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Abstract

Empirical studies on the information content of option prices have focused on exploring whether implied volatility contains useful information regarding the future fluctuation of underlying asset prices. If expectation formation in the option markets reflects all the currently available information regarding future price movements, option prices will be useful in forecasting the price fluctuation of underlying assets. This paper extends such an analytical framework to implied probability distribution as a whole and examines its information content by using Japanese stock price index option data (on a daily basis) from mid-1989 to mid-1996. To this end, the following questions are examined: (1) whether the implied probability distribution is a good forecast for the subsequently realized distribution of stock price fluctuations, and (2) whether a leads and lags relationship exists between stock price changes and changes in the shape of the implied probability distribution. The estimation results show that (1) the implied probability distribution contains some information regarding future price movements, but its forecasting ability is not superior to that of the historical distribution, and that (2) the shape of the implied probability distribution contains some information on forecasting stock price changes as well as responding to stock price fluctuations. However, it should be noted that such results are highly sensitive to the choice of sample period, suggesting that the information content depends on macroeconomic and financial market conditions. Therefore, the information contained in an implied probability distribution is difficult to interpret automatically as an information variable for monetary policy, and further studies are needed on how to make use of information contained in implied probability distributions.

Keywords: Information content of option prices, Implied probability distribution, SUR estimation, Autocorrelation, Granger causality.

JEL classification code: E44, E52, G13.

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I. Introduction

In this paper, I will examine the information content of an implied probability distribution derived from a set of option prices, by applying formal econometric procedures.

Empirical studies of the information content of option prices have focused on whether implied volatility contains useful information regarding the future fluctuation of underlying asset prices, i.e., whether or not, if expectation formation in the option markets reflects all the currently available information on future price movements, option prices will be useful in forecasting the price fluctuation of underlying assets.

For example, Lamoureux and Christopher (1993) found that the implied volatility estimated from individual stock price options was a biased forecast, but provided useful information on forecasting stock price fluctuations for three to six months ahead. Day and Lewis (1992) compared implied volatility with various GARCH models, and pointed out that these forecasts were unbiased, although their study was inconclusive on which predictor was better. By contrast, Canina and Figlewski (1993) concluded that implied volatility from stock price index options (S&P100) was not a superior indicator to historical volatility.

In the meantime, various methods have been developed to estimate the entire implied probability distribution of future values of underlying assets from a set of option prices with the same time-to-maturity, but with different exercise prices.[1] These methods enable us to obtain information on the dispersion of market expectations concerning asset price fluctuations, as well as on market participants' beliefs about the direction of market price changes and the probability of an extreme outcome in the market.

Looking at empirical studies in Japan, our previous study, by Nakamura and Shiratsuka (1999), estimated the implied probability distribution from mid-1989 to mid-1996 on a daily basis, using Nikkei 225 stock price index options and JGB futures options. In this study, we found typical patterns in the relationship between changes in stock prices and the shape of the implied probability distribution. That is, (1) the standard deviation rises when stock prices move substantially, (2) skewness moves in an opposite direction in accordance with the rise and fall of stock prices, and (3) excess kurtosis becomes highly volatile in a period of market turbulence. In addition, by examining such a typical pattern, we succeeded in revealing the impact of external

[1] See Oda and Yoshiba (1998) and Söderlind and Svensson (1997) for details on how to estimate implied probability distribution from option prices.

shocks on financial markets and the speed at which they adjust.

This paper examines empirically the information content of an implied probability distribution by using the data employed in our previous study, Nakamura and Shiratsuka (1999). In other words, this study applies the analytical framework used in the previous studies of the information content of implied volatility to implied probability distribution as a whole, and examines its information content.

The rest of the paper is constructed as follows. Section II summarizes the theoretical foundation of the estimation of an implied probability distribution from option prices. I also describe the trading framework of the Nikkei 225 stock price index options, and examine the time-series properties of summary statistics for implied probability distribution that are used in empirical analysis in the following sections. Then Section III examines whether implied probability distribution contains useful information for forecasting the subsequently realized distribution of stock price changes, compared with the historical distribution of stock price changes (for the preceding 30 business days). Section IV explores whether the shape of the implied probability distribution provides useful information for forecasting future stock price changes by estimating VAR (vector autoregression) models to check Granger causality among stock price fluctuations and summary statistics of implied probability distribution. Finally, Section V summarizes the main empirical results of this paper and discusses topics for future research.

II. The Basic Framework for Estimating Implied Probability Distribution

In this section, I explain the basic framework for estimating implied probability distributions from option prices. In addition, I describe Japanese stock price index options data, and examine the time-series properties of summary statistics of estimated implied distributions obtained on a daily basis.

A. The Estimation of Implied Probability Distributions

1. The basic framework for estimating an implied probability distribution

First, I will explain the basic framework for estimating a probability density function from option prices. Supposing a risk-neutral market player, the price of a European-type call option (C) is given by

$$C = \exp[-r(T-t)]E[\max(0, F_T - K)]$$
$$= \exp[-r(T-t)]\int_{-\infty}^{+\infty} w(F_T)\max(0, F_T - K)dF_T \ , \quad (1)$$

where F_t, K, r, and $w(F_t)$ denote the price of an underlying asset on the expiration date ($t = T$) of an option, the strike price of the option, a risk-free interest rate, and a risk-neutral probability density function for the value F_t, respectively. Since net cash settlement is executed on the expiration date, the pricing formula for a call option in equation (1) can be simplified as follows:

$$C = \int_{-\infty}^{+\infty} w(F_T)\max(0, F_T - K)dF_T \ . \quad (2)$$

In addition, if one lets C_k, and C_{kk} be the first and second derivatives of the call option price, then the call option price satisfies the following conditions:

$$C_K = -\int_{K}^{+\infty} w(F_T)dF_T \ , \quad (3)$$

$$C_{KK} = w(K). \quad (4)$$

These equations indicate that the first and second order derivatives of option prices respectively correspond to the probability density and cumulative distribution functions of risk neutral probability on the underlying assets.

Similarly, if one lets P, P_k, and P_{kk} be the European-type put option price, and the first and second derivatives of the put option price, respectively, then the following equations are derived:

$$P = \int_{-\infty}^{+\infty} w(F_T)\max(0, K - F_T)dF_T \ , \quad (5)$$

$$P_K = \int_{-\infty}^{K} w(F_T)dF_T \ , \quad (6)$$

$$P_{KK} = w(K), \quad (7)$$

In the practical application of this approach, however, a problem arises: there is only a finite number of strike prices. This implies that equations (3), (4), (6), and (7) do not hold strictly true with respect to the observed market prices because they assume that the variable C is continuous in K. Therefore, as Breeden and Litzenberger (1978) and Neuhaus (1995) have proposed, the first-order finite difference method is applied to equation (3) to obtain the $p(F_t \geq K_i)$ payoff probability that an underlying asset price on the expiration date ($t = T$) F_t exceeds a strike price K_i as follows:

$$p(F_T \geq K_i) \approx \frac{C_{i-1} - C_{i+1}}{K_{i+1} - K_{i-1}}, \tag{8}$$

where C_i indicates the option premium corresponding to the strike price K_i. Therefore, this yields the probability as

$$p(K_i) = p(F_T \geq K_i) - p(F_T \geq K_{i+1}) \approx \frac{C_{i-1} - C_{i+1}}{K_{i+1} - K_{i-1}} - \frac{C_i - C_{i+2}}{K_{i+2} - K_i}. \tag{9}$$

Analogously, from equation (6), the probabilities from the put option prices are given as

$$p(K_i) = p(F_T \geq K_{i+1}) - p(F_T \geq K_i) \approx \frac{P_{i+2} - P_i}{K_{i+2} - K_i} - \frac{P_{i+1} - P_{i-1}}{K_{i+1} - K_{i-1}}. \tag{10}$$

Considering the problem that in-the-money options tend to be priced incorrectly, our previous study, Nakamura and Shiratsuka (1999), employs the following procedure to estimate a complete probability distribution: we first calculate two different probability distributions from the option premium for out-of-the-money call and put options separately, then combine these probability distributions to the complete probability distribution.[2] In other words, we use the probability distribution derived from out-of-the-money put options in the lower range from the at-the-money strike price, and that derived from out-of-the-money call options in the upper range.

2. Summary statistics

I employ a time-series of summary statistics, such as mean, standard deviation (*Stdv*), skewness (*Skew*), and excess kurtosis (*Ex-Kurt*), for the estimated implied probability distribution to investigate its information content.

Since stock prices are positive values, the expectation distribution of future assets will approximately follow a lognormal distribution. In other words, the distribution can

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[2] Needless to say, even though they are out-of-the-money options, deep-out-of-the-money options tend to be priced incorrectly. Therefore, we excluded the observation in estimating the complete probability distribution, if the calculated relative frequency fell negative.

be expected to be skewed to the right. However, it is not convenient to use a lognormal distribution as a benchmark for evaluating the size of the above summary statistics. Therefore, we calculate these summary statistics by using the strike price converted into a logarithm, and employ a normal distribution as the benchmark for evaluating the four summary statistics.

Since I employ the both put and call options data to estimate the entire implied probability distribution, by applying equations (9) and (10), the aforementioned four summary statistics can be shown as follows:

$$\mu = \sum \frac{\ln(K_i) + \ln(K_{i+1})}{2} p(K_i) , \tag{11}$$

$$Stdv = \sqrt{\sum \left(\frac{\ln(K_i) + \ln(K_{i+1})}{2} - \mu\right)^2 p(K_i)} , \tag{12}$$

$$Skew = \sum \left(\frac{\ln(K_i) + \ln(K_{i+1})}{2} - \mu\right)^3 p(K_i) \Big/ Stdv^3 , \tag{13}$$

$$Ex - Kurt = \sum \left(\frac{\ln(K_i) + \ln(K_{i+1})}{2} - \mu\right)^4 p(K_i) \Big/ Stdv^4 - 3 . \tag{14}$$

While the mean of the estimated risk-neutral implied probability shifts in parallel with the size of the risk premium compared with the true probability distribution, the risk premium itself is difficult to estimate.[3] Thus, in the following, we focus on changes of moments higher than the first, i.e. the mean, and examine their movements over time.[4]

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[3] The model used in this paper assumes a risk-neutral world, while market participants in the actual market are not necessarily risk-neutral. In addition, it is likely that the risk preferences of market participants change over time. In that case, the (risk-neutral) implied probability distribution, estimated under the assumption of risk-neutral valuation, will differ from the true probability distribution. However, Cox and Ross (1976) have claimed that, compared with the true probability distribution, the risk-neutral implied probability distribution shifts in parallel with the size of the risk premium, and thus will not affect moments higher than the second.

