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Ippei Fujiwara* and Yuki Teranishi**

Abstract
In this paper, we investigate the relationship between real exchange rate dynamics and financial market imperfections. For this purpose, we first construct a New Open Economy Macroeconomics (NOEM) model that incorporates international staggered loan contracts as a simple form of the financial market imperfections. Recent empirical studies show that such staggered loan contracts are prevalent in the US, UK, and Japan and direct shocks to the bank lending interest rate (risk premium shocks) are major drivers of business cycle dynamics. Simulation results only with such a financial market friction and a risk premium shock can generate persistent, volatile, and realistic hump-shaped responses of real exchange rates, which have been thought very difficult to reproduce in standard NOEM models. This implies that these financial market developments can possibly be a major source of real exchange rate fluctuations.

Keywords: Financial Market Imperfections; Real Exchange Rates; Staggered Loan Contracts
JEL classification: F31, E41

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1 Introduction

Empirical studies conclude that real exchange rate dynamics are very volatile, persistent, and hump-shaped against shocks.\(^1\) So far in international finance, it has been intensively argued whether theoretical dynamic general equilibrium models can reproduce such realistic exchange rate dynamics. Chari, Kehoe, and McGrattan (2002), focusing on the first two features, insist that the New Open Economy Macroeconomics (NOEM) models may account for the volatility but not for the persistence. In response to this critique, several studies have attempted to solve these three puzzles by introducing such features as strategic complementarity, nonoptimizing monetary policy, and optimal monetary policy into otherwise standard NOEM models as in Bergin and Feenstra (2000), Benigno (2004a), and Benigno (2004b). These newly introduced mechanisms mitigate the persistence puzzle of the real exchange rate dynamics to some extent, but have not yet solved it completely. Actual persistence of real exchange rates is still higher than that simulated in those models. Furthermore, they cannot explain the significant hump-shaped responses of real exchange rates found in actual data.

While the focus has been on the monetary shock in the above studies following Chari, Kehoe, and McGrattan (2002), Steinsson (2007) first stresses the importance of generating hump-shaped responses based on his autoregressive estimation of real exchange rates\(^2\) and shows that realistic levels of volatility, persistence, and hump-shaped responses of real exchange rates can be generated with the NOEM models when the cost-push shock is added to the economy, where the home bias is very strong. The resolution by Steinsson (2007) is simple but very powerful. Because the real exchange rate is determined as the ratio

\(^1\)See, for example, Huizinga (1987), Eichenbaum and Evans (1995) and Cheung and Lai (2000).

\(^2\)The importance of reproducing the hump-shaped responses are emphasized in, for example, Christiano and Vigfusson (2003) and Vigfusson (2007).
of the marginal utility out of consumption in the home country over that in the foreign country, the volatility and persistence of real exchange rates are muted when a market is complete, as usually assumed in literatures in this field. By setting the home bias parameter very high, the home shocks have only negligible effects on foreign variables. Under such circumstances, the hump-shaped responses of real exchange rates come from responses in consumptions and so from real interest rates, because the consumption can be represented as the discounted sum of future real interest rates according to the Euler conditions. There, after a cost-push shock hits the economy, inflation and consumption can move in opposite directions while they comove to the monetary policy shock. As a result, real interest rates affected naturally by short-term nominal interest rates set through the Taylor (1993)-type rule show nonmonotonic responses.\footnote{This logic is similar to Benigno (2004a).} Therefore, we can reproduce the hump-shaped responses in real interest rates and therefore in real exchange rates.

Although the explanation by Steinsson (2007) is neat and very clear, is this the end of the conquest for the causes of real exchange rate dynamics? To this question, we answer “not necessarily.” The reasons are as follows. The empirical part of Steinsson (2007) only demonstrates the estimation results of the autoregressive models of the real exchange rates. This implies that the cost-push shock should be considered only as one possible source from numerous candidates, which are important for realistic real exchange dynamics. Particularly, recent empirical papers emphasize the role of a direct shock on the financial market in explaining the actual business cycle tendencies found in the data. For example, Christiano, Motto, and Rostagno (2007) show that according to their Bayesian estimation results of the dynamic general equilibrium model\footnote{The model is based on Christiano, Eichenbaum, and Evans (2005).} combined with the financial accelerator model by Bernanke, Gertler, and Gilchrist.
a direct shock on the net worth dynamics, namely a direct shock on the external finance premium, has been playing a very important role in US and Euro area business cycles.\(^5\) Moreover, estimation results on microlevel firm data by Levin, Natalucci, and Zakrajsek (2004) show that the external finance premium moves irrespective of the dynamics in leverages. These results suggest that the premium is fluctuating, not according to such a theory as the costly state verification in Townsend (1979), but following the shock that affects the financial market directly. In international finance, several studies emphasize the importance of such an interest rate spread shock. Uribe and Yue (2006) demonstrate through VAR analysis that country interest rate spread shocks explain about 12% of business cycles in emerging economies. Neumeyer and Perri (2005) show that country interest rate spread shocks explain 50% of output fluctuations in Argentina. Moreover, in international finance, economic theories have not yet fully explained long remaining differences between domestic and foreign interest rates. There should be international risk premiums that still cannot be explained by theories. Therefore, we examine the case of a shock directly related to the financial market; i.e., a risk premium shock.

To be able to investigate the role of such a direct shock in financial markets on real exchange rate dynamics, we construct a model with an explicit role of banks. In particular, we incorporate staggered (sticky) loan contracts examined in Teranishi (2007) into an otherwise standard NOEM model. Many studies, such as Goldfeld (1966), Slovin and Sushka (1983), Berger and Udell (1992), Cowling (2007), and Teranishi (2007), provide empirical evidence on sticky loan interest rate contracts in the US, UK, and Japan. This sticky loan contract is one form of financial market imperfection. Financial market imperfection

\(^5\)Smets and Wouters (2007) conclude that a risk premium shock has a substantial effect on US output dynamics even though such a risk premium shock is assumed to affect the consumer side through asset holdings rather than the firm side in their model.
implies the existence of a wedge between the optimal levels and the actual levels of loan interest rates. For instance, in the financial accelerator model, this wedge is determined by the time-varying leverage. However, in our model with staggered loan contracts, the wedge is determined by the Calvo (1983) parameter as in the case with a staggered price setting. We can apply direct shocks to this model related to financial market imperfections such as the cost-push shock in the standard New Keynesian models.

The contribution of this paper is that we investigate the relationship between real exchange rate dynamics and financial market developments using a NOEM model that incorporates international staggered loan contracts as a simple form of the international financial market imperfections, and risk premium shocks as a direct shock in the financial market. We show that staggered loan contracts and risk premium shocks can generate persistent, volatile, and hump-shaped responses in real exchange rates, which are considered very difficult to reproduce in standard NOEM models. An outcome that such a risk premium shock can actually generate volatile, persistent, and hump-shaped responses in real exchange rates is very intriguing. A risk premium shock itself is not a cost-push shock but is considered to be a more microfounded cost-push shock. We assume working capital loans, namely, that firms must borrow from banks in advance of production to pay wage bills, as in Christiano, Eichenbaum, and Evans (2005). Under such circumstances, loan payments and wages become the components of the marginal costs. As a result, a direct shock to loan interest rates works as if it were a cost-push shock and therefore results in hump-shaped responses in the real exchange rate by a mechanism similar to that explained in Steinsson (2007). Moreover, in contrast to Chari, Kehoe, and McGrattan (2002) and Steinsson (2007), we show that only with financial market developments such as a staggered loan contract and a risk premium shock can we reproduce real-
Figure 1: Agents in the model

This means that in a model with international staggered loan contracts, price stickiness is not necessary for replicating realistic exchange rate dynamics. Therefore, staggered loan contracts and a risk premium shock is possibly a major source of real exchange rate fluctuations.

This paper is structured as follows. In the next section, we derive a model under international staggered loan contracts. Then, Section 3 shows that realistic responses of real exchange rates are reproduced in such a model. Finally, in Section 4, we summarize our findings in this paper and discuss future extensions.

