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Financial Stability in Open Economies

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Abstract
This paper investigates the implications for monetary policy of financial markets that are internationally integrated but have intrinsic frictions. When there is no other distortion than financial market imperfections in the form of staggered international loan contracts, financial stability, which here constitutes eliminating the inefficient fluctuations of loan premiums, is the optimal monetary policy in open economies, regardless of whether policy coordination is possible. Yet, the optimality of inward-looking monetary policy requires an extra condition, in addition to those included in previous studies on the optimal monetary policy in open economies. To make allocations between cooperative and noncooperative monetary policy coincide, the exchange rate risk must be perfectly covered by the banks. Otherwise, each central bank has an additional incentive to control the nominal exchange rate to favor firms in her own country by reducing the exchange rate risk.

Keywords: optimal monetary policy; policy coordination; global banking; international staggered loan contracts

JEL classification: E50, F41

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1 Introduction

Financial globalization has been expanding quite rapidly. We can easily observe this trend from recent financial and economic developments. For example, many banks in the world now suffer from losses stemming from the US subprime loan crisis. Gadanecz (2004), McGuire and Tarashev (2006), and Lane and Milesi-Ferretti (2007, 2008) formally show that more funds from foreign countries are flowing into the domestic financial markets of many countries. Although we can find several studies investigating the implications of goods market integration for monetary policy, as summarized in Woodford (2007), very few studies have focused on monetary policy under global banking or internationally integrated financial markets.

Does international financial stability matter for central banks? How do international financial market developments alter the form of the optimal monetary policy? Should central banks conduct monetary policy cooperatively when financial markets are internationally integrated?

In order to answer these questions, we construct a new open economy macroeconomic (NOEM) model that incorporates international loan contracts by extending Fujiwara and Teranishi (2008). In our model, financial markets are characterized by staggered loan contracts following the Calvo (1983) - Yun (1996) framework. Stickiness in the loan contract rate is reported by many studies, for example Slovin and Sushka (1983) and Berger and Udell (1992) for the US economy, Sorensen and Werner (2006) and Gambacorta (2008) for the euro area economy, and Bank of Japan (2007) for the Japanese economy.\footnote{For the US, using micro level data, Slovin and Sushka (1983) and Berger and Udell (1992) show that it takes two or more quarters for the private banks to adjust the loan interest rates. For the euro area, Sorensen and Werner (2006) estimate the incompleteness in the pass-through from the policy interest rate to loan interest rates via an error correction model using macro data. They further show that the degree to which pass-through is incomplete differs significantly among countries. Gambacorta (2008) conducted a similar analysis for Germany and shows the existence of sticky adjustment in the loan interest rate. For Japan, according to BOJ (2007), the major city banks need five quarters and local banks need seven quarters to adjust their loan interest rates.} For detailed modeling of the financial market, it is popular to incorporate the financial accelerator in a dynamic stochastic general equilibrium model, where net worth as the state variable
causes the deviations of loan rates from the policy interest rate, as in Bernanke, Gertler, and Gilchrist (1999). The staggered loan contract model can be considered a simplification or another type of financial market friction. We aim to capture the dynamics of loan rates by staggered loan contracts instead of through net worth dynamics. In our model, the wedge between the loan rate and the policy rate is due to imperfect competition among banks; this follows Sander and Kleimeier (2004), Gropp, Sorensen, and Lichtenberger (2007), van Leuvensteijn, Sorensen, Bikker, and van Rixtel (2008) and Gropp and Kashyap (2009), who point out the importance of bank competition in the staggered loan rate setting. The end consequences are, however, the same irrespective of which of these models is adopted. A shock related to the financial market imperfections eventually results in an increase in the costs of goods production. The most advantageous feature of our approach is that we can analyze the nature of the optimal monetary policy analytically and therefore more intuitively.

Welfare analysis shows that the central banks should stabilize the international financial disturbance, implying the central bank should care about international financial market heterogeneity between domestic and foreign countries. Most notably, when there is no other distortion than staggered loan contracts as examined in this paper, financial stability, which in this context constitutes eliminating the inefficient fluctuation in loan premiums stemming from financial market imperfections, turns out to represent the optimal monetary policy in open economies irrespective of whether or not there is cooperation between central banks. Each central bank should aim at stabilizing the loan premium. Yet, the optimality of inward-looking, i.e., independent, monetary policy requires an additional condition on top of those included in previous studies on the optimal monetary policy in open economies. Specifically, for allocations between cooperative and noncooperative monetary

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2 Nevertheless, initial responses of loan rates to the monetary policy shock are quite different. In the financial accelerator model, the response is much larger than in our model. Where this difference comes from is not, however, a trivial question. Morozumi (2008) shows that the financial accelerator mechanism does act to amplify the responses to a monetary policy shock but does not make them persistent. To investigate which is superior for empirical accounting is left for our future research.

3 There exist models where financial market imperfections affect the aggregate TFP. See, for example, Chari, Kehoe, and McGrattan (2007).
policy to coincide, the exchange rate risk must be perfectly covered by lenders. Otherwise, each central bank has an additional incentive to control the nominal exchange rate so as to favor firms in her own country by reducing the exchange rate risk. Thus, joint management of the exchange rate through cooperative monetary policy improves global welfare when firms’ marginal costs of production are exposed to exchange rate fluctuations. These represent new findings that are not considered in previous studies that investigate the optimal monetary policy in open economies such as Obstfeld and Rogoff (2002), Clarida, Gali, and Gertler (2002), Benigno and Benigno (2003), Devereux and Engel (2003), and Corsetti and Pesenti (2005).

The structure of the paper is as follows. Section 2 describes the model used for the analyses in this paper. Then, in Section 3, we derive the loss function that the central bank should minimize. Section 4 investigates the nature of the optimal monetary policy in internationally integrated financial markets. Section 5 provides a short discussion of the results. Finally, Section 6 summarizes the findings of this paper.

2 Model

The model consists of two countries. There are four types of agent in each country—consumers, firms, private banks and the central bank—as depicted in Figure 1.

2.1 Consumers

A representative consumer has four roles: (1) to consume differentiated goods determined through two-step cost minimization problems on both home- and foreign-produced consumer goods; (2) to choose the amount of aggregate consumption, bank deposits and investment in risky assets given a deposit interest rate set by the central bank; (3) with monopolistic power over labor supply, to provide differentiated labor services that depend on whether he belongs to either domestically financially supported (DFS) or the internationally financially supported (IFS) groups, as well as to offer wages to those differentiated types of labor; and (4) to own banks and firms and to receive dividends in each period. Role (3) is crucial in staggered loan contracts. Thanks to this differentiated labor supply,
the demand for loans is differentiated without assuming any restrictions on aggregate loans or loan interest rates.\textsuperscript{4}

### 2.1.1 Cost Minimization

The utility of the representative consumer in the home country $H$ comes from the aggregate consumption index $C_t$. The consumption index that consists of bundles of differentiated goods produced by home and foreign firms is expressed as

$$
C_t = \frac{C_{H,t}^{\psi} C_{F,t}^{1-\psi}}{\psi^{\psi} (1-\psi)^{1-\psi}},
$$

where $\psi (0 \leq \psi \leq 1)$ is a preference parameter that expresses the home bias, which is set to be 0.5 in this paper, implying no home bias.\textsuperscript{5} Here, $C_{H,t}$ and $C_{F,t}$ are consumption subindices of the continuum of differentiated goods produced by firms in the home country and the foreign country, respectively. They are defined as

$$
C_{H,t} = \left[ \int_0^1 c_t(f) \frac{\sigma-1}{\sigma} df \right]^{\frac{\sigma}{\sigma-1}},
$$

\textsuperscript{4}For details, see Teranishi (2007).

\textsuperscript{5}Here, we follow Obstfeld and Rogoff (2000).
and

\[ C_{F,t} = \left[ \int_0^1 c_t(f^*) \frac{1}{\sigma} df^* \right]^{\frac{\sigma}{\sigma-1}}, \]

where \( c_t(f) \) is the demand for a good produced by firm \( f \) in the home country and \( c_t(f^*) \) is the demand for a good produced by a firm \( f^* \) in the foreign country, where the asterisk denotes foreign variables. Following the standard cost minimization problem on the aggregate consumption index of home and foreign goods as well as the consumption subindices of the continuum of differentiated goods, we can derive the consumption-based price indices:

\[ P_t = P_{H,t}^{\frac{1}{2}} P_{F,t}^{\frac{3}{2}}, \quad (2) \]

with

\[ P_{H,t} = \left[ \int_0^1 p_t(f) \frac{1}{1-\sigma} df \right]^{\frac{1}{1-\sigma}}, \]

and

\[ P_{F,t} = \left[ \int_0^1 p_t(f^*) \frac{1}{1-\sigma} df^* \right]^{\frac{1}{1-\sigma}}, \]

where \( p_t(f) \) is the price given \( c_t(f) \), and \( p_t(f^*) \) is the price given \( c_t(f^*) \). Then, we can obtain the following Hicksian demand functions for each differentiated good given the aggregate consumption:

\[ c_t(f) = \frac{1}{2} \left[ \frac{p_t(f)}{P_{H,t}} \right]^{-\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t, \quad (3) \]

and

\[ c_t(f^*) = \frac{1}{2} \left[ \frac{p_t(f^*)}{P_{F,t}} \right]^{-\sigma} \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t. \]

Here, as in other applications of the Dixit and Stiglitz (1977) aggregator, consumers’ allocations across differentiated goods at each time are optimal in terms of cost minimization.