[4] As Bates (1991) and others have pointed out, it is generally known that standard deviation decreases as the maturity date of the contract is approached. Therefore, we try to control the impact of changes in time-to-maturity on the estimated time-series of standard deviation by multiplying the square root of (360/time-to-maturity) to obtain the annual rate.

B. Data and Some Reservations

The Nikkei 225 option (a European-type option) is listed on the Osaka Stock Exchange (OSE) and began trading in June 1989. The contract months are four consecutive near-term expiration months. Five strike prices are set: 1,000 yen and 500 yen above and below the strike price closest to the central price, which is initially set as the closing price of the Nikkei 225 on the business day before the first day of trading. The last trading day is the business day before the second Friday of each expiration month, and the option can be exercised on the business day following the last trading day.

Taking into consideration data limitations, our previous study, Nakamura and Shiratsuka (1999), applied the aforementioned simple discrete approximation method to carefully sorted data. First, since there is only a very thin trading volume for in-the-money (ITM) strike prices, the reliability of price information is not entirely satisfactory. Therefore, we used price data regarding both put and call options that are at-the-money (ATM) and out-of-the-money (OTM),[5] although most of the empirical studies estimating implied probability distribution from option prices use either a put or a call option.

Second, we have excluded mispriced observations, such as those that result in a negative probability density, in estimating the complete probability distribution. This is because using closing price data does not guarantee that the option premiums of different strike prices were traded at the same time, and arbitrage may not function thoroughly.

C. Time-Series Movements of an Implied Probability Distribution

Next, I examine the time-series properties of computed summary statistics for implied probability distributions. Summary statistics are presented in Table 1.

In this table, the standard deviation of *Stdv* is very small, but large for *Skew* and *Ex-Kurt*, suggesting that these summary statistics of implied probability distributions show very volatile movements. In particular, *Ex-Kurt* easily takes an extreme value,

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[5] More precisely, we first calculate two probability distributions from the option premium for call and put options separately, then combine two probability distributions at the ATM strike price to form the complete probability distribution. In this sense, our methodology has the merit of effectively utilizing trading price information by making the ATM strike price the boundary and using both put and call options on the OTM side. Therefore, in estimating implied probability distribution, there is an advantage in using limited quotations of option premiums across different strike prices, such as in Japanese option markets.

considering that its maximum value diverges from the median and mean, and excess kurtosis is also very large.

Looking at the autocorrelation, large autocorrelation coefficients for *Stdv* persist even for the higher order of lag length, while *Skew* and *Ex-Kurt* converge to zero after the ten- and five-period lags, respectively. This indicates that the fluctuation of *Std*v is sticky or persistent but that fluctuations in *Skew* and *Ex-Kurt* are not.

Moreover, regarding the cross-correlation between summary statistics of the implied probability distribution and stock price fluctuations (both for simple and absolute changes), *Stdv* shows a positive correlation with absolute changes in stock prices and *Skew* a negative correlation with changes in stock prices.

III. The Predictability of Realized Fluctuations

In this section, I examine empirically whether the shape of the implied distribution (ID) contains information useful in predicting the subsequently realized distribution of stock price fluctuations (realized distribution, RD) compared with the historical distribution of stock price fluctuations during a certain past period (historical distribution, HD).

A. Fluctuation of Realized, Implied, and Historical Distributions

I first describe the data that is used in the empirical investigation of forecasting performance.

ID (implied distribution) is the implied probability distribution that is computed from a set of option prices (closing price) with the same time-to-maturity, but with different exercise prices. RD (realized distribution) is the subsequently realized distribution of stock price changes during the period from the option trading day to maturity day while HD (historical distribution) represents the distribution of stock price changes during the preceding 30 business days. By using summary statistics for these three distributions, I will examine the forecasting power of ID on RD, compared with HD.[6]

1. Time-series movements

Figure 1 plots the daily movement of stock price changes and summary statistics for ID, RD, and HD. In general, since RD and HD are computed from actual changes in stock prices, both move up and down substantially when stock prices fluctuate greatly, as RD

[6] Among summary statistics, standard deviation is annualized by multiplying the square root of 250.

leads to HD, according to the given definitions. By contrast, ID exhibits a relatively stable movement.

Looking at each summary statistic in turn, standard deviations of ID and RD show a similar tendency, suggesting that ID responds quickly to changes in RD, which is computed from the subsequently realized changes in stock prices. However, it should be noted that when the market level moves substantially, RD fluctuates greatly, while ID moves within a relatively narrow range. On the other hand, the skewness and excess kurtosis of ID and RD move very differently, indicating that they could provide us with different kinds of information.

2. The stability of cross-correlation over time

Next, I examine how the cross-correlation of RD with ID and HD changes over time. Figure 2 plots time-series movements of the coefficients of this cross-correlation, as well as the acceptance region of the null hypothesis for no cross-correlation at 10-percent significance in two-sided hypothesis testing, shown as a shaded area in the figure.[7]

With respect to the standard deviation, the coefficients of the cross-correlation of RD with ID and HD show a very similar tendency over time. Such correlation is generally positive: among 1,523 subsamples, positive correlation is observed in 56.6 percent for RD and ID and 67.9 percent for RD and HD, while negative correlation is 10.9 and 14.8 percent, respectively.

However, looking at this figure in detail, the correlation between RD and ID declines from end-1989 to mid-1990 by comparison with that between RD and HD. In addition, both the correlation between RD and ID and that between RD and HD turn significantly negative during the periods between end-1991 and early 1992, and in early 1995, which correspond to periods of market turbulence in the aftermath of a sharp decline in stock prices. The low correlation observed between RD and ID during these periods is consistent with a casual observation of Figure 1, where ID shows relatively stable movement and its response to market fluctuation is limited compared with RD and HD.

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[7] The critical level of the coefficients of cross-correlation (r_a) is 0.123 at ten-percent significance in two-sided test with 180 samples, based on the equation as follows:

$$r_a = t_\alpha / \sqrt{t_\alpha^2 + (n-2)}\ ,$$

where t_α, n denote two-sided α percentile of the Student's t distribution, and number of sample, respectively.

Meanwhile, regarding the skewness and excess kurtosis, the coefficients of the cross-correlation of RD with ID and HD exhibit a very different tendency over time, and such correlation is relatively weak. In particular, correlation between RD and HD for both skewness and excess kurtosis exhibit negative in more than half of the subsamples: 69.1 percent for skewness and 59.4 percent for excess kurtosis.

B. Estimation Equations and the Tested Hypothesis

Next, I conduct regression analysis to examine how well ID forecasts RD, compared with HD. The following three equations are estimated to examine whether ID or HD forecasts RD better.[8]

$$RD_t = \alpha + \beta ID_t + \varepsilon_t , \tag{15}$$

$$RD_t = \alpha + \beta HD_t + \varepsilon_t , \tag{16}$$

$$RD_t = \alpha + \beta_1 ID_t + \beta_2 HD_t + \varepsilon_t , \tag{17}$$

where RD_t, ID_t, and HD_t indicate summary statistics (standard deviation, skewness, or excess-kurtosis) for the realized distribution (stock price fluctuation between trading date to expiration date), the implied distribution (computed from a set of options with the same time-to-maturity, but with different strike prices), and the historical distribution (stock price fluctuation during the preceding 30 business days) for at the time period of t, respectively.

Regarding equations (15) and (16), if ID and HD are unbiased forecasts of RD, the estimates of α and β will respectively be close to 0 and 1. However, even if the null hypothesis of β =1 is rejected, rejection of the null hypothesis of β =0 in each equation suggests that ID and HD respectively have some predictive power for the RD. In equation (17), if the null hypothesis of either β_1 =0 or β_2 =0 is rejected, either ID or HD contains some useful information for forecasting the future fluctuation of RD. In this case, if both ID and HD independently contain useful information, β_1 and β_2 are simultaneously significantly different from zero. By contrast, either ID or HD reflects

[8] Predictability tests are conducted with the following two-step procedure: first, ID and HD are estimated, and, then, parameters for equations (15) to (17) are estimated. In this case, one should be careful for the errors-in-variables problem: that is, if the estimates in the first step are not consistent, estimates in the second step will be biased. However, Jorion (1995) shows that impact of the errors-in-variable problems is not so serious by simulation exercises. Therefore, I do not make any particular adjustment on these problems.

all the information contained in the other, in which case the estimated parameter for the encompassed indicator will be insignificantly different from zero.[9]

In addition, I conduct a *J*-test to compare the performance of non-nested models of equations (16) and (15).[10] To this end, I estimate the following equations and test the significance of the null hypothesis assuming that γ and δ in equations (18) and (19) are equal to zero.

$$RD_t = \alpha + \beta ID_t + \gamma \hat{RD}_t(HD) + \varepsilon_t, \tag{18}$$

$$RD_t = \alpha + \delta \hat{RD}_t(ID) + \beta HD_t + \varepsilon_t, \tag{19}$$

where $\hat{RD}_t(HD)$ and $\hat{RD}_t(ID)$ denote estimated summary statistics for RDs in equations (15) and (16), respectively. Possible results of the above hypothesis testing fall into one of the following four cases shown in Table 2, according to the combination of test results on the pair of non-nested null hypotheses for equations (18) and (19).

C. Full-sample Estimation Results

In the following, I first check the forecasting performance of each summary statistic in turn by estimating single equation models. Then, I examine the forecasting performance of the three summary statistics simultaneously by estimating an SUR (seemingly unrelated regression) model that takes account of the correlation among error terms. This is important because the three summary statistics, ID, RD, and HD, are jointly distributed.

