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Abstract

The first aim of the paper is to investigate a new source of economic stickiness, staggered nominal loan interest rate contracts between a private bank and a firm under the monopolistic competition. We introduce this staggered loan contract mechanism with micro-foundation based on agent's optimized behaviors into a standard New Keynesian model in a tractable way. Simulation results show that staggered loan contracts play an important role in determining both the amplitude and the persistence of economic fluctuations. The second aim of the paper is to analyze optimal monetary policy in this environment with staggered loan contracts. To this end, we derive an approximated microfounded-welfare function in the model. Unlike the loss functions derived in other New Keynesian models, this model's welfare function includes a term that measures the first order difference in loan interest rates, which requires reduction of the magnitude of policy interest rate changes in the welfare itself. We derive the optimal monetary policy rule when the central bank can commit to its policy in the timeless perspective. One implication of the optimal policy rule is that the central bank has the incentive to smooth the policy interest rate. This empirically realistic conclusion can be seen in our simulation results.

Keywords: Staggered Loan Interest Rate Contract; Optimal Monetary Policy; Economic Fluctuation

JEL classification: E32, E44, E52, G21

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1 Introduction

The subject of financial market imperfections is the focus of a considerable body of theoretical and empirical analysis in macroeconomics. Financial market imperfection means the existence of the wedge between the optimal levels and the actual levels of loan interest rates. Bernanke, Gertler and Gilchrist (1999, henceforth BGG) first stresses that a financial market imperfection has a significant influence on business cycle dynamics. In BGG model, this financial market wedge is determined by the time varying leverage, in that endogenous mechanisms in credit markets work to amplify and propagate shocks to economy (called as the financial accelerator mechanism). Christiano, Motto and Rostagno (2007) shows that according to their Bayesian estimation results on Christiano, Eichenbaum and Evans (2005) model with the financial accelerator mechanism by BGG the financial market imperfection has been playing very important role in U.S. and Euro area business cycles. Also, Ravenna and Walsh (2006) introduces a banking sector which distorts credit conditions through higher loan interest rates into a New Keynesian model as our model and analyzes an optimal monetary policy under this distorted cost channel. These papers, however, assume that loan interest rates can be changed costlessly each period and do not focus on the realistic financial market imperfection in this paper: the sticky adjustment of loan interest rates.

Several empirical studies provide strong evidence that loan interest rates do not move too frequently. For example, Slovin and Sushka (1983) shows that private banks, on average, need at least two quarters and probably more to adjust their loan rates. Berger and Udell (1992) investigates over one million individual loans in U.S. from 1977 to 1988 and they conclude that the commercial loan rate is sticky with respect to open-market rates. Also, Hülsewig, Mayery, and Wollmershäuserz (2007) assumes a similar staggered loan contract in a simple New Keynesian model as our model and empirically supports that frictions in a loan market plays an important role in the propagation of monetary policy shocks in Euro area because of incomplete pass-through from a change in policy rates to loan rates[1]. Furthermore, we can find very interesting evidence of staggered loan interest rates in Japan. Figure 1 demonstrates the movement in the Average Contracted Interest Rates on Loans (New Loans and Stock) between private banks and firms in the Japanese loan market. We can confirm that there are clear gaps between an average loan interest rate for new lending and for stock (total lending). In setting loan interest rates for new lending, private banks can arbitrarily set any interest rate. In this sense, we can interpret that loan rates for new lending are flexible. Thus, the gaps imply that it is difficult for firms and private banks to instantly adjust loan interest rates that have been already contracted and implemented. Moreover, Figure 1 implies one more interesting fact about staggered loan contracts between private banks and firms. In both cases of long-term and short-term loan contracts, the gaps gradually shrink when the call rates were fixed at almost zero by the Bank of Japan (BOJ). This implies that private banks can fully adjust all loan interest rates to an appropriate level under the special situation in which the BOJ set call rates essentially to zero for 6 years. This point is more apparent in Figure 2, which shows the average loan interest rates of major city banks. In the case of long-term loan rates (lower panel) around 2005, the lines of average loan interest rates for new loans and for stock almost cross each other after enough time of adjustments, 6 years[2]. This stickiness in loan interest rates are not only in loan contracts between firms and private banks but also in loans between consumers and private banks. For example, Figure 3 shows mortgage loan interest rates in the Japanese mortgage loan market. In this figure, we see clear evidence of the staggered macro level mortgage rate. Privates banks tend to fix the interest rates on housing loans against the call rates for some periods[3]. It is difficult to find such macro level data that shows the staggered property

[1] Their model, however, assume an ad-hoc loan demand function which is induced from a distorted aggregator on loans (money). Thus, in their model, money has different colors and has different values in different loans. Also, they do not investigate an optimal monetary policy under a staggered loan contract.

[2] Thus, by comparing Figure 1 and 2, we can say that regional banks or small banks need longer time to adjust their loan interest rates than major city banks.

[3] It should be noted that contract periods of housing loan are usually much longer than those of loans for firms, so we can not simply compare the interest rate stickinesses in housing loans and loans for firms. Moreover, when we consider a risk premium on a market rate depicted in Figure 3 according to borrower's character, a mortgage loan

of loan interest rates between firms and private banks because these loans are highly customized to the firms' risks. There, however, is no reason to assume that only loan interest rates between firms and private banks are perfectly flexible. Thus, not only other types of financial market imperfections but also staggered loan interest rate contracts seem to be very important factors.

The first contribution of this paper is to introduce a staggered nominal loan interest rate contract with microfoundation based on agent's optimized behaviors into a simple New Keynesian model in a tractable way. We assume the staggered loan contracts between a bank and a firm under the *Monopolistic Competition*. This monopolistically competitive assumption can be justified by the fact that loan interest rates are enough sticky (but not perfectly sticky) in the real economy as suggested by the empirical papers. This staggered contract mechanism highlights a new source of economic rigidity which adds to the pre-existing sources of stickiness in the literature. Moreover, by introducing a new equation regarding the financial market, i.e. the loan rate curve, we can incorporate a shock to the financial market into our model. The second contribution is to demonstrate that, in the model with staggered loan contracts, economic fluctuations persist longer and have greater amplitude than those in a model without staggered loan contracts.

In order to capture this staggered loan interest rate contract mechanism in a tractable way, we assume a situation in which (1) a bank can only re-evaluate the risks associated with financing firm's projects when it receives a random signal, or (2) a bank can negotiate loan rates with a firm's project manager only when those managers randomly close old businesses and start new ones. These situations can be justified by citing limitations on informational transactions or costs associated with re-evaluating risks of firm's businesses. We make use of the Calvo (1983) - Yun (1992) framework to introduce our staggered loan contracts under the monopolistic competition. The setting, in which (1) a private bank and a firm promise a long term contract, (2) the private bank fixes a loan interest rate for a certain period, and (3) the firm can freely borrow money from

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interest rate may be more flexible.

the private bank by the pre-agreed loan interest rate at each time, shares the properties with the *Commitment Line Contract*[4]. A commitment line contract is a contract between a private bank and a firm, which legally forces the private bank to extend a loan to the firm, whenever it requests one, up to the amount that is agreed to in advance with the specified loan interest rate. Thus, the firm obtain a loan at any time during the term of the contract, up to the amount specified in the commitment line contract. Shockley and Thakor (1997) reports that, in the United States, over 80 percent of all private bank loans to corporations are done through this commitment line contract. Even in Japan, the amount of the commitment line contract is rapidly increasing[5]. Thus, in a baseline model, we approximate the staggered-ness of loan interest rates using the simple Calvo (1983) - Yun (1992) framework. However, as mentioned in Shockley and Thakor (1997), on many commitment line contracts in the U.S., the loan interest rates are variable, since they link to a representative interest rate index, such as Prime Rate, LIBOR, Fed Fund Rate, and Treasury Bond Rate. This indexation comes after a private bank have optimally set the loan interest rates according to the risks in a firm's projects or the economic situation. We permit a private bank to adjust its loan interest rate through this indexation when the private bank does not optimally reset its loan interest rate in a later section.

The third contribution of the paper is that we derive a new objective function for the central bank given the friction in the financial market, and therefore investigate the characteristics of optimal monetary policy. We show that this approximated utility-based welfare criterion holds a specific property that is not shared by other welfare functions in previous papers such as Rotemberg and Woodford (1997), Giannoni (2000), Erceg, Henderson and Levin (2000), Aoki (2001), Steinsson (2003), Edge (2003), Benigno (2004), and Ravenna and Walsh (2006). Giannoni (2000)

[4] It is also called as *Loan Commitment* or *Credit Line*.

[5] Recently, the amount of commitment line contract is rapidly increaseing as mentined by the BOJ: *The commitment line has become widely recognized by commercial banks and their corporate clients and as a result, recent years have seen a significant increase in the amount outstanding of commitment lines extended and in the number of corporate clients. The data will enable users to identify changes in corporate financing and banks' lending behavior* (on the webwite of the BOJ).

derive second order approximation to the consumer's utility function in a model with monetary transaction costs. Erceg, Henderson and Levin (2000) derives such an approximated welfare function in a model with a staggered wage contracts. Aoki (2001) derives an approximated welfare criteria for a central bank in a model with heterogeneous price setting sectors, a flexible-price sector and a sticky-price sector. Steinsson (2003) shows an approximated welfare function for a model in which one agent behaves by following the Calvo-type price setting and the other agent sets prices according to a rule-of-thumb, which induces a hybrid Phillips curve including both forward-looking and backward-looking terms. Edge (2003) adds endogenous capital accumulation to the standard New Keynesian model by Woodford (Ch. 3, 2003a) and derives the approximated welfare criteria. Benigno (2004) extends the discussion of welfare criteria to an international macro framework. Ravenna and Walsh (2006) derives a welfare criteria and investigates an optimal monetary policy under a cost channel, flexible loan contracts between firme and private banks. In contrast to these papers, under the model with staggered loan interest rate contracts, we show that the approximated welfare function includes the first-order difference of loan interest rates, which induces reduction of the magnitude of interest-rate changes in a social objective itself to shocks such as price shocks and demand shocks. As a result, optimal monetary policy has the characteristic of interest rate smoothing. These outcomes explain the fact that a central bank changes its policy rates through a series of small adjustments in the same direction as mentioned in previous papers, such as Goodfriend (1991) and Woodford (2003b). Thus, it is the staggered property of financial market that induces the central bank to optimally smooth interest rates. However, at the same time, a central bank has to quickly change the policy interest rates to smooth the loan interest rates against financial shocks such as a loan rate shock. Thus, a central bank should change its responses to economic disturbances according to types of shocks.

The rest of the paper is organized as follows. The following section sets up a baseline model. In Section 3, we show the impulse responses of the model under a Taylor rule. In Section 4, we derive a second order approximation to the consumer's utility function and comment on its properties. In Section 5, we derive an optimal monetary policy rule and show the properties of this optimal monetary policy. In Section 6, we extend our baseline model by allowing the loan interest rate to be indexed. In Section 7, we conclude the paper.

2 Baseline Model

We introduce staggered nominal loan interest rate contracts between private banks and firms into a model based on a standard New Keynesian framework built by Clarida, Gali and Gertler (1999) and Woodford (2003a). The model consists of four agents: a consumer, a firm, a central bank, and a private bank (see Figure 4).

A representative consumer plays four roles: (1) it consumes differentiated goods determined through a cost minimization problem given an aggregate consumption level, (2) it chooses the optimal amount of aggregate consumption, bank deposit, and investment into risky assets given the deposit rate set by the central bank, (3) it provides differentiated labor services and, because of its monopolistic power, decides the wage of each differentiated type of labor, and (4) it owns both the bank and the firm, and so receives dividends in each period.

A representative firm consists of three layers: a president, a continuum project groups populated on the $[0, 1]$ interval under the president, and a continuum business units populated on the $[0, 1]$ interval in each project group. Here, we assume that the business unit h in each project group is characterized by differentiated type of labor h. The firm plays two roles: (1) the president decides the amount of differentiated workers to hire, which is determined through a cost minimization problem in which a fraction of the labor cost must be financed through an external loan from a private bank. To clarify, each project group uses all types of workers and all types of loan in the same proportion[6], (2) under the monopolistic competition (an individual demand curve on differentiated consumption goods offered by the consumer), each project manager sets a differentiated goods price and produces one good using the external loan assigned by the president to finance parts of the labor costs[7] in order to maximize its profit. We assume a staggered price setting by Calvo (1983) - Yun (1992) framework.

A representative private bank consists of two layers: a president and a continuum of working groups populated over $[0, 1]$ under the president. A private bank plays two roles: (1) the president receives the deposit from the consumer[8] and allocates the deposit to each working group, (2) under the monopolistic competition, each working group lends to the firm by setting differentiated nominal loan interest rates according to the loan's demand curve. As explained below, we assume that each working group can set the differentiated nominal loan interest rate according to the properties of the business units, characterized by the differentiated labor type. We assume staggered nominal loan contracts between the firm and the private bank in the sense that the private bank perfectly fixes loan interest rates for a certain period. In the baseline model, we replicate the staggered property of loan interest rates, through the Calvo (1983) - Yun (1992) framework without indexing the interest rates. In section 6, however, we relax this assumption. As mentioned in the last section, this relation between the private bank and the firm is a kind of commitment line contract.

Finally a central bank sets the deposit rate according to a Taylor rule. Thus, in contrast to the staggered contracts between the firm and the private bank, the contracts between the consumer

[6] Erceg, Henderson and Levin (2000) assumes the same situation on employment. In other words, all project groups solve the cost minimization problems under a same situation, especially under a same labor index in this model. About (1), we can assume a representative labor aggregator, i.e. an employment coordinator, as in Erceg, Henderson and Levin (2000), instead of a president in a firm. In this case, it is natural to assume many independent different firms that produce different goods using differentiated labor service exist instead of one firm.

[7] The same assumption is in Christiano, Motto and Rostagno (2004).

[8] The same assumption is in Bernanke, Gertler and Gilchrist (1999). We can also assume that many different private banks that lend loan to each project group in a firm or to differenr firms exist instead of one private bank. In this case, each private bank receives a deposit from consumer and lends that whole deposit to a firm. Thus, total amount of each private bank's deposit should be eventually equal to a total deposit of consumer.

and the private bank is flexible. In this sense, for the private bank, the flexibility of its contracts are different between inflow and outflow. The details of the optimization problem, derivations of the first-order conditions, and log-linearizations are in Appendix A.

2.1 Cost Minimization

In this model, we have two cost minimization problems. The first determines the optimal allocation of differentiated goods for the consumer. The second determines the optimal allocation of labor services, given the loan rates and wages for the firm's president.

For the consumer, we assume that the consumer's utility from consumption is increasing and concave in the consumption index, which is defined as a Dixit-Stiglitz aggregator as in Dixit and Stiglitz (1977), of bundles of differentiated goods $f \in [0, 1]$ produced by firm's project groups as

$$C_t \equiv \left[\int_0^1 c_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}},$$

where C_t is aggregate consumption, $c_t(f)$ is a particular differentiated good along a continuum produced by the firm's project group f, and $\theta > 1$ is the elasticity of substitution across goods produced by project groups. For the consumption aggregator, the appropriate consumption-based price index is given by

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}},$$

where P_t is aggregate price and $p_t(f)$ is the price on a particular differentiated good $c_t(f)$. As in other applications of the Dixit-Stiglitz aggregator, the consumer's allocation across differentiated goods at each time must solve a cost minimization problem. This means that the relative expenditures on a particular good is decided according to:

$$c_t(f) = C_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta}. \tag{2.1}$$

An advantage of this consumption distribution rule is to imply that the consumer's total expenditure on consumption goods is given by P_tC_t. We use this demand function for differentiated goods in the firm sector.

On the firm's side, the president optimally allocates labor services from the consumer to each project group according to another cost minimization problem. The labor index L_t is given by

$$L_t \equiv \left[\int_0^1 l_t(h)^{\frac{\epsilon-1}{\epsilon}} dh \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{2.2}$$

where $l_t(h)$ is the differentiated labor supply of type $h \in [0, 1]$ that goes to the firm's business unit h within each project group. Thus, we assume that the differentiated labor types are not perfectly substitutable. In this paper, we assume that each project group uses all types of workers and loans in the same proportion. Also, we assume that the labor index, L_t, is used for production. Given this model's setup in which the firm must finance a fraction of the labor cost of business unit h, $\gamma w_t(h) l_t(h)$ (where $0 \leq \gamma \leq 1$)[9], through loan from the working group h in the private bank, the cost minimization problem of the president is given by

$$\min_{l_t(h)} \int_0^1 (1 + \gamma r_t(h)) w_t(h) l_t(h) dh,$$

subject to Eq. (2.2), where $r_t(h)$ is the nominal loan interest rate to the business unit h, which is set by the working group h in the private bank, and $w_t(h)$ is nominal wage for labor supply to h business unit of all project groups, which is set by the consumer. Thus, we assume that the private bank can set different nominal loan interest rates for different business units, each characterized by the type of labor. Importantly, this implies that the private bank interprets the differences in types of labors as the differences in risks of business units. It is this perspective that induces the banks to set different loan rates to each business unit. In this sense, the private bank holds a

[9] In the actual economy, the ratio of the external finance should change according to the economic stuation in the individual firm levels. In terms of the macro level, however, we can think that the ratio of the external finance is almost constant at least during several years. The analysis in this paper is focusing on such a time span.

monopolistic power in deciding loan interest rates. Given this interpretation, the relative demand for each differentiated type of labor, which is decided by the firm's president, is given by:

$$l_t(h) = L_t \left[\frac{(1+\gamma r_t(h))w_t(h)}{\Omega_t} \right]^{-\epsilon}, \tag{2.3}$$

$$\Omega_t \equiv \left[\int_0^1 ((1+\gamma r_t(h))w_t(h))^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}}. \tag{2.4}$$

Then, we have

$$\int_0^1 (1+\gamma r_t(h))w_t(h)l_t(h)dh = \Omega_t L_t.$$

Using the assumption that a specific fraction of the firm's labor cost due to labor type h is financed through a loan, then the amount the firm needs to borrow per labor type is

$$q_t(h) = \gamma w_t(h) l_t(h).$$

Then, we also have

$$q_t(h) = \gamma L_t \left[\frac{(1+\gamma r_t(h))w_t(h)}{\Omega_t} \right]^{-\epsilon} w_t(h).$$

By defining $Q_t \equiv \int_0^1 q_t(h)dh$, we have

$$q_t(h) = \left[\frac{(1+\gamma r_t(h))^{-\epsilon}(w_t(h))^{1-\epsilon}}{\Omega_t^*} \right] Q_t, \tag{2.5}$$

where $\Omega_t^* \equiv \int_0^1 (1+\gamma r_t(h))^{-\epsilon}(w_t(h))^{1-\epsilon} dh$. This is the demand function for loans by business units of type h in the firm. Due to the differentiated type of labor, the demand for loans is differentiated without assuming any restrictions on aggregate loans and loan rates. In contrast to Hülsewig, Mayery, and Wollmershäuserz (2007), we derive the demand function for loan from firm's optimized behavior. We will use this demand function in the private bank side.

2.2 Consumer

We consider the representative consumer who derives utility from consumption and disutility from a supply of work. The consumer maximizes the following utility function:

$$J_t = E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T, \nu_T) - \int_0^1 V(l_T(h), \nu_T) dh \right] \right\},$$

where E_t is an expectation conditional on the state of nature at data t. The function U is increasing and concave in the consumption index as shown in the last subsection. The budget constraint of the consumer is given by

$$P_t C_t + E_t \left[X_{t,t+1} B_{t+1} \right] + D_t \leq B_t + (1 + i_{t-1}) D_{t-1} + \int_0^1 w_t(h) l_t(h) dh + \int_0^1 \Pi_t^B(h) dh + \int_0^1 \Pi_t^F(f) df, \tag{2.6}$$

where B_t is a risky asset, D_t is the amount of bank deposit, i_t is the nominal deposit rate set by the central bank from t to $t+1$, $w_t(h)$ is the nominal wage for labor supply, $l_t(h)$, to the firm's business unit of type h, $\Pi_t^B(h)$ is a nominal dividend from owning the h working group in the bank, $\Pi_t^F(f)$ is a nominal dividend from owning the f project group in the firm, and $X_{t,t+1}$ is the stochastic discount factor. We assume a complete financial market for risky assets. Thus, we can hold a unique discount factor and can characterize the relationship between the deposit rate and the stochastic discount factor:

$$\frac{1}{1+i_t} = E_t \left[X_{t,t+1} \right]. \tag{2.7}$$

Given the optimal allocation of consumption expenditure across the differentiated goods, the consumer must choose the total amount of consumption, the optimal amount of risky assets to hold, and an optimal amount to deposit in each period. Necessary and sufficient conditions are given by

$$U_C(C_t,\nu_t) = \beta(1+i_t)E_t\left[U_C(C_{t+1},\nu_{t+1})\frac{P_t}{P_{t+1}}\right], \tag{2.8}$$

$$\frac{U_C(C_t,\nu_t)}{U_C(C_{t+1},\nu_{t+1})} = \frac{\beta}{X_{t,t+1}}\frac{P_t}{P_{t+1}}.$$

Together with Eq. (2.7), we can find that the condition given by Eq. (2.8) expresses the intertemporal optimal allocation on aggregate consumption. Assuming that the market clears, so that the supply of each differentiated good equals its demand, $c_t(f) = y_t(f)$ and $C_t = Y_t$, we finally obtain the standard New Keynesian IS curve by log-linearizing Eq. (2.8):

$$x_t = E_t x_{t+1} - \sigma(\widehat{i}_t - E_t\pi_{t+1} - \widehat{r}_t^n), \tag{2.9}$$

where we name x_t the output gap, π_{t+1} inflation, and $\widehat{r}_t^n$ the natural rate of interest. $\widehat{r}_t^n$ will be an exogenous shock. Each variable is defined as the log deviation from its steady states (except x_t and π_t. Also, the log-linearized version of variable m_t is expressed by $\widehat{m}_t = \ln(m_t/\overline{m})$, where $\overline{m}$ is steady state value of m_t.).