We can derive similar optimality conditions for the foreign counterpart. For example, the demand functions for each differentiated good given the aggregate consumption are expressed as

\[ c_t^* (f) = \frac{1}{2} \left[ \frac{p_t^* (f)}{P_{H,t}^*} \right]^{-\sigma} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-1} \]

\[ C_t^*, \quad (4) \]

and

\[ c_t^* (f^*) = \frac{1}{2} \left[ \frac{p_t^* (f^*)}{P_{F,t}^*} \right]^{-\sigma} \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-1} \]

\[ C_t^*. \]
2.1.2 Utility Maximization

A representative consumer in the home country maximizes the following utility function:

\[ U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ U(C_T) - \int_0^n V([l_T(h)]) \, dh - \int_n^1 V([l_T(\bar{h})]) \, d\bar{h} \right\}, \]

where \( E_t \) is the expectations operator conditional on the state of nature at date \( t \) and \( \beta \) is the subjective discount factor. The functions \( U \) and \( V \) are increasing and concave in the consumption index and the labor supply, respectively. The disutility of the representative consumer in the home country \( H \) comes from the labor supplies \( l_T(h) \) and \( l_T(\bar{h}) \). The budget constraint of the consumer is given by

\[
P_tC_t + E_t [X_{t,t+1}B_{t+1}] + D_t \leq B_t + (1 + i_{t-1})D_{t-1} + \int_0^n w_t(h)l_t(h) \, dh \\
+ \int_n^1 w_t(\bar{h})l_t(\bar{h}) \, d\bar{h} + \Pi_t^{PB} + \Pi_t^F - T_t, \tag{5}
\]

where \( B_t \) is a set of risky asset, \( D_t \) is deposit to private banks, \( i_t \) is the nominal deposit interest rate set by a central bank from \( t-1 \) to \( t \), \( w_t(h) \) is the nominal wage for labor \( l_t(h) \) at the DFS group, \( w_t(\bar{h}) \) is the nominal wage for labor \( l_t(\bar{h}) \) at the IFS group, \( \Pi_t^{PB} = \int_0^1 \Pi_t^{PB}(h) \, dh \) is the nominal dividend stemming from the ownership of both local and international banks in the home country, \( \Pi_t^F = \int_0^1 \Pi_t^F(f) \, df \) is the nominal dividend from the ownership of the firms in the domestic country, \( X_{t,t+1} \) is the stochastic discount factor, and \( T_t \) is the lump sum tax.\(^6\) Here, because we assume a complete financial market between the two countries, the consumer in each country can conduct international purchases and sales of the state contingent securities to insure against country-specific shocks. Consequently, there exists only one unique discount factor. The relationship between the deposit interest rate and the stochastic discount factor is now expressed as

\[
\frac{1}{1 + i_t} = E_t [X_{t,t+1}]. \tag{6}
\]

Given the optimal allocation of differentiated consumption expenditure, the consumer now optimally chooses the total amount of consumption, risky assets and deposits in each

\(^6\)For simplicity, we do not explicitly include the amount of contingency claims under complete financial markets.
period. Necessary and sufficient conditions, when the transversality condition is satisfied, for those optimizations are given by

\[ U_C(C_t) = \beta (1 + i_t) E_t \left[ U_C(C_{t+1}) \frac{P_t}{P_{t+1}} \right], \]  

(7)

Together with equation (6), we see that the condition given by equation (7) defines the intertemporally optimal allocation on aggregate consumption. Then, the standard New Keynesian IS curve for the home country, by log-linearizing equation (7) around steady states, is obtained as follows:

\[ \bar{C}_t = E_t \bar{C}_{t+1} - \nu \left( \hat{i}_t - E_t \pi_{t+1} \right), \]  

(8)

where aggregate inflation in the home country is \( \pi_t \equiv \ln \frac{P_t}{P_{t-1}} \) and \( \nu \equiv -\frac{U_C}{U_C'}, \) except for \( \pi_t, \) given that \( \pi \) is the steady-state value of \( x_t. \)

In this model, the representative consumer provides all types of differentiated labor to each firm and therefore maintains some monopoly power over the determination of his own wage, as in Erceg, Henderson, and Levin (2000).\(^7\) There are two types of labor group: the DFS and the IFS. The population of workers located on \([0, n]\) belong to the DFS, while those located on \([n, 1]\) belong to the IFS.\(^8\) We assume that each firm hires all types of labor in the same proportion from the two groups. The consumer sets the wage \( w_t(h) \) for any \( h \) and \( w_t(\bar{h}) \) for any \( \bar{h} \) to maximize its utility subject to the budget constraint given by equation (5) and the labor demand functions given by equations (24) and (25) in the next section. Here, although differentiated labor supply is assumed, consumers change wages in a flexible manner. The optimality conditions for labor supply then emerge as follows:

\[ \frac{w_t(h)}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{V_t[l_t(h) \pi_t]}{U_C(C_t)}, \]  

(9)

\(^7\)The same optimal allocations are obtained even when each consumer provides differentiated labor supply to each firm.

\(^8\)Differences between these two groups comprise broader characteristics, such as whether a worker is English speaking or Japanese speaking; differences within groups comprise narrower characteristics, like whether a worker has knowledge of bank-related accountancy or the skill to assemble an automobile.
and
\[
\frac{w_t(h)}{P_t} = \frac{\varepsilon}{\varepsilon - 1} V_t[h_t(h)],
\] (10)
where \(\varepsilon\) is the elasticity of substitution among differentiated labor. As written above, thanks to this heterogeneity in labor supply, we can model the differentiated demand for loans without assuming any restrictions on aggregate loans and loan interest rates. In this paper, consumers supply their labor only to firms, not to banks.

As in the above case with cost minimization, we can derive the optimality conditions for the foreign counterpart. For example, the standard New Keynesian IS curve for the foreign country is
\[
\hat{C}_t^* = E_t \hat{C}_{t+1}^* - v \left( \pi_t^* - E_t \pi_{t+1}^* \right),
\] (11)

\subsection{Exchange Rate}

Under complete financial markets with a symmetric initial state,
\[
U_C^*(C_t^*) = \frac{S_t P_t^*}{P_t} U_C(C_t),
\]
and
\[
Q_t = \frac{S_t P_t^*}{P_t},
\]
where \(S\) is the nominal exchange rate and \(Q\) is the real exchange rate. As we will see below, because of the symmetry in the home bias parameter and because no nominal rigidities are assumed in this paper:
\[
C_t^* = C_t \equiv C_t^W,
\] (12)
where \(C_t^W\) is the world consumption and
\[
Q_t = 1.
\]

However, reflecting the nominal interest rate set by the central bank, nominal exchange rates can fluctuate according to the UIP condition as
\[
E_t \Delta S_{t+1} = \hat{r}_t - \pi_t^*,
\] (13)
which can also be expressed as
\[
\Delta \hat{S}_t = \pi_t - \pi_t^*.
\] (14)
2.2 Firms

There exists a continuum of firms populated over unit mass \([0, 1]\) in each country. Each firm plays two roles. Firstly, it decides the amount of differentiated labor to be employed from the DFS and IFS groups, through a two-step minimization of its production costs. Part of the costs of labor must be financed by external loans from banks. For example, in country \(H\), to finance the costs of hiring workers from the DFS group, the firm must borrow from local banks in the home country. However, to finance the costs of hiring workers from the IFS group, the firm must borrow from international banks in the foreign country. The grounds for such heterogeneous sources of funds are as follows. First, Gadanecz (2004), McGuire and Tarashev (2006), Lane and Milesi-Ferretti (2007), and Lane and Milesi-Ferretti (2008) show that firms tend to borrow funds from both domestic and foreign banks; \(i.e.,\), a bank lends funds to both domestic and foreign firms. Second, we also know from looking at actual project finance that firms borrow funds with many different loan interest rates at the same time depending on the nature of projects. In this paper, these project differences stem from the types of labor, which are immobile between the two countries. Since it is assumed that firms must use all types of labor, they borrow from both local and international banks.\(^9\) The structure of the exchange rate risk sharing is as follows. Domestic firms borrow \(\xi \times 100\) percent of loans in foreign currency from international banks in the foreign country. Thus, the exchange rate risk is shared by the firm in the home country and the international banks in the foreign country in the ratio \(\xi\) to \(1 - \xi\).

Secondly, each firm resets goods price to maximize her present profit in a flexible manner.

2.2.1 Cost Minimization

Firms in both the home and foreign countries optimally hire differentiated labor as price takers. This optimal labor allocation is also carried out via a two-step cost minimization. Domestic firm \(f\) hires all types of labor from both the DFS and IFS groups. When hiring from the DFS group, \(\gamma\) portion of the labor cost associated with labor type \(h\) is financed

\(^9\text{The same structure is assumed for employment in Woodford (2003).}\)
by borrowing from the local bank $h$. Then, the first-step of the cost minimization problem to determine the allocation of differentiated labor from the DFS is given by
\[
\min_{l_t(h, f)} \int_0^n [1 + \gamma r_t(h)] w_t(h) l_t(h, f) \, dh,
\]
subject to the subindex regarding labor from DFS group to firm $f$:
\[
L_t(f) = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n l_t(h, f) \frac{l_t}{\Omega_t} \, dh \right]^{1-\varepsilon},
\]
where $r_t(h)$ is the loan interest rate applied to employ a particular labor type $h$ during time $t$, and $l_t(h, f)$ denotes type of labor $h$ employed by firm $f$. The local bank $h$ has some monopoly power over setting loan interest rates. The relative demand for differentiated labor is given as follows:
\[
l_t(h, f) = \frac{1}{n} L_t \left\{ \frac{[1 + \gamma r_t(h)] w_t(h)}{\Omega_t} \right\}^{-\varepsilon},
\]
where
\[
\Omega_t = \left\{ \frac{1}{n} \int_0^n \left[ [1 + \gamma r_t(h)] w_t(h) \right]^{1-\varepsilon} \, dh \right\}^{\frac{1}{1-\varepsilon}}.
\]

Then, the first-step cost minimization problem to determine the allocation of differentiated labor from the IFS group is given by
\[
\min_{l_t(h, f)} \int_0^1 \left[ \frac{S_{t+1} - S_t}{S_t} \right] [1 + \gamma r_t(\overline{h})] w_t(\overline{h}) l_t(\overline{h}, f) \, d\overline{h},
\]
subject to the subindex regarding labor from DFS group to firm $f$:
\[
\mathcal{L}_t(f) = \left[ \left( \frac{1}{1 - n} \right)^{\frac{1}{\varepsilon}} \int_0^1 l_t(\overline{h}, f) \frac{l_t}{\Omega_t} \, d\overline{h} \right]^{1-\varepsilon},
\]
where $\xi$ is the proportion of borrowing carried out in the domestic currency.\textsuperscript{10} Through a similar cost minimization problem, we can derive the relative demand for each type of differentiated labor from the IFS group as
\[
l_t(\overline{h}, f) = \frac{1}{1 - n} \mathcal{L}_t(f) \left\{ \frac{S_{t+1} - S_t}{S_t} \right\} \left[ 1 + \gamma r_t(\overline{h}) \right] w_t(\overline{h}) \frac{\delta_t(\overline{h}, f)}{\Omega_t} \right\}^{-\varepsilon},
\]
\textsuperscript{10}We here assume either one period delivery lag for the loan repayment to the foreign country or segmented financial markets between consumers and firms.
where
\[
\overline{\Omega}_t = \left[ \frac{S_{t+1}}{S_t} \xi + (1 - \xi) \right] \left\{ \frac{1}{1 - n} \int_n^1 \left\{ \left[ 1 + \gamma r_t (\overline{h}) \right] w_t (\overline{h}) \right\}^{1-\varepsilon} d\overline{h} \right\}^{\frac{1}{1-\varepsilon}}.
\] (19)