## 2 Model

The model consists of two symmetric countries. There are four types of agent in each country, consumers, firms, private banks, and the central bank, as depicted in Figure 1.
2.1 Consumer

A representative consumer plays four roles: (1) to consume differentiated goods determined through two-step cost minimization problems on both home- and foreign-produced consumer goods; (2) to choose the amount of aggregate consumption, bank deposits, and investment in risky assets given a deposit interest rate set by the central bank; (3) with monopolistic power on labor supply, to provide differentiated labor services which belong to either the domestically financially supported (DFS) or the internationally financially supported (IFS) groups, and to offer wages to those differentiated types of labor; and (4) to own banks and firms, and to receive dividends in each period. Role (3) is crucial role in staggered loan contracts. Thanks to this differentiated labor supply, the demand for loans is differentiated without assuming any restrictions on aggregate loans and loan interest rates.\(^6\)

2.1.1 Cost minimization

The utility of the representative consumer\(^7\) in the home country \(H\) is increasing and concave in the aggregate consumption index \(C_t\). The consumption index that consists of bundles of differentiated goods produced by home and foreign firms\(^8\) is expressed as:

\[
C_t = \frac{C_{H,t}^{\phi_H} C_{F,t}^{1-\phi_H}}{\phi_H (1 - \phi_H)^{1-\phi_H}},
\]

where \(\phi_H (0 \leq \phi_H \leq 1)\) is a preference parameter that expresses the home bias. Here, \(C_{H,t}\) and \(C_{F,t}\) are consumption subindices of the continuum of differentiated goods produced by firms in the home country and the foreign country, respectively. They are defined as:

\[
C_{H,t} \equiv \left[ \int_0^1 c_t (f) \frac{x-1}{f(x)} df \right]^{\frac{x}{x-1}},
\]

\(^6\)For details, see Teranishi (2007).
\(^7\)The same optimal allocations are obtained even by assuming that each homogenous consumer provides differentiated labor supply to each firm.
\(^8\)We follow Obstfeld and Rogoff (2000).
and
\[ C_{F,t} = \left[ \int_0^1 c_t(f^*) \, \frac{e^{-\sigma}}{f^*} \, df^* \right]^{\frac{1}{1-\sigma}}, \]
where \( c_t(f) \) is the demand for a good produced by firm \( f \) in the home country and \( c_t(f^*) \) is the demand for a good produced by a firm \( f^* \) in the foreign country, where the asterisk denotes foreign variables. It is assumed that there are no trade frictions and consumers in both countries have the same preferences over the differentiated goods. Following the standard cost minimization problem on the aggregate consumption index of home and foreign goods as well as the consumption subindices of the continuum of differentiated goods, we can derive the consumption-based price indices:
\[ P_t \equiv P_{H,t}^{\phi_H} P_{F,t}^{1-\phi_H}, \]
with
\[ P_{H,t} \equiv \left[ \int_0^1 p_t(f)^{1-\sigma} \, df \right]^{\frac{1}{1-\sigma}}, \]
and
\[ P_{F,t} \equiv \left[ \int_0^1 p_t(f^*)^{1-\sigma} \, df^* \right]^{\frac{1}{1-\sigma}}, \]
where \( p_t(f) \) is the price on \( c_t(f) \), and \( p_t(f^*) \) is the price on \( c_t(f^*) \). Then, we can obtain the following Hicksian demand functions for each differentiated good given the aggregate consumption:
\[ c_t(f) = \phi_H \left( \frac{p_t(f)}{P_{H,t}} \right)^{-\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t, \]
and
\[ c_t(f^*) = (1 - \phi_H) \left[ \frac{p_t(f^*)}{P_{F,t}} \right]^{-\sigma} \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t. \]
Here, as in other applications of the Dixit and Stiglitz (1977) aggregator, consumers’ allocations across differentiated goods at each time are optimal in terms of cost minimization.
We can derive similar optimality conditions for the foreign counterpart. For example, the consumption-based price index is now expressed as:

$$P_t^* \equiv (P_{H,t}^*)^{\phi_F} (P_{F,t}^*)^{1-\phi_F},$$

and the demand functions for each differentiated good given the aggregate consumption are:

$$c_t^*(f) = \phi_F \left[ \frac{p_t^*(f)}{p_{H,t}^*} \right]^{-\sigma} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^*,$$

and

$$c_t^*(f^*) = (1 - \phi_F) \left[ \frac{p_t^*(f^*)}{p_{F,t}^*} \right]^{-\sigma} \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-1} C_t^*.$$

### 2.1.2 Utility maximization

A representative consumer in the home country maximizes the following utility function:

$$U T_t = E_t \left\{ \sum_{j=t}^{\infty} \beta^{j-t} \left[ U(C_T) - \int_{0}^{n} V(l_T(h))dh - \int_{1}^{n} V(l_T(\bar{h}))d\bar{h} \right] \right\},$$

where $E_t$ is the expectation operator conditional on the state of nature at date $t$. The budget constraint of the consumer is given by:

$$P_tC_t + E_t [X_{t+1}B_{t+1}] + D_t \leq B_t + (1 + i_{t-1})D_{t-1} + \int_{0}^{n} w_t(h)l_t(h)dh$$

$$+ \int_{1}^{n} w_t(\bar{h})l_t(\bar{h})d\bar{h} + \Pi_t^B + \Pi_t^F,$$

where $B_t$ is a risky asset, $D_t$ is the deposit to private banks, $i_t$ is the nominal deposit interest rate set by a central bank from $t - 1$ to $t$, $w_t(h)$ is the nominal wage for labor supplied from the DFS $l_t(h)$, $w_t(\bar{h})$ is the nominal wage for labor supplied from the IFS $l_t(\bar{h})$, $\Pi_t^B = \int_{0}^{1} \Pi_{t-1}^B(h)dh$ is the nominal dividend stemming from the ownership of both local and international banks in the home country, $\Pi_t^F = \int_{0}^{1} \Pi_{t-1}^F(f)df$ is the nominal dividend from the ownership of the

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For simplicity, we do not explicitly include the amount of contingency claims under complete financial markets.
firms in the domestic country, and $X_{t,t+1}$ is the stochastic discount factor. Here, because we assume a complete financial market between the two countries, the consumer in each country can internationally buy and sell the state contingent securities to insure against country specific shocks. Consequently, there only exists a unique discount factor. The relationship between the deposit interest rate and the stochastic discount factor is now expressed as:

$$\frac{1}{1+i_t} = E_t [X_{t,t+1}].$$

(6)

Given the optimal allocation of differentiated consumption expenditures, the consumer now optimally chooses the total amount of consumption, risky assets, and deposits in each period. Necessary and sufficient conditions, when the transversality condition is satisfied, for those optimizations are given by:

$$U_C(C_t) = \beta (1 + i_t) E_t \left[ U_C(C_{t+1}) \frac{P_t}{P_{t+1}} \right],$$

(7)

$$U_C(C_{t+1}) U_C(C_{t+1}) = \frac{\beta}{X_{t,t+1}} P_{t+1}. \quad (8)$$

Together with equation (6), we see that the condition given by equation (7) defines the intertemporally optimal allocation on aggregate consumption. Then, the standard New Keynesian IS curve for the home country, by log-linearizing equation (7) around steady states, is obtained as follows:

$$\tilde{C_t} = E_t \tilde{C}_{t+1} - v \left( \tilde{i_t} - E_t \pi_{t+1} \right), \quad (9)$$

where an aggregate inflation in the home country $\pi_t \equiv \ln \frac{P_t}{P_{t-1}}$ and $v \equiv -\frac{\gamma C}{\sigma C C D} > 0$. Each variable is defined as the log deviation from its steady state value, where the log-linearized version of variable $x_t$ is expressed by $\tilde{x}_t = \ln (x_t / \bar{x})$, except for $\pi_t$, given that $\bar{x}$ is the steady state value of $x_t$.

In this model, a representative consumer provides all types of differentiated labor to each firm, and therefore maintains some monopoly power over the determination of his own wage, as in Erceg, Henderson, and Levin (2000). There
are two types of labor group: the DFS and the IFS. The workers populated on \([0, n]\) belong to the DFS and other labor populated on \([n, 1]\) belong to the IFS.\(^\text{10}\)

We assume that each firm hires all types of labor in the same proportion from the two groups. The consumer sets each wage \(w_t(h)\) for any \(h\) and \(w_t(h)\) for any \(h\) to maximize its utility subject to the budget constraint given by equation (5) and the labor demand functions given by equations (18) and (19) in the next section. Here, although differentiated labor supply is assumed in this paper, consumers change wages in a flexible manner. Then we have the optimality conditions for labor supply as follows:

\[
\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{V_t[l_t(h)]}{U_C(C_t)} \tag{10}
\]

and

\[
\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{V_t[l_t(h)]}{U_C(C_t)} \tag{11}
\]

As written above, thanks to this heterogeneity in labor supply, we can model the differentiated demand for loans without assuming any restrictions on aggregate loans and loan interest rates. In this paper, consumers supply their labor only for firms, not for banks.