In this model, the consumer provides differentiated types of labor to the firm and so holds the power to decide the wage of each type of labor as in Erceg, Henderson and Levin (2000). We assume that each project group hires all types of workers in the same proportion. The consumer sets each wage $w_t(h)$ for any h in every period to maximize its utility subject to the budget constraint given by Eq. (2.6) and the demand function of labor given by Eq. (2.3)[10]. Then we have the following relation

$$\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon-1}\frac{V_l(l_t(h),\nu_t)}{U_C(C_t,\nu_t)}. \tag{2.10}$$

In this paper, we assume that the consumer supplies its labors only for the firm, not for the private bank. We use the relation given by Eq. (2.10) in the firm side.

[10] We assume a flexible wage setting in a sense that the consumer can change wage in every period.

2.3 Firm

In this paper, the representative firm consists of three layers: the president, a continuum of project groups populated over $[0, 1]$ under the president, and a continuum of business units populated over $[0, 1]$ in each project group. As explained above, firstly, we assume that the president determines the amount of workers employed through a cost minimization problem in which a fraction of the labor costs must be financed through external loans from a private bank. Secondly, in a monopolistically competitive goods market, each project manager sets a differentiated goods price and produces one good. Each project manager employs all types of workers, borrows all types of external loans, and re-sets its price with some intervals.

Under the Calvo (1983) - Yun (1992) framework, the f project manager re-sets its price with probability $1 - \alpha$ and maximizes the firm's present discounted value of profit:

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[p_t(f) y_{t,T}(f) - \Omega_T L_T(f) \right],$$

where we define $y_{t,T}(f) \equiv Y_T \left[\frac{p_t(f)}{P_T}\right]^{-\theta}$ from Eq. (2.1) under $c_t(f) = y_t(f)$ and use $C_t = Y_t$ for any t. Here we use the consumer's (shareholder's) marginal rate of substitution, $X_{t,t+1}$, for each firm's project group. Importantly, the price set by a firm's project group is independent of the loan rate chosen by a bank's working group. Then, the optimal price $p_t^*(f)$ in this Calvo environment is

$$\begin{aligned} & E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \left[\frac{\theta - 1}{\theta} \frac{p_t^*(f)}{P_T} \right] \qquad (2.11) \\ = \; & E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \left[\frac{\epsilon}{\epsilon - 1} \left\{ \int_0^1 (1 + \gamma r_T(h))^{1-\epsilon} \left(\frac{V_l(l_{t,T}(h), \nu_T)}{U_Y(Y_T, \nu_T)} \frac{\partial L_T(f)}{\partial y_{t,T}(f)} \right)^{1-\epsilon} dh \right\}^{\frac{1}{1-\epsilon}} \right], \end{aligned}$$

where we assume that the firm's production function is given by $y_t(f) = A_t f(L_t(f))$, where $f(\cdot)$ is an increasing and concave function and Eq. (2.10) has been substituted to simplify Eq. (2.11). By log-linearing Eq. (2.11), we have

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t^*(f) = E_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[\sum_{\tau=t+1}^{T}\pi_\tau + \Theta\widehat{R}_T + \widehat{mc}_{t,T}(f)\right], \tag{2.12}$$

where we define a real marginal cost as $mc_{t,T}(h,f) \equiv \frac{V_l(l_{t,T}(h),\nu_t)}{U_Y(Y_t,\nu_t)}\frac{\partial L_{t,T}(f)}{\partial y_t(f)}$ and $\widehat{mc}_{t,T}(f) \equiv \int_0^1 \widehat{mc}_{t,T}(h,f)dh$, we define $1+R_t \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1+r_t(h))dh$ and $\widehat{\widetilde{p}}_t^*(f) \equiv \frac{p_t^*(f)}{Pt}$, and Θ is a positive parameter. The chances of re-setting prices are randomly assigned to each project group with equal probability, which implies we can take the average across f. Then, Eq. (2.12) becomes

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t^* = E_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\omega_p\theta)^{-1}(\widehat{mc}_T + \Theta\widehat{R}_T) + \sum_{\tau=t+1}^{T}\pi_\tau\right],$$

where we define $(p_t^*)^{1-\theta} \equiv \int_0^1 p_t^*(f)^{1-\theta}dh$. We have also defined the average real marginal cost as $mc_t(h) \equiv \frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)}\frac{\partial L_t}{\partial Y_t}$ and $\widehat{mc}_t \equiv \int_0^1 \widehat{mc}_t(h)dh$ and we make use of the relation of $\widehat{mc}_{t,T}(f) = \widehat{mc}_T - \omega_p\theta(\widehat{\widetilde{p}}_t(f) - \sum_{\tau=t+1}^{T}\pi_\tau)$[11], where ω_p is a positive parameter. In the Calvo (1983) - Yun (1992) setting, the evolution of aggregate price index P is described by the following equation

$$\int_0^1 p_t(f)^{1-\theta}df = \alpha\int_0^1 p_{t-1}(f)^{1-\theta}df + (1-\alpha)\int_0^1 p_t^*(f)^{1-\theta}df,$$

$$\Longrightarrow P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(p_t^*)^{1-\theta}. \tag{2.13}$$

This implies that the current aggregate price is equal to a weighted average of changed and unchanged prices. Since the opportunity to reset prices are randomly assigned to each firm with equal probability, then the price change at time t should be evaluated by an average of individual price changes by all project groups. This is true even for the unchanged price. By log-linearing Eq. (2.13) and manipulating the resulting equation, we have the following relation

[11] Unit marginal cost is same for all project groups under the situation in which each project group uses all types of labor and all types of loan with a same proportion. Thus, all project groups set a same price if they have chances to reset their prices at time t. Thus, we acutually do not need to take average across f in Eq. (2.12).

$$\pi_t = \kappa x_t + \xi \widehat{R}_t + \beta E_t \pi_{t+1}, \tag{2.14}$$

where κ and ξ are positive parameters. In contrast to the standard New Keynesian Phillips curve, this augmented one includes the loan rate.

2.4 Private Bank

The representative bank consists of a continuum of working groups populated over the $[0,1]$ interval that handles the job of financing each firm under a president. Each working group manager can reset its loan rate with probability $1-\varphi$. We assume that each working group can set different loan rates that depend on the business units' labor type. In this sense, the private bank holds a monopolistic power in deciding the loan rates to each project. Under a monopolistically competitive loan market, we can define the maximization problem for the working group h, where the objective is to maximize the present discounted value of profit:

$$E_t \sum_{T=t}^{\infty} \varphi^{T-t} X_{t,T} \left[(r_t(h) - i_T) q_{t,T}(h) - z_T(h) \right], \tag{2.15}$$

where we define $q_{t,T}(h) = \left[\frac{(1+\gamma r_t(h))^{-\epsilon} (w_T(h))^{1-\epsilon}}{\Omega_T^*} \right] Q_T$ from Eq. (2.5), $r_t(h)$ is the nominal loan interest rate set by the working group h in the private bank, i_T is the deposit rate which is set by the central bank and is same for all working groups, and $z_T(h)$ is the cost associated with the bank's working group h handling the financing of the firm's business unit h. We use the consumer's (shareholder's) marginal rate of substitution, X, for each working group of the bank. For simplicity, we assume that the cost of the bank's working group is constant, specifically zero. We can interpret this fixed cost as license expenses to run banking. Also, in equilibrium, we assume that the supply of deposits equals the demand: $D_T = Q_T$. Thus, the president of private bank implicitly allocates deposit to each working group. Lastly, $r_t(h)$ is the interest rate at time

t[12]. We can transform Eq. (2.15) as follows

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} \left[q_{t,T}(h)(r_t(h) - i_T) \right].$$

Then, the optimal loan interest rate, $r_t(h)$, in this Calvo setting solves the equation

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[1 - \epsilon\gamma \frac{r_t(h) - i_T}{1 + \gamma r_t(h)} \right] = 0. \quad (2.16)$$

Working groups that are allowed to reset their loan rates will set the same loan rate, so the solution of $r_t(h)$ in Eq. (2.16) is expressed by r_t^*. On the other hand, we have the following evolution of the aggregate loan rate index R :

$$1 + R_t = \varphi(1 + R_{t-1}) + (1 - \varphi)(1 + r_t^*). \quad (2.17)$$

By log-linearizing Eq. (2.16) and Eq. (2.17), we can characterize the relationship between the loan rate and the deposit rate:

$$\widehat{R}_t = \lambda_1 E_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t, \quad (2.18)$$

where $\lambda_1 > 0$, $\lambda_2 > 0$, and $\lambda_3 > 0$[13]. Each variable is defined as the log deviation from steady state. We can call this equation the loan rate curve.

2.5 Closed System of the Model

To close the model, we have to describe the central bank. In this paper, we assume the central bank sets a deposit rate, i.e. policy interest rate, in every period by following a simple Taylor rule:

[12] If we interpret that $r_t(h)$ is the interest rate from t to $t+1$, the divident from the private bank in the consumer's budget constraint is given by $\int_0^1 \Pi_t^B(h)dh$. However, even in this case, the model does not change.

[13] After I finished writing this paper, I found that Kobayashi (2008) also have done a similar study completely independently. He assumes a complete monopoly in lending/borrowing relation between banks and firms, which can be justified in a situation where a loan market is geographically segmented.

$$\widehat{i}_t = \mu_\pi \pi_t + \mu_x x_t, \tag{2.19}$$

where $\mu_\pi > 0$ and $\mu_x > 0$. With this equation, we have completely closed the model, which consists of Eq. (2.9), Eq. (2.14), Eq. (2.18), and Eq. (2.19). The system includes the output gap, inflation rate, loan rate, and deposit rate[14].

3 Impulse Response under Taylor Rule

In this baseline model, we characterize the properties of this model with staggered loan contracts in terms of its impulse response functions.

We borrow our parameters from Rotemberg and Woodford (1997), as shown in Table 1, except γ (ratio of external finance) , φ (loan contract rigidity), and ϵ (substitutability of differentiated types of labor)[15]. In addition to shocks from the natural rate of interest, we introduce shocks to the Phillips curve, given by Eq. (2.14), and the loan rate curve, given by Eq. (2.18). Here we assume preference shocks on the parameters θ and ϵ in Eq. (2.14) and Eq. (2.18) respectively[16]. Given these shocks, we can write

$$\pi_t = \kappa x_t + \xi \widehat{R}_t + \beta E_t \pi_{t+1} + \Psi_t,$$

$$\widehat{R}_t = \lambda_1 E_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t + \Upsilon_t,$$

where Ψ_t and Υ_t, which we will call the price mark-up and loan rate shocks, respectively. A key contribution of this paper is to incorporate shocks to the financial market, specifically shocks to the loan interest rate contracts, into an otherwise standard New Keynesian model. We can interpret

[14] Behind this closed system, all variables in the model are determined.

[15] In this ϵ, we have relatively high steady state value of loan rate as $\overline{R} = 0.31$ to $\overline{i} = 0.01$. Thus, it may be better to set higher ϵ. For example, Altig, Christiano, Eichenbaum and Linde (2002) assumes $\epsilon = 21$. In this case, we have $\overline{R} = 0.11$ to $\overline{i} = 0.01$. We, however, do not have a clear diffrence in any impulse response by setting different ϵ.

[16] We can assume different sources of shocks in many ways.

the loan rate shock in two ways. First, the loan rate shock could represent a change in the bank's lending attitude, coming from more or less risk-averse behavior. The change in a bank's attitude may include contract length changes. Second, the loan rate shock could also represent the private banks' changing responses to monetary policy shocks. This interpretation implies that monetary policy shocks are not the result of only the central bank's behavior, as in Christiano, Eichenbaum and Evans (2005)[17], but rather are affected by both the central bank and private banks.

In this section, we set policy parameters in Taylor rule as $\mu_\pi = 2$ and $\mu_x = 0.25$ as in Woodford (Ch. 4, 2003a), and assume $\varphi = 0.66$ (loan contract rigidity is equal to price rigidity) and $\gamma = 1$ (all labor costs are financed by loan). A reason of $\varphi = 0.66$ is that the empirical papers suggest at least two quarters and perhaps more to adjust the loan interest rates[18]. Also we show simulations of the standard New Keynesian model[19] and compare the results to those of our loan contract model.

3.1 Loan Rate Shock

We assume an unexpected 1% positive loan rate shock with shock persistence 0.9 in AR(1) process[20]. Figure 5 shows simulation outcomes. To show effects of the staggered loan contract, we

[17] They show that a monetary policy shock is one of the most important source of economic disturbances.

[18] For example, in Japan, the report by the Bank of Japan, Financial System Report (March 2007), shows that average durations of fixing loan interest rates are about three quarters for major city banks and about five quarters for local banks.

[19] We assume the model in Woodford (Ch. 4, 2003a) as the standard New Keynesian model. It consists from three equation as

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + \Psi_t,$$

$$x_t = E_t x_{t+1} - \sigma(\widehat{i}_t - E_t \pi_{t+1} - \widehat{r}_t^n),$$

$$\widehat{i}_t = \mu_\pi \pi_t + \mu_x x_t.$$

These are Phillips curve, IS curve, and monetary policy rule, respectively. We use the parameters in Table 1.

[20] We may express it as

$$\widehat{r}_t^n = 0.9 \widehat{r}_{t-1}^n + \upsilon_t,$$

where υ_t is i.i.d shock process.

also demonstrate a case of $\varphi = 0$ and $\gamma = 1$. A shock to the loan rate increases inflation rates due to increase in cost, then raises policy interest rates. In turn, a high policy interest rate induces a negative output gap. This tendency is clearer in the case of staggered loan contracts than in that of flexible loan contracts. We can confirm that the loan rate shock induces a significantly large and persistent economic fluctuation by a staggered mechanism of loan contracts. Thus, the staggered-ness of loan contracts can play an important role in explaining economic fluctuations and in amplifying economic disturbance.

3.2 Natural Rate of Interest Shock

For our impulse responses, we assume that there is an unexpected 1% positive shock to the natural rate of interest with persistence equal to 0.9, using an AR(1) process. Figure 6 shows the simulation results. As in the standard New Keynesian model, each simulation shows that both inflation rate and the output gap increase, which leads to higher policy interest rates and loan rates. Specifically, the model with staggered loan contracts is characterized by fluctuations that have greater persistence and amplitude than the standard New Keynesian model.

3.3 Price Mark-Up Shock

In this section, we assume that there is an unexpected 1% positive price mark up shock with shock persistence 0.9 in AR(1) process. Figure 7 shows simulation results. As in the standard New Keynesian model, each simulation shows that the inflation rate goes up, which implies that the policy interest rate rises in the staggered loan contract model. In turn, these changes induce a negative output gap in the both models. Again, we can confirm that the impulse responses in the staggered loan setting are longer and larger than those in the standard New Keynesian model.

4 Analysis on Welfare Function

In this section, we first derive a second order approximation to the welfare function (all details of these derivations and explanations are in Appendix B). Second, we demonstrate that this approximated welfare function leads the central bank to smooth policy interest rates.

4.1 Approximated Welfare Function

Assuming that the goods market clears, which implies that $Y_t = C_t$ and $y_t(f) = c_t(f)$ for any f, a discounted loss of the consumer is given by

$$J_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_t \right\}, \tag{4.1}$$

where the welfare criterion U_t is given by

$$U_t = U(Y_t, \nu_t) - \int_0^1 V(l_t(h), \nu_t) dh, \tag{4.2}$$

and

$$Y_t \equiv \left[\int_0^1 y_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}},$$

where $U(Y_t; \nu_t)$ is an increasing and concave function of Y_t, $V(l_t(h); \nu_t)$ is an increasing and convex function of $l_t(h)$, and θ is a preference parameter on differentiated goods. We log-linearize Eq. (4.2) step by step to derive the approximated welfare function.

First, we log-linearize the first term of Eq. (4.2).

$$U(Y_t; \nu_t) = \overline{Y} U_c \left[\widehat{Y}_t + \frac{1}{2}(1 - \sigma^{-1}) \widehat{Y}_t^2 + \sigma^{-1} g_t \widehat{Y}_t \right] + t.i.p + Order(\| \xi \|^3), \tag{4.3}$$

where $\overline{U} \equiv U(\overline{Y}; 0)$, $\widetilde{Y}_t \equiv Y_t - \overline{Y}$, $t.i.p$ means that there are additional terms that are independent of monetary policy, $Order(\| \xi \|^3)$ indicates that there are additional terms of higher order than two, σ is the intertemporal elasticity of substitution for private expenditures, and $g_t \equiv -\frac{U_{c\nu}\nu_t}{\overline{Y}U_{cc}}$. To replace $\widetilde{Y}_t$ by $\widehat{Y}_t \equiv \ln(Y_t/\overline{Y})$, we use the Taylor series expansion on $Y_t/\overline{Y}$ in the second line as $Y_t/\overline{Y} = 1 + \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + Order(\parallel \xi \parallel^3)$.

Second, we log-linearize the second term of Eq. (4.2) in a similar way.

$$\int_0^1 V(l_t(h);\nu_t)dh = \overline{L}V_l\left[\widehat{L}_t + \frac{1}{2}(1+\nu)\widehat{L}_t^2 - \nu\widetilde{\nu}_t\widehat{L}_t + \frac{1}{2}(\nu+\frac{1}{\epsilon})var_h\widehat{l}_t(h)\right] + t.i.p + Order(\parallel \xi \parallel^3), \tag{4.4}$$

where we use the relation

$$L_t \equiv \left[\int_0^1 l_t(h)^{\frac{\epsilon-1}{\epsilon}}dh\right]^{\frac{\epsilon}{\epsilon-1}},$$

and $var_h\widehat{l}_t(h)$ is the variance of $\widehat{l}_t(h)$ across all labor types. We have also defined $\widehat{L}_t \equiv \ln(L_t/\overline{L})$; ν is an elasticity of desired real wage with respect to the quantity of demanded labor, ϵ is a preference parameter on differentiated labors, and $\widetilde{\nu}_t \equiv -\frac{V_{l\nu}\nu_t}{\overline{L}V_{ll}}$. To replace $\widehat{L}_t$ by $\widehat{Y}_t$, we invoke the market clearing condition for labor:

$$L_t = \int_0^1 L_t(f)df = \int_0^1 f^{-1}(\frac{y_t(f)}{A_t})df, \tag{4.5}$$

where we have used the production function $y_t(f) = A_t f(L_t(f))$. By log-linearizing Eq. (4.5) and substituting it into Eq. (4.4), we have

$$\begin{aligned}\int_0^1 V(l_t(h);\nu_t)dh &= \phi_h\overline{L}V_l\left[\widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t\widehat{Y} + \frac{1}{2}(1+\omega_p\theta)\theta var_f\widehat{p}_t(f) + \frac{1}{2}\phi_h^{-1}(\nu+\frac{1}{\epsilon})var_h\widehat{l}_t(h)\right] \\ &+t.i.p + Order(\parallel \xi \parallel^3),\end{aligned} \tag{4.6}$$

where q_t is a vector that includes shocks given by $\widetilde{\nu}_t$ and productivity shocks given by A_t, $var_f\widehat{p}_t(f)$ is the variance of $\widehat{p}_t(f)$ across all differentiated goods prices, ϕ_h is the inverse of the elasticity of output with respect to additional labor input, ω_p is the negative value of the elasticity of the marginal product of labor with respect to aggregate output, and ω is the sum of ω_p and the elasticity of the real wage under a flexible-wage labor supply with respect to aggregate output. Also, we use the demand function on each differentiated goods to replace $var_h \widehat{y}_t(h)$ by $var_f \widehat{p}_t(f)$, which can be derived from a consumer's cost minimization problem as

$$y_t(f) = Y_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta}. \tag{4.7}$$

By log-linearizing Eq. (4.7), we have

$$var_f \ln y_t(f) = \theta^2 var_f \ln p_t(f).$$

To evaluate $var_h \widehat{l}_t(h)$, we use the optimal condition of labor supply and the labor demand function given by Eq. (2.3), Eq. (2.10), and Eq. (2.4). By log-linearizing these equations, we finally have a following relation

$$var_h \ln l_t(h) = \Xi var_h \ln(1 + r_t(h)).$$

where Ξ is a positive parameter. Then we can transform Eq. (4.6) as

$$\begin{aligned} \int_0^1 V(l_t(h); \nu_t) dh &= \phi_h \overline{L} V_l \left[\widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t \widehat{Y} + \frac{1}{2}\eta_\pi var_f \ln p_t(f) + \frac{1}{2}\eta_r var_h \ln(1 + r_t(h)) \right] \\ &+ t.i.p + Order(\parallel \xi \parallel^3), \end{aligned} \tag{4.8}$$

where η_π and η_r are positive parameters.

