | 1.13 | 1.07 |
| rtpxav | $\ln 1 \mathrm{~d}$ | 1.61 | 1.09 | 0.77 | 1.12 |
| rtpxed | $\ln 1 d^{\prime}$ | 1.68 | 1.28 | 1.13 | 1.07 |
| tosho | ln | 1.58 | 0.92 | 1.15 | 1.09 |
| tosho | $\ln 1 d$ | 1.85 | 1.01 | 0.98 | 0.89 |
| land | $\ln 1 d$ | 6.78 | 0.98 | 0.44 | 1.07 |
| rland | $\ln 1 d$ | 3.44 | 1.31 | 0.46 | 1.13 |


| 0.96 | 1.05 | 1.01 | 1.86 |
| :--- | :--- | :--- | ---: |
| 1.41 | 1.24 | 0.60 | 2.50 |
| 1.08 | 0.70 | 0.15 | 0.88 |
| 2.17 | 1.58 | 0.41 | 2.83 |
| 1.50 | 1.04 | 0.20 | 0.85 |
| 1.79 | 1.16 | 0.64 | 2.20 |
| 1.57 | 1.16 | 1.66 | 21.61 |
| 1.66 | 2.49 | 1.02 | 32.60 |
| 2.59 | 1.06 | 0.46 | 49.87 |
| 4.43 | 0.96 | 0.30 | 25.50 |
| 1.05 | 0.83 | 1.07 | 0.72 |
| 2.69 | 1.14 | 0.73 | 3.21 |
| 1.76 | 0.94 | 0.76 | 0.65 |
| 2.07 | 1.25 | 0.31 | 2.17 |
| 1.84 | 2.19 | 1.52 | 2.22 |
| 1.18 | 1.13 | 0.99 | 1.34 |
| 1.11 | 1.12 | 0.53 | 1.68 |
| 1.70 | 1.27 | 0.49 | 1.71 |
| 1.03 | 0.87 | 0.45 | 2.51 |
| 2.21 | 1.47 | 0.58 | 3.56 |
| 1.19 | 0.79 | 0.57 | 2.22 |
| 1.94 | 1.14 | 0.31 | 2.79 |
| 1.91 | 0.48 | 1.90 | 2.04 |
| 1.14 | 1.16 | 1.20 | 1.15 |
| 1.04 | 0.99 | 1.15 | 1.18 |
| 1.11 | 0.47 | 2.80 | 2.04 |
| 1.41 | 1.14 | 1.18 | 1.15 |
| 4.63 | 1.05 | 0.94 | 0.89 |
| 5.36 | 0.96 | 0.76 | 1.22 |
| 4.30 | 1.00 | 0.96 | 1.09 |
| 5.04 | 1.07 | 0.82 | 1.26 |
| 1.98 | 1.20 | 1.05 | 0.87 |
| 0.98 | 1.03 | 1.00 | 0.95 |
| 0.95 | 0.98 | 1.00 | 1.27 |
| 3.27 | 1.05 | 0.94 | 0.90 |
| 4.69 | 0.83 | 0.98 | 1.20 |
| 3.15 | 1.06 | 0.92 | 0.96 |
| 4.65 | 0.84 | 0.99 | 1.20 |
| 2.92 | 1.11 | 0.91 | 1.00 |
| 3.19 | 1.30 | 0.71 | 0.96 |
| 3.31 | 1.09 | 0.97 | 1.25 |
| 3.24 | 1.27 | 0.74 | 1.10 |
| 3.73 | 1.06 | 0.99 | 1.17 |
| 4.30 | 1.24 | 0.82 | 0.95 |
| 4.10 | 1.03 | 0.96 | 1.17 |
| 4.43 | 1.24 | 0.87 | 1.17 |
| 1.33 | 0.77 | 0.25 | 1.31 |
| 1.23 | 1.07 | 0.29 | 1.41 |
| 1.38 | 1.83 | 1.26 | 1.15 |
| 1.19 | 1.09 | 0.89 | 1.10 |
| 0.80 | 2.23 | 0.98 | 1.00 |
| 1.34 | 0.77 | 0.26 | 1.29 |
| 1.59 | 0.97 | 0.30 | 1.40 |
| 1.23 | 0.82 | 0.27 | 1.25 |
| 0.99 | 0.96 | 0.30 | 1.29 |
| 1.22 | 0.88 | 0.27 | 1.23 |
| 1.18 | 0.89 | 0.31 | 1.28 |
| 2.67 | 1.39 | 0.71 | 1.09 |
| 1.77 | 1.10 | 0.86 | 1.05 |
| 6.34 | 1.99 | 0.24 | 1.16 |
| 4.07 | 1.61 | 0.24 | 1.13 |
|  |  |  |  |
|  |  |  |  |


| 1.05 | 1.03 | 0.98 | 3.37 |
| ---: | :--- | :--- | ---: |
| 2.35 | 1.11 | 0.54 | 4.26 |
| 1.30 | 0.91 | 0.41 | 0.65 |
| 3.43 | 2.29 | 0.26 | 4.32 |
| 1.59 | 1.04 | 0.14 | 1.38 |
| 1.60 | 1.25 | 1.06 | 4.22 |
| 2.98 | 1.37 | 2.82 | 46.55 |
| 2.03 | 2.58 | 1.29 | 65.50 |
| 3.12 | 0.96 | 0.54 | 63.68 |
| 2.26 | 1.03 | 0.43 | 48.90 |
| 1.13 | 1.08 | 0.80 | 0.71 |
| 2.49 | 1.37 | 1.64 | 3.24 |
| 1.96 | 1.18 | 1.30 | 0.46 |
| 2.21 | 0.93 | 0.82 | 2.40 |
| 2.63 | 3.44 | 2.39 | 2.61 |
| 1.25 | 0.91 | 2.60 | 1.21 |
| 1.31 | 1.10 | 0.39 | 1.85 |
| 1.47 | 1.24 | 0.31 | 3.05 |
| 0.97 | 0.92 | 0.54 | 2.75 |
| 2.89 | 2.23 | 0.77 | 4.65 |
| 1.18 | 0.94 | 0.69 | 3.04 |
| 2.89 | 1.26 | 0.17 | 4.39 |
| 1.87 | 1.29 | 2.53 | 3.68 |
| 1.68 | 1.21 | 1.03 | 0.90 |
| 1.92 | 1.01 | 1.13 | 1.22 |
| 1.49 | 0.93 | 2.88 | 3.11 |
| 1.78 | 1.16 | 1.27 | 1.83 |
| 0.74 | 0.93 | 0.96 | 0.92 |
| 0.20 | 0.92 | 0.93 | 1.01 |
| 0.60 | 1.00 | 1.00 | 0.95 |
| 0.23 | 0.86 | 0.92 | 1.09 |
| 2.08 | 1.40 | 1.00 | 0.76 |
| 0.97 | 1.01 | 0.99 | 0.99 |
| 0.92 | 1.00 | 0.97 | 1.51 |
| 0.39 | 1.01 | 1.00 | 0.98 |
| 0.27 | 0.87 | 1.13 | 1.04 |
| 0.43 | 0.97 | 1.04 | 1.04 |
| 0.27 | 0.87 | 1.15 | 1.08 |
| 0.79 | 1.06 | 0.80 | 0.80 |
| 0.21 | 1.24 | 0.81 | 0.98 |
| 0.89 | 1.05 | 0.83 | 0.86 |
| 0.26 | 1.19 | 0.81 | 1.01 |
| 0.82 | 1.03 | 0.86 | 0.75 |
| 0.19 | 1.20 | 0.80 | 0.80 |
| 0.50 | 1.06 | 0.91 | 0.81 |
| 0.28 | 1.17 | 0.81 | 0.83 |
| 1.27 | 0.71 | 0.22 | 1.29 |
| 0.98 | 0.87 | 0.28 | 1.42 |
| 1.11 | 2.68 | 2.29 | 1.09 |
| 1.24 | 1.01 | 0.90 | 0.75 |
| 1.11 | 2.14 | 1.02 | 1.02 |
| 1.35 | 0.72 | 0.23 | 1.29 |
| 1.10 | 0.83 | 0.31 | 1.43 |
| 1.25 | 0.66 | 0.27 | 1.12 |
| 1.11 | 0.72 | 0.26 | 1.23 |
| 1.33 | 0.66 | 0.27 | 1.12 |
| 1.13 | 0.73 | 0.28 | 1.23 |
| 3.68 | 3.21 | 0.69 | 1.61 |
| 1.23 | 0.91 | 0.66 | 1.43 |
| 6.62 | 8.06 | 0.26 | 0.77 |
| 4.63 | 5.97 | 0.28 | 0.74 |
|  |  |  |  |
|  |  |  |  |