In doing so, it should be noted that least-square estimates might be biased and inconsistent since error terms are serially correlated. Such serial correlation is provoked because the expiration date is fixed for each trading month and the data frequency is shorter than the life of the options, thus implying that forecast horizons inevitably

[9] Procedure to test the forecasting performance of implied probability distribution is the same as in Fair and Shiller (1990), which compared the forecasting performance of different macro econometric models.

[10] The *J*-test is one of the non-nested hypothesis testing procedures proposed by Davidson and MacKinnon (1981). See, for example, Davidson and MacKinnon (1993), for details of the *J*-test. In nested hypothesis testing, the null hypothesis is a special case of alternative hypothesis, while, in non-nested hypothesis testing, the null hypothesis is not a special case of alternative hypothesis, and they do not encompass each other.

overlap.[11] In addition, such overlapped forecast horizons vary as time-to-maturity changes. Therefore, for both the single equation estimation and the SUR estimation I apply Newey and West's (1987, 1994) procedure to adjust serial correlation in computing standard errors for estimated coefficients by automatically deciding the bandwidth of serial correlation adjustment.[12]

1. Single equation estimation

Table 3 summarizes estimation results on the forecasting power of each summary statistic separately in a single equation model.

With respect to the standard deviation, estimation results using ID or RD alone (equations (15) and (16)) strongly reject the null hypotheses of $\beta=0$, suggesting that both ID and RD contain some useful information for predicting the future realized distribution of stock price fluctuations. However, at the same time, those results also strongly reject the null hypotheses of $\beta=1$, indicating that such forecasts are biased. The estimation result of equation (17) indicates that HD has superior forecasting power to ID, since β_1 shows a negative sign and is insignificantly different from zero, while β_2 is significantly different from zero.

The above results are consistent with those in Canina and Figlewski (1993) that analyze US stock price index options, although they contradict the results of the Bank of Japan (1995) that studied Japanese stock price index options and concluded that both historical and implied volatilities jointly possess explanatory power. Here, in comparing these results, the following points should be noted. First, as I will show later, the information content of option prices is highly sensitive to sample periods. Second, the Bank of Japan (1995) employs OLS standard errors in hypothesis testing, even though there exists a significant autocorrelation among residuals.

Next, turning to the estimation results of skewness and excess kurtosis, both ID and HD are deemed to have poor forecasting power. The estimation results of equation (15) using ID as an explanatory variable for both skewness and excess kurtosis show that the null hypotheses of $\beta=0$ are not rejected at the 5-percent significance level, but that the null hypotheses of $\beta=1$ are rejected. In addition, regarding HD in equation (16),

[11] Regarding the overlapping observation problems, Hansen and Hodrick (1980) examine them in detail by studying the predictability of forward rates on future spot rates in foreign exchange markets. Analogously, Canina and Figlewski (1993) and Lamoureux and Christopher (1993) examined such problems in testing the predictability of implied volatilities in stock markets. For the analysis in foreign exchange markets see also Jorion (1995), West and Cho (1995), and Hara and Kamada (1999).

[12] All the estimations in this section were made by using GAUSS for Windows NT/95 Version 3.2.38.

coefficients are estimated to be negative and inconsistent with the expected positive sign. In estimation results for equation (17), both β_1 and β_2 are negative for excess kurtosis, and β_1 is positive but insignificant for skewness. Therefore, regarding skewness and excess kurtosis, it is confirmed that neither ID nor HD contains useful information for predicting RD.

Table 4 shows the results of the *J*-test in single equation estimation for each set of summary statistic. On the one hand, the null hypotheses of γ=0 are generally rejected: for standard deviation and excess kurtosis at the 1-percent significance level, and for skewness at the ten-percent significance level. On the other hand, the null hypotheses of δ=0 are not rejected. Therefore, HDs are deemed to be better forecasts for RDs than IDs in all the summary statistics.

2. SUR estimation

Next, I simultaneously estimate equations for three summary statistics (standard errors, skewness, and excess kurtosis) by applying an SUR (seemingly unrelated regression) model that takes into account the correlation among residuals in estimation equations for them. Estimation results are reported in Table 5.

Looking at the standard deviation, estimated values are almost same as those in the single equation estimation. Moreover, their statistical significance is unchanged, while their standard errors slightly decline. Concerning the estimates for skewness and excess kurtosis, although some estimated values differ from those in the single equation estimation shown in Table 3, estimated coefficients are negative and are inconsistent with the expected positive sign. Thus, the basic conclusion that neither ID nor RD contains useful information in predicting RD holds true.

Table 6 reports the results for the *J*-test that examines the non-nested hypothesis of the predictive power of ID and HD. Only the result for standard deviation is comparable to that for the single equation estimation shown in Table 3. That is, the estimated coefficient is statistically significant only in standard deviation of HD, and the remaining coefficients are all insignificant. Therefore, ID and HD are deemed to be poor forecasts of the subsequently realized distribution of stock price fluctuations, except for the standard deviation of HD.

D. Rolling Estimation Results

Estimation results so far might be sensitive to sample periods since the information content of option prices seems to depend highly on macroeconomic and financial market conditions. In the following, I conduct rolling regressions using a subsample of 180 business days’ data, to check the stability of estimation results with the SUR model.

First, Table 7 summarizes the results of hypothesis testing for estimated equations (15) - (17).[13] On standard deviation, the basic conclusion with respect to the full sample estimations, that is, HD is a better predictor for RD, holds. HD is superior to ID in terms of forecasting power for RD in 35 percent of cases (14.5 percent [β(HD)>0 and β_2>0] + 20.3 percent [β(ID)>0 and β(HD)>0, and β_2>0] + 0.0 percent [β(HD)>0, and β_1>0 and β_2>0]), while ID is superior to HD in 21 percent of cases (15.8 percent [β(ID)>0 and β_1>0] + 4.8 percent [β(ID)>0 and β(HD)>0, and β_1>0] + 0.1 percent [β(ID)>0, and β_1>0 and β_2>0]). Both ID and HD have predictive power, but are difficult to be ranked in 17 percent of cases (β(ID)>0 and β(HD)>0 as well as β_1>0 and β_2>0).

With respect to skewness, a significant predictive power for ID is detected in 9.0 percent of sample periods, and 0.5 percent in the case of RD, suggesting that ID might have better forecasting power than HD. However, it should be noted that effective predictive power is observed in very limited time periods, considering that significant predictive power is detected in just 10 percent of sample periods. Meanwhile, for excess kurtosis, predictive power is observed for HD in only 1.7 percent of cases.

Second, Table 8 reports the results of *J*-test, based on the rolling estimation results for equations (18) and (19).[14] Results are generally contrasting to those for the full sample estimation in Table 6, suggesting that information content of IPDs are highly sensitive to the changes in sample periods. Looking at standard deviation, the table shows that ID is better forecasts of RD in 21 percent of cases, HD in 17 percent, and both are informative but inconclusive in 56 percent, suggesting that ID is slightly better forecasts for RD than HD. Regarding skewness and excess-kurtosis, the table indicates that both ID and RD contain useful information but are hard to compare in most cases, though such results are not so reliable as that for standard deviation due to high standard errors for estimated coefficients of γ and δ.

Application of rolling regression technique reveals that predictive powers of HD and ID on RD are at best highly sensitive to the changes in sample period. Regarding skewness and excess kurtosis, predictive power is relatively low in the sense that significant and reliable predictability is detected only in very short periods. Standard

[13] By applying Newey and West's (1994) procedure, bandwidths are automatically chosen as 8 or 9 business days in most of the subsample periods: mean of the bandwidths are 8.542, 8.656, and 8.480 for equations (15) - (17), respectively.

[14] By applying Newey and West's (1994) procedure, bandwidths are also automatically chosen 8 or 9 business days in most of the subsample periods: mean of the bandwidths are 8.556 and 8.601 for equations (18) and (19), respectively.

deviation is much better predictor for the subsequently realized distribution, though such information is highly instable over time and relative superiority between HD and ID is inconclusive. Figure 3 shows the subsample periods that superior predictive power is detected in standard deviations for either HD or ID, in the sense that one of the two is proven to be better forecast in both tests in Table 7 and Table 8. In this figure, dark and light shadows indicate that HD and ID respectively have superior predictive power than the other. In fact, it is hard to tell a general regularity of their information content in relation to stock market developments.

IV. The Relationship with Stock Price Fluctuations

In this section, I explore whether the shape of the implied probability distribution could forecast future stock price fluctuations.

A. Correlation between Market Fluctuation and Implied Distribution

As we have already seen in Section II, there exist typical patterns between stock price fluctuations and summary statistics for an implied probability distribution. That is, (1) the standard deviation rises when stock prices move substantially, (2) skewness moves in the opposite direction in accordance with the rise and fall of stock prices, and (3) excess kurtosis becomes highly volatile in a period of market turbulence.[15] Let me begin my statistical analysis of these relationships by checking the stability of cross-correlation over time, and examining dynamic cross-correlation.

1. The cross-correlation with market fluctuations

In order to check the stability of the relationship between changes in market level and changes in the shape of an implied probability distribution, Figure 4 plots the coefficients of cross-correlation between stock price fluctuations and summary statistics for an implied probability distribution over time. In this figure, the upper and lower panels show the coefficients of correlation of summary statistics with proportional changes and absolute changes in stock prices, respectively. The acceptance region for

[15] See Nakamura and Shiratsuka (1999) for details of the relationship between stock price fluctuations and changes in the shape of an implied probability distribution. They examine various episodes in Japanese financial markets from 1989 to 1995 to examine the changes in market sentiment and their impact on financial markets.

the null hypothesis of no correlation is also shown as a shaded area in the figure.[16] Estimation periods are subsamples of 180 business days that begin each day on the horizontal axis in the figure.

First, with respect to the correlation with changes in stock prices, the coefficients of correlation with skewness constantly show large and negative values, and are statistically significant in all the subsample periods. However, although the coefficient stays at large negative values of around –0.6 during the period from mid-1989 to the beginning of 1991, weakening thereafter. During the period from 1991 to early 1993, the coefficient was around –0.4 and it has since declined further.