Similar to the above case with cost minimization, we can derive the optimality conditions for the foreign counterpart. For example, the standard New Keynesian IS curve for the foreign country is:

\[
\hat{C}_t^* = E_t \hat{C}_{t+1}^* - \nu^* (\hat{t}_t^* - E_t \pi_{t+1}^*), \tag{12}
\]

which is derived from the optimality condition on the foreign asset holdings:

\[
\frac{U_C^*(C_t)}{U_C^*(C_{t+1})} = \beta \left( 1 + i_t^* \right) E_t \left[ \frac{U_C^*(C_{t+1}) P_{t+1}^*}{P_t^*} \right],
\]

\[
\frac{U_C^*(C_t)}{U_C^*(C_{t+1})} = \beta \left( 1 + i_t^* \right) E_t \left[ \frac{P_{t+1}^*}{P_t^*} \right]. \tag{13}
\]

\(^\text{10}\)The difference of these two groups is characterized by somewhat wider properties of workers, like English speaking or Japanese speaking, though the differences of workers within each group are characterized by narrower properties of workers, like person who has knowledge of accounting in bank or person who has skill of making automobile in plant.
Furthermore, the optimality conditions for labor supply are expressed now as:

\[
\frac{w^*_t(h^*)}{P^*_t} = \frac{\epsilon^*}{\epsilon^* - 1} \frac{V^*_t(l^*_t(h^*))}{U^*_C(C^*_t)},
\]

and

\[
\frac{w^*_t(h^*)}{P^*_t} = \frac{\epsilon^*}{\epsilon^* - 1} \frac{V^*_t(l^*_t(h^*))}{U^*_C(C^*_t)}.
\]

### 2.2 Firms

There exists a continuum of firms populated over unit mass \([0, 1]\) in each country. Each firm plays two roles. First, each firm decides the amount of differentiated labor to be employed from both the DFS and IFS groups, through the two-step cost minimization problem on the production cost. Part of the costs of labor must be financed by external loans from banks. For example, in country \(H\), to finance the costs of hiring workers from the DFS, the firm must borrow from local banks in the home country. However, to finance the costs of hiring workers from the IFS, the firm must borrow from international banks in the foreign country. One reason of such a loan difference is that a firm borrows loans from both domestic and foreign banks, i.e. a bank lends loans to both domestic and foreign firms as shown in Gadanecz (2004). Another reason is that we also know that firms borrow many loans with different loan interest rates at the same time depending on properties of projects. We interpret that these project differences are characterized by types of labor. Labor is immobile between the two countries. Here, we assume that firms must use all types of labor and therefore borrow from both local and international banks in the same proportion.\(^{11}\)

Second, in a monopolistically competitive goods market, where individual demand curves on differentiated consumption goods are offered by consumers, each firm sets a differentiated goods price for both home and foreign countries to maximize its profit. Prices are set in a staggered manner as in

\(^{11}\)The same structure is assumed for employment in Woodford (2003).
the Calvo (1983) - Yun (1996) framework under the pricing-to-market strategy following Steinsson (2007). Under a pricing-to-market assumption, firms set different prices on the same goods in different countries.\(^{12}\)

**2.2.1 Cost minimization**

Firms in both home and foreign countries optimally hire differentiated labor as price takers. This optimal labor allocation is carried out through two-step cost minimization problems. Domestic firm \(f\) hires all types of labor from both the DFS and IFS groups. When hiring from the DFS group, \(\gamma\) portion of the labor cost associated with labor type \(h\) is financed by borrowing from the local bank \(h\). Then, the first-step cost minimization problem on the allocation of differentiated labor from the DFS is given by:

\[
\min_{l_t(h,f)} \int_0^n [1 + \gamma r_t(h)] w_t(h) l_t(h,f) dh,
\]

subject to the sub-index regarding labor from DFS to firm \(f\):

\[
L_t(f) \equiv \left[ \frac{1}{n} \int_0^n l_t(h,f) \frac{1}{\frac{1}{\gamma} + dh} \right]^\frac{1}{\gamma},
\]

where \(r_t(h)\) is the loan interest rate applied to employ a particular labor type \(h\) applied to differentiated labor supply. There \(l_t(h,f)\) denotes type of labor \(h\) employed by firm \(f\). The local bank \(h\) has some monopoly power over setting loan interest rates. Thus, we assume the monopolistic competition on the loan contracts between banks and firms. The relative demand on differentiated labor is given as follows:

\[
l_t(h,f) = \frac{1}{n} L_t \left\{ \frac{[1 + \gamma r_t(h)] w_t(h)}{\Omega_t} \right\}^{-\varepsilon}, \tag{14}
\]

where

\[
\Omega_t \equiv \left\{ \frac{1}{n} \int_0^n \left[ [1 + \gamma r_t(h)] w_t(h) \right]^{1-\varepsilon} dh \right\}^\frac{1}{1-\varepsilon}.
\]

\(^{12}\)For the theoretical rational behind pricing-to-market, see, for example, Krugman (1987) and Devereux (1997).
As a result, we can derive:

\[
\int_0^n [1 + \gamma r_t (h)] w_t (h) l_t (h, f) \, dh = \Omega_t L_t (f).
\]

Through a similar cost minimization problem, we can derive the relative demand for each type of differentiated labor from the IFS as:

\[
l_t (\bar{h}, f) = \frac{1}{1 - n} \mathcal{L}_t \left\{ \frac{[1 + \gamma r_t^* (\bar{h})] w_t (\bar{h})}{\Omega_t} \right\}^{-\epsilon}, \tag{15}
\]

where

\[
\Omega_t \equiv \left\{ \frac{1}{1 - n} \int_n^1 \left\{ [1 + \gamma r_t^* (\bar{h})] w_t (\bar{h}) \right\}^{1-\epsilon} \, d\bar{h} \right\}^{1/\epsilon}.
\]

Then:

\[
\int_n^1 [1 + \gamma r_t^* (\bar{h})] w_t (\bar{h}) l_t (\bar{h}, f) \, dh = \mathcal{L}_t \bar{L}_t (f).
\]

According to the above two optimality conditions, the firms optimally choose the allocation of differentiated workers between the two groups. Because firms have some preference \( n \) to hire workers from the DFS and \( (1 - n) \) to hire workers from the IFS, the second-step cost minimization problem describing the allocation of differentiated labor between these two groups is given by:

\[
\min_{L_t, \bar{L}_t} \Omega_t L_t (f) + \mathcal{L}_t \bar{L}_t (f),
\]

subject to the aggregate labor index:

\[
\tilde{L}_t (f) \equiv \frac{[L_t (f)]^n \left[ \bar{L}_t (f) \right]^{1-n}}{n^n (1 - n)^{1-n}}.
\]

Then, the relative demand functions for each differentiated type of labor are derived as follows:

\[
L_t (f) = n \bar{L}_t (f) \left( \frac{\Omega_t}{\Omega_t} \right)^{-1}, \tag{16}
\]

\[
\bar{L}_t (f) = (1 - n) \tilde{L}_t (f) \left( \frac{\Omega_t}{\Omega_t} \right)^{-1}, \tag{17}
\]

and

\[
\tilde{\Omega}_t \equiv \Omega_t^n \mathcal{L}_t^{1-n}.
\]
Therefore, we can obtain the following equations:

\[ \Omega_t L_t (f) + \bar{\Omega}_t \bar{L}_t (f) = \tilde{\Omega}_t \tilde{L}_t (f), \]

\[ l_t (h, f) = \left\{ \frac{[1 + \gamma r_t (h)] w_t (h)}{\Omega_t} \right\}^{-\epsilon} \left( \frac{\Omega_t}{\bar{\Omega}_t} \right)^{-1} \tilde{L}_t (f), \quad (18) \]

and

\[ l_t (\bar{h}, f) = \left\{ \frac{[1 + \gamma r_t (\bar{h})] w_t (\bar{h})}{\Omega_t} \right\}^{-\epsilon} \left( \frac{\bar{\Omega}_t}{\Omega_t} \right)^{-1} \tilde{L}_t (f), \quad (19) \]

from equations (14), (15), (16), and (17). We can now clearly see that the demand for each differentiated worker depends on wages and loan interest rates, given the total demand for labor.

Finally, by the assumption that the firms finance part of the labor costs by loans, we can derive:

\[ q_t (h, f) = \gamma w_t (h) l_t (h, f) \]

\[ = \gamma w_t (h) \left\{ \frac{[1 + \gamma r_t (h)] w_t (h)}{\Omega_t} \right\}^{-\epsilon} \left( \frac{\Omega_t}{\bar{\Omega}_t} \right)^{-1} \tilde{L}_t (f), \]

and

\[ q_t (\bar{h}, f) = \bar{\gamma} w_t (\bar{h}) l_t (\bar{h}, f) \]

\[ = \bar{\gamma} w_t (\bar{h}) \left\{ \frac{[1 + \bar{\gamma} r_t (\bar{h})] w_t (\bar{h})}{\Omega_t} \right\}^{-\epsilon} \left( \frac{\bar{\Omega}_t}{\Omega_t} \right)^{-1} \tilde{L}_t (f). \]

\[ q_t (h, f) \] and \[ q_t (\bar{h}, f) \] denote amounts of loan borrowed by firm \( f \) to the labor types \( h \) and \( \bar{h} \), respectively. These conditions demonstrate that the demands for each differentiated loan also depend on the wages and loan interest rates, given the total labor demand.