To express an approximated loss function in terms of the output gap which is defined as difference between the output and the natural rate of output, we specify the natural rate of output according to Woodford (Ch. 6, 2003a). The real marginal cost function of the firm, $m(\cdot)$, to supply the good f is given by

$$m(y_t(f), Y_t, r_t; \nu_t) = \frac{\epsilon}{\epsilon - 1}\left[\int_0^1 (1+\gamma r_t(h))^{1-\epsilon}(\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)})^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}} \frac{\partial L_t(f)}{\partial y_t(f)},$$

and the natural rate of output $Y_t^n = Y^n(\nu_t)$ is then defined by

$$m(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta - 1}{\theta} \equiv \frac{\epsilon}{\epsilon - 1}(1+\gamma\overline{R})(1-\Phi), \tag{4.9}$$

where a parameter Φ expresses the size of the distortion to the output level, induced by firm's price mark up through $\left[\int_0^1 (\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)} \frac{\partial L_t(f)}{\partial y_t(f)})^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}}$, that would exist in an economy with flexible prices and no role for monetary policy. We assume that monetary policy has no impact on the level of natural rate of output. Also, we assume that Φ is of order one, $Order(\parallel \xi \parallel)$, as in Woodford (2003a). By assuming that there is a proportional tax on sales τ with the implication that

$$m(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta - 1}{\theta}(1-\tau) \equiv \frac{\epsilon}{\epsilon - 1}(1+\gamma\overline{R})(1-\Phi),$$

we see that Φ is of order one[21]. Thus, without loss of generality, we assume that the Φ is of order one. Moreover, by log-linearizing Eq. (4.9), the natural rate of output is actually given by

$$\widehat{Y}_t^n \equiv \ln \frac{Y_t^n}{\overline{Y}} = \frac{\sigma^{-1} g_t + \omega q_t}{\sigma^{-1} + \omega}. \tag{4.10}$$

Then we can combine Eq. (4.3) and Eq. (4.8),

$$U_t = -\frac{1}{2}\overline{Y} U_c \left[(\sigma^{-1}+\omega)(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) + \eta_r var_h \ln(1+r_t(h))\right] + t.i.p + Order(\parallel \xi \parallel^3), \tag{4.11}$$

where $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$ and $x^* \equiv \ln(Y^*/\overline{Y})$. Here Y^* is called an efficient level of output as in Woodford (2003a), which follows from having no output distortion: $\left[\int_0^1 (\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)} \frac{\partial L_t(f)}{\partial y_t(f)})^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}}$. Here we use Eq. (4.10) and the assumption that Φ is of order one.

[21] So we assume that the government carries out a tax policy to realize a low distortion of the output level, induced by firm's price mark up through $\left[\int_0^1 (\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)} \frac{\partial L_t(f)}{\partial y_t(f)})^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}}$.

The remaining work to derive the approximated welfare function is to evaluate $var_h \ln p_t(h)$ and $var_h \ln(1+r_t(h))$ in Eq. (4.11). Following Woodford (2003a), we define

$$\overline{P}_t \equiv E_f \ln p_t(f),$$

$$\triangle_t \equiv var_f \ln p_t(f).$$

Then we have

$$\overline{P}_t - \overline{P}_{t-1} = (1-\alpha)E_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right],$$

and so we can have

$$\triangle_t = \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}(\overline{P}_t - \overline{P}_{t-1}), \tag{4.12}$$

where $p_t^*(f)$ is the optimal price set by the firm's project group h. It is clear that all project groups that are able to change their prices will reset their prices to the same price p_t^* at time t because the unit marginal cost of production is the same for all project groups. This implies that the law of motion for the aggregate price is

$$P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(p_t^*)^{1-\theta}.$$

Also, we have a following relation that relates $\overline{P}_t$ with P_t

$$\overline{P}_t = \ln P_t + Order(\parallel \xi \parallel^2),$$

where $Order(\parallel \xi \parallel^2)$ represents the terms that are of order higher than the first order approximation. We have made use of the definition of the price aggregator $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$. Then Eq. (4.12) can be transformed into

$$\triangle_t = \alpha \triangle_{t-1} + \frac{\alpha}{1-\alpha}\pi_t, \tag{4.13}$$

where $\pi_t \equiv \ln \frac{P_t}{P_{t-1}}$. From Eq. (4.13), we have

$$\triangle_t = \alpha^{t+1}\triangle_{-1} + \sum_{s=0}^{t}\alpha^{t-s}\left(\frac{\alpha}{1-\alpha}\right)\pi_s^2,$$

and so

$$\sum_{t=0}^{\infty}\beta^t\triangle_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p + Order(\parallel \xi \parallel^3). \tag{4.14}$$

To evaluate $var_h \ln(1+r_t(h))$, we define $\overline{R}_t$ and $\triangle_t^R$ as

$$\overline{R}_t \equiv E_h \ln(1+r_t(h)),$$

$$\triangle_t^R \equiv var_h \ln(1+r_t(h)).$$

Then we have

$$\overline{R}_t - \overline{R}_{t-1} = (1-\varphi)\left[\ln(1+r_t^*(h)) - \overline{R}_{t-1}\right], \tag{4.15}$$

and so we have

$$\triangle_t^R = \varphi\triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\overline{R}_t - \overline{R}_{t-1})^2. \tag{4.16}$$

This equation indicates that all working groups that are allowed to reset their interest rates, will optimally set the same loan interest rate r_t^* at time t because the cost of lending is same for all working groups. Thus, the law of motion governing the aggregate loan interest rates is

$$1 + R_t = \varphi(1 + R_{t-1}) + (1-\varphi)(1 + r_t^*).$$

As in the discussion on price, we have

$$\overline{R}_t = \ln(1 + R_t) + Order(\| \xi \|^2), \tag{4.17}$$

where we have made use of the definition of the aggregate loan rate $1+R_t \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1+r_t(h))dh$. Then, from Eq. (4.16) and Eq. (4.17), we have

$$\triangle_t^R = \varphi \triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\widehat{R}_t - \widehat{R}_{t-1})^2, \tag{4.18}$$

where $\widehat{R}_t \equiv \ln \frac{1+R_t}{1+\overline{R}}$. From Eq. (4.18), we have

$$\triangle_t^R = \varphi^{t+1} \triangle_{-1}^R + \sum_{s=0}^{t} \varphi^{t-s} \left(\frac{\varphi}{1-\varphi} \right) (\widehat{R}_s - \widehat{R}_{s-1})^2,$$

and so

$$\sum_{t=0}^{\infty} \beta^t \triangle_t^R = \frac{\varphi}{(1-\varphi)(1-\varphi\beta)} \sum_{t=0}^{\infty} \beta^t (\widehat{R}_t - \widehat{R}_{t-1})^2 + t.i.p + Order(\| \xi \|^3). \tag{4.19}$$

From Eq. (4.11), Eq. (4.14), and Eq. (4.19), we finally have

$$\sum_{t=0}^{\infty} \beta^t U_t \simeq -\Lambda \sum_{t=0}^{\infty} \beta^t \left(\lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_R (\widehat{R}_t - \widehat{R}_{t-1})^2 \right),$$

where Λ, λ_x, and λ_R are positive parameters. Thus, by approximating the welfare function to the second order, we have the following approximated microfounded-welfare function.

$$U_t = \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_R (\widehat{R}_t - \widehat{R}_{t-1})^2. \tag{4.20}$$

The welfare function includes a quadratic loss of the first order difference in loan rates in addition to quadratic losses of inflation and the output gap.

In the case of flexible loan contracts, the welfare function only includes quadratic losses of inflation and the output gap, i.e. $\lambda_R = 0$[22]. This welfare function is consistent with one in Ravenna and Walsh (2006). Thus, in a model with staggered loan contracts, the central bank should also pay attention to the loan rate fluctuation, specifically to the first order difference of loan rates[23].

4.2 Policy Interest Rate Smoothing

In reality, central banks often change their policy rates through a series of small adjustments in the same direction, as mentioned in previous works, such as Rudebusch (1995), Goodhart (1996), and Woodford (2003b). Woodford (2003b) suggests that optimal commitment policy can induce this gradualism in policy, i.e. the history dependent property of monetary policy. However, in Woodford's model, the central bank does not have a term that measures the change in interest rates in its objective function. We show that in a staggered loan contract setting, the central bank does indeed have this additional term, which implies that the central bank has the incentive to smooth policy rates.

As shown in Giannoni (2000) and Woodford (2003a), an interest rate term can be theoretically introduced into the central bank's objective function by assuming monetary frictions. Often, these frictions imply the loss function

$$U_t = \lambda_\pi \pi_t^2 + \lambda_x x_t^2 + \lambda_i (\widehat{i}_t - i^*)^2,$$

where λ_i is a positive parameter. This loss function includes the quadratic loss of the nominal interest rate deviation from its steady state value, but does not include a term for the change in interest rates over time. As such, this loss function is not consistent with the fact that central banks

[22] Even in the case of no loan contract between a firm and a private bank, we have $\lambda_R = 0$.

[23] The relative values of λ_R to λ_π and λ_x increase as the staggered-ness of loan rate contracts rises. Also, the relative values of λ_R to λ_π and λ_x increase as the ratio of the firm's loan finance increases.

try to smooth interest rate changes and do not try to minimize the interest rate deviations from it steady state. Discussing this difference, Woodford (2003b) refers to the *delegation problem*, which yields equilibrium paths to shocks similar to those associated with the optimal commitment policy in a regular welfare function[24] under a situation that a central bank acts as day-by-day minimizer of its assigned loss function, called a discretionary policy defined in Woodford (2003b). He shows that a central bank can achieve exactly same equilibrium brought by the optimal commitment policy in *delegation problem* by assuming a loss function that has imposed an additional term measuring the change in interest rates:

$$U_t = \lambda_\pi \pi_t^2 + \lambda_x x_t^2 + \lambda_i i_t^2 + \lambda_\triangle (\widehat{i}_t - \widehat{i}_{t-1})^2, \tag{4.21}$$

where $\lambda_\triangle$ is a positive parameter[25]. However, he shows that this desirable outcome holds only in a specific environment. Thus, in *delegation problem,* the loss function given by Eq. (4.21) can not generally induce equilibrium responses achieved by the optimal commitment policy under the regular welfare function.

In contrast to the discussion in Woodford (2003b), a model with staggered loans directly modifies the central bank's welfare function in a way that induces it to smooth interest rates over time. Using the loan rate curve given by Eq. (2.18), we can transform $(\widehat{R}_t - \widehat{R}_{t-1})^2$ in the welfare function given by Eq. (4.20) as

$$(\widehat{R}_t - \widehat{R}_{t-1})^2 = \left(\left[\lambda_1^{-1} z_1^{-1} (1 - z_3 F)^{-1} (1 - z_2 L)^{-1} \right] \left\{ \lambda_3 (\widehat{i}_t - \widehat{i}_{t-1}) + (\Upsilon_t - \Upsilon_{t-1}) \right\} \right)^2,$$

[24] This regular welfare function is given by

$$L_t = \lambda_\pi \pi_t^2 + \lambda_x x_t^2.$$

[25] We set $i^* = 0$.

where L expresses a lag operator and F expresses a forward operator. Also, we have $z_1+z_2 = \lambda_1^{-1}$, $z_1z_2 = -\lambda_1^{-1}\lambda_2$, and $z_3 = z_1^{-1}$. If there is no shock in the loan rate curve, i.e. $\Upsilon_t = 0$ for any t, stabilizing the loan rate implies that the central bank stabilizes policy interest rates. Thus, when faced with shocks to the mark-up and the natural rate of output, the central bank has the incentive to minimize the changes in the policy rates[26]. In other words, a central bank conducts monetary policy generating realistic time paths of interest rates in an environment with staggered loan contracts. This staggered property of the financial market is absent in the literature. As a result, other papers cannot introduce a term which induces the policy interest rate smoothing in the central bank's welfare function. Interestingly, on the other hands, a central bank has to react quicly to the shock in the loan rate curve. In this case, a central bank aggresively change the policy rates to smooth the loan rates. We confirm these policy properties under an optimal monetary policy rule in the next section.

5 Optimal Monetary Policy Analysis

5.1 Optimal Monetary Policy

We consider an optimal monetary policy scheme in which the central bank is credibly committed to a policy rule in the *Timeless Perspective*[27]. In this case, as shown in Woodford (2003a), the central bank conducts monetary policy in a forward looking way by paying attention to future economic variables and by taking account of the effects of monetary policy on those future variables.

The objective of monetary policy is to minimize the expected value of the loss criterion given by Eq. (4.1) and Eq. (4.20) under the standard New Keynesian IS curve given Eq. (2.9), the augmented Phillips curve given by Eq. (2.14), and the loan rate curve given by Eq. (2.18). The optimal monetary policy is expressed by the solution of the optimization problem which is

[26] If we assume shocks in the loan rate curve, policy interest rates can be more volatile, though loan rates are still stabilized enough. In this case, policy interest rates respond to loan rate shocks to minimize the changes of loan interest rates.

[27] The detailed explanations about the timeless perspective are in Woodford (2003).

represented by the following Lagrangian:

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ L_t + 2\Xi_{1t} \left[x_{t+1} - \sigma(\widehat{i}_t - \pi_{t+1} - r_t^n) - x_t \right] + 2\Xi_{2t} \left[\kappa x_t + \xi \widehat{R}_t + \beta \pi_{t+1} - \pi_t \right] \right\} \right\}$$

$$+ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ 2\Xi_{3t} \left[\lambda_1 E_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t - \widehat{R}_t \right] \right\} \right\},$$

where Ξ_1, Ξ_2, and Ξ_3 are the Lagrange multipliers associated with the IS curve constraint, the Phillips curve constraint, the loan rate curve constraint, respectively. We differentiate the Lagrangian with respect to π_t, x_t, $\widehat{R}_t$, and $\widehat{i}_t$ to obtain the first-order conditions:

$$\lambda_\pi \pi_t + \beta^{-1} \sigma \Xi_{1t-1} - \Xi_{2t} + \Xi_{2t-1} = 0, \tag{5.1}$$

$$\lambda_x (x_t - x^*) - \Xi_{1t} + \beta^{-1} \Xi_{1t-1} + \kappa \Xi_{2t} = 0, \tag{5.2}$$

$$\lambda_R (\widehat{R}_t - \widehat{R}_{t-1}) - \beta \lambda_R (E_t \widehat{R}_{t+1} - \widehat{R}_t) + \xi \Xi_{2t} - \Xi_{3t} + \beta^{-1} \lambda_1 \Xi_{3t-1} + \beta \lambda_2 E_t \Xi_{3t+1} = 0, \tag{5.3}$$

$$\Xi_{3t} - \lambda_3^{-1} \sigma \Xi_{1t} = 0. \tag{5.4}$$

These four conditions, together with the IS curve, the Phillips curve, and the loan rate curve equations, are conditions governing the loss minimization for $t \geq 0$. In other words, the sequence of interest rates determined by these conditions is the optimal path interest rate.

For simplicity, we can better understand optimal policy by reducing the number of conditions to

$$(1 - z_1 L)(1 - z_2 L) \left[\lambda_R (\triangle \widehat{R}_t - \beta E_t \triangle \widehat{R}_{t+1}) - \kappa^{-1} \xi \lambda_x (x_t - x^*) \right] \tag{5.5}$$

$$= E_t \left[-z_3 z_4 (1 - z_5 L)(1 - z_6 F)(\kappa \lambda_\pi \pi_t + \lambda_x \triangle x_t) \right],$$

where z_1, z_2, z_3, and z_4 are parameters, satisfying $z_1 + z_2 = 1 + \beta^{-1} + \kappa\sigma\beta^{-1}$, $z_1 z_2 = \beta^{-1}$ ($z_1 > 1$,

$0 < z_2 < 1)$, $z_3 = -\frac{\beta\lambda_2\sigma}{\lambda_3}$, $z_4 z_5 = \frac{1}{z_3}(\frac{\sigma}{\lambda_3} - \frac{\xi}{\kappa})$, $z_4 + z_5 = -\frac{1}{z_3}(\frac{\xi}{\beta\kappa} - \frac{\sigma\lambda_1}{\beta\lambda_3})$, and $z_6 = z_4^{-1}$. We can confirm that the central bank has the incentive to pay attention to the first order difference in loan rates, as well as the standard concerns of the output gap and the inflation rate. This property is induced by the staggered loan contracts. Moreover, as shown in Subsection 3.2, the first order difference of the loan interest rates can be replaced by the first order difference of the policy interest rates, so the central bank has the incentive to smooth changes in the policy rates. There are both forward-looking and backward-looking terms in the optimal policy. Thus, not only does the optimal rule imply history dependence, but it also has the pre-emptive property (precautionary property). This pre-emptive property comes from the inertia in the loan rate curve. In the case of flexible loan contracts, i.e. $\varphi = 0$, λ_R is zero, and so the optimal monetary policy rule reduces to

$$-\kappa^{-1}\xi\lambda_x(1 - z_1 L)(1 - z_2 L)(x_t - x^*) = E_t\left[(1 - z_3 L)(1 - z_4 F)(\kappa\lambda_\pi\pi_t + \lambda_x\triangle x_t)\right]. \tag{5.6}$$

Under flexible loan rate contracts, the central bank does not have incentive to pay attention to the loan interest rates. Thus, we can see how the staggered loan contracts changes the behavior of the central bank. In a model in which no part of the labor cost must be paid through a loan, i.e. $\gamma = 0$, the optimal monetary policy rule reduces to the optimal one in a standard New Keynesian model[28] as

$$\kappa\lambda_\pi\pi_t + \lambda_x\triangle x_t = 0.$$

This equation implies that the central bank does not have the incentive to minimize the change in the interest rate.

[28] See Footnote 19.

5.2 Impulse Response under Optimal Monetary Policy

We use the parameter values listed in Table 1. As in Section 2, we assume three types of shocks: natural rate of interest shocks, price mark up shocks, and loan rate shocks.

Here, we compare the impulse responses from two different monetary policies: the optimal monetary policy given by Eq. (5.5) and the non-optimal monetary policy given by policy Eq. (5.6). Thus, under the rule of Eq. (5.6), a central bank does not care about the staggered loan interest rate contracts[29], i.e. $\lambda_R = 0$, and so does not have an incentive to smooth loan interest rates and policy rates in the welfare function. On the other hand, under the rule of Eq. (5.5), the central bank has an incentive to smooth loan interest rates and policy rates as explained in Subsection 3.2.

5.2.1 Natural Rate of Interest Shock

We assume that there is an unexpected 1% positive natural rate of interest shock with shock persistence 0.9 in an AR(1) process. Figure 8 shows the simulation results. In the case of the optimal monetary policy, the output gap responds positively to the natural rate of interest shock, then to stabilize inflation, and the central bank sufficiently increases policy interest rates. In the case of the non-optimal monetary policy, however, the output gap responds negatively to the shock, though the shock itself has a positive effect on the output gap in IS curve thanks to a sufficient increase in policy rates. Given the higher policy rates, loan rates also increase. Thus, the two rules proscribe different responses. Specifically, as discussed in Subsection 3.2, both the loan interest rate changes and the policy interest rate changes are smaller under the optimal monetary policy rule than under the non-optimal rule[30]. The simulation results therefore support the discussion

[29] The central bank misunderstands that there is no staggered loan interest rate contract and conducts the optimal monetary policy under such a misunderstanding.