Second, regarding the coefficients of cross-correlation with absolute changes in stock prices in the lower panel, the positive relationship between standard deviation and excess-kurtosis, which is confirmed with the entire data sample shown in Table 1, is highly unstable over time.

2. Dynamic cross-correlation with market fluctuations

Next, I check the dynamic cross-correlation between market fluctuations and summary statistics for the implied probability distribution with the entire data sample. Figure 5 plots the coefficients of dynamic cross-correlation, and indicates their lead/lag relationship as follows. By locating zero in the middle of the horizontal axis as a boundary, the summary statistics of implied probability distribution lead market changes on the left side, while summary statistics lag market changes on the right side.

First, with regard to the dynamic cross-correlation with changes in stock prices (upper panel in the figure), skewness shows a maximum negative correlation at the point of simultaneity, and its negativity gradually declines in the same direction as stock prices with a definite time lag. This suggests that skewness responds to changes in stock prices and moves in the opposite direction. However, skewness shows almost no correlation on the right side of the figure, implying that it does not seem to be a leading indicator for market fluctuations.

Second, looking at the dynamic cross-correlation with absolute changes (lower panel in the figure), the standard deviation shows a positive correlation on both the left and right sides of the figure. However, a stronger correlation is observed on the left side lagging absolute changes in stock prices, suggesting the existence of the ARCH effect: that is, a large fluctuation in stock prices leads to an increase in the standard deviation. Meanwhile, although excess kurtosis indicates a positive but weak correlation at the point of simultaneity, its correlation is generally insignificant. Skewness shows a

[16] See footnote 7 for the detail.

positive but weak correlation on the right side, which seems to be caused by the declining trend of stock prices in the sample period.

B. Granger Causality Test

Next, I estimate four-variable VAR models that consist of three summary statistics (standard deviation, skewness, and excess kurtosis) of implied probability distribution and stock price changes (simple change or absolute change).[17, 18] Then, I test Granger causality to examine their lead/lag relationship.

Table 9 reports the Granger causality test with the entire sample data set. In the estimation of VAR models, six- period and seven-period lags are chosen for the VAR model with simple changes and with absolute changes in stock prices, respectively, according to minimizing AIC (Akaike's information criterion). This result supports the basic observation of a dynamic cross-correlation between stock price changes and the summary statistics of implied probability distribution. For example, absolute changes in stock prices Granger causes both standard deviation and excess kurtosis at the 1-percent statistically significance. Changes in stock prices also Granger causes skewness at the 1-percent statistical significance level.

Moreover, the multivariate model detects another lead/lag relationship as follows. In the VAR model with changes in stock prices, standard deviation and skewness Granger cause changes in stock prices at the 1- and 5-percent statistical significance levels, respectively. In the VAR model with absolute changes in stock prices, standard deviation and skewness Granger cause absolute changes in stock prices at the 5- and 1-percent statistically significance levels, respectively. Yet, a lead/lag relationship between excess kurtosis and market fluctuations is not detected in either VAR models.

The above results indicate that the shape of the implied probability distribution not only responds to changes in market level, but also, at least as far as standard deviation and skewness are concerned, contains some information that is useful for forecasting market fluctuations.[19]

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[17] I used RATS for Windows (version 4.30) for the estimation in this section.

[18] Results of unit root test indicate that five variables used in the VAR estimation (simple and absolute changes in stock prices, standard deviation, skewness, and excess kurtosis of implied probability distribution) are stationary at least 5 percent statistical significance. Data for changes in stock prices is adjusted for the trading-day effects by the web-based program of DECOMP.

[19] It should be noted that the lead/lag relationship from skewness to changes and absolute changes in stock prices may possibly be detected because the data sample mainly covers the period after the

I also estimate VAR models using summary statistics for historical distribution (the preceding 30 business days). Table 10 summarizes the results. According to the test results, both changes and absolute changes in stock prices Granger cause standard deviation at the 1--percent statistically significance level. In the reverse direction, standard deviation and excess kurtosis Granger cause absolute changes in stock prices.

C. Granger Causality in Rolling Regressions

Above Granger causality relationship might be sensitive to sample periods since the information content of option prices seems to depend highly on macroeconomic and financial market conditions. In the following, I conduct rolling regressions on the aforementioned four-variable VAR models with subsamples of 180 business days' data in order to check the robustness of Granger causality among stock price fluctuations and the shape of the implied probability distribution over time.

Figure 6 reports *F*-values for Granger causality tests for a relationship between the standard deviation and absolute changes in stock prices (upper panel), between skewness and changes in stock prices (middle panel), and between excess kurtosis and absolute changes in stock prices (lower panel), respectively. According to these figures, although Granger causality from absolute changes to standard deviation is fairly stable, other relationships are highly sensitive to the choice of sample period.

However, Granger causality from absolute changes to standard deviation becomes temporarily insignificant during the subsample periods ending at the second half of 1992, mid-1993, and the periods after end-1994. Indeed, *F*-values for Granger causality tests fall below the 10-percent statistically significant level during these periods. These periods correspond to periods when stock prices plunged and stock markets were turbulent. During market stress, since market participants tend to overstate risk of price fluctuations, the standard deviation stays high, thus making the lead/lag relationship between stock price fluctuation and standard deviation unstable.

The above estimation results suggest that the relationship between the shape of the implied probability distribution and stock price fluctuations depends closely on the developments of the economy and financial markets. In our previous study, Nakamura and Shiratsuka (1999), we point out the relationship between implied probability distribution and market changes as follows. On the one hand, we find typical patterns of change in the shape of the implied probability distribution in response to stock price fluctuations. On the other hand, by comparing such typical patterns to examine the size

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collapse of asset price bubbles and contains more observations for the period of stock price decline.

and persistence of response of summary statistics to market fluctuations, we succeed in evaluating the impacts of external shocks and their adjustment speeds. Therefore, because of such a time-varying relationship between implied probability distribution and stock price changes, empirical evidence is likely to be unstable over time, reflecting the magnitude of stress in the stock market.

V. Conclusions

In this paper, I empirically analyzed the information content of implied probability distribution estimated from a set of option prices for stock price indices in the following two ways.

First, I examined whether implied probability distribution contains useful information for forecasting the subsequently realized distribution of stock price changes, compared with the historical distribution of stock price changes. The estimation results suggest that the implied probability distribution contains some information regarding future price movements, but that it is not superior to the historical distribution.

Second, I explored whether the shape of the implied probability distribution produces useful information for forecasting future stock price changes. To this end, I estimated VAR models to check Granger causality among stock price fluctuations and summary statistics of implied probability distribution. The results of Granger causality tests confirmed that absolute changes in stock prices Granger causes standard deviation and excess kurtosis, and that changes in stock prices Granger causes skewness. In addition, the results indicate that the shape of the implied probability distribution contains some information for forecasting stock price changes, at least concerning standard deviation and skewness.

However, it should be noted that empirical evidence concerning the information content of implied probability distribution is highly sensitive to the choice of sample period, as I confirmed by applying rolling regression techniques. This is because this relationship varies according to the development of the economy and the state of financial markets. In particular, when stock prices decline substantially and markets become turbulent, such information content is likely to become unstable. The empirical results in this paper suggest that it is difficult to extract useful information automatically from the shape of an implied probability distribution to be used for the conduct of monetary policy. Thus, it seems very important to accumulate know-how on how to extract such information through such case studies as conducted by Nakamura and Shiratsuka (1999).

Moreover, one should be careful in treating the tails of the implied probability distribution since the range of strike prices that are actually traded is very limited and the tails of the estimated implied probability distribution vary depending on the procedure employed. Therefore, it might not be an appropriate strategy to employ summary statistics that utilize information regarding an entire distribution. In this sense, it seems important to devise new and more stable indicators that exclude outlier information in the very tails of implied probability distribution.

Appendix: Heteroskedasticity and Autocorrelation Robust Standard Errors in SUR Estimation

This appendix explains procedures to compute the heteroskedasticity and autocorrelation robust standard errors in SUR estimation.

The SUR model in general can be written as follows:

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon ,$$

where $\mathbf{Y}$, $\mathbf{X}$, and ε denote the vector of independent variables, the matrix of dependent variables, the vector of estimated coefficients, and the vector of error terms, respectively. In the case of M-order simultaneous equations, the above equation can be rewritten as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X}_M \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix} .$$

The estimated coefficients and the variance-covariance matrix for this SUR model can be obtained by applying the GLS estimation procedure as follows:

$$\beta = [\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}]^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y} ,$$
$$Var[\beta] = [\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}]^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y} ,$$

where

$$\mathbf{V} = \begin{bmatrix} s_{11}\mathbf{I} & s_{21}\mathbf{I} & \cdots & s_{M1}\mathbf{I} \\ s_{12}\mathbf{I} & s_{22}\mathbf{I} & \cdots & s_{M2}\mathbf{I} \\ \vdots & \vdots & \ddots & \vdots \\ s_{1M}\mathbf{I} & s_{2M}\mathbf{I} & \cdots & s_{MM}\mathbf{I} \end{bmatrix}.$$

In addition, s_{ij} is computed by using the residual vector $\mathbf{e}_i$ as follows:

$$s_{ij} = \frac{\mathbf{e}_i'\mathbf{e}_j}{T}.$$

However, if there is a significant autocorrelation among the error terms, the above estimates will not produce an unbiased estimator for the variance-covariance matrix. Therefore, in this paper, I extend the procedure proposed in Newey and West (1987) to simultaneous equations, and adjust the effects of heteroskedasticity and autocorrelation as follows:

$$s_{ij} = \sum_{t=1}^{n} e_{it} e_{jt} \mathbf{x}_{it} \mathbf{x}'_{jt} + \frac{1}{n} \sum_{l=1}^{r} \sum_{t=l+1}^{n} w(l) e_{it} e_{j,t-l} [\mathbf{x}_{it} \mathbf{x}'_{j,t-l} + \mathbf{x}_{i,t-l} \mathbf{x}'_{jt}],$$

where $w(l)$ is the Bartlett kernel that is defined by $w(l) = 1 - l/(r+1)$. The bandwidth r is determined by following the guideline in den Haan and Levin (1996).