We can obtain similar conditions for the foreign country. For example:

\[ l_t (h^*, f^*) = \left\{ \frac{[1 + \gamma r_t^* (h^*)] w_t^* (h^*)}{\Omega_t^*} \right\}^{-\epsilon^*} \left( \frac{\Omega_t^*}{\bar{\Omega}_t^*} \right)^{-1} \tilde{L}_t^* (f^*), \]

and

\[ l_t (\bar{h}^*, f^*) = \left\{ \frac{[1 + \bar{\gamma} r_t^* (\bar{h}^*)] w_t^* (\bar{h}^*)}{\Omega_t^*} \right\}^{-\epsilon^*} \left( \frac{\bar{\Omega}_t^*}{\Omega_t^*} \right)^{-1} \tilde{L}_t^* (f^*), \]
where

\[ L_t^*(f^*) \equiv \left( \frac{1}{n} \right)^{\frac{1}{n}} \int_0^n l_t^*(h^*, f) \frac{dh^*}{\tilde{h}^*} \right)^{\frac{1}{1-n}}, \]

\[ \tilde{L}_t^*(f^*) \equiv \left( \frac{1}{1-n} \right)^{\frac{1}{n}} \int_0^1 l_t^* \left( \tilde{h}^*, f \right) \frac{d\tilde{h}^*}{\tilde{h}^*} \right)^{\frac{1}{1-n}}, \]

\[ \tilde{\Omega}_t^* \equiv \frac{\left[ L_t^*(f^*) \right]^n \left[ \tilde{L}_t^*(f^*) \right]^{n-1}}{n^n (1-n)^{1-n}}, \]

\[ \Omega_t^* \equiv \left\{ \left( \frac{1}{n} \right)^{\frac{1}{n}} \int_0^n \left\{ \left( 1 + \gamma^* r_t^* (h^*) \right) w_t^* (h^*) \right\} ^{1-\epsilon} dh^* \right\} ^{\frac{1}{1-n}}, \]

\[ \Omega_t^* \equiv \left\{ \left( \frac{1}{1-n} \right)^{\frac{1}{n}} \int_0^1 \left\{ \left( 1 + \tau^* r_t (\tilde{h}^*) \right) w_t (\tilde{h}^*) \right\} ^{1-\epsilon} d\tilde{h}^* \right\} ^{\frac{1}{1-n}}, \]

and

\[ \tilde{\Omega}_t^* \equiv (\Omega_t^*)^n \left( \tilde{\Omega}_t^* \right)^{1-n}. \]

Furthermore, loan demand conditions are:

\[ q_t^* (h^*, f^*) = \gamma^* w_t^* (h^*) l_t^* (h^*) = \gamma^* w_t^* (h^*) \left\{ \frac{1 + \gamma^* r_t^* (h^*)}{\Omega_t^*} \right\} ^{-\epsilon} \left( \frac{\Omega_t^*}{\Omega_t^*} \right) ^{-1} L_t^* (f^*), \]

and

\[ q_t^* (\tilde{h}^*, f^*) = \tau^* w_t^* (\tilde{h}^*) l_t^* (\tilde{h}^*) = \tau^* w_t^* (\tilde{h}^*) \left\{ \frac{1 + \tau^* r_t (\tilde{h}^*)}{\Omega_t^*} \right\} ^{-\epsilon} \left( \frac{\Omega_t^*}{\Omega_t^*} \right) ^{-1} L_t^* (f^*). \]

As aggregate labor demand conditions, we can obtain:

\[ \tilde{L}_t = \int_0^1 \tilde{L}_t (f) df, \]

and

\[ \tilde{L}_t^* = \int_0^1 \tilde{L}_t^* (f^*) df^*. \]
2.2.2 Price setting (profit maximization)

As is standard in the New Keynesian model following the Calvo (1983) - Yun (1996) framework, each firm \( f \) resets its price with probability \((1 - \alpha)\) and maximizes the present discounted value of profit, which is given by:

\[
E_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[ p_t(f) c_{t,T} (f) + S_T p^*_t (f) c^*_t;T (f) - \tilde{\Omega}_T \tilde{L}_T (f) \right],
\]  

(20)

where we use equations (2) and (4) for any time \( t \). Here, the firm sets \( p_t(f) \) and \( p^*_t (f) \) separately under the pricing-to-market assumption. There \( S_t \) is the nominal exchange rate. The present discounted value of the profit given by equation (20) is further transformed into:

\[
E_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left\{ \phi_{TP_t}(f) \left[ \frac{p_t(f)}{P_{H,T}} \right]^{-\sigma} \left[ \frac{P_{H,T}}{P_T} \right]^{-1} C_T \right. \\
+ \phi_{PS_T} p^*_t(f) \left[ \frac{p^*_t(f)}{P_{H,T}} \right]^{-\sigma} \left[ \frac{P_{H,T}}{P_T} \right]^{-1} C^*_T - \tilde{\Omega}_T \tilde{L}_T (f) \left\}
\]

It should be noted that price setting is independent of the loan interest rate setting of private banks.

The optimal price setting of \( \bar{p}_t(f) \) under the situation in which managers can reset their prices with probability \((1 - \alpha)\) is given by:

\[
E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} U_C(C_T) y_{t,T} (f) \left[ \frac{\sigma - 1}{\sigma} \bar{p}_t(f) - \tilde{\Omega}_T \frac{\partial \tilde{L}_T (f)}{\partial y_{t,T} (f)} \right] = 0,
\]  

(21)

where we substitute equation (8). By further substituting equations (10) and (11) into equation (21), equation (21) can be now rewritten as:

\[
E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} U_C(C_T) y_{t,T} (f) \left[ \frac{\sigma - 1}{\sigma} \bar{p}_t(f) \frac{P_{H,T}}{P_T} \frac{P_{H,t}}{P_{H,T}} - \frac{\epsilon}{\epsilon - 1} Z_{t,T} (f) \right] = 0,
\]  

(22)

where

\[
Z_{t,T} (f) = \left\{ \left( \frac{1}{n} \right) \int_0^n \left[ 1 + \gamma r_t (h) \right] \left[ \frac{P_T}{P_{H,T}} \frac{V_l [l_T (h)]}{U_C(C_t)} \right] \frac{dh}{\tilde{\Omega}_T \tilde{L}_T (f)} \right\}^{1-\epsilon} \\
\times \left\{ \left( \frac{1}{1 - n} \right) \int_1^n \left[ 1 + \pi r_t (\tilde{h}) \right] \left[ \frac{P_T}{P_{H,T}} \frac{V_l [l_T (\tilde{h})]}{U_C(C_t)} \right] \frac{dh}{\tilde{\Omega}_T \tilde{L}_T (f)} \right\}^{1-\epsilon}.
\]
By log-linearizing equation (22), we derive:

\[
\frac{1}{1 - \alpha\beta} \hat{p}_t (f) = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[ \sum_{\tau=t+1}^{T} \pi_{H,T} + \Theta_1 \hat{R}_{H,T} + \Theta_2 \hat{R}_{H,T} + \tilde{mc}_{H,t,T} (f) \right],
\]

(23)

where \( \Theta_1 \equiv n \frac{\gamma (1 + \pi_H)}{1 + \gamma R_H} \) and \( \Theta_2 \equiv (1 - n) \frac{\pi (1 + \pi_H)}{1 + \pi_R} \) are positive parameters, and we define the real marginal cost as:

\[
\tilde{mc}_{H,t,T} (f) \equiv \int_0^n \tilde{mc}_{H,t,T} (h, f) \, dh + \int_n^1 \tilde{mc}_{H,t,T} (\bar{h}, f) \, d\bar{h},
\]

where

\[
m_{H,t,T} (h, f) \equiv \frac{Pr \cdot V_t [I_T] \partial \tilde{L}_{t,T} (f)}{P_{H,T} \cdot U_Y (C_T) \partial y_{t,T} (f)},
\]

and

\[
m_{H,t,T} (\bar{h}, f) \equiv \frac{Pr \cdot V_t [I_T] \partial \tilde{L}_{t,T} (f)}{P_{H,T} \cdot U_Y (C_T) \partial y_{t,T} (f)}.
\]