[30] In this case, the differences in interest rates are not so large, but we can make larger differences by assuming bigger ϵ or φ. For example, Altig, Christiano, Eichenbaum and Linde (2002) assumes $\epsilon = 21$. Moreover, in a more realistic case in which the indexational loan interest rate change is assumed, we can find clearer differences in interest rates.

in Subsection 3.2.

5.2.2 Price Mark Up Shock

We assume that there is an unexpected 1% positive price mark up shock with shock persistence 0.9 in an AR(1) process. Figure 9 shows the simulation results. The inflation rates increase because of the shock, but the policy rates and loan rates decrease. The reason why policy interest rates fall is that the real interest rate is still positive[31], which implies that the output gap is negative. The biggest differences between the two monetary policy rules occurs in the responses of the policy rates and the loan interest rate smoothing.

5.2.3 Loan Rate Shock

We assume that there is an unexpected 1% positive loan interest rate shock with shock persistence 0.9 in an AR(1) process. Figure 10 shows the simulation results. The shock to the loan interest rate raises the inflation rate due to the increase in production costs. Then, the output gap responds to the shock in different directions in the first few periods according to real interest rates under different monetary policy rules. The fluctuations in the policy rate are larger in the case of the optimal monetary policy than those in the case of the non-optimal monetary policy as suggested in Subsection 4.2. Thus, we can confirm that the central bank has to aggresively react to the loan rate shocks to smooth the loan interest rates.

6 Model with Indexational Loan Rate Change

In the baseline model, we assumed that each working group of the private bank can reset loan rates with probability $1 - \varphi$ and that the working groups not selected to change loan rates keep the same loan rates as the previous period. However, since many commitment line contracts link the loan interest rates to a representative interest rate index, it is more natural to assume that

[31] By assuming slightly smaller κ or slightly larger ω_p, the policy interest rate positively responds to the price mark up shock.

the private bank ties its loan rates to the movements of aggregate loan rates, even though they do not actively re-evaluate the risks of projects[32]. Thus, in this section, we assume that each working group of the bank optimally resets its loan rates with probability $1 - \varphi$ and with probability φ automatically revises its rate given the change in the index value. Thus, there is a constant margin of difference between a private bank's rate (when it is not allowed to reset its rate) and the market interest rate index. When working groups of the private bank optimally change their loan interest rates, they change the size of that constant margin. This adjustment scheme is standard in a commitment line contract, as reported in Shockley and Thakor (1997). However, in the sense that the bank does not optimally reset its offering loan interest rates but merely follows the index movements, we maintain our perspective that the loan interest rate contracts between the firm and the private bank are still staggered.

Specifically, we assume that the private bank adjusts its loan interest rates according to one lag of changes in aggregate loan rates. Thus, we assume that publicly known interest rate indices, such as Prime Rate, LIBOR, Fed Fund Rate, and Treasury Bond Rate, in a financial market are determined by financial transactions in the previous period. This assumption is quite natural because publicly known market interest rates can be determined only after some actual transactions among agents have been done. Moreover, if all private banks can optimally reset their loan interest rates in every period just refering to the changes of aggregate loan interest rates from last period, who wants to reset the loan interest rate by paying costs of re-evaluating risks of firm's project instead of indexation?

6.1 Augmented Model

With probability φ, the working group h is not chosen to optimally reset its loan rate. However, these working groups, using an index as their guidelines, do follow a systematic rule in revising

[32] Woodford (Ch. 3, 2003a) introduces an indexational price change on firm's price setting behavior and derives the Hybrid-Phillips curve.

their rates. This rule is written below (see the details of derivation in Appendix C).

$$\ln(1+r_t(h)) = \ln(1+r_{t-1}(h)) + \psi(\ln(1+R_{t-1}) - \ln(1+R_{t-2})). \tag{6.1}$$

where ψ $(0 \leq \psi \leq 1)$ is a measure of the degree of indexation[33] to the most available information on aggregate loan rates[34]. In this case, the optimization problem of the working group h is given by

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} \left[q_{t,T}(h)((1+r_t(h)) \left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1 - i_T) \right].$$

where the working group h sets $r_t(h)$ to maximize the discounted value of profit. We have defined $q_{t,T}(h) = \left[\frac{(1+\gamma((1+r_t(h))\left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1))^{-\epsilon}(w_T(h))^{1-\epsilon}}{\Omega_T^*} \right] Q_T$ and $Q_T \equiv \int_0^1 q_{t,T}(h)dh$. Then, the optimal loan rate, $r_t(h)$, is endogenously determined in the equation

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} \left[1 - \epsilon\gamma \frac{(1+r_t(h))\left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1 - i_T}{1+\gamma((1+r_t(h))\left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1)} \right] = 0. \tag{6.2}$$

Just as before, all working groups that are allowed to reset their loan rates will set the same loan rate, so the solution of $r_t(h)$ in Eq. (6.2) is expressed by r_t^*. This yields the following law of motion for the aggregate loan rate index R

$$1+R_t = \varphi(1+R_{t-1})\left[\frac{1+R_{t-1}}{1+R_{t-2}}\right]^{\psi} + (1-\varphi)(1+r_t^*). \tag{6.3}$$

By log-linearizing Eq. (6.2) and Eq. (6.3) and manipulating the result, we have a following relation

[33] The loan interest rate contract is still staggered evwn when $\psi = 1$ thanks to the lag indexation.

[34] To simplify the derivation, we assume Eq. (6.1). But we can assume another rule of indexational change as

$$\ln r_t(h) = \ln r_{t-1}(h) + \psi(\ln R_{t-1} - \ln R_{t-2}).$$

$$\widehat{R}_t = \lambda_1^* E_t \widehat{R}_{t+1} + \lambda_2^* \widehat{R}_{t-1} + \lambda_3^* \widehat{R}_{t-2} + \lambda_4^* \widehat{i}_t. \tag{6.4}$$

Thus, under indexational changes of loan rates according to changes of aggregate loan rates, we have introduced a new term, $\widehat{R}_{t-2}$, into the loan rate curve given by Eq. (2.18). Additionally, the sensitivities of the variables are different from ones in Eq. (2.18). When $\psi = 0$, the augmented loan rate curve reduces to the original loan rate curve[35].

6.2 Impulse Response under Taylor Rule

In this model, the augmented loan rate curve is given by the fourth order difference equation of loan rates. Thus, we can presume that the new loan rate curve implies that economic fluctuations will last longer and so makes the economy more unstable.

We use the parameter values listed in Table 1. In this case, the loan rate curve is given by

$$\widehat{R}_t = \lambda_1^* E_t \widehat{R}_{t+1} + \lambda_2^* \widehat{R}_{t-1} + \lambda_3^* \widehat{R}_{t-2} + \lambda_4^* \widehat{i}_t + \Upsilon_t.$$

As in the previous sections, we set the policy parameters in Taylor rule to $\mu_\pi = 2$ and $\mu_x = 0.25$

[35] We may think a different way of indexational loan rate change. In the case in which each working group of the bank indexationally reset loan rate according to the change of policy rate (deposit rate) i_t with probability φ, the working group h that is not selected to optimally change the loan rate changes the loan rate in a following way.

$$\ln(1 + r_t(h)) = \ln(1 + r_{t-1}(h)) + \psi(\ln(1 + i_{t-1}) - \ln(1 + i_{t-2})).$$

By taking a similar procedure as shown above, we have another augmented loan rate curve.

$$\widehat{R}_t = \lambda_1 E_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t + \lambda_4 \triangle \widehat{i}_t + \lambda_5 \triangle \widehat{i}_{t-1},$$

where $\lambda_4 \equiv -\frac{\varphi\beta}{1+\varphi^2\beta}\psi$, and $\lambda_5 \equiv \frac{\varphi}{1+\varphi^2\beta}\psi$. Thus, under indexational change of loan rate according to policy rate change, two adjustment terms, $\triangle \widehat{i}_t$ and $\triangle \widehat{i}_{t-1}$, are introduced into the loan rate curve given by Eq. (2.18). Again, when $\psi = 0$, the augmented loan rate curve turns back to the original loan rate curve.

Moreover, if we assume another indexation of

$$\ln(1 + r_t(h)) = \ln(1 + r_{t-1}(h)) + \psi(\ln(1 + i_t) - \ln(1 + i_{t-1})),$$

then we have a different loan rate curve.

$$\widehat{R}_t = \lambda_1 E_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t + \lambda_4 E_t \triangle \widehat{i}_{t+1} + \lambda_5 \triangle \widehat{i}_t.$$

In this case, we may assume that the policy interest rate is available for any agents without any cost.

and assume $\varphi = 0.66$ (so that the loan contract rigidity equals the price rigidity) and $\gamma = 1$ (so that the entire labor cost must be financed by a loan). We show the results of the simulations when $\psi = 0$ (no indexational loan rate change) and $\psi = 0.05$[36].

6.2.1 Loan Rate Shock

We assume that there is an unexpected 1% positive loan rate shock with shock persistence 0.9 in an AR(1) process. Figure 11 shows the simulation results. As discussed in the last section, the shock to the loan rate increases the inflation rate due to rises in the production costs, which then raises policy interest rates. In turn, a high policy interest rate induces a negative output gap. This response is more clear in this indexed environment compared to one without indexation.

6.2.2 Natural Rate of Interest Shock

We assume that there is an unexpected 1% positive natural rate of interest shock with shock persistence 0.9 in an AR(1) process. Figure 12 shows the simulation results. As discussed in the last section, each simulation outcome shows that both the inflation rate and the output gap increase with the shock, which implies that the policy interest rate and the loan rate also rise. Specifically, we can see that, except for the output gap, all impulse responses with the indexational loan rate are larger and longer than those without this indexation. For the output gap, we can see that the responses are quite different between with and without indexation.

6.2.3 Price Mark Up Shock

We assume that there is an unexpected 1% positive price mark up shock with shock persistence 0.9 in an AR(1) process. Figure 13 shows the simulation results. Each simulation outcome shows that the inflation rate goes up with the shock, which induces an increase in both the policy interest rate and the loan rate. In turn, these changes induce a negative output gap. Thus, the biggest

[36] Here, we are not sure a proper value of ψ, but at least we should set larger values of ψ from the actual economic situation. For larger ψ, however, the impulse responses become more fluctuated under the assumed Taylor rule and it is difficult to keep determiacy.

difference between the impulse responses in a model with indexed loan rate contracts compared to one without, is that the indexed loan rate contracts induce longer and larger fluctuations in the key variables.

6.3 Analysis on Welfare Function

Under the augmented loan rate curve, the working group h that is not selected to optimally change its loan rates resets loan rates in the following way

$$1 + r_t(h) = (1 + r_{t-1}(h)) \left[\frac{1 + R_{t-1}}{1 + R_{t-2}}\right]^{\psi}.$$

By log-linearizing this equation and combining it with Eq. (4.15), Eq. (4.16), and Eq. (4.17), Eq. (4.18) then (see the details of derivation in Appendix D)

$$\triangle_t^R = \varphi \triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\triangle \widehat{R}_t - \psi \triangle \widehat{R}_{t-1})^2.$$

In this case, we have the following approximation to the microfounded-welfare function.

$$U_t = \lambda_\pi \pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_R (\triangle \widehat{R}_t - \psi \triangle \widehat{R}_{t-1})^2. \quad (6.5)$$

Looking at this approximation, we see that a new term, $\psi \triangle \widehat{R}_{t-1}$, has been added to the original welfare function given by Eq. (4.20). This implies that a central bank has a stronger incentive to smooth policy interest rates.

6.4 Optimal Monetary Policy Analysis

6.4.1 Optimal Monetary Policy

Given the approximation to the welfare function, we now solve for the optimal monetary policy when the central bank is credibly committed to a policy rule in the timeless perspective. The objective of monetary policy is to minimize the expected loss function given by Eq. (4.1) and Eq.

(6.5) under the standard New Keynesian IS curve given Eq. (2.9), the augmented Phillips curve given by Eq. (2.14), and the augmented loan rate curve given by Eq. (6.4). In this case, the optimal conditions given by Eq. (5.3) and Eq. (5.4) are replaced with the following equations

$$\lambda_R n_t - \beta(1+\psi)\lambda_R E_t n_{t+1} + \psi\beta^2\lambda_R E_t n_{t+2} + \xi\Xi_{2t} - \Xi_{3t} + \beta^{-1}\lambda_1^*\Xi_{3t-1} + \beta\lambda_2^* E_t\Xi_{3t+1} + \beta^2\lambda_3^* E_t\Xi_{3t+2} = 0, \tag{6.6}$$

$$\lambda_4^*\Xi_{3t} - \sigma\Xi_{1t} = 0, \tag{6.7}$$

where $n_t \equiv \triangle\widehat{R}_t - \psi\triangle\widehat{R}_{t-1}$. Then, the four optimal conditions given by Eq. (5.1), Eq. (5.2), Eq. (6.6), and Eq. (6.7), the IS curve, the Phillips curve, and the augmented loan rate curve are the conditions that govern the loss minimization for $t \geq 0$. In order to understand the properties of the optimal monetary policy, we can simplify the conditions to

$$(1 - z_1^* L)(1 - z_2^* L)\left[\lambda_R(n_t - \beta(1+\psi)E_t n_{t+1} + \beta^2\psi E_t n_{t+2}) - \kappa^{-1}\xi\lambda_x(x_t - x^*)\right]$$

$$= E_t\left[z_3^* z_7^* z_8^*(1 - z_6^* L)(1 - z_7^* F)(1 - z_8^* F)(\kappa\lambda_\pi\pi_t + \lambda_x\triangle x_t)\right],$$

where z_1, z_2, z_3, and z_4 are parameters satisfying $z_1^* + z_2^* = 1 + \beta^{-1} + \kappa\sigma\beta^{-1}$, $z_1^* z_2^* = \beta^{-1}$ ($z_1 > 1$, $0 < z_2 < 1$), $z_3^* = -\frac{\beta^2\lambda_3^*\sigma}{\lambda_4^*}$, $z_4^* z_5^* z_6^* = -\frac{1}{z_3^*}(\frac{\xi}{\kappa\beta} - \frac{\sigma\lambda_1^*}{\beta\lambda_4^*})$, $z_4^* + z_5^* + z_6^* = \frac{1}{z_3^*}\frac{\sigma\beta\lambda_2^*}{\lambda_4^*}$, $z_4^* z_5^* + z_5^* z_6^* + z_4^* z_6^* = \frac{1}{z_3^*}(\frac{\sigma}{\lambda_4^*} - \frac{\xi}{\kappa})$, $z_7^* = (z_4^*)^{-1}$, and $z_8^* = (z_5^*)^{-1}$. This equation helps us confirm that the central bank has an increased incentive to smooth the policy interest rate. This additional incentive to stabilize the loan rate is induced by the indexational loan rate changes. Moreover, as shown in Subsection 4.2, the first order difference in the loan rates can be substituted by the first order difference in policy rates, which means that the central bank has more incentive to smooth policy rate changes. The history dependent and pre-emptive properties on the policy rule are stronger in this case. When we do not have an indexational loan rate changes, i.e. $\psi = 0$, this augmented optimal monetary policy is reduced to one given by Eq. (5.5).

6.4.2 Impulse Response under Optimal Monetary Policy

Again, we use the parameter values listed in Table 1. As shown in the last section, we assume three types of shocks: natural rate of interest shocks, price mark up shocks, and loan rate shocks and assume two different monetary policy rules, the optimal monetary policy given by Eq. (5.5) and the non-optimal monetary policy given by Eq. (5.6). Here we assume that $\psi = 0.1$, which is slightly bigger than one in the last subsection.

Natural Rate of Interest Shock We assume that there is an unexpected 1% positive natural rate of interest shock with shock persistence 0.9 in an AR(1) process. Figure 14 shows the simulation results. In the case of the optimal monetary policy, the output gap positively responds to the shock, then to stabilize inflation, the central bank increases its policy rate. In the case of the non-optimal monetary policy, however, the output gap falls with the shock, even though the shock itself has a positive effect on the output gap through the IS curve, because of the increase in the policy rate. Since the policy rate increases, the loan rate also rises. There are key differences between the two cases. As discussed in Subsection 3.2, both the loan interest rate changes and the policy interest rate changes are smaller under the optimal monetary policy rule than those under the non-optimal monetary policy rule.

Price Mark Up Shock We assume that there is an unexpected 1% positive price mark up shock with shock persistence 0.9 in an AR(1) process. Figure 15 shows the simulation results. With the shock, the inflation rate increases, but the policy interest rate and the loan rates fall. This occurs because the real interest rate remains positive, and so the output gap is negative. The main differences between the two monetary policy rules can be seen in the policy interest rate and the loan interest rate smoothing. Both the loan interest rate changes and the policy interest rate changes are smaller under the optimal monetary policy rule than those under the non-optimal monetary policy rule.

Loan Rate Shock We assume that there is an unexpected 1% positive loan interest rate shock with shock persistence 0.9 in an AR(1) process. Figure 16 shows the simulation results. The shock to the loan rate increases inflation because of the rise in the production cost. Then, the output gap response to the shock depends upon the real interest rate. Again, we can confirm that the central bank has to aggresively change the policy rates to smooth the loan interest rates to the loan rate shocks as suggested by Subsection 4.2.

7 Concluding Remarks

In this paper, we introduce staggered nominal loan interest rate contracts between a private bank and a firm into the standard New Keynesian model in a tractable way. Simulation results of the model show that the staggered loan contracts effectively increase the amplitude and the persistence of economic fluctuations. This means that previous papers that do not include this financial friction may fail to introduce an important source of economic stickiness that cannot be captured through price stickiness, wage stickiness, adjustment cost of investment, and necessary time to build capital.

The question as to what the central bank should seek to accomplish is a primary purpose of this paper. We show that a second order approximation to the consumer's welfare function includes a first order difference term in the loan interest rate. This is a novel contribution of this paper. This property implies that the central bank wants to smooth the policy interest rate over time to the shocks from real economy such as price and demand shocks. In reality, the central bank tends to adjust the policy rate through a series of small adjustments in the same direction, and it is the staggered property of the loan rate contracts that implies that such small adjustments are optimal theoretically. However, at the same time, a central bank has to quickly change the policy interest rates to smooth the loan interest rates against financial shocks such as a loan rate shock. Thus, a central bank should change its responses to economic disturbances according to types of shocks. These findings may provide some explanations to quick responses by the Federal Reserve Board to the sub-prime mortgage problem, which can be interpreted as a shock in financial market, occured after fall of 2007 in US.

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Appendix

A Baseline Model

Except x_t and π_t, log-linearized version of variable m_t is expressed by $\widehat{m}_t = \ln(m_t/\overline{m})$, where $\overline{m}$ is steady state value of m_t.

A.1 Consumer

A cost minimization problem of consumer on differentiated consumption bundle is given by

$$\min_{c_t(f)} \int_0^1 c_t(f) p_t(f) df,$$

subject to

$$C_t \equiv \left[\int_0^1 c_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}.$$

By defining a following consumption-based price index

$$P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}},$$

we can derive a relative expenditure on (demand for) differentiated goods as follows

$$c_t(f) = C_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta}.$$

Then the consumer maximizes the objective function

$$U_t = E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T, \nu_T) - \int_0^1 V(l_T(h), \nu_T) dh \right] \right\},$$

subject to the budget constraint

$$P_tC_t + E_t\left[X_{t,t+1}B_{t+1}\right] + D_t \leq B_t + (1+i_{t-1})D_{t-1} + \int_0^1 w_t(h)l_t(h)dh + \int_0^1 \Pi_t^B(h)dh + \int_0^1 \Pi_t^F(f)df.$$

The consumer chooses C_t, B_{t+1}, D_t, and $w_t(h)$ in every period under given optimal allocation of differentiated goods, then we have following relations:

$$\frac{U_C(C_t,\nu_t)}{U_C(C_{t+1},\nu_{t+1})} = \frac{\beta}{X_{t,t+1}}\frac{P_t}{P_{t+1}}, \tag{A.1}$$

$$U_C(C_t,\nu_t) = \beta(1+i_t)E_t\left[U_C(C_{t+1},\nu_{t+1})\frac{P_t}{P_{t+1}}\right], \tag{A.2}$$

$$\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon-1}\frac{V_l(l_t(h),\nu_t)}{U_C(C_t,\nu_t)}.$$

Under assumption of Eq. (2.7), we can find that the conditions given by Eq. (A.1) and the one given by Eq. (A.2) are same. Thus we use the relation given by Eq. (A.2). Before log-linearization, under equilibrium $C_t = Y_t$ for any t, we interpret Eq. (A.2) as

$$U_Y(Y_t - g_t) = \beta(1+i_t)E_t\left[U_Y(Y_{t+1}-g_{t+1})\frac{P_t}{P_{t+1}}\right], \tag{A.3}$$

where g_t expresses the disturbance ν_t. Under the definitions of $\pi_t \equiv \ln P_t/P_{t-1}$ and $\widehat{i}_t \equiv \ln(1+i_t)/(1+\bar{i})$, we log-linearize Eq. (A.3), then we have

$$x_t = E_tx_{t+1} - \sigma(\widehat{i}_t - E_t\pi_{t+1} - \widehat{r}_t^n),$$

where $\widehat{r}_t^n \equiv \sigma^{-1}\left[(g_t - \widehat{Y}_t^n) - E_t(g_{t+1} - \widehat{Y}_{t+1}^n)\right]$ and $\sigma \equiv -\frac{U_Y}{U_{YY}\overline{Y}} > 0$ (See definitions of x_t and $\widehat{Y}_t^n$ in the next subsection.).