Notes: The value in the first row is the root mean squared forecast error (RMSFE). The second row and onwards are the relative MSFE. The shaded sections are for a relative MSFE of less than one. Note that the forecast data for 1983-85 is missing for series regarding economic activity levels and asset values that start between 1971 and 1973, so 1983-86 uses a mean that does not include the data for this period.

## Chart 7: Variables used in multivariate forecasts

(a) CPI

| Real economic |
| :--- |
| activities |
| (react) |
| kijmi lnld |
| kijhi $\quad$ lnld |
| kenjal lnld |
| kenjmi lnld |
| kenjhi lnld |
| kkeall lnld |
| ukekun lnld |
| ukechi lnld |
| ckhime lnld |
| expt lnld |



| $\begin{aligned} & \text { Money } \\ & \text { (money) } \end{aligned}$ |
| :---: |
| mon2 ln1d |
| rmon2 lnld |
| lended $m 11$ d |
| rlended ln 1 d |
| velo ln $1 d$ |
| rnote lnid |


| Asset prices <br> (asset) |
| :--- |
| sprded lev |
| ratest |
| ratev |
| ratekr |
| nikav |
| nikkr |
| ln1d |
| rnikav |
| tpxav |
| rtpxav |
| rn1d |
| tosho |
| ln1d |

Notes: The partial forecast combination (licatall) is a series that combines the above 31 series at once. The overall forecast combination (liall) is a series that combines all (148) of the bivariate forecasts in Chart 4.
(b) GDP

| Real economic activities (react) |  |
| :---: | :---: |
| сари | ln1d |
| kijmi | ln1d |
| kijhi | ln1d |
| kenjal | lnld |
| kenjmi | lnld |
| juckko | gap |
| juckme | gap |
| car | gap |
| lcexpt | gap |


| Prices/wage/ market prices (pr_wa) |  |
| :---: | :---: |
| wpi | $\ln 1 d$ |
| wpiined | ln1d |
| ipiav | lnld |
| ipisoav | ln1d |
| reued | ln1d |
| rreued | lnld |
| glded | ln1d |
| rglded | $\ln 1 d$ |


| $\begin{aligned} & \text { Money } \\ & (\text { money }) \end{aligned}$ | $\begin{array}{l}\text { Asset prices } \\ \text { (asset) }\end{array}$ |
| :---: | :---: |
| rmon0 ln1d | rateav ln1d |
| mon2 lnld | rrateav lnld |
| rmon2 $m$ ld | efratav lnld |
| lended lnld | nikav lnld |
| rlended lnld | niksk lev |
| velo lnld | rnikav lnld |
|  | tpxav lnld |
|  | rtpxav lndd |
|  | tosho lnld |

Notes: The partial forecast combination (licatall) is a series that combines the above 31 series at once. The overall forecast combination (liall) is a series that combines all (147) of the bivariate forecasts in Chart 6.

Chart 8: Relative MSFE of multivariate forecasts (CPI forecasts)