We also define:

\[
R_{H,t} \equiv \frac{1}{n} \int_0^n r_t (h) \, dh,
\]

\[
R_{H,t}^* \equiv \frac{1}{1 - n} \int_n^1 r_t^* (\bar{h}) \, d\bar{h},
\]

\[
\hat{p}_t (f) \equiv \frac{P_{H,T}}{P_{H,t}},
\]

and

\[
\pi_{H,t} \equiv \frac{P_{H,t+1}}{P_{H,t}}.
\]

where \( R_{H,t} \) is the aggregate loan interest rate by local banks in the home country, \( R_{H,t}^* \) is the aggregate loan interest rate by international banks in the home country, and \( \pi_{H,t} \) is inflation of goods produced and consumed in the home country. Then, equation (23) can be transformed into:

\[
\frac{1}{1 - \alpha\beta} \hat{p}_t (f) = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[ (1 + \eta_2 \sigma)^{-1} \left( \tilde{mc}_{H,T} + \Theta_1 \hat{R}_{H,T} + \Theta_2 \hat{R}_{H,T} \right) + \sum_{\tau=t+1}^{T} \pi_{H,T} \right],
\]

(24)

where we make use of the relationship:

\[
\tilde{mc}_{H,t,T} (f) = \tilde{mc}_{H,t,T} - \eta_2 \sigma \left[ \hat{p}_t (f) - \sum_{\tau=t+1}^{T} \pi_{H,T} \right],
\]

\(17\)
where \( \eta_2 \equiv - \frac{I^{\gamma-1}_X (Y)}{Y} \). We further denote the average real marginal cost as:

\[
\bar{m}c_{H,T} \equiv \int_0^u \bar{m}c_{H,T} (h) \, dh + \int_u^1 \bar{m}c_{H,T} (h) \, dh,
\]

where

\[
m_{cH,T} (h) \equiv \frac{P_T \cdot V_l [l_T (h)]}{P_{H,T} \cdot U_Y (C_T \cdot Y)} \frac{\partial \bar{L}_T}{\partial Y_{H,T}},
\]

and

\[
m_{cH,T} (\bar{h}) \equiv \frac{P_T \cdot V_l [l_T (\bar{h})]}{P_{H,T} \cdot U_Y (C_T \cdot Y)} \frac{\partial \bar{L}_T}{\partial Y_{H,T}}.
\]

The point is that unit marginal cost is the same for all firms in the situation where each firm uses all types of labor and loans with the same proportion. Thus, all firms set the same price if they have a chance to reset their prices at time \( t \).

In the Calvo (1983) - Yun (1996) setting, the evolution of the aggregate price index \( P \) is described by the following law of motion:

\[
\int_0^1 p_t (f)^{1-\sigma} \, df = \alpha \int_0^1 p_{t-1} (f)^{1-\sigma} \, df + (1 - \alpha) \int_0^1 \bar{p}_t (f)^{1-\sigma} \, df,
\]

\[
\implies P^{1-\sigma}_{H,t} = \alpha P^{1-\sigma}_{H,t-1} + (1 - \alpha) (\bar{p}_t)^{1-\sigma},
\]

where

\[
P^{1-\sigma}_{H,t} \equiv \int_0^1 p_t (f)^{1-\sigma} \, df,
\]

and

\[
\bar{p}^{1-\sigma}_{t} \equiv \int_0^1 \bar{p}_t (f)^{1-\sigma} \, df.
\]

The current aggregate price is given by the weighted average of changed and unchanged prices. Because the chances of resetting prices are randomly assigned to each firm with equal probability, an aggregate price change at time \( t \) should be evaluated by an average of price changes by all firms. By log-linearizing equation (25), together with equation (24), we can derive the following New Keynesian Phillips curve:

\[
\pi_{H,t} = \kappa \left( \bar{m}c_{H,t} + \Theta_1 \bar{R}_{H,t} + \Theta_2 \bar{R}^*_t \right) + \beta E_t \pi_{H,t+1},
\]

(26)
where the slope coefficient $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\gamma\sigma)}$ is a positive parameter. This is quite similar to the standard New Keynesian Phillips curve, but contains loan interest rates as cost components.

Similarly, regarding the optimal price setting of $P^*_f(f)$, we can derive:

$$\pi^*_{H,t} = \kappa \left( \tilde{m}c^*_{H,t} + \Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}^*_{H,t} - \hat{S}_t \right) + \beta E_t \pi^*_{H,t+1}, \quad (27)$$

where $\tilde{m}c^*_{H,t}$ is given by replacing $P$ by $P^*$ in $\tilde{m}c_{H,t}$. $\pi^*_{H,t}$ is inflation of goods produced in the home country and consumed in the foreign country.

Furthermore, from optimal price settings by firm $f^*$ in the foreign country, the New Keynesian Phillips curves are derived as:

$$\pi^*_{F,t} = \kappa^* \left( \tilde{m}c^*_{F,t} + \Theta_1^* \hat{R}^*_{F,t} + \Theta_2^* \hat{R}^*_{F,t} + \hat{S}_t \right) + \beta E_t \pi^*_{F,t+1}, \quad (28)$$

$$\pi^*_{F,t} = \kappa^* \left( \tilde{m}c_{F,t} + \Theta_1^* \hat{R}^*_{F,t} + \Theta_2^* \hat{R}^*_{F,t} + \hat{S}_t \right) + \beta E_t \pi^*_{F,t+1}, \quad (29)$$

where $\kappa^* \equiv \frac{(1-\alpha^*)(1-\alpha^*\beta^*)}{\alpha^*(1+\gamma^2\sigma)} > 0$, $\Theta_1^* \equiv \eta^* \left( 1 + \frac{\gamma^* \left( 1 + \bar{P}^* \right)}{1 + \gamma^* \bar{R}^*} \right) > 0$, $\Theta_2^* \equiv (1 - \eta^*) \left( 1 + \frac{\gamma^* \left( 1 + \bar{P}^* \right)}{1 + \gamma^* \bar{R}^*} \right) > 0$, and $\eta^* \equiv -\frac{\bar{R}^*}{\bar{P}^*}$. $R^*_{F,t}$ is the aggregate loan interest rate by international banks in the foreign country, $\hat{R}^*_{F,t}$ is the aggregate loan interest rate by local banks in the foreign country, $\pi^*_{F,t}$ is inflation of goods produced and consumed in the foreign country, and $\pi^*_F$ is inflation of goods produced in the foreign country and consumed in the home country.

As for CPI inflation rates, from equations (1) and (3), we can derive the following log-linearized relations as:

$$\pi_t = \phi_H \pi^*_{H,t} + (1 - \phi_H) \pi^*_F,$$

and

$$\pi^*_t = (1 - \phi_H) \pi^*_H + \phi_H \pi^*_F,$$

under the assumption of $\phi_H = 1 - \phi_F$; namely, symmetric home bias. Then, by considering the weighted average of equations (26) and (29) and the weighted
average of equations (27) and (28), respectively, we can finally obtain the following two New Keynesian Phillips curves for consumer prices:

\[
\pi_t = \phi_H \kappa \left( \Theta_1 \tilde{R}_{H,t} + \Theta_2 \tilde{R}_{H,t} \right) + (1 - \phi_H) \kappa^* \left( \Theta_1^* \tilde{R}_{F,t} + \Theta_2^* \tilde{R}_{F,t} \right) + \pi_{t-1} + \beta E_t \pi_{t+1},
\]

\[
\pi_t^* = \left(1 - \phi_H\right) \kappa \left( \Theta_1 \tilde{R}_{H,t} + \Theta_2 \tilde{R}_{H,t} \right) + \phi_H \kappa^* \left( \Theta_1^* \tilde{R}_{F,t} + \Theta_2^* \tilde{R}_{F,t} \right) + \pi_{t-1} + \beta E_t \pi_{t+1},
\]

where \( \pi \equiv \kappa \left( \eta_1 \mu_1^{-1} + \eta_2 + \nu^{-1} \right) > 0, \pi^* \equiv \kappa^* \left[ \eta_1^* \left( \mu_1^* \right)^{-1} + \eta_2^* + (\nu^*)^{-1} \right] > 0, \)
\( \kappa \equiv 2 \kappa \phi_H (1 - \phi_H), \) and \( \kappa^* \equiv 2 \kappa^* \phi_F (1 - \phi_F), \) where \( \mu_1 \equiv \frac{f_1 (L) L}{f(L)} > 0, \eta_1 \equiv \frac{V_\mu^*}{\nu^*} > 0, \) and:

\[
\tilde{C}_t^{WA} = \phi_H \hat{\tilde{C}}_t + (1 - \phi_H) \hat{\tilde{C}}_t^*,
\]

and

\[
\tilde{C}_t^{WA*} = \phi_F \hat{\tilde{C}}_t + (1 - \phi_F) \hat{\tilde{C}}_t^*.
\]