A.2 Firm

As explained, the demand function of loans by a firm is given by

$$q_t(h) = \left[\frac{(1+\gamma r_t(h))^{-\epsilon}(w_t(h))^{1-\epsilon}}{\Omega_t^*} \right] Q_t. \tag{A.4}$$

Under given optimal allocation of loans by a president, the h project manager uses loan to finance a part of wage, re-sets its price, $p_t(h)$, with probability $1-\alpha$ to maximize present discounted value of profit given by

$$E_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[p_t(f) y_{t,T}(f) - \int_0^1 (1+\gamma r_T(h)) w_T(h) l_T^f(h) dh \right],$$

$$\Longrightarrow E_t \sum_{T=t}^{\infty} \alpha^{T-t} X_{t,T} \left[p_t(f) \left[\frac{p_t(f)}{P_T} \right]^{-\theta} Y_T - \Omega_T L_T(f) \right],$$

where we use the outcome from the cost minimization problem and use the demand function on differentiated goods $y_{t,T}(f) \equiv Y_T \left[\frac{p_t(f)}{P_T} \right]^{-\theta}$ from Eq. (2.1) under $c_t(f) = y_t(f)$ and use $C_t = Y_t$ for any t. Here we use consumer's (shareholder's) marginal rate of substitution, $X_{t,t+1}$, as given discount rate for each firm's project group. For specifying the derivation, we put f on $l_t(h)$ and L_t. In this case, in the relation of L_t, we may have

$$L_t = \int_0^1 L_t(f) df.$$

It notes that the price setting of firm's project group is independent from the loan rate setting of bank's working group. Then, we can transform the present discounted value of profit as

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} \left[p_t(f) \left[\frac{p_t(f)}{P_T} \right]^{-\theta} Y_T - \Omega_T L_T(f) \right].$$

We can find the optimal price setting, $p_t^*(f)$, in a following first-order condition:

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{U_Y(Y_T, \nu_T)}{P_T} \left[(1-\theta) y_{t,T}(f) - \left[\int_0^1 ((1+\gamma r_T(h)) w_T(h))^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}} \frac{\partial L_T(f)}{\partial p_t(f)} \right] = 0.$$

$$\begin{aligned} \Longrightarrow \quad & E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \left[\frac{\theta - 1}{\theta} \frac{p_t^*(f)}{P_T} \right] \\ = \quad & E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \frac{\epsilon}{\epsilon - 1} \left[\int_0^1 (1+\gamma r_T(h))^{1-\epsilon} \left(\frac{V_l(l_T(h), \nu_T)}{U_Y(Y_T, \nu_T)} \frac{\partial L_T(f)}{\partial y_{t,T}(f)} \right)^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}}, \end{aligned} \tag{A.5}$$

due to Eq. (2.10). Here we assume that the firm's production functions is given by $y_t(f) = A_t f(L_t(f))$, where $f(\cdot)$ is an increasing and concave function. Then we can transform Eq. (A.5) again as

$$\begin{aligned} & E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \left[\frac{\theta - 1}{\theta} \frac{p_t^*(f)}{P_t} \frac{P_t}{P_{t+1}} \frac{P_{t+1}}{P_{t+2}} \cdots \frac{P_{T-1}}{P_T} \right] \qquad \text{(A.6)} \\ = \quad & E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_C(C_T, \nu_T) y_{t,T}(f) \frac{\epsilon}{\epsilon - 1} \left[\int_0^1 (1+\gamma r_T(h))^{1-\epsilon} (mc_{t,T}(h,f))^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}}, \end{aligned}$$

where we define real marginal cost as $mc_{t,T}(h,f) \equiv \frac{V_l(l_{t,T}(h), \nu_t)}{U_Y(Y_t, \nu_t)} \frac{\partial L_{t,T}(f)}{\partial y_t(f)}$. By log-linearizing Eq. (A.6), we have a following equation

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\widehat{\widetilde{p}}_t^*(f) - \sum_{\tau=t+1}^{T} \pi_\tau - \Theta \widehat{R}_T - \widehat{mc}_T + \omega_p \theta (\widehat{\widetilde{p}}_t^*(f) - \sum_{\tau=t+1}^{T} \pi_\tau) \right] = 0, \tag{A.7}$$

where we define $1+R_t \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1+r_t(h))dh$, $\widehat{r}_t(h) \equiv \ln(1+r_t(h))/(1+\overline{r})$, and $\widehat{R}_t \equiv \ln(1+R_t)/(1+\overline{r})$, and so we have $\widehat{R}_t \equiv \int_0^1 \widehat{r}_t(h)dh$ and $\Theta \equiv \frac{\gamma(1+\overline{r})}{1+\gamma\overline{r}}$. Also, we define $\widehat{mc}_t(f) \equiv \int_0^1 \widehat{mc}_t(h,f)dh$, $\widehat{mc}_t(h,f) \equiv \ln(mc_t(h,f)/\overline{mc})$, $\widetilde{p}_t^*(f) \equiv \frac{p_t^*(f)}{Pt}$, and $\widehat{\widetilde{p}}_t^*(f) \equiv \ln(\widetilde{p}_t^*(f)/\overline{\widetilde{p}}_t^*)$. It notes that log-linearized average real marginal cost is given by $mc_t(h) \equiv \frac{V_l(l_t(h), \nu_t)}{U_Y(Y_t, \nu_t)} \frac{\partial L_t}{\partial Y_t}$ and $\widehat{mc}_t \equiv \int_0^1 \widehat{mc}_t(h)dh$, and we make use of the relation of $\widehat{mc}_{t,T}(f) = \widehat{mc}_T - \omega_p \theta (\widehat{\widetilde{p}}_t(f) - \sum_{\tau=t+1}^{T} \pi_\tau)$, where ω_p is the elasticity of $\frac{\partial L_t(f)}{\partial y_t(f)}$ with respect to y. By transforming Eq. (A.7), we have

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t^*(f) = E_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\omega_p\theta)^{-1}(\widehat{mc}_T + \Theta\widehat{R}_T) + \sum_{\tau=t+1}^{T}\pi_\tau\right]. \tag{A.8}$$

Thus, all project groups which change prices at time t set the same price. Then, by taking average of f, Eq. (A.8) can be transformed to

$$\frac{1}{1-\alpha\beta}\widehat{\widetilde{p}}_t^* = E_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\omega_p\theta)^{-1}(\widehat{mc}_T + \Theta\widehat{R}_T) + \sum_{\tau=t+1}^{T}\pi_\tau\right], \tag{A.9}$$

where $(p_t^*)^{1-\theta} \equiv \int_0^1 p_t^*(f)^{1-\theta}df$, and so $\widehat{\widetilde{p}}_t^* = \int_0^1 \widehat{\widetilde{p}}_t^*(f)df$. In the Calvo (1983) - Yun (1992) setting, the evolution of aggregate price index P is described by the following motion

$$\int_0^1 p_t(f)^{1-\theta}df = \alpha\int_0^1 p_{t-1}(f)^{1-\theta}df + (1-\alpha)\int_0^1 p_t^*(f)^{1-\theta}df,$$

$$\Longrightarrow P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(p_t^*)^{1-\theta}. \tag{A.10}$$

By log-linearizing Eq. (A.10), we have

$$\widehat{\widetilde{p}}_t^* = \frac{\alpha}{1-\alpha}\pi_t. \tag{A.11}$$

After substituting Eq. (A.11) into Eq. (A.9), we have a following relation

$$\frac{\alpha}{1-\alpha}\pi_t = (1-\alpha\beta)E_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[(1+\omega_p\theta)^{-1}(\widehat{mc}_T + \Theta\widehat{R}_T) + \sum_{\tau=t+1}^{T}\pi_\tau)\right]. \tag{A.12}$$

Then, by considering of $\frac{\alpha}{1-\alpha}\pi_t - \alpha\beta E_t\frac{\alpha}{1-\alpha}\pi_{t+1}$ in Eq. (A.12), we finally have the augmented Phillips curve.

$$\pi_t = \chi(\widehat{mc}_t + \Theta\widehat{R}_t) + \beta E_t\pi_{t+1},$$

where $\chi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega_p\theta)}$.

On the other hand, according to the discussion in Woodford (2003a), we define the natural rate of output Y_T^n from Eq. (A.5) as

$$\frac{\theta-1}{\theta}-(1+\gamma\overline{R})\frac{\epsilon}{\epsilon-1}\left[\int_0^1(\frac{V_l(l_t^n(h),\nu_t)}{U_Y(Y_t^n,\nu_t)})^{1-\epsilon}dh\right]^{\frac{1}{1-\epsilon}}\frac{\partial f^{-1}(Y_t^n/A_t)}{\partial Y_t^n}=0, \qquad \text{(A.13)}$$

where, under the natural rate of output, we assume a flexible price setting, $p_t^*(f)=P_t$, and assume no impact of monetary policy, $\widehat{r}_t(h)=\overline{R}$, and so hold $y_t(f)=Y_t^n$. Also, $l_t^n(h)$ is the amount of labor type h employed under Y_t^n. Thus shocks induced by ν_t is absorbed by the natural rate of output. The definition of the natural rate of output is slightly different from one defined in Friedman(1968)[37] and Woodford (2003a) in terms of treatment of loan rates in Eq. (A.13). We assume that the natural rate of output is independent from monetary policy[38]. Then, we have

$$\widehat{mc}_t=(\omega+\sigma^{-1})(\widehat{Y}_t-\widehat{Y}_t^n),$$

where $\widehat{Y}_t\equiv\ln(Y_t/\overline{Y})$, and $\widehat{Y}_t^n\equiv\ln(Y_t^n/\overline{Y})$, and $\omega\equiv\omega_p+\omega_w$[39]. Here ω_w is the elasticity of marginal disutility of work with respect to output increase in $\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)}$. Then, by defining $x_t\equiv\widehat{Y}_t-\widehat{Y}_t^n$, we finally have

$$\pi_t=\kappa x_t+\xi\widehat{R}_t+\beta E_t\pi_{t+1},$$

where $\kappa\equiv\chi(\omega+\sigma^{-1})$ and $\xi\equiv\chi\Theta$.

A.3 Private Bank

Then under given demand function of loan set by Eq. (A.4), each working group of private bank re-sets its loan rates, $r_t(h)$, with probability $1-\varphi$ to maximize present discounted value of profit given by

[37] Friedman, M. "The role of monetary policy." *American Economic Review*, Vol. 58, 1968, pp1–17.

[38] This is because I assume that the monetary policy can work for short run events and can not work for long run events, such as change of productivity, technological growth, and transition of core parameters of economy.

[39] We can see more detailed derivation in Woodford (Ch. 3, 2003).

$$E_t \sum_{T=t}^{\infty} \varphi^{T-t} X_{t,T} \left[(r_t(h) - i_T) q_{t,T}(h) - z_T(h) \right], \tag{A.14}$$

where we define $q_{t,T}(h) = \left[\frac{(1+\gamma r_t(h))^{-\epsilon}(w_T(h))^{1-\epsilon}}{\Omega_T^*} \right] Q_T$ from Eq. (2.5) and $Q_T \equiv \int_0^1 q_{t,T}(h)dh$, i_T is deposit rates which is set by a central bank and is same for all working groups, and $z_T(h)$ is a cost of bank's working group h to handle finance to firm's project group h. We assume $z_T(h)$ is zero, $z(h) = 0$ and $D_T = Q_T$ in equilibrium. Then, we can transform Eq. (A.14) as

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} \left[q_{t,T}(h)(r_t(h) - i_T) \right].$$

Then an optimal loan rate setting of $r_t(h)$ under the situation in which managers can re-set their loan rates with probability $1 - \varphi$ is given by

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[1 - \epsilon\gamma \frac{r_t(h) - i_T}{1 + \gamma r_t(h)} \right] = 0. \tag{A.15}$$

By log-linearizing Eq. (A.15), we have a following equation

$$\frac{\gamma(1-\epsilon)(1+\bar{r})}{\beta\varphi - 1} \widehat{r}_t(h) = E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \left[\gamma\epsilon(1+\bar{i})\widehat{i}_T \right]. \tag{A.16}$$

Here working groups that are allowed to change their loan rates will set the same loan rate, so the solution of $r_t(h)$ in Eq. (A.15) is expressed by r_t^*, and so the solution of $\widehat{r}_t(h)$ in Eq. (A.16) is expressed by $\widehat{r}_t^*$. On the other hand, we have the following evolution of aggregate loan rate index R

$$1 + R_t = \varphi(1 + R_{t-1}) + (1 - \varphi)(1 + r_t^*). \tag{A.17}$$

By log-linearizing Eq. (A.17), we have

$$\widehat{r}_t^* = \frac{1}{1-\varphi} \widehat{R}_t - \frac{\varphi}{1-\varphi} \widehat{R}_{t-1}.$$

Then, by considering of $\widehat{r}_t^* - \varphi\beta E_t \widehat{r}_{t+1}^*$ in Eq. (A.16), we finally have a loan rate curve

$$\widehat{R}_t = \lambda_1 E_t \widehat{R}_{t+1} + \lambda_2 \widehat{R}_{t-1} + \lambda_3 \widehat{i}_t,$$

where $\lambda_1 \equiv \frac{\varphi\beta}{1+\varphi^2\beta}$, $\lambda_2 \equiv \frac{\varphi}{1+\varphi^2\beta}$, and $\lambda_3 \equiv \frac{1-\varphi\beta}{1+\bar{r}} \frac{\epsilon}{\epsilon-1} \frac{1-\varphi}{1+\varphi^2\beta}(1+\bar{i})$.

B Derivation of Approximated Welfare Function

In derivation of approximated welfare function, we basically follow the way of Woodford (2003a). Except x_t and π_t, log-linearized version of variable m_t is expressed by $\widehat{m}_t = \ln(m_t/\overline{m})$, where $\overline{m}$ is steady state value of m_t[40]. Under the situation in which goods supply matches goods demand in every level, $Y_t = C_t$ and $y_t(f) = c_t(f)$ for any f, the welfare criterion of consumer is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_t \right\},$$

where

$$U_t = U(Y_t, \nu_t) - \int_0^1 V(l_t(h), \nu_t) dh, \tag{B.1}$$

and

$$Y_t \equiv \left[\int_0^1 y_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}.$$

We log-linearize Eq. (B.1) step by step to derive an approximated welfare function. Firstly, we log-linearize the first term of Eq. (B.1).

[40] You can see Woodford (2003) about how to log-linearize a function.

$$
\begin{aligned}
U(Y_t;\nu_t) &= \overline{U} + U_c\widetilde{Y}_t + U_\nu\nu_t + \frac{1}{2}U_{cc}\widetilde{Y}_t^2 + U_{c\nu}\widetilde{Y}_t + \frac{1}{2}\nu_t' U_{\nu\nu}\nu_t + Order(\parallel \xi \parallel^3) \\
&= \overline{U} + \overline{Y}U_c(\widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2) + U_\nu\nu_t + \frac{1}{2}U_{cc}\overline{Y}^2\widehat{Y}_t^2 + \overline{Y}U_{c\nu}\nu_t\widehat{Y}_t + \frac{1}{2}\nu_t' U_{\nu\nu}\nu_t + Order(\parallel \xi \parallel^3) \\
&= \overline{Y}U_c\widehat{Y}_t + \frac{1}{2}\left[\overline{Y}U_c + \overline{Y}^2U_{cc}\right]\widehat{Y}_t^2 - \overline{Y}^2U_{cc}g_t\widehat{Y}_t + t.i.p + Order(\parallel \xi \parallel^3) \\
&= \overline{Y}U_c\left[\widehat{Y}_t + \frac{1}{2}(1-\sigma^{-1})\widehat{Y}_t^2 + \sigma^{-1}g_t\widehat{Y}_t\right] + t.i.p + Order(\parallel \xi \parallel^3), \qquad \text{(B.2)}
\end{aligned}
$$

where $\overline{U} \equiv U(\overline{Y};0)$, $\widetilde{Y}_t \equiv Y_t - \overline{Y}$, $t.i.p$ means the terms that are independent from monetary policy, $Order(\parallel \xi \parallel^3)$ expresses order terms higher than the second order approximation, $\sigma^{-1} \equiv -\frac{\overline{Y}U_{cc}}{U_c} > 0$, and $g_t \equiv -\frac{U_{c\nu}\nu_t}{\overline{Y}U_{cc}}$. To replace $\widetilde{Y}_t$ by $\widehat{Y}_t \equiv \ln(Y_t/\overline{Y})$, we use the Taylor series expansion on $Y_t/\overline{Y}$ in the second line as

$$
Y_t/\overline{Y} = 1 + \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + Order(\parallel \xi \parallel^3).
$$

Secondly, we log-linearize the second term of Eq. (B.1) by a similar way.

$$
\begin{aligned}
\int_0^1 V(l_t(h);\nu_t)dh &= V_l\overline{L}(E_h\widehat{l}_t(h) + \frac{1}{2}E_h(\widehat{l}_t(h))^2) + \frac{1}{2}V_{ll}\overline{L}^2E_h(\widehat{l}_t(h))^2 + V_{l\nu}\overline{L}\nu_tE_h\widehat{l}_t(h) + t.i.p + Order(\parallel \xi \parallel^3) \\
&= \overline{L}V_l\left[\widehat{L}_t + \frac{1}{2}(1+\nu)\widehat{L}_t^2 - \nu\widetilde{\nu}_t\widehat{L}_t + \frac{1}{2}(\nu + \frac{1}{\epsilon})var_h\widehat{l}_t(h)\right] + t.i.p + Order(\parallel \xi \parallel^3) \\
&= \phi_h\overline{L}V_l\left[\begin{array}{c} \widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t\widehat{Y}_t + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ +\frac{1}{2}\phi_h^{-1}(\nu + \frac{1}{\epsilon})var_h \ln l_t(h) \end{array}\right] \\
&\quad + t.i.p + Order(\parallel \xi \parallel^3) \\
&= \overline{Y}U_c\left[\begin{array}{c} (1-\Phi)\widehat{Y}_t + \frac{1}{2}(1+\omega)\widehat{Y}_t^2 - \omega q_t\widehat{Y}_t + \frac{1}{2}(1+\omega_p\theta)\theta var_f \ln p_t(f) \\ +\frac{1}{2}\phi_h^{-1}(\nu + \frac{1}{\epsilon})var_h\widehat{l}_t(h) \end{array}\right] \\
&\quad + t.i.p + Order(\parallel \xi \parallel^3), \qquad \text{(B.3)}
\end{aligned}
$$

where $\widetilde{\nu}_t \equiv -\frac{V_{l\nu}\nu_t}{\overline{L}V_{ll}}$, $\nu \equiv \frac{\overline{L}V_{ll}}{V_l}$, $\phi_h \equiv \frac{\overline{Y}}{\overline{L}f'}$, $\omega_p \equiv \frac{ff''}{(f')^2}$, $q_t \equiv (1+\omega^{-1})a_t + \omega^{-1}\nu\widetilde{\nu}_t$, $a_t \equiv \ln A_t$, $var_h\widehat{l}_t(h)$ is the variance of $\widehat{l}_t(h)$ across all types of labor, and $var_f\widehat{p}_t(f)$ is the variance of $\widehat{p}_t(f)$ across all differentiated good prices. Here the definition of labor aggregator is given by

$$L_t \equiv \left[\int_0^1 l_t(h)^{\frac{\epsilon-1}{\epsilon}} dh \right]^{\frac{\epsilon}{\epsilon-1}},$$

and so we have $\widehat{L}_t = E_h \widehat{l}_t(h) + \frac{1}{2}\frac{\epsilon-1}{\epsilon} var_h \widehat{l}_t(h) + Order(\| \xi \|^3)$ in the second order approximation. We use this relation in the second line. From the second line to the third line, we use the condition that the demand of labor is equal to the supply of labor as

$$L_t = \int_0^1 L_t(f)df = \int_0^1 f^{-1}(\frac{y_t(f)}{A_t})df,$$

where the production function is given by $y_t(f) = A_t f(L_t(f))$, where $f(\cdot)$ is an increasing and concave function. By taking the second order approximation, we have

$$\widehat{L}_t = \phi_h(\widehat{Y}_t - a_t) + \frac{1}{2}(1 + \omega_p - \phi_h)\phi_h(\widehat{Y}_t - a_t)^2 + \frac{1}{2}(1 + \omega_p \theta)\theta var_f \widehat{p}_t(f) + Order(\| \xi \|^3),$$

where we log-linearize the demand function on differentiated goods to derive the relation $var_f \ln y_t(f) = \theta^2 var_f \ln p_t(f)$, which can be derived from the consumer's cost minimization problem under Dixit-Stiglitz aggregator, as

$$y_t(f) = Y_t \left[\frac{p_t(f)}{P_t} \right]^{-\theta},$$

where the aggregate price index is given by $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}}$. Also, we use the relation of $\phi_h \nu = \omega_w$ and $\omega = \omega_p + \omega_w$, where ω_w is an elasticity of real wage under the flexible-wage labor supply with respect to aggregate output. To the forth line, we replace $\phi_h \overline{L} V_l$ by $(1 - \Phi)\overline{Y} U_c$. Here, we use the assumption that distortion of the output level Φ, induced by firm's price mark up through $\left[\int_0^1 (\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)} \frac{\partial L_t(f)}{\partial y_t(f)})^{1-\epsilon} dh \right]^{\frac{1}{1-\epsilon}}$, which would exist under flexible price and no role of monetary policy is of order one as in Woodford (2003a)[41]. Thus, in terms of the natural rate of

[41] We assume that the monetary policy has no impact on the level of the natural rate of output.

output, we actually assume that real marginal cost function of firm $m(\cdot)$ in order to supply a good f is given by

$$m(y_t(f), Y_t, r_t; \nu_t) = \frac{\epsilon}{\epsilon - 1}\left[\int_0^1 (1+\gamma r_t(h))^{1-\epsilon}(\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)})^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}} \frac{\partial L_t(f)}{\partial y_t(f)},$$

then the natural rate of output $Y_t^n = Y^n(\xi_t)$ is given by

$$m(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta - 1}{\theta} \equiv \frac{\epsilon}{\epsilon - 1}(1+\gamma\overline{R})(1-\Phi), \tag{B.4}$$

where a parameter Φ expresses the distortion of output level and is of order one[42].