| $h=2$ |  |  |  |  |  | $h=4$ |  |  |  | $h=8$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 83-86 | 87-90 | 91-94 | 95-99 | 83-86 | 87-90 | 91-94 | 95-99 | 83-86 | 87-90 | 91-94 | 95-99 |
| react | mean | 0.95 | 1.12 | 0.94 | 0.92 | 0.95 | 1.08 | 0.86 | 0.64 | 0.98 | 1.05 | 0.90 | 0.62 |
| react | median | 0.98 | 1.05 | 0.95 | 0.96 | 0.92 | 1.10 | 0.92 | 0.77 | 1.03 | 1.07 | 0.94 | 0.71 |
| react | tr.mean | 0.93 | 1.10 | 0.94 | 0.98 | 0.96 | 1.10 | 0.92 | 0.67 | 1.03 | 1.06 | 0.94 | 0.65 |
| react | TM_-2 | 0.96 | 1.08 | 0.93 | 0.96 | 0.96 | 1.09 | 0.92 | 0.72 | 1.12 | 1.04 | 0.94 | 0.68 |
| react | ridge $k=0.25$ | 0.69 | 1.75 | 0.90 | 0.73 | 0.65 | 2.90 | 1.01 | 0.42 | 0.91 | 6.80 | 0.57 | 0.36 |
| react | ridge $k=0.5$ | 0.69 | 1.70 | 0.89 | 0.72 | 0.65 | 2.88 | 0.94 | 0.41 | 0.91 | 6.54 | 0.58 | 0.35 |
| react | ridge $k=1$ | 0.69 | 1.64 | 0.87 | 0.72 | 0.65 | 2.75 | 0.89 | 0.41 | 0.91 | 6.10 | 0.59 | 0.36 |
| react | ridge $k=10$ | 0.77 | 1.25 | 0.85 | 0.80 | 0.73 | 1.51 | 0.77 | 0.50 | 0.91 | 2.87 | 0.68 | 0.46 |
| react | ridge $k=100$ | 0.89 | 1.12 | 0.91 | 0.90 | 0.85 | 1.07 | 0.83 | 0.61 | 0.91 | 1.21 | 0.85 | 0.59 |
| react | ridge $k=500$ | 0.92 | 1.12 | 0.93 | 0.92 | 0.88 | 1.07 | 0.85 | 0.63 | 0.91 | 1.07 | 0.89 | 0.61 |
| react | fctr | 1.52 | 1.25 | 1.07 | 1.19 | 1.02 | 1.53 | 0.89 | 0.65 | 0.96 | 1.49 | 0.83 | 0.41 |
| $p r \_w a$ | mean | 0.77 | 0.87 | 0.63 | 0.66 | 0.73 | 0.87 | 0.66 | 0.54 | 0.71 | 0.69 | 0.71 | 0.53 |
| $p r$ _wa | median | 0.95 | 0.84 | 0.66 | 0.74 | 0.94 | 0.79 | 0.67 | 0.58 | 0.77 | 0.69 | 0.80 | 0.57 |
| pr_wa | tr.mean | 0.92 | 0.84 | 0.62 | 0.71 | 0.89 | 0.80 | 0.67 | 0.54 | 0.76 | 0.70 | 0.79 | 0.56 |
| $p r \_w a$ | TM_-2 | 0.95 | 0.84 | 0.66 | 0.74 | 0.94 | 0.79 | 0.67 | 0.58 | 0.77 | 0.69 | 0.80 | 0.57 |
| pr_wa | ridge $k=0.25$ | 0.32 | 1.38 | 0.75 | 0.54 | 0.14 | 2.22 | 0.88 | 0.33 | 0.41 | 5.25 | 0.43 | 0.09 |
| $p r \_w a$ | ridge $k=0.5$ | 0.33 | 1.34 | 0.73 | 0.52 | 0.13 | 2.09 | 0.84 | 0.33 | 0.41 | 4.67 | 0.42 | 0.11 |
| $p r$ _wa | ridge $k=1$ | 0.33 | 1.28 | 0.70 | 0.52 | 0.14 | 1.90 | 0.79 | 0.33 | 0.43 | 3.99 | 0.39 | 0.13 |
| pr_wa | ridge $k=10$ | 0.50 | 0.94 | 0.58 | 0.59 | 0.35 | 1.05 | 0.58 | 0.43 | 0.56 | 1.27 | 0.43 | 0.32 |
| pr_wa | ridge $k=100$ | 0.72 | 0.87 | 0.62 | 0.65 | 0.66 | 0.87 | 0.63 | 0.52 | 0.68 | 0.70 | 0.65 | 0.49 |
| pr_wa | ridge $k=500$ | 0.76 | 0.87 | 0.63 | 0.66 | 0.71 | 0.87 | 0.65 | 0.54 | 0.70 | 0.69 | 0.69 | 0.52 |
| $p r \_w a$ | fctr | 0.66 | 1.21 | 0.67 | 1.34 | 0.70 | 1.08 | 0.53 | 1.40 | 0.47 | 0.83 | 0.19 | 0.61 |
| money | mean | 0.87 | 1.22 | 0.93 | 0.87 | 0.85 | 1.86 | 0.68 | 0.61 | 0.38 | 3.26 | 0.48 | 0.34 |
| money | median | 0.84 | 1.02 | 0.93 | 0.91 | 0.74 | 1.63 | 0.66 | 0.66 | 0.38 | 2.76 | 0.51 | 0.32 |
| money | tr.mean | 0.85 | 1.12 | 0.94 | 0.86 | 0.83 | 1.66 | 0.67 | 0.60 | 0.37 | 2.92 | 0.48 | 0.31 |
| money | TM_-2 | 0.84 | 1.02 | 0.93 | 0.91 | 0.74 | 1.63 | 0.66 | 0.66 | 0.38 | 2.76 | 0.51 | 0.32 |
| money | ridge $k=0.25$ | 0.40 | 1.92 | 0.85 | 0.72 | 0.32 | 2.72 | 1.02 | 0.45 | 0.16 | 4.72 | 0.46 | 0.21 |
| money | ridge $k=0.5$ | 0.44 | 1.79 | 0.86 | 0.72 | 0.33 | 2.50 | 0.99 | 0.46 | 0.17 | 3.88 | 0.43 | 0.21 |
| money | ridge $k=1$ | 0.48 | 1.67 | 0.86 | 0.73 | 0.35 | 2.29 | 0.93 | 0.46 | 0.18 | 3.21 | 0.40 | 0.21 |
| money | ridge $k=10$ | 0.66 | 1.31 | 0.84 | 0.80 | 0.55 | 1.71 | 0.67 | 0.54 | 0.28 | 2.32 | 0.36 | 0.27 |
| money | ridge $k=100$ | 0.83 | 1.23 | 0.91 | 0.86 | 0.79 | 1.82 | 0.67 | 0.60 | 0.36 | 3.05 | 0.46 | 0.32 |
| money | ridge $k=500$ | 0.86 | 1.22 | 0.93 | 0.86 | 0.83 | 1.85 | 0.67 | 0.61 | 0.38 | 3.21 | 0.48 | 0.33 |
| money | fctr | 0.81 | 1.68 | 1.02 | 0.84 | 0.78 | 2.71 | 0.71 | 0.62 | 0.33 | 6.47 | 0.54 | 0.28 |
| asset | mean | 1.08 | 1.03 | 1.07 | 1.00 | 1.08 | 0.91 | 1.03 | 0.72 | 0.83 | 0.86 | 1.10 | 0.92 |
| asset | median | 1.22 | 1.12 | 1.04 | 1.20 | 1.32 | 0.94 | 1.06 | 0.75 | 0.88 | 0.92 | 1.08 | 1.03 |
| asset | tr.mean | 1.13 | 1.06 | 1.08 | 1.05 | 1.16 | 0.93 | 1.08 | 0.75 | 0.86 | 0.94 | 1.11 | 0.97 |
| asset | TM_-2 | 1.19 | 1.07 | 1.08 | 1.10 | 1.25 | 0.91 | 1.09 | 0.78 | 0.85 | 0.90 | 1.11 | 0.99 |
| asset | ridge $k=0.25$ | 0.54 | 1.63 | 0.81 | 0.74 | 0.60 | 2.91 | 0.88 | 0.46 | 0.91 | 6.84 | 0.80 | 0.51 |
| asset | ridge $k=0.5$ | 0.52 | 1.61 | 0.80 | 0.74 | 0.60 | 2.85 | 0.85 | 0.47 | 0.91 | 6.37 | 0.74 | 0.51 |
| asset | ridge $k=1$ | 0.52 | 1.56 | 0.79 | 0.75 | 0.61 | 2.69 | 0.83 | 0.48 | 0.91 | 5.73 | 0.71 | 0.52 |
| asset | ridge $k=10$ | 0.72 | 1.19 | 0.86 | 0.86 | 0.70 | 1.43 | 0.81 | 0.58 | 0.91 | 2.41 | 0.83 | 0.70 |
| asset | ridge $k=100$ | 0.97 | 1.04 | 1.03 | 0.97 | 0.82 | 0.94 | 0.97 | 0.69 | 0.91 | 1.00 | 1.04 | 0.88 |
| asset | ridge $k=500$ | 1.02 | 1.03 | 1.06 | 0.99 | 0.84 | 0.91 | 1.02 | 0.72 | 0.91 | 0.88 | 1.09 | 0.91 |
| asset | fctr | 1.04 | 1.14 | 1.21 | 1.32 | 0.99 | 0.90 | 1.20 | 0.68 | 0.90 | 0.89 | 1.18 | 0.74 |
| licatall | mean | 0.84 | 1.03 | 0.87 | 0.80 | 0.80 | 1.07 | 0.80 | 0.54 | 0.65 | 1.04 | 0.81 | 0.56 |
| licatall | median | 0.95 | 1.01 | 0.92 | 0.92 | 0.89 | 1.06 | 0.91 | 0.62 | 0.75 | 1.04 | 0.94 | 0.83 |
| licatall | tr.mean | 0.85 | 1.03 | 0.88 | 0.83 | 0.82 | 1.04 | 0.82 | 0.55 | 0.67 | 1.05 | 0.82 | 0.57 |
| licatall | TM_-2 | 0.87 | 1.02 | 0.89 | 0.86 | 0.83 | 1.03 | 0.83 | 0.57 | 0.69 | 1.02 | 0.83 | 0.59 |
| licatall | TM_. 15 | 0.89 | 1.01 | 0.89 | 0.88 | 0.85 | 1.02 | 0.84 | 0.59 | 0.71 | 0.99 | 0.85 | 0.63 |
| licatall | ridge $k=0.25$ | 0.65 | 1.43 | 0.69 | 0.57 | 0.64 | 2.66 | 0.87 | 0.52 | 0.98 | 7.18 | 0.71 | 0.23 |
| licatall | ridge $k=0.5$ | 0.65 | 1.51 | 0.71 | 0.59 | 0.64 | 2.75 | 0.85 | 0.43 | 0.98 | 6.96 | 0.64 | 0.24 |
| licatall | ridge $k=1$ | 0.66 | 1.56 | 0.73 | 0.61 | 0.64 | 2.79 | 0.84 | 0.40 | 0.98 | 6.65 | 0.60 | 0.27 |
| licatall | ridge $k=10$ | 0.70 | 1.39 | 0.75 | 0.67 | 0.68 | 2.13 | 0.76 | 0.41 | 0.98 | 4.39 | 0.55 | 0.36 |
| licatall | ridge $k=100$ | 0.83 | 1.07 | 0.81 | 0.75 | 0.80 | 1.11 | 0.74 | 0.50 | 0.98 | 1.49 | 0.70 | 0.49 |
| licatall | ridge $k=500$ | 0.89 | 1.03 | 0.86 | 0.79 | 0.86 | 1.06 | 0.78 | 0.53 | 0.98 | 1.10 | 0.78 | 0.54 |
| licatall | fctr | 1.13 | 1.12 | 1.29 | 0.96 | 1.10 | 0.92 | 1.22 | 0.48 | 1.34 | 1.22 | 1.11 | 0.35 |
| liall | mean | 0.53 | 1.01 | 0.85 | 1.04 | 0.55 | 1.22 | 0.76 | 1.07 | 0.62 | 1.72 | 0.77 | 1.22 |
| liall | median | 0.72 | 1.01 | 0.89 | 0.97 | 0.74 | 1.10 | 0.84 | 0.92 | 0.80 | 1.15 | 0.87 | 1.03 |
| liall | tr.mean | 0.54 | 1.01 | 0.85 | 1.02 | 0.56 | 1.23 | 0.76 | 1.01 | 0.64 | 1.67 | 0.77 | 1.18 |
| liall | TM_-2 | 0.55 | 1.01 | 0.85 | 1.02 | 0.57 | 1.22 | 0.77 | 0.97 | 0.65 | 1.66 | 0.78 | 1.15 |
| liall | TM_. 15 | 0.63 | 1.01 | 0.87 | 1.01 | 0.65 | 1.18 | 0.80 | 0.92 | 0.72 | 1.39 | 0.82 | 1.09 |
| liall | ridge $k=0.25$ | 0.63 | 1.55 | 0.71 | 0.74 | 0.42 | 2.40 | 1.31 | 1.25 | 0.78 | 7.16 | 0.70 | 0.46 |
| liall | ridge $k=0.5$ | 0.60 | 1.47 | 0.70 | 0.79 | 0.42 | 2.40 | 1.28 | 1.14 | 0.78 | 7.14 | 0.67 | 0.29 |
| liall | ridge $k=1$ | 0.56 | 1.41 | 0.71 | 0.81 | 0.42 | 2.40 | 1.21 | 1.00 | 0.78 | 7.10 | 0.63 | 0.23 |
| liall | ridge $k=10$ | 0.49 | 1.32 | 0.74 | 0.84 | 0.42 | 2.33 | 0.82 | 0.75 | 0.78 | 6.32 | 0.55 | 0.50 |
| liall | ridge $k=100$ | 0.49 | 1.10 | 0.75 | 0.91 | 0.45 | 1.52 | 0.66 | 0.86 | 0.78 | 3.82 | 0.64 | 0.92 |
| liall | ridge $k=500$ | 0.53 | 1.01 | 0.81 | 0.99 | 0.50 | 1.20 | 0.70 | 0.99 | 0.78 | 2.29 | 0.72 | 1.12 |