According to the above two optimality conditions, firms optimally choose the allocation of differentiated workers between these two groups. Because firms have some preference \( n \) to hire workers from the DFS group and \((1 - n)\) to hire workers from the IFS group, the second-step cost minimization to determine the allocation of differentiated labor between these two groups is given by

\[
\min_{L_t, \overline{L}_t} \Omega_t L_t (f) + \overline{\Omega}_t \overline{L}_t (f),
\]
subject to the aggregate labor index:

\[
\tilde{L}_t (f) = \frac{[L_t (f)]^n [\overline{L}_t (f)]^{1-n}}{n^n (1 - n)^{1-n}}.
\] (20)

Then, the relative demand functions for each differentiated type of labor are derived as follows:

\[
L_t (f) = n \tilde{L}_t (f) \left( \frac{\Omega_t}{\overline{\Omega}_t} \right)^{-1},
\] (21)

\[
\overline{L}_t (f) = (1 - n) \tilde{L}_t (f) \left( \frac{\overline{\Omega}_t}{\Omega_t} \right)^{-1},
\] (22)

and

\[
\tilde{\Omega}_t = \Omega_t^n \overline{\Omega}_t^{1-n}.
\] (23)

Therefore, we can obtain the following equations:

\[
l_t (h, f) = \left\{ \frac{[1 + \gamma r_t (h)] w_t (h)}{\Omega_t} \right\}^{-\varepsilon} \left( \frac{\Omega_t}{\overline{\Omega}_t} \right)^{-1} \tilde{L}_t (f),
\] (24)

and

\[
l_t (\overline{h}, f) = \left\{ \frac{[1 + \gamma r_t (\overline{h})] w_t (\overline{h})}{\overline{\Omega}_t} \right\}^{-\varepsilon} \left( \frac{\overline{\Omega}_t}{\Omega_t} \right)^{-1} \tilde{L}_t (f),
\] (25)

from equations (16), (18), (21), and (22). We can now clearly see that the demand for each differentiated worker depends on wages and loan interest rates, given the total demand for labor.

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Finally, from the assumption that firms finance part of their labor costs by loans, we can derive

\[ q_t(h,f) = \gamma w_t(h) l_t(h,f) = \gamma w_t(h) \left\{ \frac{1 + \gamma r_t(h)}{\Omega_t} w_t(h) \right\}^{-\varepsilon} \left( \frac{\Omega_t}{\Omega_t} \right)^{-1} \tilde{L}_t(f), \]

and

\[ q_t(\overline{h},f) = \gamma w_t(\overline{h}) \left\{ \frac{S_{t+1} \xi + (1 - \xi)}{\Omega_t} \left[ 1 + \gamma r^*_t(\overline{h}) \right] w_t(\overline{h}) \right\}^{-\varepsilon} \left( \frac{\Omega_t}{\Omega_t} \right)^{-1} \tilde{L}_t(f). \]

\( q_t(h,f) \) and \( q_t(\overline{h},f) \) denote the amounts borrowed by firm \( f \) to finance labor costs accruing from labor types \( h \) and \( \overline{h} \), respectively. These conditions demonstrate that the demands for each differentiated loan also depend on the wages and loan interest rates, given the total labor demand.

We can obtain similar conditions for the foreign country.

### 2.2.2 Price Setting (Profit Maximization)

In this paper, where the focus is on understanding the role of international staggered loan contracts, we assume no price rigidities. Therefore, each firm \( f \) resets its price \( p_t(f) \) and \( p^*_t(f) \) to maximize present profit given by

\[
(1 + \tau) p_t(f) c_t(f) + (1 + \tau) S_t p^*_t(f) c^*_t(f) - \tilde{\Omega}_t \tilde{L}_t(f),
\]

where \( \tau \) is the rate of subsidy, \( S_t \) is the nominal exchange rate and is the sales subsidy to eliminate the monopolistic rents in the steady state.\(^{11}\) By substituting equations (3) and (4), we can obtain

\[
(1 + \tau) p_t(f) \left\{ \frac{p_t(f)}{P^*_t} \right\}^{-\sigma} \left( \frac{P_{t H, t}}{P_t} \right)^{-1} C_t + (1 + \tau) S_t p^*_t(f) \left\{ \frac{P^*_t}{P^*_t} \right\}^{-\sigma} \left( \frac{P^*_t}{P^*_t} \right)^{-1} C^*_t - \tilde{\Omega}_t \tilde{L}_t(f).
\]

The optimal price setting is given by

\[
(1 + \tau) \frac{\sigma - 1}{\sigma} p_t(f) = \tilde{\Omega}_t \frac{\partial \tilde{L}_t(f)}{\partial c_t(f)},
\]

\(^{11}\) As is standard with New Keynesian models, fiscal policy eliminates the steady-state markup from goods production.
where we use equation (3). By further substituting equations (9), (10), (17), (19) and (23), the above optimality condition can be now rewritten as

$$1 = (1 + \tau) \frac{\sigma}{\sigma - 1} MC_t,$$

where the marginal cost $MC_t$ is given by

$$MC_t = \left\{ \frac{1}{n} \int_0^n \left\{ [1 + \gamma r_t (h)] \frac{\varepsilon}{\varepsilon - 1} \frac{V_t [l_t (h)]}{U_C (C_t)} \frac{P_t}{p_t (f)} \frac{\partial \tilde{L}_t (f)}{\partial c_t (f)} \right\}^{1-\varepsilon} dh \right\}^{\frac{1-n}{1-\varepsilon}},$$

because without nominal rigidities,

$$P_{H,t} = p_t (f).$$

By log-linearizing equation (26), we can derive

$$\hat{mc}_t = \Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t} + \hat{mc}_t (1 - n) \xi \left( \hat{i}_t - \hat{i}_t \right) = 0,$$

where we use equation (13) and, $\Theta_1 \equiv n \frac{\gamma (1 + R_{SS})}{1 + \gamma R_{SS}}$ and $\Theta_2 \equiv (1 - n) \frac{\gamma (1 + R_{SS})}{1 + \gamma R_{SS}}$ are positive parameters, and under symmetric equilibrium, the marginal cost with respect to labor input is given by

$$\hat{mc}_t = \int_0^n \hat{mc}_t (h) dh + \int_n^1 \hat{mc}_t (h) dh,$$

where

$$\hat{mc}_t (h) = \frac{P_t}{P_{H,t}} \frac{V_t [l_t (h)]}{U_C (C_t)} \frac{\partial \tilde{L}_t (f)}{\partial c_t (f)},$$

and

$$\hat{mc}_t (h) = \frac{P_t}{P_{H,t}} \frac{V_t [l_t (h)]}{U_C (C_t)} \frac{\partial \tilde{L}_t (f)}{\partial c_t (f)}.$$
Similarly, regarding the optimal price setting of $p_t(f)$, we can derive
\[ \bar{mc}_t = \bar{mc}_t^* + \Theta_1^* R_{F,t} + \Theta_2^* R_{F,t} - (1 - n^*) \xi^* (\hat{i}_t - \hat{i}_t^*) = 0, \]
where we use equation (13) and $\Theta_1^* \equiv n^* \frac{1 + R_{SS}}{1 + \gamma R_{SS}} > 0$ and $\Theta_2^* \equiv (1 - n^*) \frac{1}{1 + \gamma R_{SS}} > 0$. $R_{F,t}$ is the aggregate loan interest rate by international banks in the foreign country and $R_{F,t}^*$ is the aggregate loan interest rate by local banks in the foreign country.

### 2.2.3 Marginal Cost with respect to Labor Input

Here, we derive the equations for $\bar{mc}_t$ and $\bar{mc}_t^*$. By linearizing equation (20) under symmetric equilibrium, we can obtain
\[ \hat{\bar{L}}_t \equiv n\hat{\bar{L}}_t + (1 - n)\hat{\bar{L}}_t. \]
Because the production function of each firm is assumed to be
\[ y_t(f) = f \left[ \bar{L}_t(f) \right], \]
where $f(\cdot)$ is an increasing and concave function. The aggregate domestic production function is now expressed as
\[ Y_{H,t} = f \left( \bar{L}_t \right). \]
By log-linearization, this can be transformed into
\[ \hat{Y}_{H,t} = \mu \left[ n\hat{\bar{L}}_t(\bar{h}) + (1 - n)\hat{\bar{L}}_t(\bar{h}) \right], \]
where $\mu \equiv \frac{\bar{L}_t \bar{h}}{f'}. \text{Now, by using equations (29) (30), and (33), equation (28) is transformed into} \]
\[ \bar{mc}_t = \left( \theta + \frac{\eta}{\mu} \right) \hat{Y}_{H,t} + \frac{1}{\nu} \hat{C}_t - \hat{p}_{H,t}, \]
where
\[ p_{H,t} = \frac{P_{H,t}}{P_t}. \]
and $\eta \equiv \frac{V_H}{V_I}$ and $\theta \equiv -\frac{f_V \mu}{f_I \nu}$. Without nominal rigidities, because the country size is the same, the demand function is given by

$$Y_{H,t} = (1 + \tau) p_{H,t} C_t^W,$$

where we use equation (12). This can be linearly approximated as

$$\hat{Y}_{H,t} = \hat{C}_t^W - \hat{p}_{H,t}.$$

At the same time, from equation (2),

$$\hat{p}_{H,t} = -\frac{1}{2} \hat{T}oT_t,$$

where we define

$$T_oT_t = \frac{P_{F,t}}{P_{H,t}}.$$

Therefore, we can rewrite equation (34) as

$$\tilde{mc}_t = \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \hat{C}_t^W + \left( \theta + \frac{\eta}{\mu} + 1 \right) \frac{1}{2} \hat{T}oT_t$$

$$= \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \hat{Y}_{H,t} + \left( 1 - \frac{1}{v} \right) \frac{1}{2} \hat{T}oT_t,$$

where we use the relation of $\hat{C}_t^W = \hat{Y}_{H,t} - \frac{1}{2} \hat{T}oT_t$.