Then we can combine Eq. (B.2) and Eq. (B.3),

$$\begin{aligned} U_t &= \overline{Y}U_c\left[\Phi\widehat{Y}_t - \frac{1}{2}(\sigma^{-1}+\omega)\widehat{Y}_t^2 + (\sigma^{-1}g_t + \omega q_t)\widehat{Y}_t - \frac{1}{2}\eta_\pi var_f \ln p_t(f) - \frac{1}{2}\eta_l var_h \ln l_t(h)\right] \\ +t.i.p + Order(&\parallel \xi \parallel^3) \\ &= -\frac{1}{2}\overline{Y}U_c\left[(\sigma^{-1}+\omega)(x_t - x^*)^2 + \eta_\pi var_f \ln p_t(f) + \eta_l var_h \ln l_t(h)\right] \\ +t.i.p + Order(&\parallel \xi \parallel^3), \end{aligned}$$

where $\eta_\pi \equiv \theta(1+\omega_p\theta)$, $\eta_l \equiv \phi_h^{-1}(\nu+\epsilon^{-1})$, $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n$, and $x^* \equiv \ln(Y^*/\overline{Y})$. Here Y^* is a solution in Eq. (B.4) when $\Phi = 0$, which is called as an efficient level of output as defined in Woodford (2003a). In the second line, we use the log-linearization of Eq. (B.4) as

$$\widehat{Y}_t^n \equiv \ln(Y_t^n/\overline{Y}) = \frac{\sigma^{-1}g_t + \omega q_t}{\sigma^{-1}+\omega},$$

and the relation as

[42] By assuming a proper proportional tax on sales τ as

$$m(Y_t^n, Y_t^n, \overline{R}; \nu_t) = \frac{\theta - 1}{\theta}(1-\tau) \equiv \frac{\epsilon}{\epsilon - 1}(1+\gamma\overline{R})(1-\Phi),$$

we can induce $\Phi = 0$ as in Rotemberg and Woodford (1997).

$$\ln(Y_t^n/Y_t^*) = -(\sigma^{-1}+\omega)\Phi + Order(\| \xi \|),$$

which is given by the relation between the efficient level of output and the natural rate of output in terms of $\left[\int_0^1 (\frac{V_l(l_t(h),\nu_t)}{U_Y(Y_t,\nu_t)}\frac{\partial L_t(f)}{\partial y_t(f)})^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}}$. This expresses that the percentage difference between Y_t^n and Y_t^* is independent from shocks in the first order approximation. It again notes that we assume that Φ is of order one. To evaluate $var_h \widehat{l_t}(h)$, we use the optimal condition of labor supply and the labor demand function given by following equations

$$l_t(h) = L_t \left[\frac{(1+\gamma r_t(h))w_t(h)}{\Omega_t}\right]^{-\epsilon},$$

$$\frac{w_t(h)}{P_t} = \frac{\epsilon}{\epsilon-1}\frac{V_l(l_t(h),\nu_t)}{U_C(C_t,\nu_t)}.$$

where

$$\Omega_t \equiv \left[\int_0^1 ((1+\gamma r_t(h))w_t(h))^{1-\epsilon} dh\right]^{\frac{1}{1-\epsilon}}.$$

By log-linearizing these equations, we finally have a following relation

$$var_h \ln l_t(h) = \Xi var_h \ln(1+r_t(h)) + Order(\| \xi \|^3).$$

where $\Xi \equiv \epsilon^2\Theta^2(\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2}+1)$ and $\Theta \equiv \frac{\gamma(1+\bar{r})}{1+\gamma\bar{r}}$. Then, Eq. (B.4) is transformed into

$$U_t = -\frac{1}{2}\overline{Y}U_c\left[(\sigma^{-1}+\omega)(x_t-x^*)^2 + \eta_\pi var_f \ln p_t(f) + \eta_r var_h \ln(1+r_t(h))\right] + t.i.p + Order(\| \xi \|^3),$$

where $\eta_r \equiv \Xi\eta_l = \epsilon\phi_h^{-1}(1+\nu\epsilon)\Theta^2(\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2}+1)$.

The remaining work to derive the approximated welfare function is to evaluate $var_h \ln p_t(f)$ and $var_h \ln(1+r_t(h))$ in Eq. (4.11). Following Woodford (2003a), we define

$$\overline{P}_t \equiv E_f \ln p_t(f),$$

$$\triangle_t \equiv var_f \ln p_t(f).$$

Then we can make following relations

$$\begin{aligned}
\overline{P}_t - \overline{P}_{t-1} &= E_f\left[\ln p_t(f) - \overline{P}_{t-1}\right] \\
&= \alpha E_f\left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right] + (1-\alpha)E_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right] \\
&= (1-\alpha)E_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right], \qquad \text{(B.5)}
\end{aligned}$$

and we also have

$$\begin{aligned}
\triangle_t &= var_f\left[\ln p_t(f) - \overline{P}_{t-1}\right] \\
&= E_f\left\{\left[\ln p_t(f) - \overline{P}_{t-1}\right]^2\right\} - (E_f \ln p_t(f) - \overline{P}_{t-1})^2 \\
&= \alpha E_f\left\{\left[\ln p_{t-1}(f) - \overline{P}_{t-1}\right]^2\right\} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)E_f\left\{\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]^2\right\} - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + (1-\alpha)(var_f(\ln p_t^*(f) - \overline{P}_{t-1}) + \left\{E_f\left[\ln p_t^*(f) - \overline{P}_{t-1}\right]\right\}^2) - (\overline{P}_t - \overline{P}_{t-1})^2 \\
&= \alpha\triangle_{t-1} + \frac{\alpha}{1-\alpha}(\overline{P}_t - \overline{P}_{t-1}), \qquad \text{(B.6)}
\end{aligned}$$

where we use Eq. (B.5) and $p_t^*(f)$ is an optimal price setting by the agent f following the Calvo (1983) - Yun (1992) framework. It notes that all project groups re-set the same price at time t when they are selected to change prices, because the unit marginal cost of production is same for all project groups. Also, we have a following relation that relates $\overline{P}_t$ with P_t

$$\overline{P}_t = \ln P_t + Order(\parallel \xi \parallel^2),$$

where $Order(\| \xi \|^2)$ is order terms higher than the first order approximation. Here we make use of the definition of price aggregator $P_t \equiv \left[\int_0^1 p_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}$. Then Eq. (B.6) can be transformed as

$$\triangle_t = \alpha \triangle_{t-1} + \frac{\alpha}{1-\alpha} \pi_t, \tag{B.7}$$

where $\pi_t \equiv \ln \frac{P_t}{P_{t-1}}$. From Eq. (B.7), we have

$$\triangle_t = \alpha^{t+1} \triangle_{-1} + \sum_{s=0}^{t} \alpha^{t-s} \left(\frac{\alpha}{1-\alpha}\right) \pi_s^2,$$

and so

$$\sum_{t=0}^{\infty} \beta^t \triangle_t = \frac{\alpha}{(1-\alpha)(1-\alpha\beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 + t.i.p + Order(\| \xi \|^3). \tag{B.8}$$

To evaluate $var_h \ln(1 + r_t(h))$, we define $\overline{R}_t$ and $\triangle_t^R$ as

$$\overline{R}_t \equiv E_h \ln(1 + r_t(h)),$$

$$\triangle_t^R \equiv var_h \ln(1 + r_t(h)).$$

Then, we can make following relations

$$\begin{aligned} \overline{R}_t - \overline{R}_{t-1} &= E_h \left[\ln(1 + r_t(h)) - \overline{R}_{t-1}\right] \\ &= \varphi E_h \left[\ln(1 + r_{t-1}(h)) - \overline{R}_{t-1}\right] + (1-\varphi) \left[\ln(1 + r_t^*) - \overline{R}_{t-1}\right] \\ &= (1-\varphi) \left[\ln(1 + r_t^*(h)) - \overline{R}_{t-1}\right], \end{aligned} \tag{B.9}$$

and

$$\begin{aligned}
\triangle_t^R &= var_h\left[\ln(1+r_t(h)) - \overline{R}_{t-1}\right] \\
&= E_h\left\{\left[\ln(1+r_t(h)) - \overline{R}_{t-1}\right]^2\right\} - (E_h \ln(1+r_t(h)) - \overline{R}_{t-1})^2 \\
&= \varphi E_h\left\{\left[\ln(1+r_{t-1}(h)) - \overline{R}_{t-1}\right]^2\right\} + (1-\varphi)\left[\ln(1+r_t^*) - \overline{R}_{t-1}\right]^2 - (\overline{R}_t - \overline{R}_{t-1})^2 \\
&= \varphi \triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\overline{R}_t - \overline{R}_{t-1})^2, \qquad \text{(B.10)}
\end{aligned}$$

where we use Eq. (B.9). Also, as in the discussion on price, we have

$$\overline{R}_t = \ln(1+R_t) + Order(\parallel \xi \parallel^2), \qquad \text{(B.11)}$$

where we make use of the definition of the aggregate loan rates $1+R_t \equiv \int_0^1 \frac{q_t(h)}{Q_t}(1+r_t(h))dh$. Then, from Eq. (B.10) and Eq. (B.11), we have

$$\triangle_t^R = \varphi \triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\widehat{R}_t - \widehat{R}_{t-1})^2,$$

where $\widehat{R}_t \equiv \ln \frac{1+R_t}{1+\overline{R}}$. From Eq. (4.18), we have

$$\triangle_t^R = \varphi^{t+1}\triangle_{-1}^R + \sum_{s=0}^{t}\varphi^{t-s}\left(\frac{\varphi}{1-\varphi}\right)(\widehat{R}_s - \widehat{R}_{s-1})^2,$$

and so

$$\sum_{t=0}^{\infty}\beta^t \triangle_t^R = \frac{\varphi}{(1-\varphi)(1-\varphi\beta)}\sum_{t=0}^{\infty}\beta^t(\widehat{R}_t - \widehat{R}_{t-1})^2 + t.i.p + Order(\parallel \xi \parallel^3).$$

From Eq. (4.11), Eq. (4.14), and Eq. (4.19), we finally have

$$\sum_{t=0}^{\infty}\beta^t U_t \simeq -\Lambda\sum_{t=0}^{\infty}\beta^t\left(\lambda_\pi \pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_R(\widehat{R}_t - \widehat{R}_{t-1})^2\right),$$

where $\Lambda \equiv \frac{1}{2}\overline{Y}u_c$, $\lambda_\pi \equiv \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta(1+\omega_p\theta)$, $\lambda_x \equiv (\sigma^{-1}+\omega)$, and $\lambda_R \equiv \epsilon\phi_h^{-1}(1+\nu\epsilon)(\frac{\gamma(1+\overline{r})}{1+\gamma\overline{r}})^2(\frac{\epsilon^2}{(\nu^{-1}+\epsilon)^2}+1)\frac{\varphi}{(1-\varphi)(1-\varphi\beta)}$.

C Model with Indexational Loan Rate Change

In the case in which each working group of a bank indexationally re-set loan rates according to the changes of aggregate loan rates with probability φ, the working group h that is not selected to optimally change loan rates re-sets loan rates in a following way.

$$\ln(1+r_t(h)) = \ln(1+r_{t-1}(h)) + \psi(\ln(1+R_{t-1}) - \ln(1+R_{t-2})),$$

where ψ $(0 \leq \psi \leq 1)$ is a measure of the degree of indexation to the most available information on aggregate loan rates. Corresponding to this indexational loan rate change rule, under the given demand function of loan set by Eq. (A.4) and under the situation in which the managers can optimally re-set their loan rates with probability $1-\varphi$ and automatically change their loan rates according to indexation of aggregate loan rates with probability φ, each working group of private bank re-sets its loan rates, $r_t(h)$ to maximize present discounted value of profit given by

$$E_t \sum_{T=t}^{\infty} \varphi^{T-t} X_{t,T} \left[q_{t,T}(h)((1+r_t(h)) \left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1 - i_T) - z_T(h) \right], \tag{C.1}$$

where we define $q_{t,T}(h) = \left[\frac{(1+\gamma((1+r_t(h))\left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1))^{-\epsilon}(w_T(h))^{1-\epsilon}}{\Omega_T^*} \right] Q_T$ from Eq. (2.5), i_T is deposit rates which is set by a central bank and is same for all working groups, $q_{t,T}(h)$ is the amount of loan to firm's project group h, and $z_T(h)$ is a cost of bank's working group h to handle finance to the firm's project group h. We assume $z_T(h)$ is zero, $z(h) = 0$, and $D_T = Q_T$ in equilibrium. Then, we can transform Eq. (C.1) as

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} \left[q_{t,T}(h)((1+r_t(h)) \left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1 - i_T) \right].$$

Then, the optimal loan rate setting of $r_t(h)$ is given by

$$E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \frac{P_t}{P_T} \frac{U_C(C_T, \nu_T)}{U_C(C_t, \nu_t)} q_{t,T}(h) \left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} \left[1 - \epsilon\gamma \frac{(1+r_t(h)) \left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1 - i_T}{1 + \gamma((1+r_t(h)) \left[\frac{1+R_{T-1}}{1+R_{t-1}}\right]^{\psi} - 1)}\right] = 0. \tag{C.2}$$

By log-linearizing Eq. (C.2), we have a following equation

$$\frac{\gamma(1-\epsilon)(1+\bar{r})}{\beta\varphi - 1} \widehat{r}_t(h) - \frac{\gamma(1-\epsilon)(1+\bar{r})}{\beta\varphi - 1} \psi \widehat{R}_{t-1} = E_t \sum_{T=t}^{\infty} (\varphi\beta)^{T-t} \left[\gamma(1-\epsilon)(1+\bar{r})\psi \widehat{R}_{T-1} + \gamma\epsilon(1+\bar{i})\widehat{i}_T\right]. \tag{C.3}$$

Here working groups that are allowed to change their loan rates will set the same loan rates, so the solution of $r_t(h)$ in Eq. (C.2) is expressed by r_t^*, and so the solution of $\widehat{r}_t(h)$ in Eq. (C.3) is expressed by $\widehat{r}_t^*$. On the other hand, we have a following evolution of the aggregate loan rate index R

$$1 + R_t = \varphi(1 + R_{t-1}) \left[\frac{1+R_{t-1}}{1+R_{t-2}}\right]^{\psi} + (1-\varphi)(1 + r_t^*). \tag{C.4}$$

By log-linearizing Eq. (C.4), we have

$$\widehat{r}_t^* = \frac{1}{1-\varphi} \widehat{R}_t - \frac{\varphi}{1-\varphi}(1+\psi)\widehat{R}_{t-1} + \frac{\varphi}{1-\varphi}\psi \widehat{R}_{t-2}. \tag{C.5}$$

Then, by considering of $\widehat{r}_t^* + \psi\widehat{R}_{t-1} - \varphi\beta(E_t\widehat{r}_{t+1}^* + \psi\widehat{R}_t)$ in Eq. (C.3) and by substituting Eq. (C.5) into it, we finally have the loan rate curve

$$\widehat{R}_t = \lambda_1^* E_t \widehat{R}_{t+1} + \lambda_2^* \widehat{R}_{t-1} + \lambda_3^* \widehat{R}_{t-2} + \lambda_4^* \widehat{i}_t,$$

where $\lambda_1^* \equiv \frac{\varphi\beta}{1+(1+\psi)\varphi^2\beta+\psi\beta\varphi(1-\varphi)}$, $\lambda_2^* \equiv \frac{\psi(1+\varphi)+\varphi(1+\psi)+\psi(1+\varphi)(\varphi\beta-1)+\varphi^2\beta\psi}{1+(1+\psi)\varphi^2\beta+\psi\beta\varphi(1-\varphi)}$, $\lambda_3^* \equiv -\frac{\varphi\psi}{1+(1+\psi)\varphi^2\beta+\psi\beta\varphi(1-\varphi)}$, and $\lambda_4^* \equiv \frac{1+\bar{i}}{1+\bar{r}} \frac{\epsilon}{\epsilon-1} \frac{1-\varphi}{1+(1+\psi)\varphi^2\beta+\psi\beta\varphi(1-\varphi)}(1-\varphi\beta)$.