Notes: 1. The shaded sections show that the relative MSFE is less than one.
2. Charts in bold show the combination method with the smallest forecast error when comparing combinations that use a simple mean, various weighted means (median, trimmed mean, ridge regression), or principal component analysis for a certain sample period, forecast horizon and combination variable type. Columns 1, 2, 3, and 4 correspond to 1983-86, 1987-90, 1991-94, and 1995-99.
3. We combine without using the data in time with missing data for economic activity level, asset price, partial combination and overall combination data for 1983-86.

Chart 9: Relative MSFE of multivariate forecasts (GDP forecasts)

| $h=2$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 83-86 | 87-90 | 91-94 | 95-99 |
| react | mean | 1.07 | 0.82 | 0.82 | 0.87 |
| react | median | 1.13 | 0.84 | 0.85 | 0.92 |
| react | tr.mean | 1.11 | 0.86 | 0.83 | 0.88 |
| react | TM_-2 | 1.11 | 0.85 | 0.84 | 0.90 |
| react | ridge $k=0.25$ | 1.66 | 0.84 | 0.84 | 0.80 |
| react | ridge $k=0.5$ | 1.69 | 0.87 | 0.86 | 0.82 |
| react | ridge $k=1$ | 1.66 | 0.89 | 0.87 | 0.83 |
| react | ridge $k=10$ | 1.22 | 0.86 | 0.86 | 0.86 |
| react | ridge $k=100$ | 1.07 | 0.83 | 0.83 | 0.87 |
| react | ridge $k=500$ | 1.07 | 0.82 | 0.82 | 0.87 |
| react | fctr | 1.33 | 0.76 | 0.85 | 1.00 |
| pr_wa | mean | 1.56 | 1.05 | 0.99 | 1.05 |
| pr_wa | median | 1.34 | 1.07 | 1.02 | 1.03 |
| pr_wa | tr.mean | 1.46 | 1.07 | 1.01 | 1.04 |
| $p r \_w a$ | TM_-2 | 1.37 | 1.08 | 1.02 | 1.04 |
| pr_wa | ridge $k=0.25$ | 2.73 | 1.53 | 0.78 | 0.94 |
| pr_wa | ridge $k=0.5$ | 2.50 | 1.50 | 0.78 | 0.94 |
| pr_wa | ridge $k=1$ | 2.24 | 1.46 | 0.79 | 0.95 |
| pr_wa | ridge $k=10$ | 1.41 | 1.23 | 0.89 | 1.00 |
| pr_wa | ridge $k=100$ | 1.48 | 1.08 | 0.97 | 1.04 |
| $p r \_w a$ | ridge $k=500$ | 1.54 | 1.06 | 0.98 | 1.05 |
| pr_wa | fctr | 1.55 | 1.12 | 1.01 | 0.95 |
| money | mean | 1.04 | 0.82 | 0.38 | 0.80 |
| money | median | 1.16 | 0.79 | 0.37 | 0.85 |
| money | tr.mean | 1.09 | 0.82 | 0.36 | 0.81 |
| money | TM_-2 | 1.16 | 0.79 | 0.37 | 0.85 |
| money | ridge $k=0.25$ | 2.46 | 1.08 | 0.39 | 0.80 |
| money | ridge $k=0.5$ | 2.34 | 1.08 | 0.38 | 0.79 |
| money | ridge $k=1$ | 2.16 | 1.06 | 0.38 | 0.79 |
| money | ridge $k=10$ | 1.25 | 0.92 | 0.38 | 0.79 |
| money | ridge $k=100$ | 1.05 | 0.83 | 0.38 | 0.80 |
| money | ridge $k=500$ | 1.04 | 0.82 | 0.38 | 0.80 |
| money | fctr | 1.24 | 0.94 | 0.35 | 1.60 |
| asset | mean | 1.83 | 0.99 | 0.59 | 1.00 |
| asset | median | 1.53 | 1.02 | 0.96 | 1.04 |
| asset | tr.mean | 1.81 | 1.00 | 0.61 | 1.01 |
| asset | TM_-2 | 1.68 | 1.00 | 0.63 | 1.02 |
| asset | ridge $k=0.25$ | 2.82 | 0.94 | 0.78 | 1.04 |
| asset | ridge $k=0.5$ | 2.87 | 0.97 | 0.75 | 1.04 |
| asset | ridge $k=1$ | 2.89 | 0.99 | 0.73 | 1.03 |
| asset | ridge $k=10$ | 2.40 | 1.00 | 0.66 | 1.01 |
| asset | ridge $k=100$ | 2.01 | 0.99 | 0.60 | 1.00 |
| asset | ridge $k=500$ | 1.96 | 0.99 | 0.59 | 1.00 |
| asset | fctr | 3.27 | 1.05 | 0.65 | 1.10 |
| licatall | mean | 1.24 | 0.90 | 0.60 | 0.90 |
| licatall | median | 1.13 | 0.97 | 0.83 | 0.99 |
| licatall | tr.mean | 1.22 | 0.92 | 0.63 | 0.91 |
| licatall | TM_-2 | 1.19 | 0.92 | 0.65 | 0.92 |
| licatall | TM_. 15 | 1.15 | 0.93 | 0.70 | 0.94 |
| licatall | ridge $k=0.25$ | 2.91 | 0.73 | 0.50 | 0.81 |
| licatall | ridge $k=0.5$ | 2.72 | 0.78 | 0.52 | 0.83 |
| licatall | ridge $k=1$ | 2.58 | 0.83 | 0.57 | 0.85 |
| licatall | ridge $k=10$ | 2.21 | 0.91 | 0.68 | 0.90 |
| licatall | ridge $k=100$ | 1.64 | 0.90 | 0.63 | 0.90 |
| licatall | ridge $k=500$ | 1.50 | 0.90 | 0.61 | 0.90 |
| licatall | fctr | 2.72 | 0.95 | 0.66 | 0.83 |
| liall | mean | 1.12 | 0.95 | 0.81 | 1.11 |
| liall | median | 1.07 | 0.99 | 0.94 | 1.01 |
| liall | tr.mean | 1.13 | 0.95 | 0.81 | 1.07 |
| liall | TM_-2 | 1.13 | 0.95 | 0.82 | 1.04 |
| liall | TM_. 15 | 1.06 | 0.96 | 0.86 | 1.01 |
| liall | ridge $k=0.25$ | 2.85 | 0.74 | 0.85 | 2.86 |
| liall | ridge $k=0.5$ | 2.67 | 0.79 | 0.84 | 2.50 |
| liall | ridge $k=1$ | 2.51 | 0.83 | 0.86 | 2.06 |
| liall | ridge $k=10$ | 2.24 | 0.93 | 0.97 | 1.27 |
| liall | ridge $k=100$ | 1.79 | 0.93 | 0.93 | 1.13 |
| liall | ridge $k=500$ | 1.43 | 0.93 | 0.85 | 1.12 |