Similarly, we can obtain the linearized equation for the foreign marginal cost with respect to labor input as

$$\tilde{mc}_t^* = \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \hat{C}_t^W - \left( \theta + \frac{\eta}{\mu} + 1 \right) \frac{1}{2} \hat{T}oT_t$$

$$= \left( \frac{1}{v} + \theta + \frac{\eta}{\mu} \right) \hat{Y}_{F,t} - \left( 1 - \frac{1}{v} \right) \frac{1}{2} \hat{T}oT_t,$$

where we use the relation of $\hat{C}_t^W = \hat{Y}_{F,t} + \frac{1}{2} \hat{T}oT_t$, which can be derived under the specification of the Cobb-Douglas aggregator in equation (1).

### 2.3 Private Banks

There exists a continuum of private banks located over $[0, 1]$. There are two types of bank in each country: local banks that populate the interval $[0, n)$ and international banks that populate the interval $[n, 1]$. Each private bank has two roles: (1) to collect the deposits
from consumers in its country, and (2) under the monopolistically competitive loan market, to set differentiated nominal loan interest rates according to their individual loan demand curves, given the amount of their deposits. We assume that each bank sets the differentiated nominal loan interest rate according to the types of labor force as examined in Teranishi (2007). Staggered loan contracts between firms and private banks produce a situation in which the private banks fix the loan interest rates for a certain period. A local bank lends only to firms when they hire labor from the DFS group. However, an international bank only provides loans to firms when they hire labor from the IFS group. The lending structure is shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Local bank</th>
<th>International bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home country</td>
<td>for f to hire h</td>
<td>for $f^<em>$ to hire $h^</em>$</td>
</tr>
<tr>
<td>Foreign country</td>
<td>for $f^<em>$ to hire $h^</em>$</td>
<td>for f to hire $h$</td>
</tr>
</tbody>
</table>

First, we describe the optimization problem for an international bank in the home country. In this case, the exchange rate risk is shared by the firm in the foreign country and the international banks in the domestic country according to $\xi^*$ and $1 - \xi^*$. Each international bank has the opportunity to reset loan interest rates with probability $1 - \tilde{\phi}^*$ following the Calvo (1983) – Yun (1996) framework.\footnote{The staggered loan contracts in this paper work in the same way as the staggered wage contracts model in Erceg, Henderson, and Levin (2000), the crucial difference being the presence of international linkages in the international staggered loan contracts setting used here. In our model, the sticky loan interest rate is the only source of economic distortion. However, as shown in Obstfeld and Rogoff (2002), the implications for monetary policy are still valid.} Within the segmented environment stemming from differences in labor supply, private banks can set different loan interest rates depending on the types of labor. As a consequence, private banks hold some monopoly power over the loan interest rate offered to firms.

Therefore, the international bank $\tilde{h}^*$ chooses the loan interest rate $r_t(\tilde{h}^*)$ to maximize
the present discounted value of profit:
\[
E_t \sum_{T=t}^{\infty} \left( \phi \right)^{T-t} X_{t,T} q_{T} \left( \bar{h}, f^* \right) \left\{ \left[ \frac{S_{T+1}}{S_T} (1 - \xi^*) + \xi^* \right] \left[ 1 + r_{T} (\bar{h}) \right] - (1 + i_{T}) \right\}.
\]
The optimal loan condition is now given by
\[
E_t \sum_{T=t}^{\infty} \left( \phi \beta \right)^{T-t} \frac{P_t}{P_T} U_C \left(C_T \right) q^*_T \left( \bar{h} \right) \left\{ \varepsilon \gamma S_T (1 + i_T) \right\} = 0.
\]
Because the international private banks that have the opportunity to reset their loan interest rates will set the same loan interest rate, the solution of \( r_t \) in equation (39) is simply \( r_t \). In this case, the evolution of the aggregate loan interest rate index for loans offered by international banks in the home country is described by:
\[
1 + R_{F,t} = \bar{\phi}^* (1 + R_{F,t-1}) + \left( 1 - \bar{\phi}^* \right) (1 + \bar{r}_t).
\]
By log-linearizing equations (39) and (40), we can determine the relationship between the loan and deposit interest rate as follows:
\[
\hat{R}_{F,t} = \hat{X}_1 E_t \hat{R}_{F,t+1} + \hat{X}_2 \hat{R}_{F,t-1} + \hat{X}_3 \left[ i_t - (1 - \xi^*) E_t \Delta S_{t+1} \right] + \hat{u}_t, \tag{41}
\]
where we use equation (13), \( \hat{X}_1^* \equiv -\frac{\bar{\phi} \beta}{1 + \left( \bar{\phi} ^* \right)^{-1}} \), \( \hat{X}_2^* \equiv -\frac{\bar{\phi}^*}{1 + \left( \bar{\phi}^* \right)^{-1}} \), and \( \hat{X}_3^* \equiv -\frac{1}{1 + \left( \bar{\phi}^* \right)^{-1}} \frac{\left( 1 - \beta \bar{\phi}^* \right) (1 + i_{SS})}{1 + R_{SS}} \) are positive parameters, \( \hat{u}_t^* \) is the shock to this loan rate curve, and \( i_{SS} \) and \( R_{SS} \) denote steady state nominal interest rates and loan rates respectively. This equation describes the foreign country’s loan interest rate (supply) curve for loans offered by the international bank in the home country.\(^{13}\)

Similarly, from the optimization problem of a local bank \( h \) in the home country, we can obtain the relationship between the loan and deposit interest rates as follows:
\[
\hat{R}_{H,t} = \lambda_1 E_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{i}_t + u_t, \tag{42}
\]
where \( \lambda_1 \equiv \frac{\phi \beta}{1 + \phi \beta} \), \( \lambda_2 \equiv \frac{\phi}{1 + \phi \beta} \), and \( \lambda_3 \equiv \frac{1 - \phi}{1 + \phi \beta} \frac{e + \frac{1 - \beta \phi}{1 + \beta \phi} i_{SS}}{1 + R_{SS}} \) are positive parameters, and \( u_t \) is the shock to this loan rate curve. This equation describes the home country’s loan interest rate curve.

\(^{13}\)We assume that this shock is a purely nominal shock, which does not alter the allocations under the flexible price equilibrium.
loan interest rate (supply) curve for loans offered by the local bank in the home country. Note that the two loan interest rates, $\hat{R}_{H,t}$ and $\hat{R}_{F,t}$, are the same when $\xi^* = 1$, $\lambda_1 = \bar{\lambda}_1^*$, $\lambda_2 = \bar{\lambda}_2^*$, and $\lambda_3 = \bar{\lambda}_3^*$, and $u_t = \pi_t^*$. In this case the law of one price is seen operating in the loan interest rates set by domestic private banks.

For international banks in the foreign country, we can derive the following loan interest rate curve:

$$\hat{R}_{H,t}^* = \bar{\lambda}_1 E_t \hat{R}_{H,t+1}^* + \bar{\lambda}_2 \hat{R}_{H,t-1}^* + \bar{\lambda}_3^* \left( (1 - \xi) \hat{\lambda}_t + \xi \hat{\lambda}_t^* \right) + u_t, \quad (43)$$

where we use equation (13), $\bar{\lambda}_1 \equiv \frac{\delta \beta}{1+\phi \beta}$, $\bar{\lambda}_2 \equiv \frac{\delta}{1+\phi \beta}$ and $\bar{\lambda}_3 \equiv \frac{1-\delta}{1+\phi \beta} \frac{\epsilon}{\beta - 1} \frac{(1-\delta \beta)(1+i_{ss})}{1+R_{ss}}$ are positive parameters, and $u_t$ is the shock to this loan rate curve. This equation describes the home country’s loan interest rate (supply) curve for loans offered by the international bank in the foreign country. Similarly, for local banks in the foreign country, we can obtain

$$\hat{R}_{F,t}^* = \lambda_1^* E_t \hat{R}_{F,t+1}^* + \lambda_2^* \hat{R}_{F,t-1}^* + \lambda_3^* \hat{\lambda}_t + u_t^*, \quad (44)$$

where $\lambda_1^* \equiv \frac{\phi \beta}{1+(\phi^*)^2 \beta}$, $\lambda_2^* \equiv \frac{\phi^*}{1+(\phi^*)^2 \beta}$ and $\lambda_3^* \equiv \frac{1-\phi^*}{1+(\phi^*)^2 \beta} \frac{\epsilon^*}{\beta - 1} \frac{(1-\delta \beta)(1+i_{ss})}{1+R_{ss}}$ are positive parameters, and $u_t^*$ is the shock to this loan rate curve. This equation describes the foreign country’s loan interest rate (supply) curve for loans offered by the local bank in the foreign country. It should be noted that the four different types of private bank (depending on whether they are local or international, operating at home or abroad) can have different probabilities of resetting their loan interest rates.

2.4 System of Equation

The linearized system of equations consists of eight equations: (27), (31), (37), (38), (41), (42), (43), (44), and two optimal monetary policies derived in the following sections for 10 endogenous variables: $\bar{C}^W$, $\bar{ToT}$, $\bar{lmc}$, $\bar{lmc}^*$, $\hat{R}_F$, $\hat{R}_H$, $\hat{R}_H^*$, $\hat{R}_F^*$, $\hat{i}$ and $\hat{i}^*$. Except for the two optimal monetary policies $\hat{i}$ and $\hat{i}^*$, the variables are summarized in Table 2.