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Table 1: Summary Statistics

(1) Time-series properties

	Summary statistics for IPDs			Market fluctuation	
	Stdv	*Skew*	*Ex-kurt*	*Change*	*Abs-change*
Mean	0.169	-0.277	-0.591	-0.000	0.011
Median	0.163	-0.277	-0.761	-0.000	0.008
Max	0.397	1.859	6.869	0.124	0.124
Min	0.018	-2.542	-1.918	-0.068	0.000
Standard deviation	0.061	0.606	0.912	0.015	0.010
Skewness	0.375	0.150	1.956	0.420	2.514
Excess kurtosis	0.324	0.581	6.962	5.172	12.806
Autocorrelation					
lag = 1	0.898	0.828	0.678	0.031	0.203
lag = 2	0.844	0.750	0.600	-0.069	0.231
lag = 3	0.792	0.700	0.545	-0.002	0.183
lag = 4	0.756	0.653	0.493	0.023	0.237
lag = 5	0.727	0.606	0.428	-0.019	0.234
lag = 10	0.651	0.472	0.293	0.016	0.157
lag = 25	0.547	0.181	0.182	0.070	0.130
lag = 50	0.421	0.025	0.120	-0.041	0.023
lag = 100	0.189	0.072	0.172	-0.007	0.007

(2) Cross-correlation

	Stdv	*Skew*	*Ex-kurt*	*Change*	*Abs-change*
Stdv	1.000	0.077	0.131	0.047	0.205
Skew	0.077	1.000	-0.392	-0.357	0.170
Ex-kurt	0.131	-0.392	1.000	0.091	0.086
Change	0.047	-0.357	0.091	1.000	0.043
Abs-change	0.205	0.170	0.086	0.043	1.000

Notes: 1. Sample period is from June 21, 1989 to May 31, 1996, and number of samples is 1674.

2. *Stdv*, *Skew*, and *Ex-kurt* indicate standard deviation, skewness, and excess kurtosis of implied probability distribution, respectively. *Change*, and *Abs-change* indicate daily changes and daily absolute changes in stock prices, respectively.

Table 2: Interpretation of Results of J-Test

Null hypothesis: $\delta = 0$	Null hypothesis: $\gamma = 0$	
	Not rejected	Rejected
Not rejected	Both ID and RD contain useful information independently, and cannot be compared with each other.	RD contains all information contained in ID, as well as additional information on RD.
Rejected	ID contains all information contained in RD, as well as additional information on RD.	Neither ID nor RD contains useful information.

Table 3: Single Equation Estimation

	Summary statistics for IPDs					
	Stdv		*Skew*		*Ex-kurt*	
Equation (15): Implied probability distribution (ID)						
α	0.119	(0.024)	0.210	(0.065)	0.639	(0.194)
β	0.555	(0.124)	-0.041	(0.090)	0.042	(0.133)
t-val (β=0)	4.458	[0.000]	-0.455	[0.324]	0.316	[0.376]
t-val (β=1)	-3.580	[0.000]	-11.610	[0.000]	-7.219	[0.000]
Adj. R^2	0.102		0.000		0.000	
B-P Test	0.197	[0.657]	3.077	[0.079]	17.904	[0.000]
L-B Q(25)	13,345.8	[0.000]	6,231.9	[0.000]	7,303.2	[0.000]
Bandwidth	28		25		24	
Equation (16): Historical distribution (past 30 business days, HD)						
α	0.092	(0.019)	0.244	(0.063)	0.680	(0.175)
β	0.557	(0.096)	-0.143	(0.066)	-0.069	(0.056)
t-val (β=0)	5.800	[0.000]	-2.175	[0.015]	-1.242	[0.107]
t-val (β=1)	-4.612	[0.000]	-17.431	[0.000]	-19.195	[0.000]
Adj. R^2	0.260		0.018		0.005	
B-P Test	288.473	[0.000]	1.562	[0.211]	0.056	[0.813]
L-B Q(25)	9,126.5	[0.000]	6,880.1	[0.000]	7,610.4	[0.000]
Bandwidth	27		26		24	
Equation (17): ID + HD						
α	0.088	(0.020)	0.221	(0.067)	0.719	(0.210)
β_1	0.044	(0.179)	-0.096	(0.094)	0.060	(0.135)
β_2	0.541	(0.140)	-0.166	(0.066)	-0.072	(0.057)
t-val (β_1=0)	0.243	[0.404]	-1.026	[0.153]	0.447	[0.327]
t-val (β_2=0)	3.855	[0.000]	-2.501	[0.006]	-1.257	[0.104]
Adj. R^2	0.260		0.023		0.005	
B-P Test	367.577	[0.000]	2.615	[0.271]	17.467	[0.000]
L-B Q(25)	9,145.0	[0.000]	6,821.6	[0.000]	7,566.6	[0.000]
Bandwidth	27		26		23	

Notes: 1. Sample period is from June 21, 1989 to May 31, 1996. Number of samples is 1,670, since the period with time-to-maturity less than 3-business days are excluded.

2. Figures in parentheses and brackets are standard errors and *p*-values, respectively. Standard errors are adjusted for heteroskedasticity and autocorrelation by Newey and West's (1987) procedure. Bandwidths are decided by following the procedure in Newy and West (1994).

3. B-P Test indicates Breusch and Pagan's (1979) diagnostic test on heteroskedasticity, and test statistics that follows χ^2-distribution (degree of freedom is equal to the number of explanatory variables). L-B Q(25) indicates Ljung and Box's (1978) diagnostic test on autocorrelation (until 25-period lags), and test statistics that follows χ^2-distribution (degree of freedom is equal to 25).

Table 4: Results of *J*-Test: Single Equation Estimation

	Summary statistics for IPDs					
	Stdv		*Skew*		*Ex-kurt*	
γ	0.971	(0.252)	1.161	(0.463)	1.041	(0.829)
t-val	3.855	[0.000]	2.506	[0.006]	1.255	[0.105]
Bandwidth	27		25		25	
δ	0.079	(0.323)	2.352	(2.292)	1.436	(3.221)
t-val	0.243	[0.404]	1.026	[0.153]	0.446	[0.328]
Bandwidth	27		26		25	

Notes: 1. Sample period is from June 21, 1989 to May 31, 1996. Number of samples is 1,670, since the period with time-to-maturity less than 3-business days are excluded.

2. Figures in the parentheses and brackets are standard errors and *p*-values, respectively. Standard errors are heteroskedasticity and autocorrelation robust estimators applying Newey and West's (1987) procedure. Bandwidths are decided by following the procedure in Newy and West (1994).

Table 5: SUR Estimation (Full Sample)

	Summary statistics for IPDs					
	Stdv		*Skew*		*Ex-kurt*	
Equation (15): Implied probability distribution (ID)						
α	0.120	(0.016)	0.208	(0.174)	0.602	(0.802)
β	0.552	(0.056)	-0.048	(0.079)	-0.021	(0.554)
t-val (β=0)	9.876	[0.000]	-0.609	[0.271]	-0.039	[0.485]
t-val (β=1)	-8.020	[0.000]	-13.197	[0.000]	-1.845	[0.033]
Adj. R^2	0.102		0.000		-0.001	
Bandwidth	28					
Equation (16): Historical distribution (past 30 business days, HD)						
α	0.089	(0.013)	0.240	(0.169)	0.640	(0.683)
β	0.570	(0.027)	-0.116	(0.046)	-0.027	(0.198)
t-val (β=0)	21.084	[0.000]	-2.510	[0.006]	-0.138	[0.445]
t-val (β=1)	-15.878	[0.000]	-24.124	[0.000]	-5.184	[0.000]
Adj. R^2	0.260		0.017		0.003	
Bandwidth	27					
Equation (17): ID + HD						
α	0.087	(0.015)	0.218	(0.173)	0.624	(0.946)
β_1	0.025	(0.030)	-0.088	(0.076)	-0.022	(0.573)
β_2	0.558	(0.020)	-0.135	(0.038)	-0.023	(0.216)
t-val (β_1=0)	0.821	[0.206]	-1.161	[0.123]	-0.037	[0.485]
t-val (β_2=0)	28.012	[0.000]	-3.547	[0.000]	-0.108	[0.457]
Adj. R^2	0.260		0.022		0.002	
Bandwidth	27					

Notes: 1. Sample period is from June 21, 1989 to May 31, 1996. Number of samples is 1,670, since the period with time-to-maturity less than 3-business days are excluded.

2. Figures in parentheses and brackets are standard errors and *p*-values, respectively. Standard errors are adjusted for heteroskedasticity and autocorrelation by Newey and West's (1987) procedure. Bandwidths are decided by following the procedure in Newy and West (1994).

Table 6: Results of *J*-Test: SUR estimation

	Summary statistics for IPDs					
	Stdv		*Skew*		*Ex-kurt*	
γ	0.979	(0.166)	1.159	(3.660)	0.853	(106.922)
t-val	5.888	[0.000]	0.317	[0.376]	0.008	[0.497]
Bandwidth	27					
δ	0.045	(0.752)	1.826	(34.405)	1.001	(965.145)
t-val	0.060	[0.476]	0.053	[0.479]	0.001	[0.500]
Bandwidth	27					

Notes: 1. Sample period is from June 21, 1989 to May 31, 1996. Number of samples is 1,670, since the period with time-to-maturity less than 3-business days are excluded.

2. Figures in parentheses and brackets are standard errors and *p*-values, respectively. Standard errors are heteroskedasticity and autocorrelation robust estimators applying Newey and West's (1987). Bandwidths are decided by following the procedure in Newy and West (1994).

Table 7: Results of Hypothesis Testing: Rolling Regressions

(%)

		Equation (17)								
		Standard deviation			Skewness			Excess kurtosis		
		β_1>0 & β_2>0	β_1>0	β_2>0	β_1>0 & β_2>0	β_1>0	β_2>0	β_1>0 & β_2>0	β_1>0	β_2>0
Equation (15)(16)	β(ID)>0 & β(HD)>0	17.4	4.8	20.3	0.0	0.0	0.0	0.0	0.0	0.0
	β(ID)>0	0.1	15.8	0.1	0.7	9.0	0.0	0.0	0.0	0.0
	β(HD)>0	0.0	0.0	14.5	0.0	0.0	0.5	0.0	0.0	1.7

Notes: 1. Subsample periods are all 180 business days. Estimations are repeatedly conducted 1,491 times for full sample period from June 21, 1989 to May 31, 1996.