| 83-86 | 87-90 | 91-94 | 95-99 |
| :---: | :---: | :---: | :---: |
| 1.07 | 0.84 | 0.60 | 0.93 |
| 1.02 | 0.88 | 0.58 | 0.94 |
| 1.04 | 0.88 | 0.59 | 0.92 |
| 1.05 | 0.88 | 0.59 | 0.92 |
| 1.86 | 1.00 | 0.76 | 0.91 |
| 1.84 | 0.97 | 0.74 | 0.90 |
| 1.78 | 0.96 | 0.73 | 0.90 |
| 1.34 | 0.89 | 0.66 | 0.91 |
| 1.10 | 0.85 | 0.61 | 0.93 |
| 1.07 | 0.85 | 0.60 | 0.93 |
| 1.66 | 0.73 | 0.47 | 1.04 |
| 1.32 | 1.00 | 0.81 | 0.97 |
| 1.15 | 1.00 | 0.88 | 0.96 |
| 1.25 | 1.00 | 0.83 | 0.96 |
| 1.17 | 1.00 | 0.85 | 0.95 |
| 2.79 | 1.48 | 0.76 | 0.88 |
| 2.65 | 1.45 | 0.75 | 0.87 |
| 2.46 | 1.42 | 0.75 | 0.87 |
| 1.45 | 1.19 | 0.78 | 0.91 |
| 1.28 | 1.03 | 0.81 | 0.96 |
| 1.31 | 1.01 | 0.81 | 0.96 |
| 2.29 | 1.08 | 1.02 | 1.26 |
| 0.99 | 0.85 | 0.16 | 0.80 |
| 1.06 | 0.87 | 0.19 | 0.93 |
| 1.05 | 0.87 | 0.17 | 0.80 |
| 1.06 | 0.87 | 0.19 | 0.93 |
| 2.55 | 1.00 | 0.16 | 0.84 |
| 2.58 | 1.05 | 0.16 | 0.82 |
| 2.46 | 1.06 | 0.16 | 0.81 |
| 1.35 | 0.95 | 0.16 | 0.80 |
| 1.02 | 0.86 | 0.16 | 0.80 |
| 1.00 | 0.85 | 0.16 | 0.80 |
| 1.03 | 0.88 | 0.10 | 1.34 |
| 1.37 | 0.83 | 0.35 | 0.99 |
| 1.24 | 0.95 | 0.68 | 1.05 |
| 1.36 | 0.83 | 0.37 | 0.99 |
| 1.30 | 0.84 | 0.40 | 1.01 |
| 1.87 | 0.90 | 0.42 | 1.21 |
| 1.80 | 0.93 | 0.43 | 1.16 |
| 1.73 | 0.95 | 0.45 | 1.11 |
| 1.47 | 0.89 | 0.41 | 1.02 |
| 1.34 | 0.83 | 0.36 | 1.00 |
| 1.32 | 0.83 | 0.35 | 0.99 |
| 2.38 | 0.66 | 0.25 | 1.08 |
| 1.07 | 0.85 | 0.41 | 0.84 |
| 1.02 | 0.94 | 0.52 | 0.93 |
| 1.07 | 0.86 | 0.44 | 0.84 |
| 1.06 | 0.87 | 0.46 | 0.86 |
| 1.05 | 0.89 | 0.49 | 0.90 |
| 1.81 | 1.06 | 0.38 | 0.93 |
| 1.75 | 1.06 | 0.40 | 0.90 |
| 1.69 | 1.06 | 0.44 | 0.87 |
| 1.48 | 1.01 | 0.51 | 0.84 |
| 1.21 | 0.89 | 0.45 | 0.84 |
| 1.14 | 0.86 | 0.42 | 0.84 |
| 2.88 | 0.76 | 0.27 | 0.64 |
| 1.10 | 0.95 | 0.58 | 1.19 |
| 1.08 | 0.96 | 0.74 | 1.04 |
| 1.09 | 0.95 | 0.58 | 1.14 |
| 1.09 | 0.95 | 0.58 | 1.11 |
| 1.07 | 0.96 | 0.65 | 1.07 |
| 1.94 | 1.33 | 0.69 | 1.41 |
| 1.88 | 1.27 | 0.67 | 1.41 |
| 1.79 | 1.24 | 0.67 | 1.37 |
| 1.52 | 1.16 | 0.76 | 1.19 |
| 1.28 | 1.05 | 0.71 | 1.16 |
| 1.11 | 0.98 | 0.63 | 1.17 |