A very straightforward explanation is possible for this system. Equations (41) to (44) determine the cost of borrowing, and these combined define the marginal costs in equations

\[ ^{14}\text{If we further add equations (8), (11), (12) and (14), we can derive the optimal responses in } \pi, \pi^*, \text{ and } S \text{ as shown in figures below.} \]
Table 2: System of Equations

Eq. (27): \( \Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* + \hat{mc}_t + (1 - n) \xi \left( \hat{i}_t - \hat{i}_t^* \right) = 0 \)

Eq. (31): \( \Theta_1 \hat{R}_{F,t} + \Theta_2 \hat{R}_{F,t} + \hat{mc}_t^* - (1 - n^*) \xi^* \left( \hat{i}_t - \hat{i}_t^* \right) = 0 \)

Eq. (37): \( \hat{mc}_t = \left( \frac{1}{v} + \theta + \frac{n}{\mu} \right) \hat{C}_W + \frac{1}{2} \left( \theta + \frac{n}{\mu} + 1 \right) \hat{T}_{oT} \)

Eq. (38): \( \hat{mc}_t^* = \left( \frac{1}{v} + \theta + \frac{n}{\mu} \right) \hat{C}_W^* - \frac{1}{2} \left( \theta + \frac{n}{\mu} + 1 \right) \hat{T}_{oT} \)

Eq. (41): \( \hat{R}_{F,t} = \hat{X}_1 e_{t} \hat{R}_{F,t+1} + \hat{X}_3 \hat{R}_{F,t-1} + \hat{X}_3 \left[ \xi^* \hat{i}_t + (1 - \xi^*) \hat{i}_t^* \right] + \bar{w}_t \)

Eq. (42): \( \hat{R}_{H,t} = \lambda_1 e_{t} \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{i}_t + u_t \)

Eq. (43): \( \hat{R}_{F,t}^* = \lambda_1 e_{t} \hat{R}_{F,t+1}^* + \lambda_2 \hat{R}_{F,t-1}^* + \lambda_3 \left[ (1 - \xi) \hat{i}_t + \xi \hat{i}_t^* \right] + \bar{w}_t \)

Eq. (44): \( \hat{R}_{F,t}^* = \lambda_1 e_{t} \hat{R}_{F,t+1}^* + \lambda_2 \hat{R}_{F,t-1}^* + \lambda_3 \hat{i}_t^* + u_t^* \)

(27) and (31). The aggregate consumption and the terms of trade are solely determined by these marginal costs as in equations (37) and (38).

3 Welfare Analysis

3.1 Preference

We assume that \( U(\cdot), U^*(\cdot), V(\cdot) \) and \( V^*(\cdot) \) are isoelastic functions as

\[
U(X) = U^*(X) = \frac{X^{1-\frac{1}{v}}}{1-v},
\]

and

\[
V(X) = V^*(X) = \frac{X^{1+\eta}}{1+\eta},
\]

where \( v \) is the intertemporal elasticity of substitution in consumption and \( \eta \) is the Frisch elasticity of labor supply.\(^{15}\) In the following analysis, we assume \( v = 1 \), namely the log utility, and the linear production function as \( Y_{H,t} = \tilde{L}_t \) and \( Y_{F,t} = \tilde{L}_t^* \).

We choose this parametric assumption since we would like to focus solely on the implications for monetary policy of an internationally integrated financial market and its intrinsic frictions. As already shown in Obstfeld and Rogoff (2002), Clarida, Galí, and Gertler (2002), Benigno and Benigno (2003), and Corsetti and Pesenti (2005), under the

\(^{15}v \equiv -\frac{\nu_v}{\nu_{ccc}} \text{ and } \eta \equiv \frac{\nu_{il}}{\nu_{u}}.\)
assumption of log utility together with the Cobb-Douglas aggregator in equation (1), the optimal allocations under cooperative and noncooperative monetary policy coincide when there are no international loan contracts. Furthermore, an inward-looking monetary policy that responds only to the domestic variable becomes optimal and there are no gains from targeting the exchange rate. The reasoning behind this optimality of independent and inward-looking monetary policy is as follows. There exist no direct effects from foreign activities on the domestic marginal cost since the terms of trade and risk sharing effects cancel. Mathematically, with a log utility function where $\phi = 1$, the terms of trade disappear in equations (37) and (38). As a result, no central bank has any incentive to manipulate the exchange rate, \( i.e., \) the terms of trade, so that it can shift the burden of production to the foreign country. Hence, by making the parametric assumptions above, we can investigate whether the newly introduced international financial market imperfections have any new previously unstudied implications for monetary policy cooperation and exchange rate targeting.

3.2 Noncooperative Allocation

We derive a second-order approximation of the welfare function for each country following Woodford (2003). To eliminate the linear term in the quadratic approximation in the noncooperative allocation stemming from the difference between consumption and output in open economies, we follow Clarida, Galí, and Gertler (2002), where output and the policy interest rate in the foreign country are assumed to be given for the home central bank and the fiscal authority sets the optimal subsidy in a noncooperative manner.\(^{16}\) Furthermore, as is standard for cost push shocks in New Keynesian models, we assume that the shocks to the loan interest rates do not alter the output in the flexible price equilibrium. The details of the derivation are shown in the Appendix.

\(^{16}\)This problem does not occur under the strict parametric assumptions employed in Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2005) where an analytical solution of the optimal monetary policy is available. Another method to eliminate the linear term in the quadratic approximation is found in Benigno and Benigno (2003). We will show that under some special conditions, since financial stability becomes the optimal independent monetary policy, we can derive the optimal noncooperative monetary policy following Benigno and Benigno (2003).
The consumer welfare in the home country is given by
\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \int_0^n l_t(h)^{1+\eta} \frac{1}{1+\eta} \, dh - \int_n^1 l_t(\bar{h})^{1+\eta} \frac{1}{1+\eta} \, d\bar{h} \right].
\]
Then, we have a second-order approximated loss function for the home country as follows:
\[
L_t \simeq \lambda_Y H \bar{Y}_H,t \bar{Y}_H,t + \lambda_H \left( \bar{R}_{H,t} - \bar{R}_{H,t-1} \right)^2 + \lambda^*_H \left( \bar{R}_{H,t}^* - \bar{R}_{H,t-1}^* \right)^2
+ \lambda_{HH} \left( \Theta_1 \bar{R}_{H,t} - \Theta_2 \bar{R}_{H,t}^* - \xi \bar{i} \right)^2,
\]
where \( \lambda_Y H \equiv \frac{n+1}{2} \), \( \lambda_H \equiv n \left[ \frac{1}{1+\gamma R_{SS}} \right]^2 \frac{\varepsilon}{1+\eta \phi (1-\phi) (1-\phi \beta)} \), \( \lambda^*_H \equiv (1-n) \left[ \frac{1}{1+\gamma R_{SS}} \right]^2 \frac{\varepsilon}{1+\eta \phi (1-\phi) (1-\phi \beta)} \), and \( \lambda_{HH} \equiv n (1-n) \frac{1}{2(1+\eta)} \).

There are several intriguing points to be noted. First, the central bank has to stabilize the financial market as captured by the last three terms in equation (45). The central bank dislikes any dispersion in loan rates whether from home or foreign banks, as can be seen from the second and third terms dealing with loan rate fluctuations. \( \left( \Theta_1 \bar{R}_{H,t} - \Theta_2 \bar{R}_{H,t}^* - \xi \bar{i} \right)^2 \) implies that the home central bank should seek to minimize the credit spread, \( i.e., \) the relative marginal cost dispersion. Second, the presence of the policy interest rate in the last term also implies that the central bank has an incentive to control the nominal exchange rate and to lower the marginal cost to favor domestic firms if the firms are not free from the exchange rate risk, \( i.e., \) \( \xi > 0 \). Third, the heterogeneity in the financial market complicates monetary policy. The central bank faces a trade-off in an international financial market with different speeds of loan rate adjustment. For example, when there is no asymmetry in the loan rates faced by domestic firms with respect to structural parameters and the size of shocks, namely \( \lambda_1 = \lambda^*_1 \), \( \lambda_2 = \lambda^*_2 \), \( \lambda_3 = \lambda^*_3 \), \( n = 0.5 \), and \( u_t = \bar{u}_t \), and domestic firms are free from the exchange rate risk, \( i.e., \) \( \xi = 0 \), the credit spread term disappears and the loss function in equation (45) is reduced to
\[
L_t \simeq \lambda_Y H \bar{Y}_H,t + \lambda \left( \hat{R}_t - \hat{R}_{t-1} \right)^2,
\]
where \( \hat{R}_t = \hat{R}_{H,t} = \hat{R}_{H,t}^* \) and \( \lambda = \lambda_H = \lambda^*_H \). Fourth, the central bank in the home country needs to monitor the lending behavior of private banks in the foreign country. As the second term in equation (45) shows, a change in the speed with which loan rates are adjusted by foreign private banks affects the optimal path of the policy interest rate set.
by the central bank in the home country. When there are no international loan contracts, 
\textit{i.e.}, \( n = 1 \), the central bank does not take account of the loan rates set by foreign private 
banks.

The optimal monetary policy in this situation aims at minimizing the home loss function 
subject to equations (27), (42), and (43) as in the closed economy model. We will come 
back to this point in the following section.

Through a similar procedure, we can derive a second-order approximated loss function 
for the foreign country as follows:

\[
L^*_t \simeq \lambda_{Y_F} \hat{Y}_{F,t}^2 + \lambda_{F} \left( \hat{R}_{F,t} - \hat{R}_{F,t-1} \right)^2 + \lambda_{F}^* \left( \hat{R}_{F,t}^* - \hat{R}_{F,t-1}^* \right)^2 
\]

(46)

\[
+ \lambda_{F}^* \left( \Theta_1^* \hat{R}_{F,t} - \Theta_2^* \hat{R}_{F,t}^* + \xi^* \hat{i}^* \right)^2,
\]

where \( \lambda_{Y_F} \equiv \frac{n+1}{2} \), \( \lambda_{F} \equiv n^* \left[ \frac{\gamma(1+\gamma)}{1+\gamma \Phi_{SS}} \right]^2 \frac{\varepsilon}{1+\eta \Phi^{*}(1-\Phi^{*})}, \)

\[
\lambda_{F}^* \equiv (1-n^*) \left[ \frac{\gamma(1+\gamma \Phi_{SS})}{1+\gamma \Phi_{SS}} \right]^2 \frac{\varepsilon}{1+\eta \Phi^{*}(1-\Phi^{*})}, \)

and

\[
\lambda_{F}^* \equiv n^*(1-n^*) \frac{1}{2(1+\eta)}. \]

The optimal monetary policy in the foreign country minimizes this foreign loss function subject to equations (31), (41), and (44).