D Approximated Welfare Function under Indexational Loan Rate Change

Even under indexational loan rate change, the derivation of second orderly approximated welfare function eventually does not change up to Eq. (B.8). The equation corresponding to Eq. (B.9), however, is given by

$$
\begin{aligned}
\overline{R}_t - \overline{R}_{t-1} &= E_h\left[\ln(1+r_t(h)) - \overline{R}_{t-1}\right] \\
&= \varphi E_h\left[\ln(1+r_{t-1}(h)) + \psi(\ln(1+R_{t-1}) - \ln(1+R_{t-2})) - \overline{R}_{t-1}\right] + (1-\varphi)\left[\ln(1+r_t^*) - \overline{R}_{t-1}\right] \\
&= (1-\varphi)\left[\ln(1+r_t^*) - \overline{R}_{t-1}\right] + \psi\varphi\left[\ln(1+R_{t-1}) - \ln(1+R_{t-2})\right].
\end{aligned}
$$

This is because the motion of aggregate loan rates is given by

$$1 + R_t = \varphi(1+R_{t-1})\left[\frac{1+R_{t-1}}{1+R_{t-2}}\right]^{\psi} + (1-\varphi)(1+r_t^*),$$

where ψ is a parameter associated with indexational loan interest rate changes. From Eq. (B.11), we finally have

$$\overline{R}_t - \overline{R}_{t-1} = (1-\varphi)\left[\ln(1+r_t^*) - \overline{R}_{t-1}\right] + \psi\varphi\left[\overline{R}_{t-1} - \overline{R}_{t-2}\right]. \tag{D.1}$$

Also we have

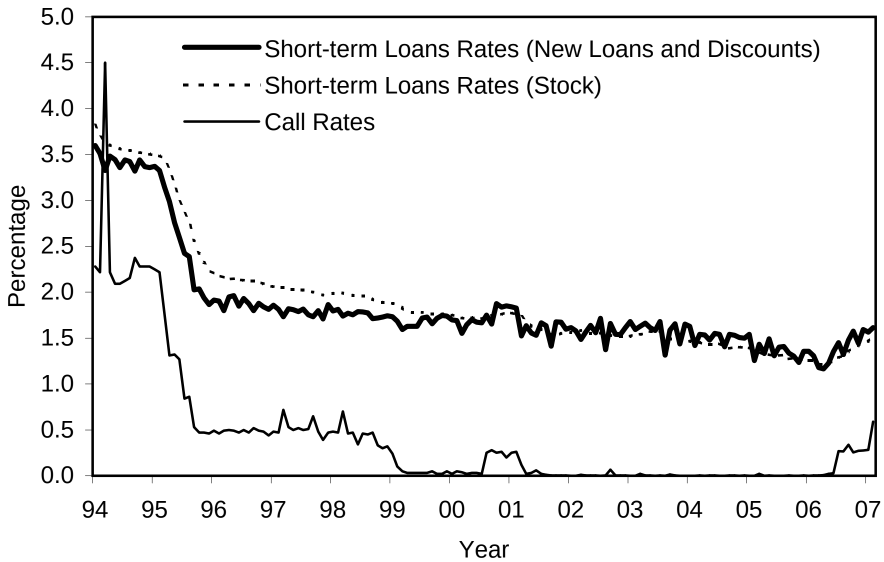
$$
\begin{aligned}
\triangle_t^R &= var_h\left[\ln(1+r_t(h)) - \overline{R}_{t-1}\right] \\
&= E_h\left\{\left[\ln(1+r_t(h)) - \overline{R}_{t-1}\right]^2\right\} - (E_h \ln(1+r_t(h)) - \overline{R}_{t-1})^2 \\
&= \varphi E_h\left\{\left[\ln(1+r_{t-1}(h)) + \psi(\ln(1+R_{t-1}) - \ln(1+R_{t-2})) - \overline{R}_{t-1}\right]^2\right\} \\
&\quad +(1-\varphi)\left[\ln(1+r_t^*) - \overline{R}_{t-1}\right]^2 - (\overline{R}_t - \overline{R}_{t-1})^2 \\
&= \varphi\triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\triangle\overline{R}_t - \psi\triangle\overline{R}_{t-1})^2 \\
&= \varphi\triangle_{t-1}^R + \frac{\varphi}{1-\varphi}(\triangle\widehat{R}_t - \psi\triangle\widehat{R}_{t-1})^2,
\end{aligned}
$$

where we use Eq. (B.11) and Eq. (D.1). Then, by taking the same procedure as shown in Appendix A, we finally have

$$
\sum_{t=0}^{\infty}\beta^t U_t \simeq -\Lambda\sum_{t=0}^{\infty}\beta^t\left(\lambda_\pi\pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_R(\triangle\widehat{R}_t - \psi\triangle\widehat{R}_{t-1})^2\right).
$$

Table 1: Parameter Values

Parameters	Values	Explanation
β	0.99	Discount factor
σ	6.25	Elasticity of the output gap to real interest rate
κ	0.032	Elasticity of inflation to the output gap
α	0.66	Probability of price change
φ	0.66	Probability of loan rate change
θ	7.66	Substitutability of differentiated consumption goods
ϵ	7.66	Substitutability of differentiated labors
γ	0.5	Ratio of external finance
ω	0.47	Total elasticity of marginal cost with respect to y
ω_p	0.33	Elasticity of marginal cost with respect to y regarding to production
ϕ_h	1.33	Inverse elasticity of the output to additional labor input
ν	0.11	Elasticity of the desired real wage to the quantity of labor demanded
x^*	0	Output distortion from efficient level of output



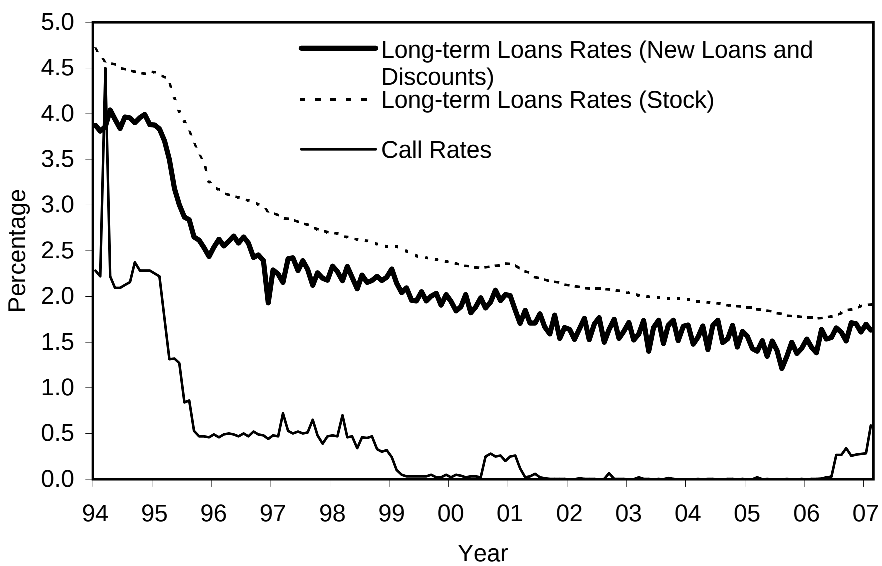
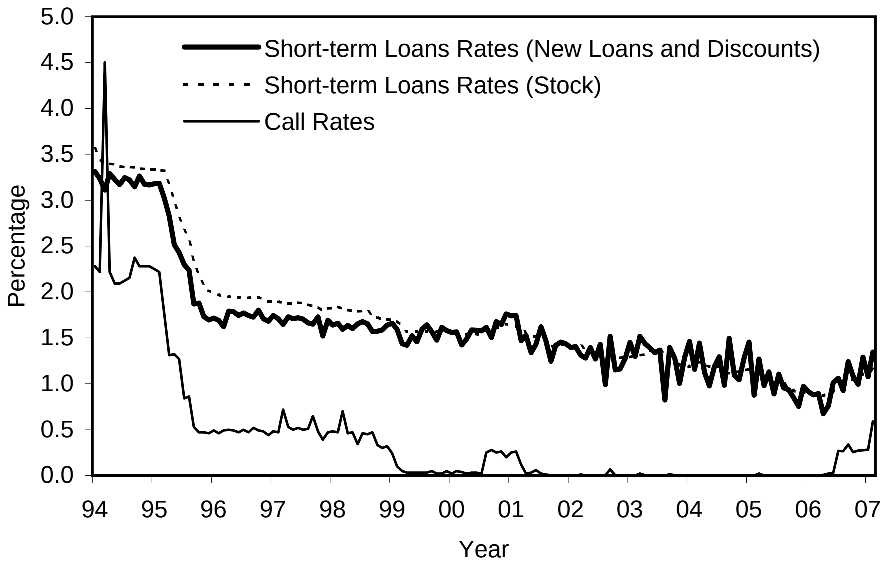


Figure 1: Call Rates (Monthly Average) and Average Contracted Interest Rates on Loans and Discounts (Total, Monthly)
Source: BOJ, Financial and Economic Statistics, Financial Markets, Lending Rates



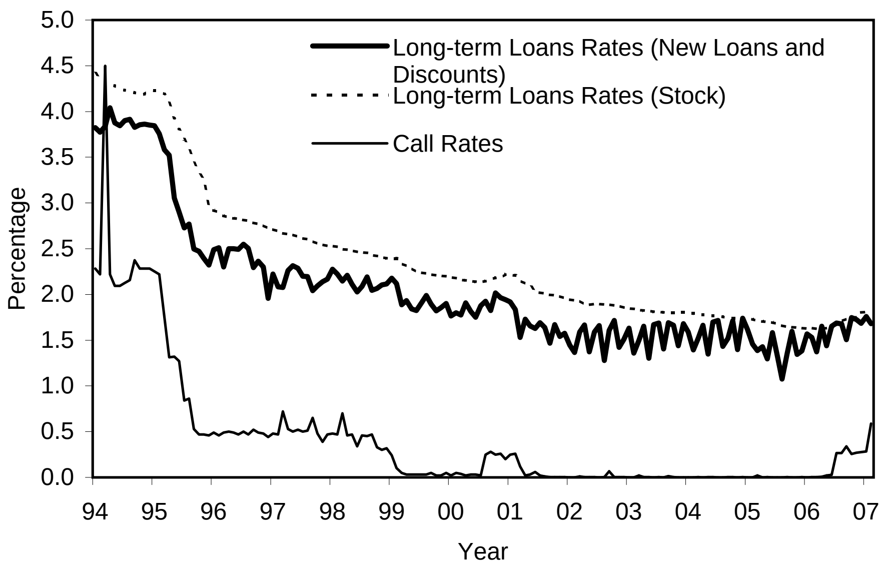


Figure 2: Call Rates (Monthly Average) and Average Contracted Interest Rates on Loans and Discounts (City Banks, Monthly)
Source: BOJ, Financial and Economic Statistics, Financial Markets, Lending Rates

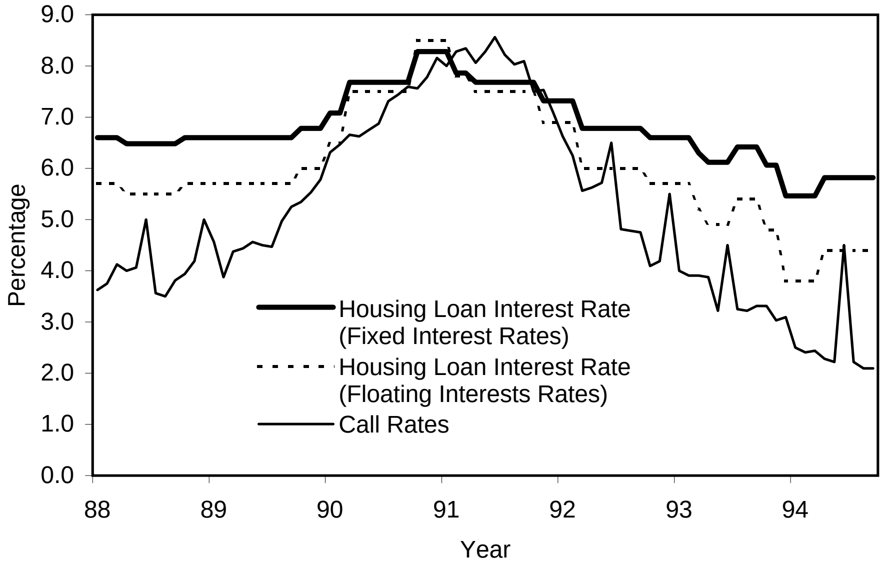


Figure 3: Interest Rates on Housing Loans (Monthly)
Source: BOJ, Financial and Economic Statistics, Financial Markets, Lending Rates