When deriving equations (30) and (31), we assume that marginal cost elasticities of productions are zero, namely that the production function is linear,\(^{13}\) and \( \kappa = \kappa^* \) and \( \nu = \nu^* \), as will be demonstrated in Table 3. Furthermore, in this transformation, we use the optimality conditions on bond holdings:

\[
\pi_t^* - \pi_t + \Delta \tilde{S}_t = \Delta \tilde{q}_t,
\]

and

\[
\tilde{C}_t - \tilde{C}_t^* = \nu \tilde{q}_t.
\]

Equation (34) demonstrates the log deviation of the real exchange rate:

\[
Q_t \equiv S_t \frac{P_t^*}{P_t},
\]

\(^{13}\eta_2 = 0, \eta_1 = 0, \eta_2^* = 0, \) and \( \eta_1^* = 0. \) If the marginal cost elasticity of production is nonzero, the Phillips curves become more complicated. However, the qualitative outcomes of simulations do not change even under a positive marginal cost elasticity of production.
from its steady state value. Equation (35) is obtained from two Euler equations in the domestic and foreign countries, namely, equations (8) and (13), under the internationally complete financial market.\footnote{Following the convention, we assume $Q_0 = 0$.}

### 2.3 Private banks

There exists a continuum of private banks populated over $[0, 1]$. There are two types of banks in each country; local banks populate over $[0, n)$ and international banks populate over $[n, 1]$. Each private bank plays two roles: (1) to collect the deposits from consumers in its country, and (2) under the monopolistically competitive loan market, to set differentiated nominal loan interest rates according to their individual loan demand curves, given the amount of their deposits. We assume that each bank sets the differentiated nominal loan interest rate according to the types of labor force as examined in Teranishi (2007). Staggered loan contracts between firms and private banks produce a situation in which the private banks fix the loan interest rates for a certain period.

A local bank lends only to firms when they hire labor from the DFS. However, an international bank only provides a loan to firms when they hire labor from the IFS. The lending structure is shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Local bank</th>
<th>International bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home country</td>
<td>for $f$ to hire $h$</td>
<td>for $f^<em>$ to hire $h^</em>$</td>
</tr>
<tr>
<td>Foreign country</td>
<td>for $f^<em>$ to hire $h^</em>$</td>
<td>for $f$ to hire $h$</td>
</tr>
</tbody>
</table>

First, we describe the optimization problem of an international bank in the home country. Here, the international bank takes on the exchange rate risk inherent in its loans. Each international bank can reset loan interest rates with probability $(1 - \overline{\pi})$ following the Calvo (1983) - Yun (1996) framework. Under
the segmented environment stemming from differences in labor supply, private banks can set different loan interest rates depending on the types of labor. As a consequence, the private bank holds some monopoly power over the loan interest rate to firms. Therefore, the international bank \( h \) chooses the loan interest rate \( r_t \left( \bar{h}^* \right) \) to maximize the present discounted value of profit:

\[
E_t \sum_{T=t}^{\infty} (\varphi)^{T-t} X_t T q_{t,T} \left( \bar{h}^*, f^* \right) \left\{ S_{T+1} \left[ 1 + r_t \left( \bar{h}^* \right) \right] - S_T \left( 1 + i_T \right) \right\}.
\]

The optimal loan condition is now given by:

\[
E_t \sum_{T=t}^{\infty} (\varphi)^{T-t} P_t \frac{U_C(C_T)}{U_G(C_I)} q_{t,T} \left( \bar{h}^* \right) \left\{ -e^* \tau_t S_{T+1} \left[ 1 + \tau_t \left( \bar{h}^* \right) \right] - S_T \left( 1 + i_T \right) \right\} = 0.
\]

Because the international private banks that have the opportunity to reset their loan interest rates will set the same loan interest rate, the solution of \( r_t \left( \bar{h}^* \right) \) in equation (36) is expressed only with \( \tau_t \). In this case, we have the following evolution of the aggregate loan interest rate index by international banks in the home country:

\[
1 + R_{F,t} = \varphi^* (1 + R_{F,t-1}) + (1 - \varphi^*) (1 + \tau_t).
\]

By log-linearizing equations (36) and (37), we can determine the relationship between the loan and deposit interest rate as follows:

\[
\hat{R}_{F,t} = \lambda_1 E_t \hat{R}_{F,t+1} + \lambda_2 \hat{R}_{F,t-1} + \lambda_3 \left( \hat{t}_t - E_t \hat{S}_{t+1} \right),
\]

where \( \lambda_1 \equiv \frac{\varphi \beta}{1 + (\varphi^*)^{1-\beta}}, \lambda_2 \equiv \frac{\varphi^* \beta}{1 + (\varphi^*)^{1-\beta}} \), and \( \lambda_3 \equiv \frac{1 - \varphi^* \beta}{1 + (\varphi^*)^{1-\beta}} \frac{(1 - \beta \varphi^*) (1 + \tau)}{1 + \hat{R}} \) are positive parameters. This equation describes the foreign country’s loan interest rate (supply) curve by the international bank in the home country.

Similarly, from the optimization problem of a local bank \( h \) in the home country, we can obtain the relationship between loan and deposit interest rates as follows:

\[
\hat{R}_{H,t} = \lambda_1 E_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{t}_t,
\]
where $\lambda_1 \equiv \frac{\delta}{1+\delta}, \lambda_2 \equiv \frac{\phi}{1+\phi},$ and $\lambda_3 \equiv \frac{1-\phi}{1+\phi} \frac{(1-\beta)(1+\gamma)}{1+R_H}$ are positive parameters. This equation describes the home country’s loan interest rate (supply) curve by the local bank in the home country.

The domestic market loan clearing conditions are expressed as:

$$q_{t,T}(h) = \int_0^1 q_{t,T}(h,f)df,$$

$$q_{t,T}^* (\bar{h}^*) = \int_0^1 q_{t,T}^* (\bar{h}^*, f^*) df^*,$$

$$\int_0^1 q_{t,T}(h)dh = nD_T,$$

and

$$S_T \int_0^1 q_{t,T}^* (\bar{h}^*) d\bar{h}^* = (1-n) D_T.$$

For international banks in the foreign country, we can derive the following loan interest rate curve:

$$\hat{R}_{H,t}^* = \lambda_1 E_t \hat{R}_{H,t+1}^* + \lambda_2 \hat{R}_{H,t-1}^* + \lambda_3 \left( E_t \Delta \hat{S}_{t+1} + \hat{i}_t \right),$$  \hspace{1cm} (40)

where $\lambda_1 \equiv \frac{\delta}{1+\delta}, \lambda_2 \equiv \frac{\phi}{1+\phi},$ and $\lambda_3 \equiv \frac{1-\phi}{1+\phi} \frac{(1-\beta)(1+\gamma)}{1+R_H}$ are positive parameters. This equation describes the home country’s loan interest rate (supply) curve by the international bank in the foreign country. Similarly, for local banks in the foreign country, we can obtain:

$$\hat{R}_{F,t}^* = \lambda_1^* E_t \hat{R}_{F,t+1}^* + \lambda_2^* \hat{R}_{F,t-1}^* + \lambda_3^* \hat{i}_t,$$  \hspace{1cm} (41)

where $\lambda_1^* \equiv \frac{\delta^*}{1+\delta^*}, \lambda_2^* \equiv \frac{\phi^*}{1+\phi^*},$ and $\lambda_3^* \equiv \frac{1-\phi^*}{1+\phi^*} \frac{(1-\beta^*)(1+\gamma^*)}{1+R_F}$ are positive parameters. This equation describes the foreign country’s loan interest rate (supply) curve by the local bank in the foreign country. It should be noted that the four types of private bank in both the home and foreign countries can have different probabilities for resetting their loan interest rates. The foreign market loan clearing conditions are expressed as:

$$q_{t,T}^* (h^*) = \int_0^1 q_{t,T}^* (h^*, f^*) df^*,$$
\[
q_{t,T} (\bar{h}) = \int_0^1 q_{t,T}^* (\bar{h}, f) df,
\]

\[
\int_0^n q_{t,T}^* (h^*) dh^* = nD_T^*,
\]

and

\[
\frac{1}{S_T} \int_{-n}^1 q_{t,T} (\bar{h}) d\bar{h} = (1 - n) D_T^*.
\]

### 2.4 Central banks

To close the model, the central banks in both countries set the deposit interest rates following the Taylor (1993)-type rules as:

\[
\hat{\iota}_t = (1 - \rho_i) \Phi_1 \pi_t + (1 - \rho_i) \Phi_2 \hat{C}_t + \rho_i \hat{\iota}_{t-1},
\]

(42)

and

\[
\hat{\iota}_t = (1 - \rho_i^*) \Phi_1^* \pi_t^* + (1 - \rho_i^*) \Phi_2^* \hat{C}_t^* + \rho_i^* \hat{\iota}_{t-1}^*,
\]

(43)

where \(\Phi_1, \Phi_2, \Phi_1^*, \Phi_2^*, \rho_i, \) and \(\rho_i^*\) are positive policy parameters.