\subsection{3.3 Cooperative Allocation}

Similarly, we can derive the world loss function which central banks coordinating their 
policies would aim to minimize.\(^{17}\) In the case of noncooperative monetary policy, we follow 
Clarida, Galí, and Gertler (2002) and compute the optimal subsidy under cooperative fiscal 
policy. The derived loss function is given by

\[
L^W_t = L_t + L^*_t = \lambda_{Y_H} \hat{Y}_{H,t}^2 + \lambda_{Y_F} \hat{Y}_{F,t}^2 
\]

(47)

\[
+ \lambda_{H} \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \lambda_{H} \left( \hat{R}_{H,t} - \hat{R}_{H,t-1}^* \right)^2 + \lambda_{H} \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \hat{i} \right]^2 
\]

\[
+ \lambda_{F} \left( \hat{R}_{F,t} - \hat{R}_{F,t-1} \right)^2 + \lambda_{F} \left( \hat{R}_{F,t} - \hat{R}_{F,t-1}^* \right)^2 + \lambda_{F}^* \left[ \Theta_1^* \hat{R}_{F,t} - \Theta_2^* \hat{R}_{F,t}^* + \xi^* \hat{i} \right]^2.
\]

Note that nothing is given in this loss function under cooperative monetary policy. The 
cooperating central banks aim at minimizing the world loss function subject to equations 
(27), (42), (43), (31), (41), and (44).

\(^{17}\)For details, see Appendix.
In contrast to the noncooperative allocation, \[\left[\Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{F,t} - \xi (\hat{i} - \hat{i}^*)\right]^2\] means that the home and foreign central banks should seek jointly to minimize the loan rate difference. This implies that central banks have an incentive to jointly manage the nominal exchange rate.

### 3.4 Welfare Weight

Here, we show how the weights, namely \(\lambda_H\) and \(\lambda_{HH}\) as well as the ratio \(\lambda_{Y_H}\), in the social loss functions given by equations (45), (46), and (47) change as the parameters for financial openness \(n\) and loan rate stickiness \(\phi\) are altered. The aim is to determine whether financial market integration with a heterogeneous degree of financial market imperfection alters the nature of the optimal monetary policy. We use the parameters in Table 3, most of which are from Woodford (2003).

#### Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>1</td>
<td>Dependence on external finance</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>7.66</td>
<td>Elasticity of substitution among differentiated labor</td>
</tr>
<tr>
<td>(\eta)</td>
<td>1</td>
<td>Elasticity of the desired real wage to the quantity of labor demanded</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0</td>
<td>Elasticity of marginal cost with respect to (y) regarding production</td>
</tr>
<tr>
<td>(\mu)</td>
<td>1</td>
<td>Elasticity of the output to additional labor input</td>
</tr>
<tr>
<td>(\nu)</td>
<td>1</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>7.66</td>
<td>Elasticity of substitution among differentiated goods</td>
</tr>
<tr>
<td>(\phi, \phi^<em>, \overline{\phi}, \overline{\phi}^</em>)</td>
<td>0.5</td>
<td>Calvo parameters for loan interest rates</td>
</tr>
<tr>
<td>(n, n^*)</td>
<td>0.5</td>
<td>Preference for DFS labor</td>
</tr>
</tbody>
</table>

Figure 2 shows the case with changing \(n\). Here a larger \(n\) means lower financial openness. Under symmetric assumptions except for the altered parameters between the two countries, \(\lambda_{Y_H}\) does not move with changes in \(n\) and \(\phi\). \(\lambda_H\), which measures the importance of the welfare loss stemming from the loan rate stickiness of the domestic (foreign) banks’ loans to
domestic firms, naturally increases (decreases) as the financial dependency on the domestic (foreign) banks becomes larger (smaller). A similar discussion is applied for $\lambda_H^*$. Although the loss from the credit spread measured by $\lambda_{HH}$ is very small under the assumption of $\phi = \phi^* = \phi_{HH} = 0.5$, the response for the changes in $n$ is non-monotonic. The term $\left(\Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t} - \xi \hat{i}^*\right)^2$ and $\left[\Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t} - \xi \left(\hat{i} - \hat{i}^*\right)\right]^2$ require central banks to stabilize the loan rate difference between domestic and foreign banks. In extreme cases where $n = 1$ or 0, there is no such dispersion. When $n$ lies between 0 and 1, there emerges some marginal cost dispersion stemming from borrowing, which peaks when $n = 0.5$. This distortion becomes relatively important when there is less stickiness in the loan contracts as Figure 3 below shows.

Figure 3 illustrates what happens when the loan rate stickiness in domestic banks’ lending increases. Naturally, $\lambda_H$ becomes larger as the loan rate stickiness at domestic banks increases, because this increases the relative loan rate dispersion among domestic firms. These results hold for $\lambda_{Y_F}$, $\lambda_F$, $\lambda_{F_F}^*$, and $\lambda_{F_F}^*$. An important implication of this exercise is that asymmetry in the loan rate stickiness between domestic and foreign banks
alters the weights in the social loss functions and may have significant implications for the optimal conduct of monetary policy cooperation.

4 International Financial Stability as Optimal Monetary Policy

We investigate the properties of the optimal monetary policy when financial markets are internationally integrated. As equations (45), (46), and (47) show, financial stability involves minimizing the dispersion among loan rates. By minimizing this dispersion, the central bank acts to reduce markup fluctuations and hence consumers’ disutility from labor. Thus, as a general principle, in the absence of distortions other than staggered loan contracts, we have

**Proposition 1** Regardless of whether cooperation is possible, central banks aim at achieving financial stability.
Nevertheless, whether financial stability is the sole target of the central bank depends on other assumptions affecting the model’s structure, namely parameters and shocks. Another interesting question is whether we can obtain the standard NOEM results regarding the optimality of inward-looking and independent monetary policy with a flexible exchange rate.

Before moving on to other propositions, for convenience, we rewrite the optimality conditions in Table 2 using lag (L) and forward (F) operators and substituting them into the requisite loss functions. Then, equations (45) and (46) become

\[
L_t = \lambda_{YH} \left\{ \frac{(1 + \eta)^{-1} \Theta_1 (1 - \lambda_1 F - \lambda_2 L) \lambda_3 \left( \hat{i}_t + u_t \right)}{1 - \lambda_1 F - \lambda_2 L} \right. \\
+ \left. (1 + \eta)^{-1} \Theta_2 (1 - \lambda_1 F - \lambda_2 L) \lambda_3 \left[ (1 - \xi) \hat{i}_t + \xi \hat{i}_t^* + \pi_t \right] \right\}^2 \\
+ \lambda_H \left\{ \frac{(1 - L) \lambda_3 \left( \hat{i}_t + u_t \right)}{1 - \lambda_1 F - \lambda_2 L} \right\}^2 \\
+ \lambda_H \left( \frac{1 - L}{1 - \lambda_1 F - \lambda_2 L} \right) \lambda_3 \left[ (1 - \xi) \hat{i}_t + \xi \hat{i}_t^* + \pi_t \right] \\
+ \lambda_{HH} \left\{ \Theta_1 \frac{\lambda_3 \left( \hat{i}_t + u_t \right)}{1 - \lambda_1 F - \lambda_2 L} - \Theta_2 \frac{(1 - L) \lambda_3 \left[ (1 - \xi) \hat{i}_t + \xi \hat{i}_t^* + \pi_t \right]}{1 - \lambda_1 F - \lambda_2 L} \right\}^2 \\
, \]

where \( \hat{i}_t^* \) is considered to be given. Furthermore, naturally,

\[
L_t^* = \lambda_{YF} \left\{ \frac{(1 + \eta)^{-1} \Theta_1^* (1 - \lambda_1^* F - \lambda_2^* L) \lambda_3^* \left( \hat{i}_t^* + u_t^* \right)}{1 - \lambda_1^* F - \lambda_2^* L} \right. \\
+ \left. (1 + \eta)^{-1} \Theta_2^* (1 - \lambda_1^* F - \lambda_2^* L) \lambda_3^* \left[ (1 - \xi^*) \hat{i}_t^* + (1 - \xi^*) \hat{i}_t^* + \pi_t^* \right] \right\}^2 \\
+ \lambda_F \left\{ \frac{(1 - L) \lambda_3^* \left( \hat{i}_t^* + u_t^* \right)}{1 - \lambda_1^* F - \lambda_2^* L} \right\}^2 \\
+ \lambda_F \left( \frac{1 - L}{1 - \lambda_1^* F - \lambda_2^* L} \right) \lambda_3^* \left[ (1 - \xi^*) \hat{i}_t^* + (1 - \xi^*) \hat{i}_t^* + \pi_t^* \right] \\
+ \lambda_{FF} \left\{ \Theta_1^* \frac{\lambda_3^* \left( \hat{i}_t^* + u_t^* \right)}{1 - \lambda_1^* F - \lambda_2^* L} - \Theta_2^* \frac{(1 - L) \lambda_3^* \left[ (1 - \xi^*) \hat{i}_t^* + (1 - \xi^*) \hat{i}_t^* + \pi_t^* \right]}{1 - \lambda_1^* F - \lambda_2^* L} \right\}^2 \\
, \]

where \( \hat{i}_t^* \) is considered to be given. Furthermore, naturally,

\[
L_t^W = L_t + L_t^*, \]

where no endogenous variables are considered to be given. This transformation enables us to analyze the nature of the optimal monetary policy with internationally integrated financial markets more intuitively.
Proposition 2 \textit{Even when financial markets are internationally integrated and banks lend both at home and abroad, there is no gain from cooperation among central banks if the exchange rate risks are completely covered by banks, i.e., } \xi = \xi^* = 0. \\

When \(\xi = \xi^* = 0\), the international banks take on all the risk stemming from exchange rate fluctuations. As a result, the foreign policy interest rate falls out of equation (48). Then, no central bank has any incentive to manipulate the welfare of counterpart country. Therefore, in this situation, as long as we assume log utility and the Cobb-Douglas aggregator as in Obstfeld and Rogoff (2002), and Corsetti and Pesenti (2005), the existence of financial market imperfections does not alter the optimality of independent monetary policy. It is worth mentioning the reason why the domestic central bank does not need any assistance from the foreign central bank, even though some portion of lending comes from foreign banks whose cost is the policy rate in the foreign country. This is because of the UIP condition. The cost for the foreign international bank lending to the home country firms, including all the risks from exchange rate fluctuations, is simply the domestic policy interest rate, as equations (42) and (43) illustrate. Even under the kind of complicated financing arrangements we see today as long as the exchange rate risks are completely covered by the lending banks and the UIP condition holds, the domestic central bank can completely control the loan rates offered by foreign international banks. Thus, we also have

Proposition 3 \textit{The optimal monetary policy is inward-looking if the exchange rate risks are completely covered by banks, i.e., } \xi = \xi^* = 0. \textit{Each central bank manipulates the policy interest rate so as to stabilize only the loan rates applied to firms in her country.}

Consequently, as long as \(\xi = \xi^* = 0\), we can derive the standard theoretical prescriptions on the optimal monetary policy in open economies, namely independent policy and a flexible exchange rate.