| $h=8$ |
| :--- |
| $83-86$ |
| $87-90$ |


| 83-86 | 87-90 | 91-94 | 95-99 |
| :---: | :---: | :---: | :---: |
| 0.96 | 0.83 | 0.76 | 1.03 |
| 1.05 | 0.96 | 0.79 | 0.91 |
| 0.97 | 0.85 | 0.77 | 1.02 |
| 1.01 | 0.90 | 0.79 | 1.00 |
| 0.77 | 1.06 | 1.17 | 0.98 |
| 0.77 | 1.03 | 1.14 | 0.92 |
| 0.78 | 1.01 | 1.10 | 0.90 |
| 0.81 | 0.90 | 0.93 | 0.95 |
| 0.85 | 0.84 | 0.79 | 1.01 |
| 0.85 | 0.83 | 0.77 | 1.02 |
| 1.58 | 0.65 | 0.37 | 1.90 |
| 0.87 | 0.95 | 0.90 | 0.76 |
| 1.03 | 0.96 | 0.94 | 0.78 |
| 0.92 | 0.96 | 0.91 | 0.77 |
| 1.00 | 0.96 | 0.93 | 0.79 |
| 1.91 | 1.36 | 1.08 | 0.60 |
| 1.87 | 1.34 | 1.07 | 0.62 |
| 1.78 | 1.31 | 1.06 | 0.63 |
| 1.18 | 1.12 | 0.97 | 0.69 |
| 0.90 | 0.98 | 0.91 | 0.74 |
| 0.87 | 0.96 | 0.90 | 0.75 |
| 1.13 | 0.96 | 1.09 | 0.90 |
| 1.26 | 1.01 | 0.29 | 0.78 |
| 1.23 | 0.99 | 0.27 | 0.73 |
| 1.25 | 1.00 | 0.27 | 0.71 |
| 1.23 | 0.99 | 0.27 | 0.73 |
| 1.97 | 1.36 | 0.39 | 0.91 |
| 1.94 | 1.35 | 0.39 | 0.87 |
| 1.88 | 1.32 | 0.38 | 0.84 |
| 1.47 | 1.13 | 0.33 | 0.79 |
| 1.29 | 1.02 | 0.29 | 0.78 |
| 1.26 | 1.01 | 0.29 | 0.78 |
| 1.11 | 0.99 | 0.17 | 1.03 |
| 1.06 | 0.69 | 0.42 | 0.81 |
| 1.24 | 0.75 | 0.60 | 0.98 |
| 1.17 | 0.69 | 0.44 | 0.80 |
| 1.19 | 0.70 | 0.47 | 0.80 |
| 0.81 | 0.86 | 0.85 | 1.25 |
| 0.81 | 0.84 | 0.84 | 1.10 |
| 0.81 | 0.81 | 0.82 | 0.98 |
| 0.81 | 0.68 | 0.62 | 0.83 |
| 0.81 | 0.67 | 0.45 | 0.81 |
| 0.81 | 0.68 | 0.43 | 0.81 |
| 0.85 | 0.55 | 0.24 | 1.02 |
| 0.95 | 0.82 | 0.52 | 0.68 |
| 1.06 | 0.94 | 0.71 | 0.75 |
| 0.97 | 0.84 | 0.55 | 0.69 |
| 0.99 | 0.86 | 0.59 | 0.71 |
| 1.02 | 0.89 | 0.62 | 0.73 |
| 0.86 | 1.36 | 1.06 | 0.96 |
| 0.86 | 1.33 | 1.01 | 0.86 |
| 0.86 | 1.31 | 0.99 | 0.76 |
| 0.86 | 1.12 | 0.88 | 0.64 |
| 0.86 | 0.86 | 0.63 | 0.66 |
| 0.86 | 0.83 | 0.55 | 0.67 |
| 0.58 | 0.50 | 0.28 | 0.56 |
| 1.04 | 1.05 | 0.70 | 1.14 |
| 1.07 | 0.99 | 0.85 | 1.00 |
| 1.04 | 1.03 | 0.70 | 1.08 |
| 1.04 | 1.02 | 0.71 | 1.05 |
| 1.07 | 0.99 | 0.79 | 1.02 |
| 0.96 | 1.76 | 1.20 | 1.82 |
| 0.96 | 1.73 | 1.25 | 1.66 |
| 0.96 | 1.71 | 1.29 | 1.48 |
| 0.96 | 1.64 | 1.35 | 1.04 |
| 0.96 | 1.31 | 1.09 | 1.01 |
| 0.96 | 1.10 | 0.83 | 1.09 |

Notes: 1. The shaded sections show that the relative MSFE is less than one
2. Charts in bold show the combination method with the smallest forecast error when comparing combinations that use a simple mean, various weighted means (median, trimmed mean, ridge regression), or principal component analysis for a certain sample period, forecast horizon and combination variable type. Columns 1, 2, 3, and 4 correspond to 1983-86, 1987-90, 1991-94, and 1995-99.
3. We combine without using the data in time with missing data for economic activity level, asset value, partial combination and overall combination data for 1983-86.

Chart 10: Decomposition of forecast combination's reduction effect
(a) CPI forecast

(b) GDP forecast


Note: 'Combin.' is the relative MSFE of each forecast combination. 'Average' is the average of the individual forecasts' relative MSFE that make up each forecast combination. ' $1 s t t^{\prime}$, ' $2 n d^{\prime}$ ', ' $3 r r^{\prime}$ ', which are the leveling of forecast bias, leveling of forecast error fluctuation, and forecast error fluctuation canceling each other out as shown in Equation (5).

## Chart 11: Optimal combination groupings

(a) Optimal number of combinations and forecast error for a given initial variable


Note: The horizontal axis is the performance of the individual forecast chosen as the initial series (in terms of the performance ranking of the initial series among all series). The right vertical axis is the number of combination series and the left vertical axis is the relative MSFE. The points ( $\rangle$ and $\square$ ) for the number of series (right axis) show the number of groupings with the smallest error for a particular initial series. Of these series, $\square$ shows the optimal number of combinations. The forecast error (left axis, relative MSFE) is the value when using the grouping with the smallest value for a given initial variable. It should be noticed that the process of finding the optimal forecast combination can be shown as a surface (see Chart below) in the space made up of $(\mathrm{X}, \mathrm{Y}, \mathrm{Z})=$ (number of series included in the combination, performance ranking of the initial variable, forecast combination error). The optimal ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) grouping is the "trough" in the entire surface, and the "optimum for a given initial variable" is the "trough" in the X-Z plane created by cutting the surface at Y.
(Reference Chart) Image of stepwise optimization

(b) Composition of grouping

|  |  | number of Combin. | rel.MSFE | Breakdown of combination series |  |  |  | Performance ranking of individual series |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | $h=2$ 1987-99 | 3 | 0.640 | $n g d p$ | rwage | sprded |  | 1 | 2 | 24 |  |
| P | $h=4$ 1987-99 | 4 | 0.527 | $n g d p$ | wage | ratest | tosho | 1 | 4 | 8 | 7 |
| I | $h=8 \quad 1987-99$ | 3 | 0.295 | $n g d p$ | wage | tosho |  | 1 | 3 | 15 |  |
| G | $h=2 \quad 1987-99$ | 2 | 0.555 | ipisoav | lended |  |  | 1 | 6 |  |  |
| D | $h=4 \quad 1987-99$ | 4 | 0.435 | kijmi | mon2 | rlended | nikav | 11 | 1 | 8 | 3 |
| P | $h=8 \quad 1987-99$ | 4 | 0.348 | juckme | rmon2 | lended | nikav | 29 | 6 | 10 | 4 |

## Chart 12: Optimal combination

(a) CPI

(b) GDP


Note: The horizontal axis is the performance of the individual forecast chosen as the initial series (in terms of the performance ranking of the initial series among all series). The right vertical axis is the number of combination series and the left vertical axis is the relative MSFE. The points ( $\langle$ and $\square$ ) for the number of series (right axis) show the number of groupings with the smallest error for a particular initial series. Of these series, $\square$ shows the optimal number of combinations. The forecast error (left axis, relative MSFE) is the value when using the grouping with the smallest value for a given initial variable.