### 2.5 System of equations

The linearized system of equations consist of 14 equations: (9), (12), (30), (31), (32), (33), (34), (35), (38), (39), (40), (41), (42) and (43) for 14 endogenous variables: \(\hat{C}, \hat{C}^*, \pi, \pi^*, \hat{C}^{WA}, \hat{C}^{WA*}, \hat{q}, \hat{S}, \hat{R}_F, \hat{R}_H, \hat{R}_F^*, \hat{R}_H^*, \hat{i}, \) and \(\hat{\iota}^*\). They are summarized in Table 2 below.

### 3 Simulation Results

The parameters are calibrated in Table 3. \(v, v^*, \kappa, \kappa^*, \alpha, \alpha^*, \phi_H, \Phi_1, \Phi_1^*, \Phi_2, \Phi_2^*, \rho_i\) and \(\rho_i^*\) are from Steinsson (2007). \(\theta, \epsilon\) and \(\epsilon^*\) are from Rotemberg and Woodford (1997). It should be noted that the elasticities of output with respect to real interest rates are set to be low enough to match the relative volatilities of the real exchange rates and consumption as in Steinsson (2007). Slovin and Sushka (1983) claim that private banks, on average, need at least two quarters
where a risk premium on loan interest rates:
be depicted when, for example, international banks maximize profit considering
time-varying exogenous international risk premium on loan interest rates from
loan interest rate curve as in equation (38), so this shock can be interpreted as a
cost-push shocks. The risk premium shock is a shock directly given to the
supply are the same between the DFS and IFS groups.
price duration in Rotemberg and Woodford (1997). We assume that the ratio
duration of loan interest rates is set to be three quarters which is the average

| Eq. (31): \( \pi_t = (1 - \phi_H)\kappa(\Theta_1 \hat{R}_{H,t} + \Theta_2 \hat{R}_{F,t}) + (1 - \phi_H)\kappa^*(\Theta_1^* \hat{R}_{H,t}^* + \Theta_2^* \hat{R}_{F,t}^*) \) |
| Eq. (32): \( \hat{C}_{t}^W = \phi_H C_t + \chi \hat{q}_t \) |
| Eq. (33): \( \hat{C}_{t}^{WA} = \phi_F \hat{C}_t + (1 - \phi) \hat{C}_t^* \) |
| Eq. (34): \( \pi_t + \Delta \hat{S}_t = \hat{q}_t \) |
| Eq. (35): \( \hat{C}_t - \hat{C}_t^* = \nu \hat{q}_t \) |
| Eq. (36): \( \hat{R}_{F,t} = \lambda_1 E_t \hat{R}_{F,t+1} + \lambda_2 \hat{R}_{F,t-1} + \lambda_3 \hat{q}_t \) |
| Eq. (37): \( \hat{R}_{H,t} = \lambda_1 E_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{i}_t \) |
| Eq. (38): \( \hat{R}_{H,t} = \lambda_1 E_t \hat{R}_{H,t+1} + \lambda_2 \hat{R}_{H,t-1} + \lambda_3 \hat{q}_t \) |
| Eq. (39): \( \hat{R}_{F,t} = \lambda_1 E_t \hat{R}_{F,t+1} + \lambda_2 \hat{R}_{F,t-1} + \lambda_3 \hat{i}_t \) |
| Eq. (40): \( \hat{R}_{F,t} = \lambda_1 E_t \hat{R}_{F,t+1} + \lambda_2 \hat{R}_{F,t-1} + \lambda_3 \hat{q}_t + \phi \hat{i}_t \) |
| Eq. (41): \( \phi = (1 - \rho) \Phi_1 \pi_t + (1 - \rho) \Phi_2 \hat{C}_t + \rho \hat{i}_t \) |
| Eq. (42): \( \hat{i}_t = (1 - \rho) \Phi_1 \pi_t + (1 - \rho) \Phi_2 \hat{C}_t + \rho \hat{i}_t \) |
| Eq. (43): \( \hat{i}_t = (1 - \rho) \Phi_1 \pi_t + (1 - \rho) \Phi_2 \hat{C}_t + \rho \hat{i}_t \) |

Table 2: System of equations

and perhaps more to adjust loan interest rates. Thus, the average contract
duration of loan interest rates is set to be three quarters which is the average
price duration in Rotemberg and Woodford (1997). We assume that the ratio
of the dependency on external finance is unity and preferences regarding labor
supply are the same between the DFS and IFS groups.

We impose two types of shock on the model. They are the risk premium and
the cost-push shocks. The risk premium shock is a shock directly given to the
loan interest rate curve as in equation (38), so this shock can be interpreted as a
time-varying exogenous international risk premium on loan interest rates from
the home country to the foreign country. More precisely, such a situation can
be depicted when, for example, international banks maximize profit considering
a risk premium on loan interest rates:

\[
E_t \sum_{T=t}^{\infty} (\bar{\pi}_t)^{T-t} X_{t,T} q_t (\bar{R}_t) \left[ S_{T+1} M_T (1 + r_t (\bar{R}_t)) - S_T (1 + i_T) - z_T (\bar{R}_t) \right],
\]

where \( M_t \) is an exogenous risk factor that international banks face. Then, we
can derive a loan interest rate curve with a disturbance $m_t$ as a risk premium:

$$
\hat{R}_{F,t} = \lambda_1 E_t \hat{R}_{F,t+1} + \lambda_2 \hat{R}_{F,t-1} + \lambda_3 (\hat{I}_t - E_t \Delta \hat{S}_{t+1}) + m_t,
$$

where $m_t = -\ln M_t$. Recent studies, such as Neumeyer and Perri (2005) and Uribe and Yue (2006), insist that international risk premium shocks can induce large business cycles. Moreover, Marston (1995) and Uribe and Yue (2006) state that interest rate spreads are quite persistent. Marston (1995) demonstrates that interest rate differences among countries are quite persistent while Uribe and Yue (2006) conclude that dynamics in country spreads show enough persistence in South American emerging countries by employing VAR analysis. Pages (1999) shows that term spreads of LIBOR from one- to 12-month maturities demonstrate strong persistence, which implies the persistence of risk premium shocks. This is the reason why we employ a very persistent risk premium with an AR(1) parameter of 0.9. We can express such a shock as:

$$
m_t = 0.9 m_{t-1} + v_t,
$$

where $v_t$ is an i.i.d shock process. As for the cost-push shock dynamics, we set the same persistence for goods produced by the home country as Steinsson

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\nu, \nu^*$</td>
<td>0.2</td>
<td>Elasticity of output with respect to real interest rate</td>
</tr>
<tr>
<td>$\kappa, \kappa^*$</td>
<td>0.43</td>
<td>Elasticity of inflation with respect to output</td>
</tr>
<tr>
<td>$\alpha, \alpha^*$</td>
<td>0.75</td>
<td>Probability of price change</td>
</tr>
<tr>
<td>$\varphi, \varphi^<em>, \overline{\varphi}, \overline{\varphi}^</em>$</td>
<td>0.66</td>
<td>Probability of loan interest rate change</td>
</tr>
<tr>
<td>$\theta$</td>
<td>7.66</td>
<td>Substitutability of differentiated consumption goods</td>
</tr>
<tr>
<td>$\epsilon, \epsilon^*$</td>
<td>7.66</td>
<td>Substitutability of differentiated laborers</td>
</tr>
<tr>
<td>$\gamma, \gamma^<em>, \tau, \tau^</em>$</td>
<td>1</td>
<td>Ratio of external finance to total finance</td>
</tr>
<tr>
<td>$n$</td>
<td>0.5</td>
<td>Preference for laborers in the DFS</td>
</tr>
<tr>
<td>$\phi_H$</td>
<td>0.94</td>
<td>Preference for goods produced in the home country</td>
</tr>
<tr>
<td>$\Phi_{11}, \Phi_{11}^*$</td>
<td>2</td>
<td>Coefficient on inflation rate in the Taylor rule</td>
</tr>
<tr>
<td>$\Phi_{22}, \Phi_{22}^*$</td>
<td>0.5</td>
<td>Coefficient on the output gap in the Taylor rule</td>
</tr>
<tr>
<td>$\rho_{1}, \rho_{1}^*$</td>
<td>0.85</td>
<td>Lag parameter in the Taylor rule</td>
</tr>
</tbody>
</table>
Table 4 shows the detailed simulation outcomes of the real exchange rate dynamics. We report five statistics for real exchange rates: (1) UL (up-life), (2) UL/HL (up-life over half-life), (3) QL-HL (quarter-life minus half-life), (4) AR(1) persistence, and (5) relative standard deviation of real exchange rates to consumption. UL denotes the duration until the impulse response falls below the top (maximum) point, while HL shows the duration until the impulse response falls below half of the top point. QL denotes the duration until the impulse response falls below a quarter of the top point. These are the measures for how hump-shaped the impulse responses functions are. For example, if the impulse responses are monotonically decreasing, UL and UL/HL should be zero. If the impulse responses demonstrate persistent hump-shaped dynamics, UL/HL and QL-HL become larger. Here, we examine three cases: (1) no loan contracts \(\gamma = \gamma^* = \tau = \tau^* = 0\); (2) flexible loan contracts \(\varphi = \varphi^* = \varpi = \varpi^* = 0\); and (3) staggered loan contracts \(\varphi = \varphi^* = \varpi = \varpi^* = 0.66\).