Another intriguing issue is whether the complete stabilization of loan interest rates is possible. In other words, can monetary policy achieve zero social loss? Equations (48) and (49) clarify this point. By setting the policy interest rates as

\[ i_t = -u_t = -\bar{u}_t, \]
and

\[ i_t^* = -u_t^* = -\bar{u}_t^*, \]

the social losses in both countries become zero. The exchange rate is expected to move in accordance with the above two monetary policy prescriptions as a result of the UIP condition in equation (13). This, however, does not cause any welfare deterioration since movements in nominal exchange rates have no impact on the marginal costs in either country when \( \xi = \xi^* = 0 \). Then, we have

**Proposition 4** When the exchange rate risks are completely covered by banks, \( \xi = \xi^* = 0 \), and the economic structures (parameters) are the same in the two countries, complete stabilization becomes possible regardless of whether monetary policy is noncooperative or cooperative if firms in one country face the same size of loan rate shocks, i.e., \( u_t = \bar{u}_t \) or \( u_t^* = \bar{u}_t^* \).

When \( 0 < \xi, \xi^* \leq 1 \), international banks and firms share the risks arising from exchange rate fluctuations. Interestingly, although with the exception of \( 0 < \xi, \xi^* \leq 1 \) the other parameter settings in this paper are the same as in previous studies of the optimality of independent and inward-looking monetary policy, there exist gains from cooperation in our economy. Both equations (48) and (49) contain the policy interest rate set by the other country’s central bank, which is outside their own control. Since monetary policy cooperation enables all policy interest rates to be internalized, higher social welfare can be achieved in both countries than when two independent monetary policies are pursued. The proposition below therefore represents very much a new feature in the literature on the optimal monetary policy in open economies.

**Proposition 5** When the risks arising from exchange rate fluctuations are shared between international banks and firms, i.e., \( 0 < \xi, \xi^* \leq 1 \), there exist gains from cooperation.

When \( 0 < \xi, \xi^* \leq 1 \), firms suffer from future exchange rate fluctuations and this acts to raise their marginal cost relative to when they are free from exchange rate risk. In order

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\(^{18}\) In this case, since attaining complete financial stability is optimal and possible, we can also derive the optimal noncooperative policy following Benigno and Benigno (2003).
to lower the marginal cost and increase social welfare, in the absence of cooperation the central bank faces a trade-off stabilizing between the financial market imperfections and the nominal exchange rate. The mechanism is similar to that discussed in the context of fixed exchange rates with local currency pricing in Devereux and Engel (2003) and Corsetti and Pesenti (2005). With local currency pricing, since exporting firms face the exchange rate risk, they set higher markups than in the case of producer currency pricing. Although in both set-ups firms end up with higher markups due to exchange rate fluctuations, the exchange rate risk affects the marginal cost through the demand channel in our paper whereas it acts through the supply channel in the case of local currency pricing. As a result, we also have

**Proposition 6** When the risks from exchange rate fluctuations are shared between international banks and firms, i.e., \(0 < \xi, \xi^* \leq 1\), there exist gains from joint nominal exchange rate management.

5 **Discussion**

There have been many empirical studies showing that firms borrow in foreign currency, i.e., that they hold foreign currency denominated debt, even though the ratios of foreign currency denominated debts to total debts are different among countries. Specifically, Kedia and Mozumdar (2003) suggest that about 1% of US firms’ debt was foreign currency denominated in 1998. Gray, Harjes, Jobst, Laxton, Tamirisa, and Stavrev (2007) report that, for East Asian countries, about 5% of firm’s loans were held in foreign currencies in 2005. For emerging countries, Jeanne (2003) reports much larger proportions of foreign-currency denominated firm debt. The ratio of foreign currency borrowing to total debt was around 60% in Argentina, 40% in Mexico, and 20% in Brazil in the 1990s. Rosenberg and Tirpak (2008) show that the new euro member states also rely heavily on foreign currency borrowing. Surprisingly, the ratio of foreign currency debt to GDP is 70% in Latvia and Estonia and 30% even in Hungary and Bulgaria, for example. These empirical facts support the assumption that \(\xi > 0\) and \(\xi^* > 0\) even though values of these parameters should differ among countries.
Based on the calibration in Table 3 with varying settings of $\xi = \xi^* = 0$, $\xi = \xi^* = 0.25$, $\xi = \xi^* = 0.5$, and $\xi = \xi^* = 0.75$, we compute welfare gains from cooperative instead of noncooperative monetary policy. We give a 1% positive loan rate shock with 0.9 AR(1) persistence to the domestic currency loan rate set by the local bank in the home country, i.e., to $u_t$. We assume the commitment optimal monetary policies in the two countries following Woodford (2003). Figure 4 shows the ratio between world welfare under cooperative monetary policy and world welfare under noncooperative monetary policy.

In this figure, a value less than unity indicates that the cooperative monetary policy is superior, which is clearly seen to be the case. As domestic firms become vulnerable to the exchange rate risk, namely as $\xi$ increases, the cooperative monetary policy becomes more beneficial. This implies that incentives for cooperative monetary policy in developed countries with low ratios of foreign currency denominated debts are weaker than in developing countries with high ratios of foreign currency denominated debt.

Moreover, surprisingly, central bank cooperation achieves much higher welfare gains when loan contracts are less sticky. This is because, when monetary policy is noncooper-
tive, nominal exchange rate fluctuations are larger under flexible loan contracts than under sticky loan contracts. As a result, the welfare gains from cooperation are more substantial, since joint management of the exchange rate enables central banks to reduce the exchange rate fluctuations which are detrimental to domestic firms’ marginal costs.

6 Conclusion

In this paper we have constructed a NOEM model with international financial frictions and have analyzed the nature of the optimal monetary policy when financial markets are internationally integrated. We demonstrate that, within this economic setting and in the absence of any other nominal rigidities, the main aim of the central bank is to achieve the financial stability which means eliminating the inefficient fluctuations of loan interest rate. Yet, at the same time, the heterogeneity in international financial markets makes the optimal conduct of monetary policy very complicated, suggesting that central banks face a trade-off unrevealed by previous studies. We show that if the exchange rate risk is partially shared among goods-producing firms, the central bank should aim to stabilize the nominal exchange rate in achieving financial stability. This is because the fluctuations in the nominal exchange rate increase the average markup set by firms which is detrimental to welfare.

One possible challenge for our future research is to incorporate sticky prices in open economies as in Clarida, Galí, and Gertler (2002) and Benigno and Benigno (2003) and to estimate such a model. This would enable a quantitative investigation of the policy trade-off between stabilizing distortions in goods and financial markets. It would also enable us to obtain robust policy prescriptions for an economy operating within a global banking system. Another direction is to examine the role of fiscal policy in addition to monetary policy under internationally integrated financial markets.

A Appendix: Derivation of the Loss Function

In this section, we derive a second-order approximation to the welfare function following Woodford (2003).
A.1 Noncooperative case

The consumer welfare in the home country is given by

$$\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ U(C_{t}) - \int_{0}^{n} V[l_{t}(h)] \, dh - \int_{n}^{1} V[l_{t}(\bar{h})] \, d\bar{h} \right\}. \tag{50}$$

The first term of equation (50) can be approximated up to the second order as

$$U(C_{t}) = \overline{CU}_{c} \left[ \hat{C}_{t} + \frac{1}{2} (1 - \nu^{-1}) \hat{C}_{t}^{2} \right] + \text{t.i.p.} + \text{Order} \left( \| \xi \|^{3} \right), \tag{51}$$

where $\text{Order} \left( \| \xi \|^{3} \right)$ expresses higher-order terms than the second-order approximation.