## Chart 13: Performance of best combination (1987-99)



Note: 'Combin.' is the relative MSFE of each forecast combination. 'Average' is the average of the individual forecasts' relative MSFE that make up each forecast combination. ' $1 s t t^{\prime}$, ' $2 n d$ ', ' $3 r d$ ', which are the leveling of forecast bias, leveling of forecast error fluctuation, and forecast error fluctuation canceling each other out as shown in Equation (5). 'best' is the performance of the optimal grouping shown in Chart 11 above.

## Chart 14: Performance of best combinations (by sample period)

(a) CPI

(b) GDP




Chart 15: Improvement due to addition of variables and its causes (1987-99)


Note: The horizontal axis is the number of variables included in combination, and the vertical axis is the improvement to the forecast error of forecast combination when second to eighth variables are added.

## Chart 16: Improvement due to addition of variables and its causes (by sample period)


(b) GDP








91-94

Note: The horizontal axis is the number of variables included in combination, and the vertical axis is the improvement to the forecast error of forecast combination when second to eighth variables are added.

## Chart 17: Optimal combination grouping by sample period

(a) Grouping


Notes: 'Best' and '2nd best' is the optimal grouping for each period and the entire period, respectively. Both of values are in the relative MSFE. The value in the parentheses below ' 2 nd best' shows the difference between the best and second best. The shaded cells are variables that the best and second best do not have in common.
(b) Comparison of improvements



[^0]:    * Institute for Monetary and Economic Studies, Bank of Japan (E-mail: tomiyuki.kitamura@boj.or.jp)
    ** Institute for Monetary and Economic Studies, Bank of Japan (E-mail: ryouji.koike@boj.or.jp)

[^1]:    ${ }^{1}$ This paper focuses on forecasting of future inflation rates and real GDP growth rates (i.e. pinpoint forecasting). On the other hand, forecasting of "turning points" in price movements or the economy is also an important theme in economic forecasting. Recent examples of research that emphasizes forecasting of turning points include Honda and Matsuoka [2001] and Kasuya and Shinki [2001].
    ${ }^{2}$ In this paper, the term "information variable" is defined as "a financial or economic indicator with correlation to and precedence over the final target." See Kato [1991] for this definition.
    ${ }^{3}$ Okina and Shiratsuka [2002] discuss this point in relation with the monetary policy management based upon the experiences of the bubble period in the eighties.
    ${ }^{4}$ Other than those mentioned here, variables that have been investigated for their predictive content for inflation and real output include yield spread (i.e. the difference between long and short term interest rates), default spread (i.e. the difference between CP and government bond rates), and the exchange rate. Stock and Watson [2001] survey this large literature on various variables including these. As for recent research in Japan focusing on individual variables, Hirata and Ueda [1998] examine the predictive content

[^2]:    of yield spread for the economic activity, and both Mio [2000] and Fukuda and Keida [2001] investigate the forecast performance of the Phillips curve for inflation.
    5 These results are obtained through more direct empirical analyses in Stock and Watson [1996], Cecchetti, Chu and Steindel [2000], and Stock and Watson [2001].
    ${ }^{6}$ In addition to these, there are approaches where macroeconometric models are used, or many explanatory variables are included directly in forecast models such as vector autoregression (VAR) or state space models. Some examples of recent studies undertaken in Japan that use these approaches include Ban and Saito [2001] for the former and Kitagawa and Kawasaki [2001] for the latter. Also, the successive approximation method, which is widely used in practical economic forecasting, can be seen as an approach that includes as much information as possible into forecasts. Under the successive approximation method, a forecaster responsible for each aspect of the economy such as production or consumption establishes an outlook for that aspect, and each outlook is repeatedly adjusted to be consistent with the whole. This method could be considered to be the antithesis of statistical methods.
    ${ }^{7}$ The dynamic factor model assumes that common factors exist behind multiple individual series and it is these factors that dynamically affect individual series. See Stock and Watson [1998] for details.

[^3]:    ${ }^{8}$ In the field of forecating theory, this approach has been examined for long time. One of the earliest seminal papers is Bates and Granger [1969].
    ${ }^{9}$ Another recent example of studies that adopts the approach of combining forecast is Marcellino [2002]. Also, in Japan, Oyama [2001] calculates a total of five real GDP forecast series, one of which is forecasted using the accumulation format, and the other of which were forecasted using each of four series such as the index of total industry activity as a information variable individually. A forecast combination series was created based on these five series and it was reported to show high forecast performance.
    ${ }^{10}$ Stock and Watson [1998] show that when there are a large number of variables, under certain technical assumptions the principal component extracted in principal component analysis is a consistent estimator for the factor in the dynamic factor model. This is the backdrop to the fact that they used principal component analysis in Stock and Watson [1999]. The adoption of principal component analysis in section 3 of this paper is also based on this fact.
    ${ }^{11}$ See section 3 for a forecasting method using ridge regression.

[^4]:    12 According to Watson [2000], the marginal improvement in forecast performance achieved by the addition of information variables in terms of the coefficient of determination rapidly decreases with the increase of the total number of information variables.

[^5]:    ${ }^{13}$ For equation (1), we select an order of lag by Akaike's information criterion (AIC) in performing a rolling estimation with the sample being the past 40 quarters. Stock and Watson [2001] fix the starting point of the sample and conduct recursive estimation that uses all subsequent data, but we suspect that this recursive estimation is more subject to the effects of changes in economic structure and in the information contained by variables, as the sample length increases. In fact, when we performed recursive estimation on the AR model used as a benchmark below, the forecast performance generally worsened from that of rolling estimation. We adopt AIC rather than Bayesian information criterion (BIC) for a similar reason: when BIC was used to select the order of lag, the forecast performance of the AR model generally worsened when compared to AIC.
    14 Similar to Stock and Watson [2001], we employ the AR model as a benchmark. However, the limitations of AR models that rely on only information from past explained variables must be noted.

[^6]:    ${ }^{15}$ When the sample period used for estimation in a forecast model is from period 0 to period $T$, "insample forecast" indicates the forecast values between period 0 and period $T$. In contrast, the "out-ofsample forecast," dealt with in this paper, refers to the forecast values for $T+l$ and later. Out-of-sample forecast is a forecast calculated with only the information available at the time of the forecast, and is more appropriate for the evaluation of the relative merits of forecast models. It should be also noted that although all of the data used in this paper are final revisions, this kind of data is usually unavailable at the time of forecast. For this reason, a precise description of the out-of-sample forecasting in this paper should be "simulated out-of-sample forecasting".
    ${ }^{16}$ We use final revisions of data in the following analysis. Bernanke and Boivin [2001] apply the method in Stock and Watson [2001] both to real-time data and to data sets consisting of only final revisions, showing that there was no significant difference in the forecast performance of the two.
    ${ }^{17}$ For variables that are directly affected by consumption tax (nominal/real GDP, GDP deflator, CPI, new car registrations, sales of large-scale retail stores, sales of department stores), the effect of consumption tax was removed using the X12-ARIMA seasonal adjustment option. On the other hand, not only the effect of consumption tax but summer power prices is excluded from domestic wholesale prices.