2. *t*-values for hypothesis testing are computed with standard errors that are heteroskedasticity and autocorrelation robust estimators applying Newey and West's (1987) procedure. Bandwidths are decided by following the procedure in Newy and West (1994), and 8 or 9 business days are chosen in most of the subsample periods.

Table 8: Results of *J*-Test: Rolling Regressions

(%)

Null hypothesis		Standard deviation	Skewness	Excess kurtosis
γ=0	δ=0			
Not rejected	Not rejected	55.8	86.2	98.3
Not rejected	Rejected	21.0	3.5	0.0
Rejected	Not rejected	17.3	9.7	1.7
Rejected	Rejected	5.9	0.7	0.0

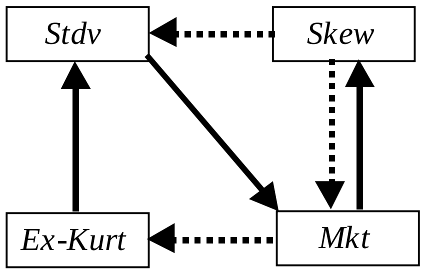
Notes: 1. Subsample periods are all 180 business days. Estimations are repeatedly conducted 1,491 times for full sample period from June 21, 1989 to May 31, 1996.

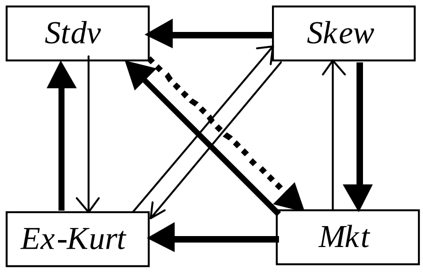
2. *t*-values for hypothesis testing are computed with standard errors that are heteroskedasticity and autocorrelation robust estimators applying Newey and West's (1987) procedure. Bandwidths are decided by following the procedure in Newy and West (1994), and 8 or 9 business days are chosen in most of the subsample periods.

Table 9: Granger Causality Tests: Implied Distributions

Dependent variable	Independent variables *Stdv*	*Skew*	*Ex-Kurt*	*Mkt*
Daily market change (lag = 6)				
Stdv	1,134.1	2.2	6.2	0.6
	(0.000)	(0.037)	(0.000)	(0.695)
Skew	0.6	371.9	1.4	3.2
	(0.699)	(0.000)	(0.197)	(0.004)
Ex-Kurt	0.8	1.6	206.7	2.3
	(0.589)	(0.154)	(0.000)	(0.033)
Mkt	3.1	2.5	1.1	2.7
	(0.005)	(0.019)	(0.346)	(0.014)
Daily absolute market change (lag = 7)				
Stdv	734.6	3.0	4.2	11.0
	(0.000)	(0.004)	(0.000)	(0.000)
Skew	0.4	437.3	1.8	1.9
	(0.887)	(0.000)	(0.076)	(0.066)
Ex-Kurt	1.2	1.8	184.1	5.9
	(0.292)	(0.092)	(0.000)	(0.000)
Mkt	2.4	4.9	0.8	19.9
	(0.019)	(0.000)	(0.589)	(0.000)

(Illustration)





Notes: 1. Data for changes in stock prices is adjusted for the trading-day effects by the web-based program of DECOMP.

2. Lag length is decided by AIC criteria.

3. Figures in the table indicate *F*-test statistics for the null hypothesis that the estimated coefficients for each dependent variable is equal to zero. Figures in parentheses are *p*-values.

4. Arrows in the illustration of the result of the Granger causality tests:

1% significant level: leads lags

5% significant level: leads lags

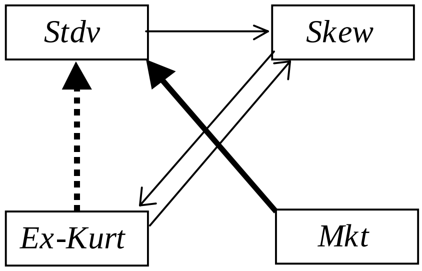
10% significant level: leads lags

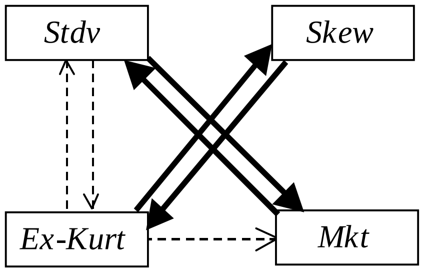
20% significant level: leads lags

Table 10: Granger Causality Tests: Historical Distributions

Dependent variable	Independent variables: *Stdv*	*Skew*	*Ex-Kurt*	*Mkt*
Daily market change (lag = 4)				
Stdv	26,534.3	0.2	3.1	13.2
	(0.000)	(0.921)	(0.014)	(0.000)
Skew	1.7	3,326.0	2.2	0.4
	(0.149)	(0.000)	(0.070)	(0.802)
Ex-Kurt	0.3	2.2	3,075.5	0.9
	(0.857)	(0.066)	(0.000)	(0.451)
Mkt	1.4	0.6	1.2	2.2
	(0.222)	(0.657)	(0.328)	(0.071)
Daily absolute market change (lag = 6)				
Stdv	7,807.0	1.1	1.5	3.4
	(0.000)	(0.350)	(0.189)	(0.002)
Skew	0.8	2,311.9	4.5	1.3
	(0.538)	(0.000)	(0.000)	(0.276)
Ex-Kurt	1.5	3.1	1,922.8	1.1
	(0.187)	(0.005)	(0.000)	(0.354)
Mkt	5.1	1.4	1.5	7.0
	(0.000)	(0.210)	(0.192)	(0.000)

(Illustration)





Notes: 1. Data for changes in stock prices is adjusted for the trading-day effects by the web-based program of DECOMP.

2. Lag length is decided by AIC criteria.

3. Figures in the table indicate *F*-test statistics for the null hypothesis that the estimated coefficients for each dependent variable is equal to zero. Figures in parentheses are *p*-values.

4. Arrows in the illustration of the result of the Granger causality tests:

1% significant level: leads lags

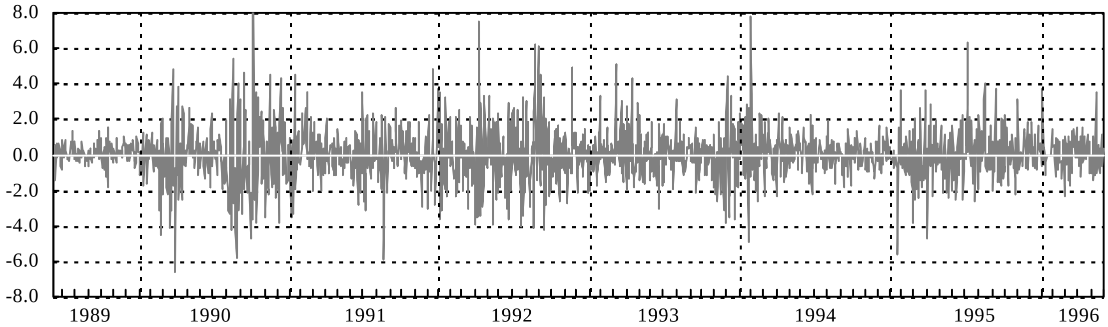
5% significant level: leads lags

10% significant level: leads lags

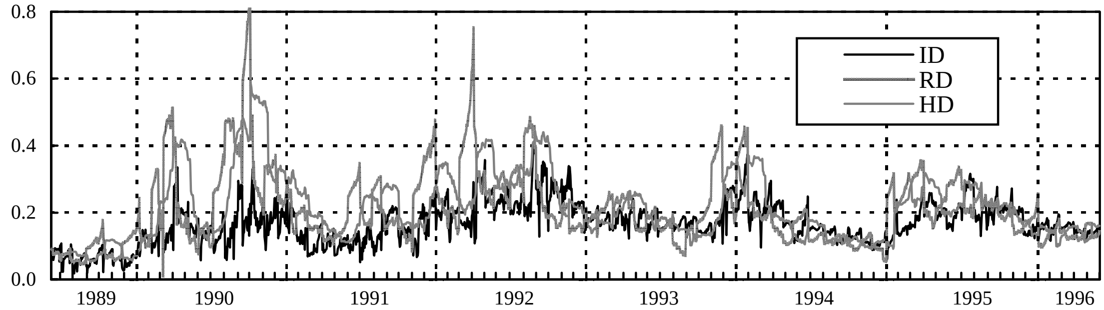
20% significant level: leads lags

Figure 1: Time-series Movement of Summary Statistics

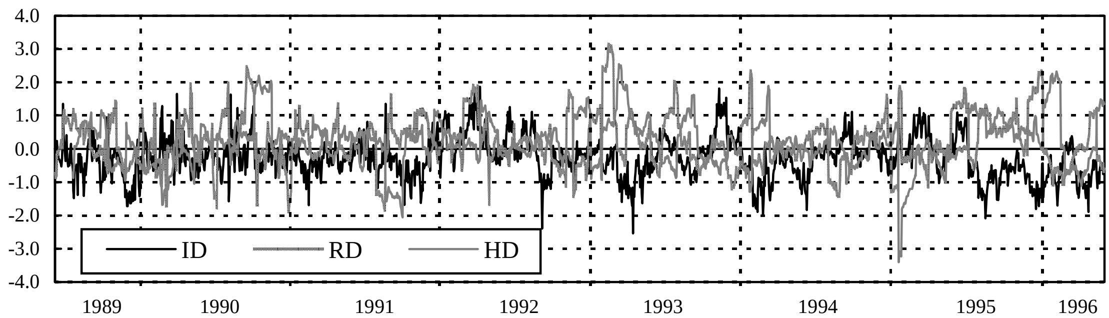
(1) Daily change in stock prices (Nikkei 225)