The second and third terms of equation (50) are also approximated as

$$\frac{1}{n} \int_{0}^{n} V[l_{t}(h)] \, dh = \overline{LV}_{t} \left[ \hat{L}_{t} + \frac{1}{2} (1 + \eta) \hat{L}_{t}^{2} + \frac{1}{2} \left( \eta + \frac{1}{\varepsilon} \right) \var_{h} \hat{L}_{t}(h) \right] + \text{t.i.p.} + \text{Order} \left( \| \xi \|^{3} \right), \tag{52}$$

and

$$\frac{1}{1 - n} \int_{n}^{1} V[l_{t}(\bar{h})] \, d\bar{h} = \overline{LV}_{t} \left[ \hat{L}_{t} + \frac{1}{2} (1 + \eta) \hat{L}_{t}^{2} + \frac{1}{2} \left( \eta + \frac{1}{\varepsilon} \right) \var_{\bar{h}} \hat{L}_{t}(\bar{h}) \right] + \text{t.i.p.} + \text{Order} \left( \| \xi \|^{3} \right). \tag{53}$$

Here, we use the labor aggregator as in equation (15) in the second-order approximation such as

$$\hat{L}_{t} = \text{E}_{h} \hat{L}_{t}(h) + \frac{1}{2} \frac{\varepsilon - 1}{\varepsilon} \var_{h} \hat{L}_{t}(h) + \text{Order} \left( \| \xi \|^{3} \right).$$

This combined with equations (52) and (53) yields

$$\int_{0}^{n} V[l_{t}(h)] \, dh + \int_{n}^{1} V[l_{t}(\bar{h})] \, d\bar{h} = \overline{LV}_{t} \left[ \hat{L}_{t} + \frac{1 + \eta}{2} \hat{L}_{t}^{2} + n - n \left( \frac{1 + \eta}{2} \right) \hat{L}_{t} + \frac{1 - n}{2} \left( \eta + \frac{1}{\varepsilon} \right) \var_{h} \hat{L}_{t}(h) \right] + \text{t.i.p.} + \text{Order} \left( \| \xi \|^{3} \right), \tag{54}$$

where we use the approximation for equation (20). From equation (32), the condition that the demand of labor is equal to the supply of labor is given by

$$\tilde{L}_{t} = \int_{0}^{1} \tilde{L}_{t}(f) \, df = \int_{0}^{1} f^{-1} [y_{t}(f)] \, df,$$
whose second-order approximation becomes

\[ \hat{L}_t = \frac{1}{\mu} \left( \hat{Y}_{H,t} - a_t \right) + \frac{1}{2} \left( 1 + \theta - \frac{1}{\mu} \right) \frac{1}{\mu} \left( \hat{Y}_{H,t} - a_t \right)^2 + \text{Order} \left( \| \xi \|^2 \right). \]

By substituting this, we can now transform equation (54) into

\[ \int_0^n V \left[ l_t(h) \right] dh + \int_n^1 V \left[ l_t(\bar{h}) \right] d\bar{h} = \mathcal{C} U_c \left[ \begin{array}{c} \hat{Y}_{H,t} + \frac{1}{2} \left( 1 + \theta + \frac{2}{\mu} \right) \hat{Y}_{H,t}^2 \\ + n \left( 1 - n \right) \frac{1}{2(1 + \eta)} \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{\xi} - \hat{\xi}^* \right) \right]^2 \end{array} \right] + \text{t.i.p.} + \text{Order} \left( \| \xi \|^2 \right), \]

where we use the following:

\[ \hat{L}_t - \hat{n} = \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{\xi} - \hat{\xi}^* \right) \right] - \eta \left( \hat{L}_t - \hat{n} \right), \]

which are derived using equations (9), (10), (16), (17), (18), (19), (21), and (22). Furthermore, following Clarida, Galí, and Gertler (2002), we replace \( \frac{1}{n} LV_i \) by \( \frac{1}{2} C U_c \) thanks to the social planner’s optimization problem.\(^{19}\)

Then we can combine equation (51) and equation (55) as

\[ U_t = \mathcal{C} U_c \left[ \begin{array}{c} \frac{1}{2} (1 - v^{-1}) \hat{C}_t^2 + \hat{C}_t \\ \hat{Y}_{H,t} - \frac{1}{2} \left( 1 + \theta + \frac{2}{\mu} \right) \hat{Y}_{H,t}^2 + \left( \theta + \frac{2}{\mu} \right) q_t \hat{Y}_{H,t} \\ - n \left( 1 - n \right) \frac{1}{2(1 + \eta)} \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{\xi} - \hat{\xi}^* \right) \right]^2 \end{array} \right] + \text{t.i.p.} + \text{Order} \left( \| \xi \|^2 \right), \]

where \( \eta_t \equiv \mu \left( \theta + \varepsilon^{-1} \right) \). To transform \( \text{var}_h \hat{l}_t(h) \) and \( \text{var}_{\hat{n}} \hat{h}(\bar{h}) \) further, we use the optimal conditions of labor supply and demand functions given by equations (9), (10), (24) and (25). Approximation of these equations leads to

\[ \text{var}_h \ln \hat{l}(h) \equiv \mathcal{C} \text{var}_h \ln \left[ 1 + r_t(h) \right] + \text{Order} \left( \| \xi \|^2 \right), \]

and

\[ \text{var}_h \ln \hat{h}(\bar{h}) \equiv \mathcal{C} \text{var}_{\hat{n}} \ln \left[ 1 + r_t(\bar{h}) \right] + \text{Order} \left( \| \xi \|^2 \right), \]

\(^{19}\)The social planner optimizes the following problem:

\[ \max_{C, \hat{L}} U(C) - V(L) \text{ s.t. } C = \hat{L}^{-\frac{1}{2}} \left( Y_F \right)^{\frac{1}{2}}, \text{ where } Y_F \text{ is exogenously given.} \]
where $\Xi \equiv \Theta^2 \frac{\varepsilon^2}{(1+\eta)^2}$ and $\Xi^* \equiv (\Theta^*)^2 \frac{\varepsilon^2}{(1+\eta)^2}$. Then, equation (56) is further transformed into

$$
U_t = -\frac{1}{2} \mathcal{C} U_c \left[ -\frac{1}{2} (1 - v^{-1}) \hat{C}_t^2 - \hat{C}_t + \hat{Y}_{H,t} + \frac{1}{2} \left( 1 + \theta + \frac{2}{n} \right) \hat{Y}_{H,t}^2 \\
+ n (1 - n) \left[ \frac{1}{2(1+\eta^2)} \left( \Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{H,t}^* - \xi \left( i - \hat{i}^* \right) \right) \right]^2 \\
+ n \eta_r \var_h \ln \left[ 1 + r_t(h) \right] + \left( 1 - n \right) \eta_r^* \var_{\pi} \ln \left[ 1 + r_t(\bar{h}) \right] \right] \\
+ \text{t.i.p. + Order (}\| \xi \| ^3 \text{),}
$$

(57)

where $\eta_r \equiv \Xi \eta_l = \mu \Theta^2 \frac{\varepsilon^2}{(1+\eta)^2}$ and $\eta_r^* \equiv \Xi^* \eta_l = \mu \Theta^* \frac{\varepsilon^2}{(1+\eta^2)}$. Here, we also assume that the central bank aims at stabilizing the deviations from the nonstochastic efficient steady state. The remaining important part is to transform $\var_h \ln \left[ 1 + r_t(h) \right]$ in equation (57).

Following Woodford (2003), we define $\bar{R}_{H,t}$ and $\Delta_i^R$ as

$$
\bar{R}_{H,t} \equiv E_h \ln \left[ 1 + r_t(h) \right],
$$

and

$$
\Delta_i^R \equiv \var_h \ln \left[ 1 + r_t(h) \right].
$$

Then, we can derive

$$
\Delta_i^R = \phi \Delta_{i-1}^R + \frac{\phi}{1 - \phi} \left( \bar{R}_{H,t} - \bar{R}_{H,t-1} \right)^2.
$$

(58)

Furthermore, the following is also derived from the log-linear approximation:

$$
\bar{R}_{H,t} = \ln (1 + R_{H,t}) + \text{Order (}\| \xi \| ^2),
$$

(59)

where we make use of the definition of the aggregate loan rates:

$$
1 + R_{H,t} \equiv \int_0^1 \frac{q_t(h)}{Q_t} [1 + r_t(h)] dh.
$$

Then, from equations (58) and (59) we obtain

$$
\Delta_i^R = \phi \Delta_{i-1}^R + \frac{\phi}{1 - \phi} \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2,
$$

(60)

where

$$
\hat{R}_{H,t} \equiv \ln \frac{1 + \hat{R}_{H,t}}{1 + \hat{R}_H}.
$$
The forward iteration of equation (60) leads to
\[ \Delta_t^R = \phi^{t+1} \Delta_{t-1}^R + \sum_{s=0}^{t} \phi^{t-s} \left( \frac{\phi}{1-\phi} \right) \left( \hat{R}_{H,s} - \hat{R}_{H,s-1} \right)^2 . \]

Then, we have
\[ \sum_{t=0}^{\infty} \beta^t \Delta_t^R = \frac{\phi}{(1-\phi)(1-\phi \beta)} \sum_{t=0}^{\infty} \beta^t \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \text{t.i.p.} + \text{Order}\left(\| \xi \|^3 \right). \] (61)

Then, we have
\[ U_t = -\Lambda \left[ \lambda_{YH} \hat{Y}_{H,t}^2 + \lambda_H \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \lambda_{H}^* \left( \hat{R}_{H,t}^* - \hat{R}_{H,t-1}^* \right)^2 + \lambda_{HH} \left( \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{t} - \hat{t}^* \right) \right)^2 \right] + \text{t.i.p.} + \text{Order}\left(\| \xi \|^3 \right), \]

where we assume parameters in Table 3 and the given foreign output and policy rate. We have similar procedures to derive the loss function of the foreign country under noncooperative allocation.

A.2 Cooperative case

In this case, the following conditions are newly necessary.

(1) the output in abroad is not given for two countries,

(2) following Clarida, Galí, and Gertler (2002), we replace \( \frac{1}{\mu} \bar{Y}_1 \) by \( \bar{C}U_c \) thanks to the social planner’s optimization problem.\(^{20}\)

Then, the world loss function \( L_t^W = L_t + L_t^* \) is given by
\[
L_t^W = L_t + L_t^* = \lambda_{YH} \hat{Y}_{H,t}^2 + \lambda_{YF} \hat{Y}_{F,t}^2
+ \lambda_H \left( \hat{R}_{H,t} - \hat{R}_{H,t-1} \right)^2 + \lambda_{H}^* \left( \hat{R}_{H,t}^* - \hat{R}_{H,t-1}^* \right)^2
+ \lambda_{HH} \left[ \Theta_1 \hat{R}_{H,t} - \Theta_2 \hat{R}_{H,t}^* - \xi \left( \hat{t} - \hat{t}^* \right) \right]^2
+ \lambda_{FF} \left[ \Theta_1 \hat{R}_{F,t} - \Theta_2 \hat{R}_{F,t}^* + \xi \left( \hat{t} - \hat{t}^* \right) \right]^2 .
\]

\(^{20}\) The social planner optimizes the following problem:
\[
\max_{c,\bar{L},\bar{Y}} \frac{1}{2} \left( U(c) - V(\bar{L}) \right) + \frac{1}{2} \left( U(c) - V(\bar{L}) \right) \text{ s.t. } C = \bar{L}^{\mu^*} \left( \bar{L} \right)^{1-\mu^*}.
\]
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