[^7]:    ${ }^{18}$ Hereafter we define italic $\operatorname{word}(s)$ in parenthesis as the abbreviating code(s) of variable, transformation, or both. See Charts 1 and 2 for the codes corresponding to each variable.
    ${ }^{19}$ Looking at the individual sample periods, it appears that some bivariate forecasts including the nominal GDP ( $n g d p$ ) and wage (wage) in prices/wage, etc. outperformed the AR forecasts. Moreover, M2+CD (mon2) and real bank lending (rlended) are effective in all periods after the collapse of the bubble. However, performance deteriorates for all of these if the sample period or number of forecast periods is changed.
    ${ }^{20}$ On average, nominal GDP minimizes the relative MSFE across all forecast horizons and sample periods, but even in this case the MSFE exceeds one in the two-quarter ahead forecast for 1995-99 and eight-quarter ahead forecast for 1987-90, with forecast performance worse than the AR model.

[^8]:    ${ }^{21}$ With regard to monetary aggregates, in four-quarter and eight-quarter ahead forecasts, monetary base (mon0 ln1d) and M2+CD (mon2 ln1d) show improvements over longer periods than other variables. Moreover, with regard to asset prices, forecasts improves over two consecutive sample periods in fourquarter and eight-quarter ahead forecasts for nominal exchange rates (rateav lnld), effective exchange rates (efrateav gap), the Nikkei average (nikav lev), TOPIX (tpxav ln1d), etc.

    22 The only individual variable that continually improved forecasts from the bubble formation period to the post-bubble period is M2+CD (mon2 ln1d). However, the relative MSFE for this variable in the 1983-86 period is consistently greater thanone, and in four-quarter ahead forecasts the relative MSFE also exceeded one in the 1987-90 sample period.

[^9]:    23 As mentioned in footnote 10, forecasts using principal components can be interpreted as forecasts based on dynamic factor models when there are a large number of variables.
    ${ }^{24}$ As defined in footnote 18, the italic word in parenthesis refers to the code of transformation.
    25 We have also examined forecast models that contained the first to fourth principal components in the regression equation extending Equation (1), but many of these show inferior performance to the model that only uses the first principal component (The same result was found in Stock and Watson [1999]). For this reason, only results for the model using the first principal component are shown in this paper.

[^10]:    ${ }^{26}$ Following Stock and Watson [1999], we employ a modified form of ridge regression in which the each weight in $w_{t}$ converges on $1 / n$ as the parameter $k$ increases. A weighted mean based on this ridge regression approaches the simple mean as $k$ increases. For example, the weighted mean is approximately $50 \%$ closer to the simple mean when $k=1$.
    ${ }^{27}$ For weighting estimation using ridge regression, the estimation results destabilize due to the decreased number of samples if only data for the most recent 40 quarters is used. As forecast performance was actually reduced, we use the data from start to finish.
    ${ }^{28}$ Stock and Watson [1999] state that performance is best for $k=1$, and curtail the results using other parameters.
    ${ }^{29}$ We select the variables used in forecast combination by variable type according to the following procedure. (We also use the same groups as the group of variables used in forecast using the principal component.)

[^11]:    ${ }^{32}$ Note that for two-and eight-quarter ahead forecasts of economic activity (react) and money (money), the performance of ridge regression is better than the performance of other forecast combinations in the periods of 1991-94 and 1995-99, but these all perform extremely poorly compared to AR forecasts in 1987-90.
    ${ }^{33}$ The relative MSFE was 1.00-1.01 for some series (mean, tr.mean) in eight-quarter ahead forecasts for 1987-90, showing that forecasts were not improved in some cases.

[^12]:    ${ }^{34}$ This view is contrary to the conclusion in Stock and Watson [1999] that the best forecast performance could be obtained using the principal component. However, it must be noted that the difference in forecast performance between combination methods was not large even in their results and that they restricted their analysis to one year forecasts of inflation. Moreover, our results suggest that a trimmed

[^13]:    mean is not necessarily the most preferable method of forecast combination while Stock and Watson [2001] focus on the trimmed mean that is less susceptible to outliers. This finding, we suspect, is consistent with the limitation on forecast combination where improvement in forecast performance stops after two to four variables. We discuss this point in detail in the following section.
    35 However, it is easy to envision that a similar mechanism to the one shown below is also working in multivariate forecasts other than simple averages. As the combination of information comes before the creation of forecast series in the method using principal component analysis, the argument below is not directly applicable, but insofar as noise is leveled, a mechanism similar to the one described below can be considered to be in place.

[^14]:    ${ }^{36}$ It should be noted that because $e_{i t}$ is the out-of-sample forecast error, the average does not always equal zero.
    ${ }^{37}$ Letting $f_{i t}$ denote the bivariate forecast using the individual variable $i$, the forecast value of forecast combination using a simple mean is $1 / n \sum_{i=1}^{n} f_{i t}$. If $y_{t}$ is the realized value of the variable to be forecast, then $e_{i t}=f_{i t}-y_{t}$ and the forecast error for forecast combination using a simple mean is $1 / n \sum_{i=1}^{n} f_{i t}-y_{t}=1 / n \sum_{i=1}^{n}\left(f_{i t}-y_{t}\right)=1 / n \sum_{i=1}^{n} e_{i t}$.

[^15]:    38 The necessary and sufficient condition for equality in Equation (5) is that all of the terms on the righthand side of Equation (5) are zero. This is equivalent to the original forecast values being equal in each of the periods. ( $e_{1 t}=\ldots=e_{n t}\langle t=1, \ldots, T\rangle$ )
    ${ }^{39}$ For this reason, even if individual original forecasts are unstable, the performance of forecasts that combine these could possibly be stabilized.
    ${ }^{40}$ Incidentally, when the optimum weights are used in combination,

    $$
    M S F E_{\text {com }, n} \leq \min \left\{M S F E_{1}, \ldots, M S F E_{n}\right\} .
    $$

    holds true (see, for example, Clements and Hendry [1998]), and this forecast combination outperforms all original forecasts. However, as shown by the forecast results for weighted averages in the previous section, it is difficult to know the "optimum weights" at the time the forecast is made. The result shown above means that, even if the "optimum weights" cannot be calculated, simple mean is enough to guarantee better performance of the forecast combination than the performance of a simple mean.

[^16]:    ${ }^{41}$ The stepwise optimization used in this paper does not completely cover all combination possibilities, and it cannot be assured that the optimum grouping has been reached. However, the number of groupings is approximately 2.1 billion $\left(=2^{31}-1\right)$ for CPI and 4.3 billion $\left(=2^{32}-1\right)$ for GDP forecasts, and it is not realistic to consider all of these because it requires unmanageable amount of calculation. For more realistic exercise, we have checked the complete forecast performance by using combinations using up to 8 series. In fact, as shown in the analysis below, the optimal combination grouping found using a stepwise method is a group of two to four series. This suggests that eight are large enough to check whether the stepwise method gives us the optimum. The results of that exercise confirmed that these results are completely consistent with the stepwise results.

[^17]:    ${ }^{42}$ The four-quarter ahead CPI forecast for 1987-90 was the only case that no forecast combinations could improve upon the best bivariate forecast.

[^18]:    ${ }^{43}$ Here, we are focusing on the combination reached using the stepwise optimization procedure. There are some cases where improvements can be seen for up to eight to nine variables depending on the initial variable, but cases such as these are always inferior to the stepwise optimum.
    ${ }^{44}$ Incidentally, more than $90 \%$ of the forecast error of all bivariate forecasts is explained by up to the second principal component for most forecast horizons when principal component analysis is performed.

[^19]:    This fact appears to be consistent with the results for when the limit on series to be combined is two to four.

    45 It should be noted that if the sample period is divided up, this obviously leads to a relaxing of the restrictions on optimization, and the performance of the best grouping always exceeds that of the "second best."

    46 Of course, there are some cases in which the gap between the best and second best is not large despite having few variables in common. Thus, the presence of common variables is not necessarily important in all cases.