(2) Standard deviation



(3) Skewness



(4) Excess kurtosis

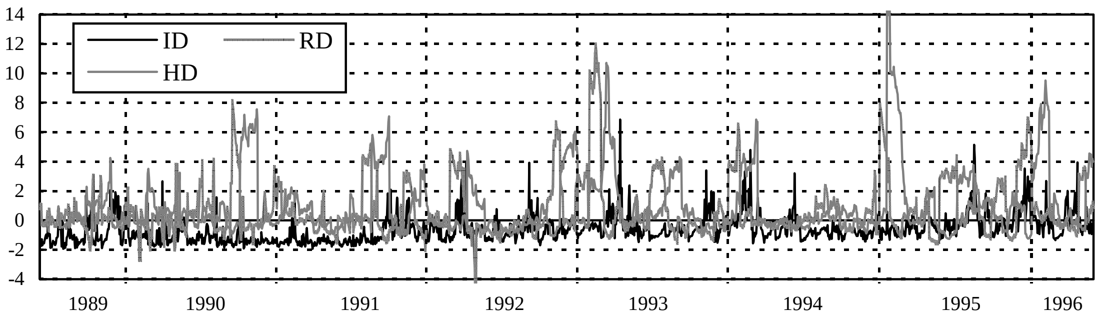
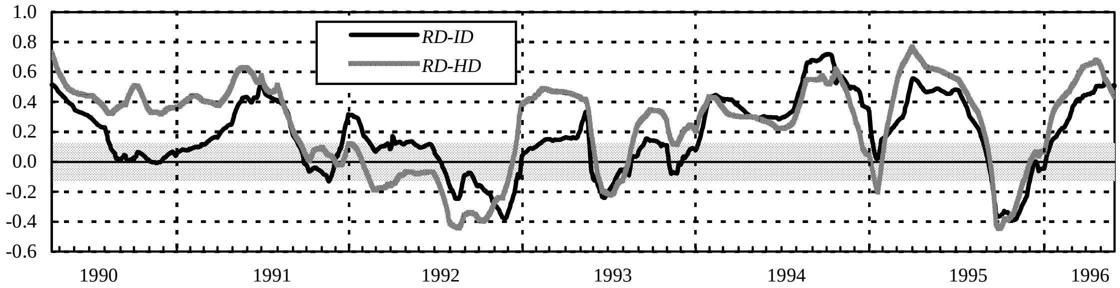
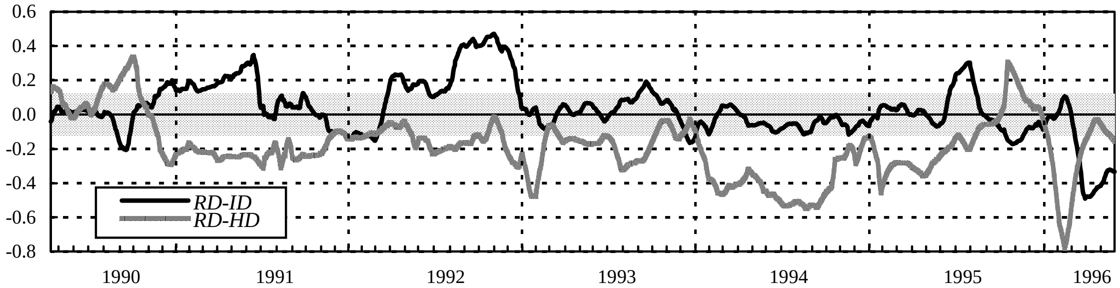


Figure 2: Stability of Cross-correlation over Time

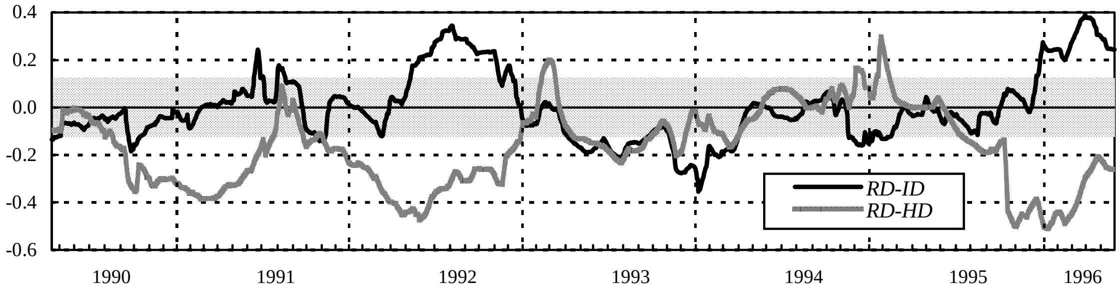
(1) Standard deviation



(2) Skewness



(3) Excess kurtosis



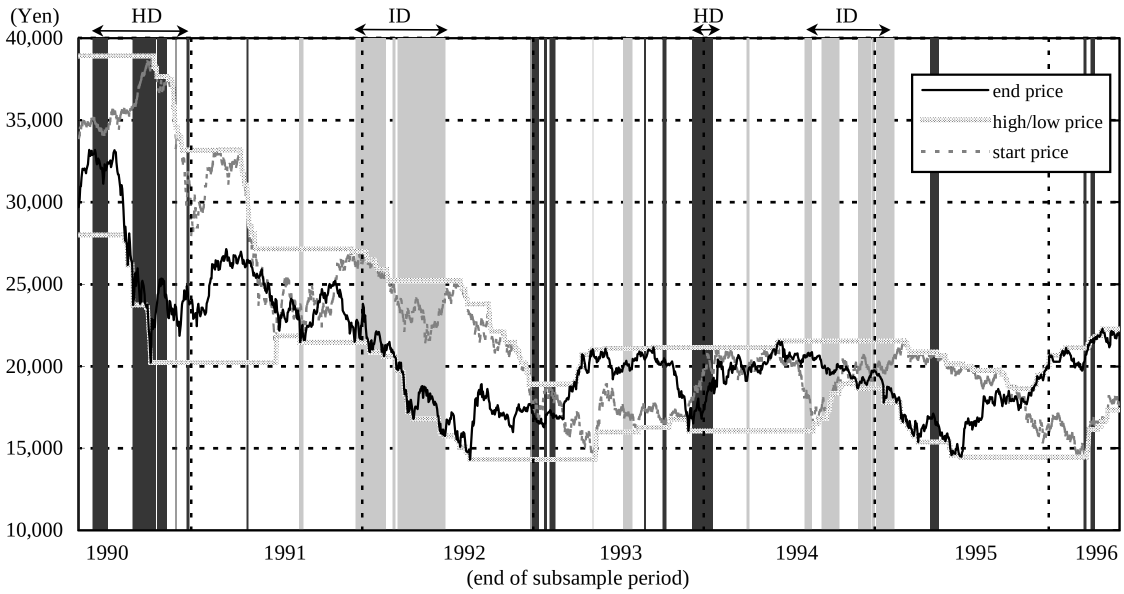
Notes: 1. Sample period for computing cross-correlation is 180 business days ending at each date on the horizontal axis.

2. Shaded area indicates a rejection interval for 10-percent significance in two-side hypothesis testing. Probability that coefficients of cross-correlation are significantly different from zero is as follows.

Unit: percent

Correlation between	*Stdv*		*Skew*		*Ex-Kurt*	
	positive	negative	positive	negative	positive	negative
RD-ID	56.6	10.9	25.0	9.4	18.3	19.4
RD-HD	67.9	14.8	6.2	69.1	3.4	59.4

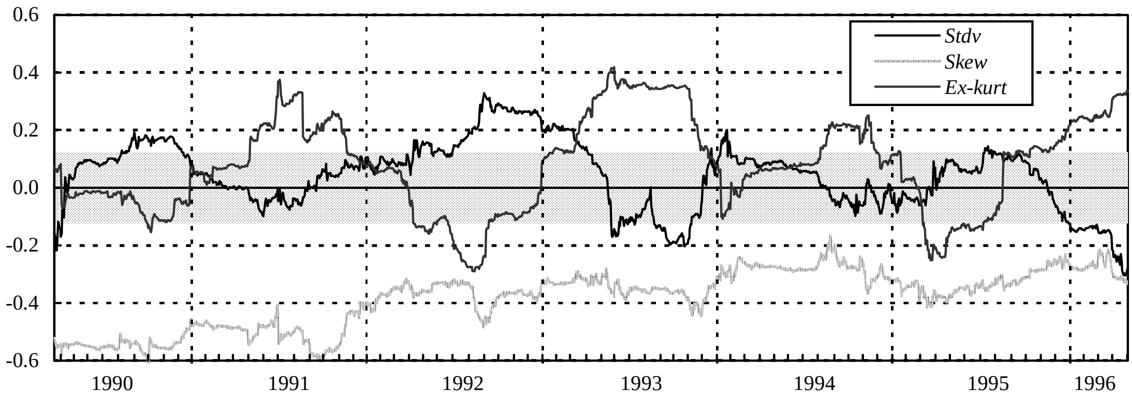
Figure 3: Predictive Power of Standard Deviation in Rolling Regression



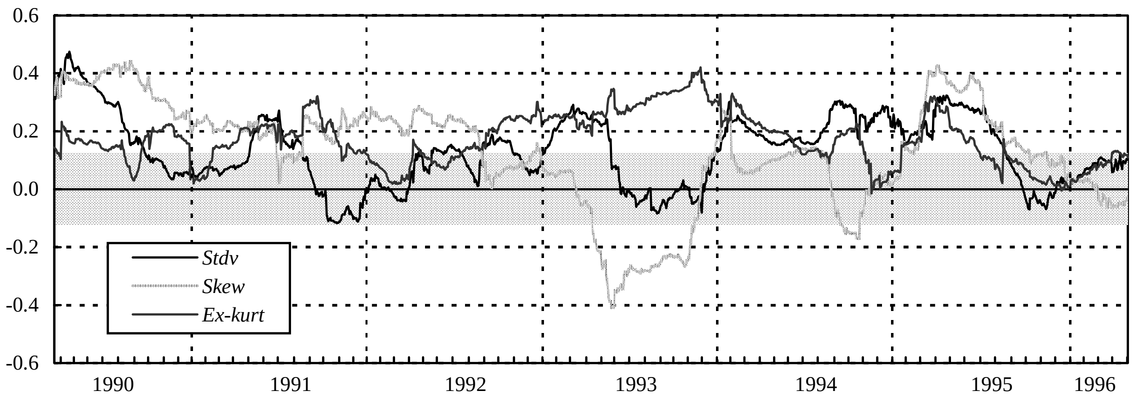
Note: Dark and light shadows indicate that HD and ID respectively have superior predictive power than the other.