The first row in Table 4 reports the key empirical features of real exchange rates in developed countries, which are estimated in Steinsson (2007).\(^\text{15}\)

The second row reports outcomes from the model without international loan contracts under the cost-push shock. This simulation setting is close to the one in the fifth row of Table 4 examined in Steinsson (2007). We confirm the finding in Steinsson (2007) that the cost-push shock can produce hump-shaped dynamics in the real exchange rates.\(^\text{16}\)

\(^{15}\)HL expresses half-life (measured in years), UL/HL expresses up-life over half-life, QL-HL expresses quarter-life minus half-life, AR(1) expresses the first order autocorrelation of the HP-filtered series and Std(q)/Std(C) expresses the standard deviation of HP-filtered q (real exchange rate) divided by the standard deviation of HP-filtered C (consumption). The point estimates for AR(1) and Std(q)/Std(C) are calculated by simulating 1000 data series from each model, in which each data length is 127, and the point estimate is the median value of the resulting distribution as in Steinsson (2007). NL, FL, and SL mean no, flexible, and staggered loan contracts, respectively. SP and FP mean staggered and flexible price settings, respectively. PS and LRS mean price and loan interest rate shocks, respectively.

\(^{16}\)We set different parameters in our model, so the simulation outcomes are not exactly the same as in Steinsson (2007).
Table 4: Properties of the real exchange rate (persistence of shock = 0.9)

<table>
<thead>
<tr>
<th>Setting</th>
<th>HL</th>
<th>UL/HL</th>
<th>QL-HL</th>
<th>AR(1)</th>
<th>Std(q)/Std(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical values</td>
<td>3.7</td>
<td>0.44</td>
<td>1.9</td>
<td>0.78</td>
<td>3.3</td>
</tr>
<tr>
<td>NL, SP, and PS</td>
<td>2.8</td>
<td>0.18</td>
<td>1.5</td>
<td>0.83</td>
<td>5</td>
</tr>
<tr>
<td>FL, SP, and PS</td>
<td>2.8</td>
<td>0.18</td>
<td>1.75</td>
<td>0.83</td>
<td>5</td>
</tr>
<tr>
<td>SL, SP, and PS</td>
<td>2.8</td>
<td>0.27</td>
<td>1.75</td>
<td>0.83</td>
<td>5</td>
</tr>
<tr>
<td>FL, SP, and LRS</td>
<td>2.8</td>
<td>0.18</td>
<td>1.75</td>
<td>0.83</td>
<td>4.6</td>
</tr>
<tr>
<td>SL, SP, and LRS</td>
<td>3.5</td>
<td>0.29</td>
<td>1.75</td>
<td>0.86</td>
<td>4.7</td>
</tr>
<tr>
<td>SL, FP, and LRS</td>
<td>3.3</td>
<td>0.31</td>
<td>1.75</td>
<td>0.9</td>
<td>4.7</td>
</tr>
</tbody>
</table>

The third and fourth rows show the outcomes from the model with international loan contracts under the cost-push shock. The third row shows the case with flexible loan contracts, while the fourth row demonstrates the case with staggered loan contracts. Significant differences from those in the second row can be observed. The UL/HL ratio and QL-HL increase and become closer to the empirical values especially in the case with staggered loan contracts. Interestingly, the degrees of staggeredness on loan contracts can change how hump-shaped the real exchange rate dynamics are in response to the cost-push shock. The more staggered the loan contract becomes, the more hump-shaped the responses in the real exchange rate dynamics become. This is similar to the recent arguments in the New Keynesian literature on the necessity of strategic complementarity in marginal cost to have realistically persistent responses of economic variables.

The fifth and sixth rows show outcomes from the model with international loan contracts under the risk premium shock. Numbers in the fifth row are obtained by assuming flexible loan contracts while those in the sixth row are computed by assuming staggered loan contracts. With flexible loan contracts, the simulation outcomes are similar to those obtained in the previous studies.

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17 In these cases, the standard deviations of consumption of the home country are replaced by that of the foreign country in calculating relative standard deviation values Std(q)/Std(C).
18 The importance of strategic complementarity in price setting is first pointed out by Kimball (1995).
that stress the importance of price stickiness. The case with staggered loan contracts, however, shows longer and more realistic HL, UL, and QL than others and demonstrates improvements in the UL/HL ratio. Contrary to the cost-push shock examined in Steinsson (2007), the importance of the risk premium shock on business cycle dynamics is strongly emphasized in recent empirical studies. Thus, together with the success in replicating the persistent hump-shaped responses, this risk premium shock should be naturally interpreted as the candidate for the main driver of real exchange rate dynamics.

The bottom row shows outcomes from a model with staggered international loan contracts but with flexible price settings under the risk premium shock. In this case, two New Keynesian Phillips curves in equations (30) and (31) are transformed into:

\[
0 = \phi_H \left( \Theta_1 \tilde{R}_{H,t} + \Theta_2 \tilde{R}^*_H \right) + (1 - \phi_H) \left( \Theta_1^* \tilde{R}^*_{F,t} + \Theta_2^* \tilde{R}_{F,t} \right) \\
+ u^{-1} \left[ \phi_H \tilde{C}^W_A + (1 - \phi_H) \tilde{C}^W_A^* \right] + 2\phi_H (1 - \phi_H) q_t,
\]

and

\[
0 = (1 - \phi_H) \left( \Theta_1 \tilde{R}_{H,t} + \Theta_2 \tilde{R}^*_H \right) + \phi_H \left( \Theta_1^* \tilde{R}^*_{F,t} + \Theta_2^* \tilde{R}_{F,t} \right) \\
+ (v^*)^{-1} \left[ \phi_F \tilde{C}^W_A + (1 - \phi_F) \tilde{C}^W_A^* \right] - 2\phi_F (1 - \phi_F) q_t.
\]

Surprisingly, we can clearly see that stickiness of prices does not change the hump-shaped dynamics of real exchange rates at all. This strongly implies that the staggered price mechanism is not very important for determining real exchange rate dynamics, though the observed price stickiness itself is essential in economy. In other words, only the financial market frictions and shocks can explain the persistent hump-shaped dynamics of real exchange rates through

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19 As shown in Teranishi (2007), if we assume indexed changes in loan interest rates, real exchange rate dynamics become more persistent and more volatile.

20 This exercise can be compared with the case of flexible price and sticky wage examined in Obstfeld and Rogoff (2000), Obstfeld and Rogoff (2002) and Obstfeld (2002).
the persistent dynamics in the marginal cost. Thus, the financial market developments can be the most important factor for the real exchange rate dynamics. This conclusion is quite contrary to Chari, Kehoe, and McGrattan (2002) and Steinsson (2007), which show the necessity of the strong price stickiness for replicating the persistent hump-shaped dynamics of real exchange rates.

4 Concluding Remarks

Empirical papers imply nontrivial roles for financial market developments through a staggered loan contract and a risk premium shock on business cycles. We introduce these factors into an otherwise standard NOEM model in a tractable manner. Simulation results with such staggered loan contracts and risk premium shocks can generate persistent, volatile, and hump-shaped responses in real exchange rates, which have been very difficult to reproduce in standard NOEM models. This implies that financial market developments can possibly be a major source of persistent hump-shaped real exchange rate dynamics.

The analysis of this paper suggests several directions for future research. First, it would be of interest to see more quantitative analysis on the roles of staggered loan contracts and risk premium shocks on business cycles through, say, Bayesian estimation techniques. Second, we should examine the effects of these two factors on other aspects of the economy, such as investment and consumption. In particular, the extension of incorporating investment whose finance relies on a staggered loan contract is of great interest. In that case, risk premium shocks may produce larger effects on business cycles than those in our model. Finally, we would like to analyze the property of optimal policy coordination in the presence of staggered loan contracts and risk premium shocks.
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