Figure 4: Stability of Cross-Correlation over Time

(1) Cross-correlation with market changes



(2) Cross-correlation with absolute market changes



Notes: 1. Cross-correlation is computed with data for 180 business days ending at the each date on the horizontal axis.

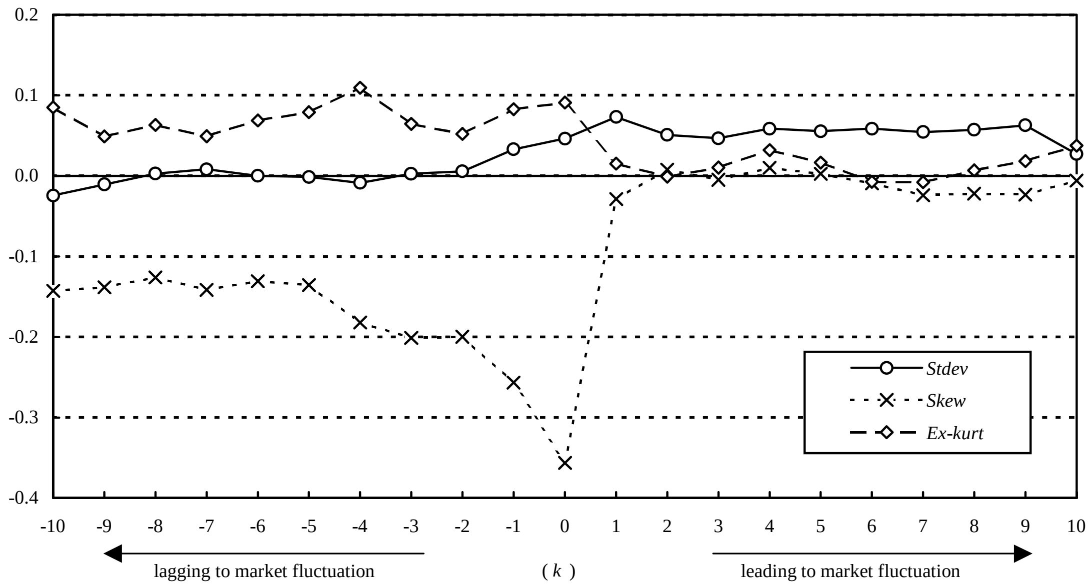
2. Shaded area indicates a rejection interval for 10-percent significance in two-side hypothesis testing. Probability that coefficients of cross-correlation are significantly different from zero is as follows.

Unit: percent

Correlation	*Stdv*		*Skew*		*Ex-Kurt*	
with	positive	negative	positive	negative	positive	negative
Mkt	21.8	11.9	0.0	100.0	39.2	11.6
Abs-Mkt	53.0	0.0	81.8	11.0	69.1	0.0

Figure 5: Dynamic cross-correlation

(1) Dynamic cross-correlation with market changes



(2) Dynamic cross-correlation with absolute market changes

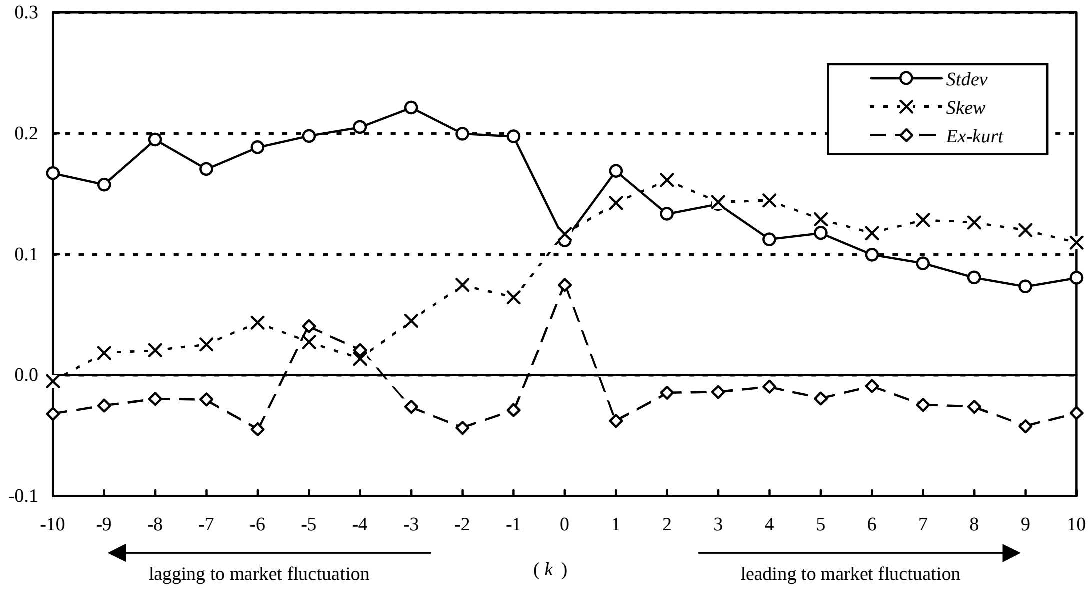
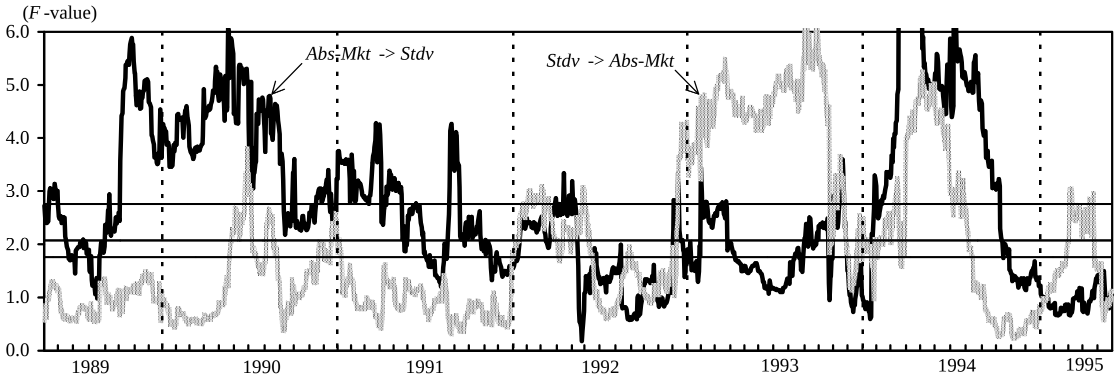
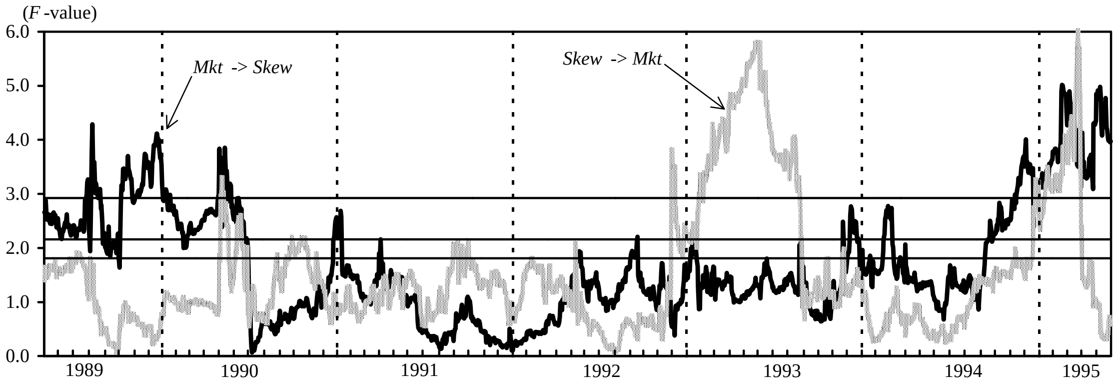


Figure 6: Granger Causality over Time

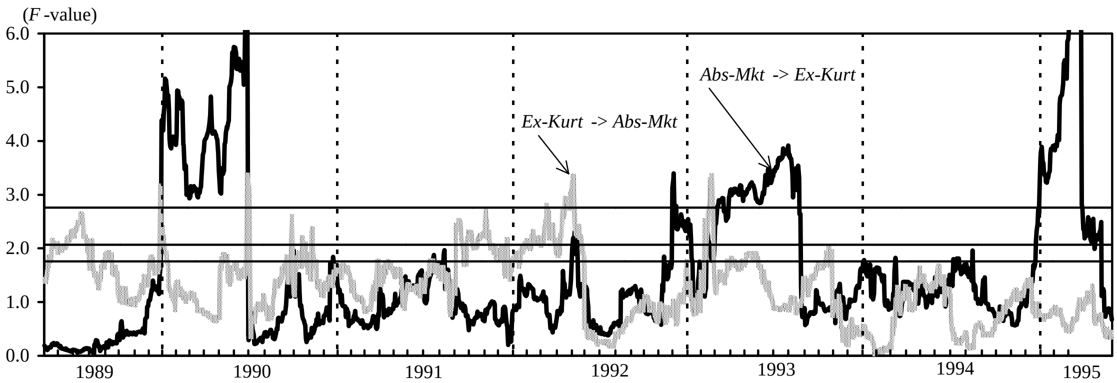
(1) Standard deviation



(2) Skewness



(3) Excess kurtosis



Notes: 1. The VAR model for Granger causality testing is the same as that for full sample estimation. Sample period is 180 business days ending at the each date on the vertical axis.

2. Data for changes in stock prices is adjusted for the trading-day effects by the web-based program of DECOMP.

3. Vertical lines in the figure are *F*-values for the 1%, 5%, and 10% significance from top to bottom, respectively.