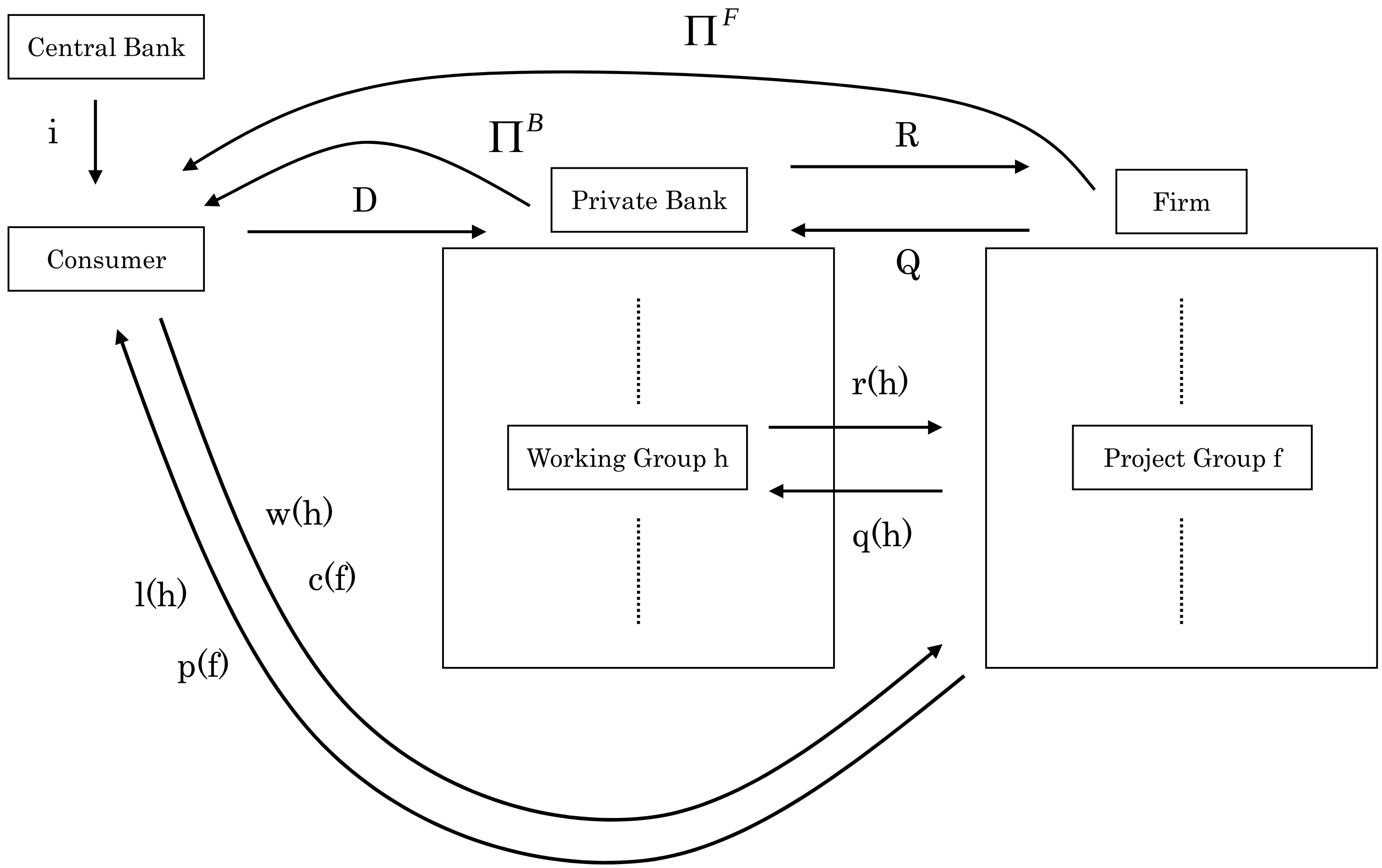


Figure 4: Consumer, Private Bank, Central Bank, and Firm

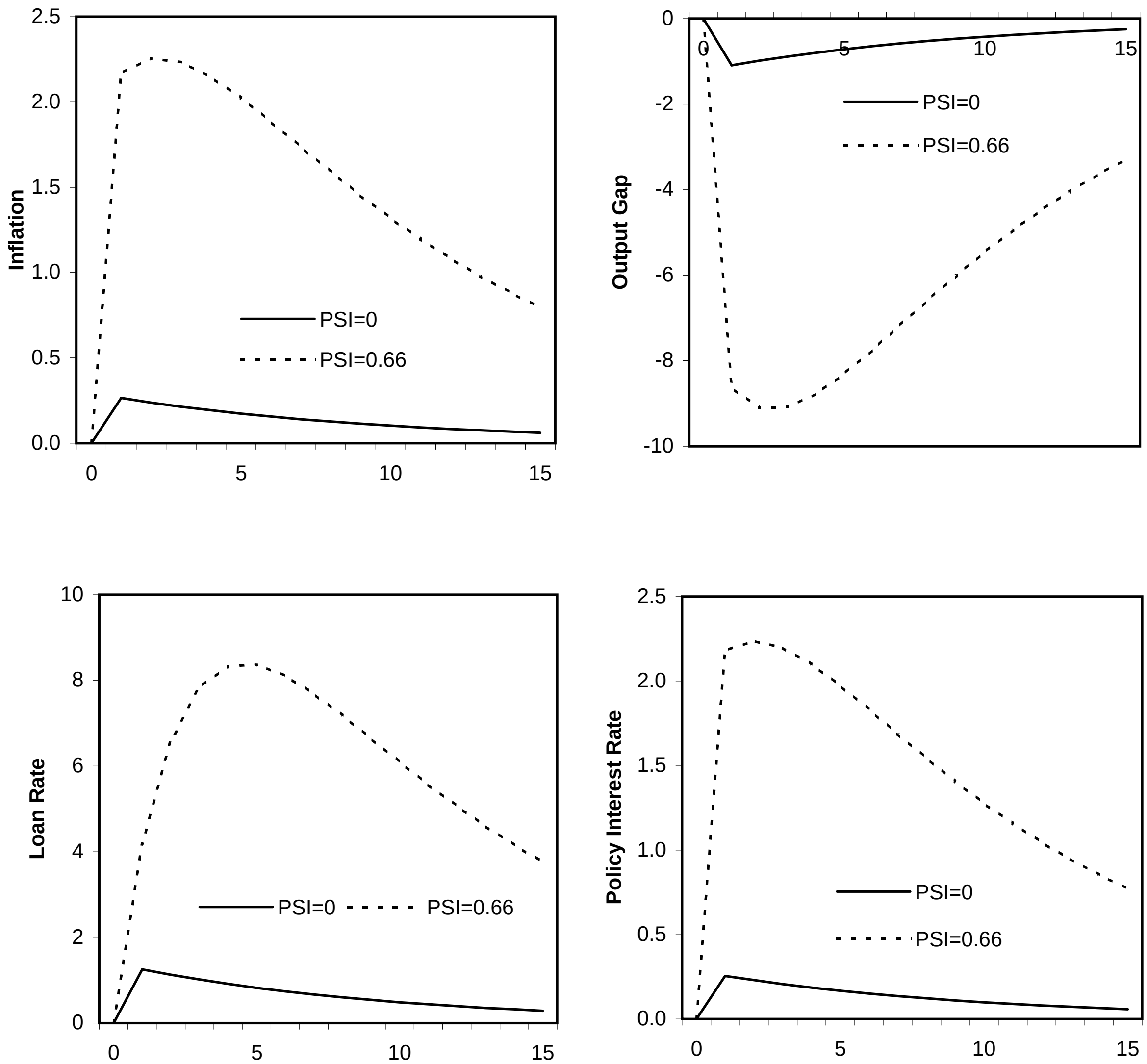


Figure 5: Loan Rate Shock under Taylor Rule

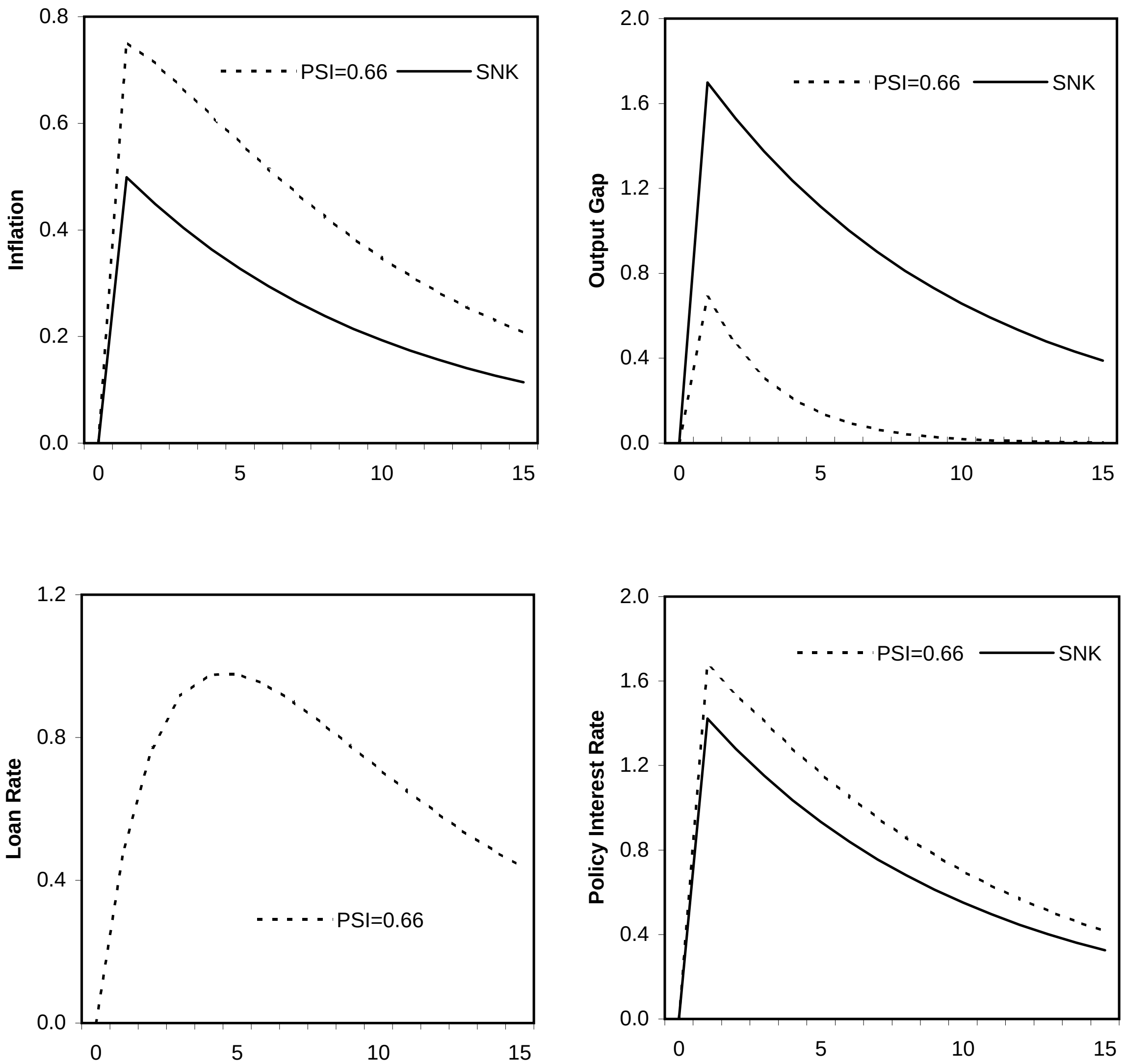


Figure 6: Natural Rate of Interest Shock under Taylor Rule

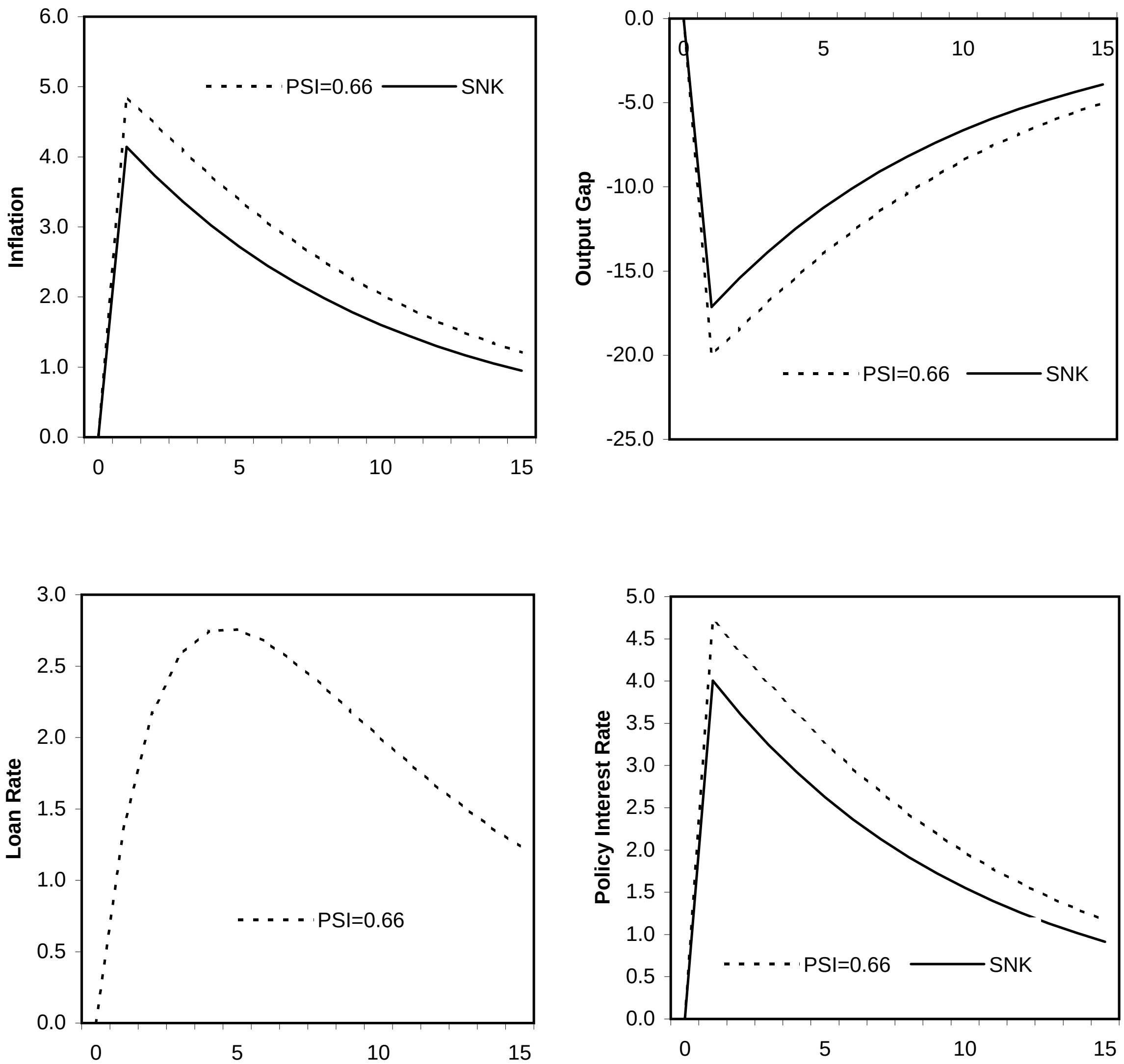


Figure 7: Price Mark Up Shock under Taylor Rule

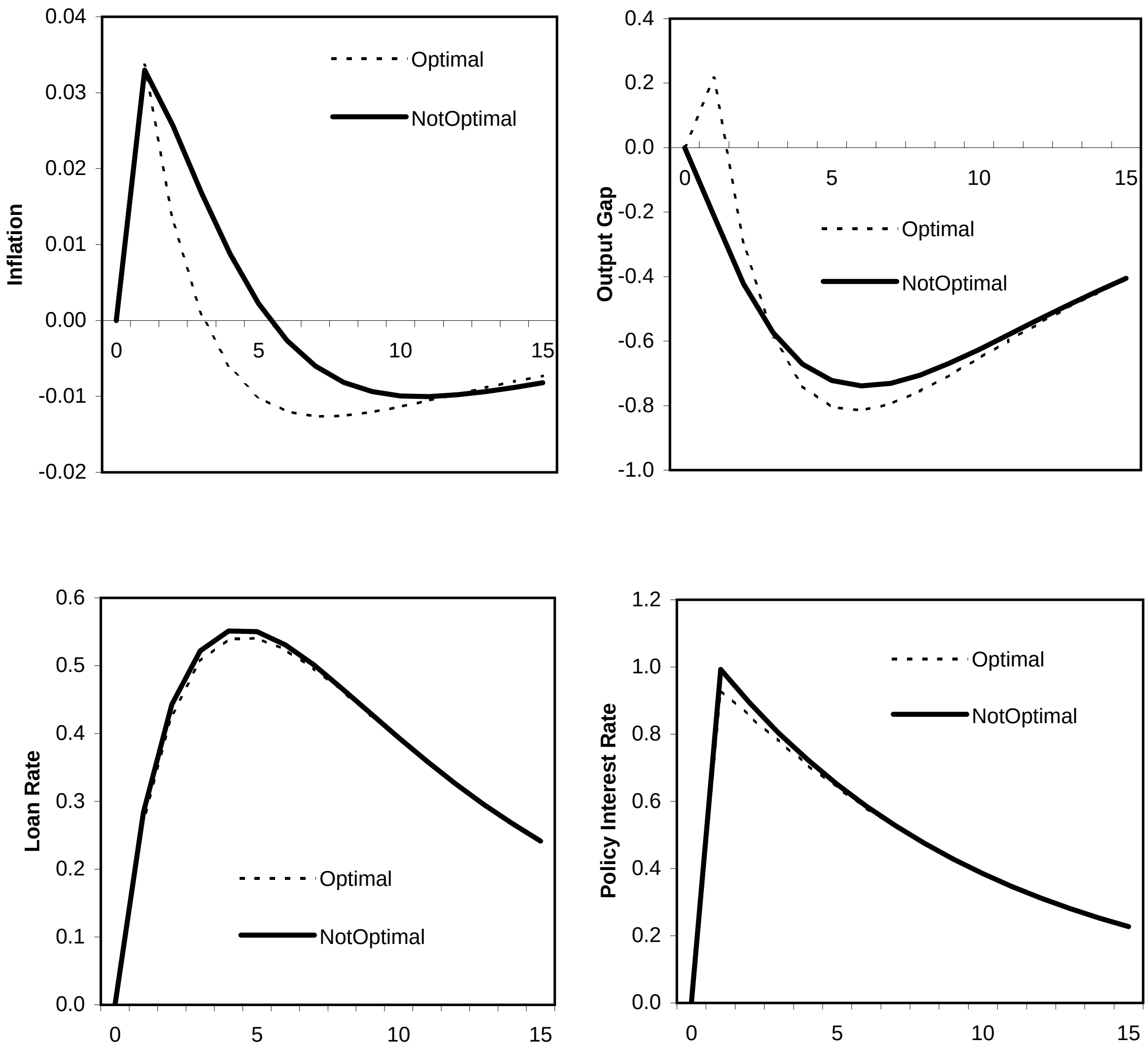


Figure 8: Natural Rate of Interest Shock under Optimal Policy

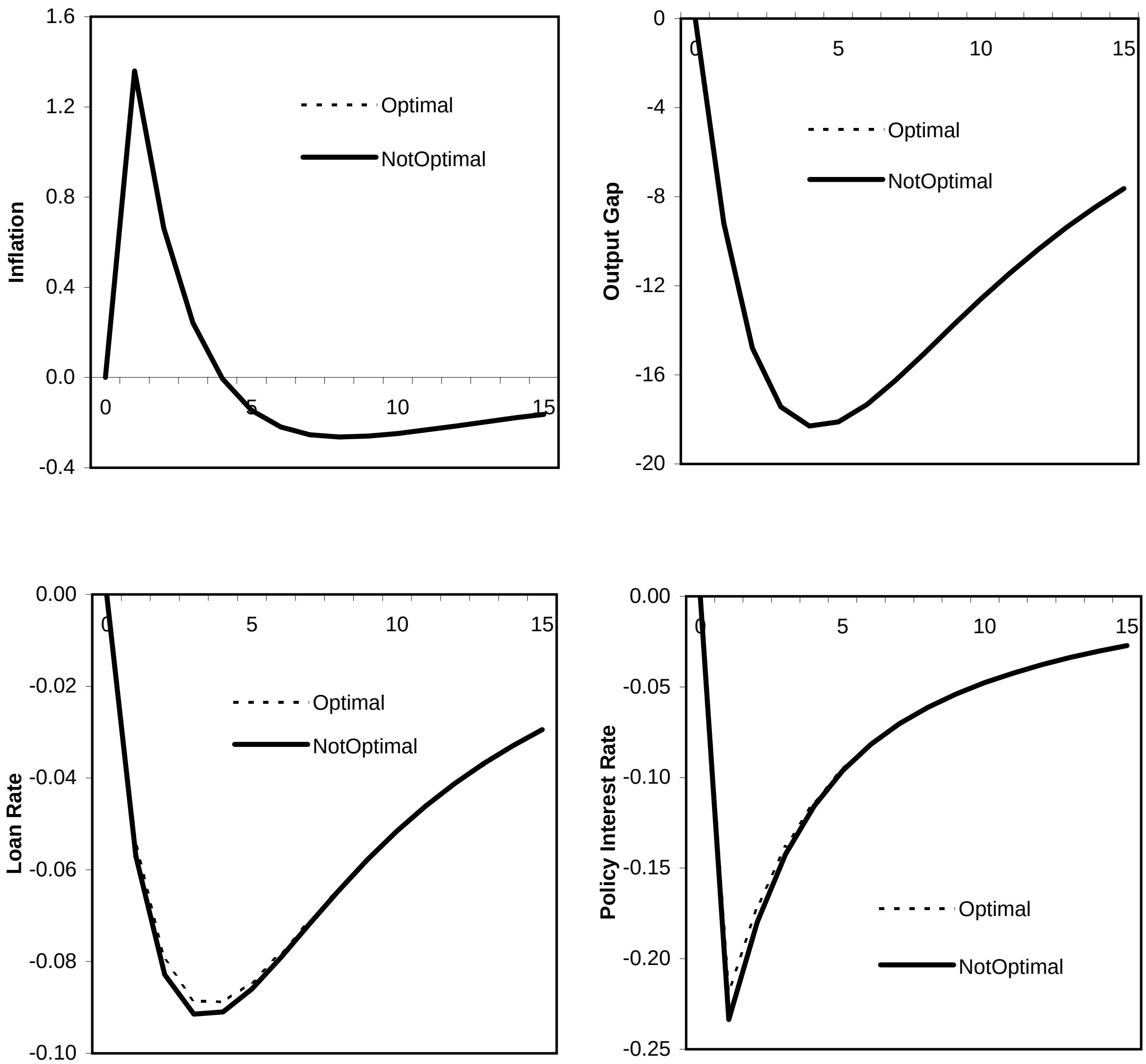


Figure 9: Price Mark Up Shock under Optimal Policy

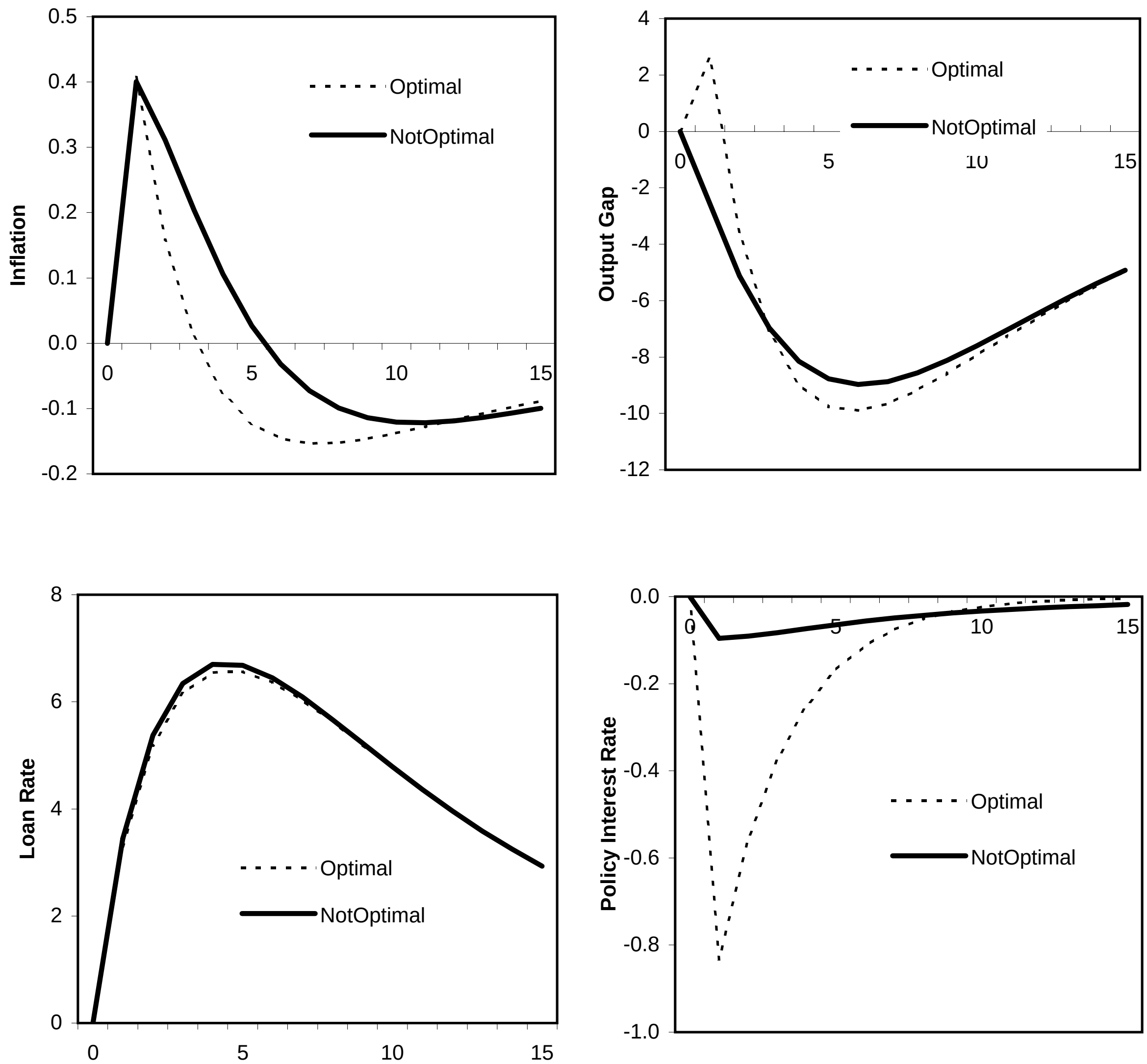


Figure 10: Loan Rate Shock under Optimal Policy

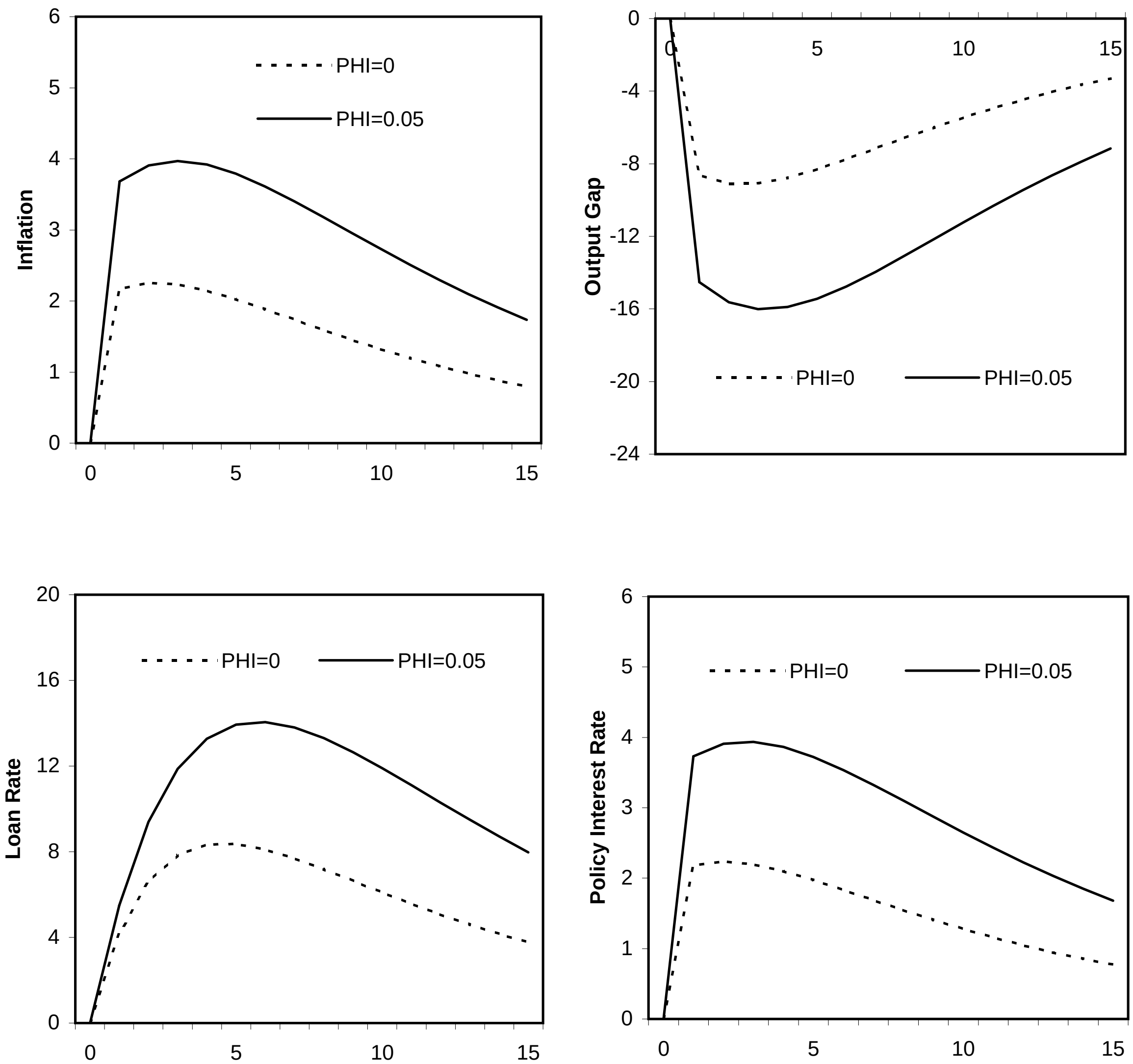


Figure 11: Loan Rate Shock under Taylor Rule and Loan Rate Indexation

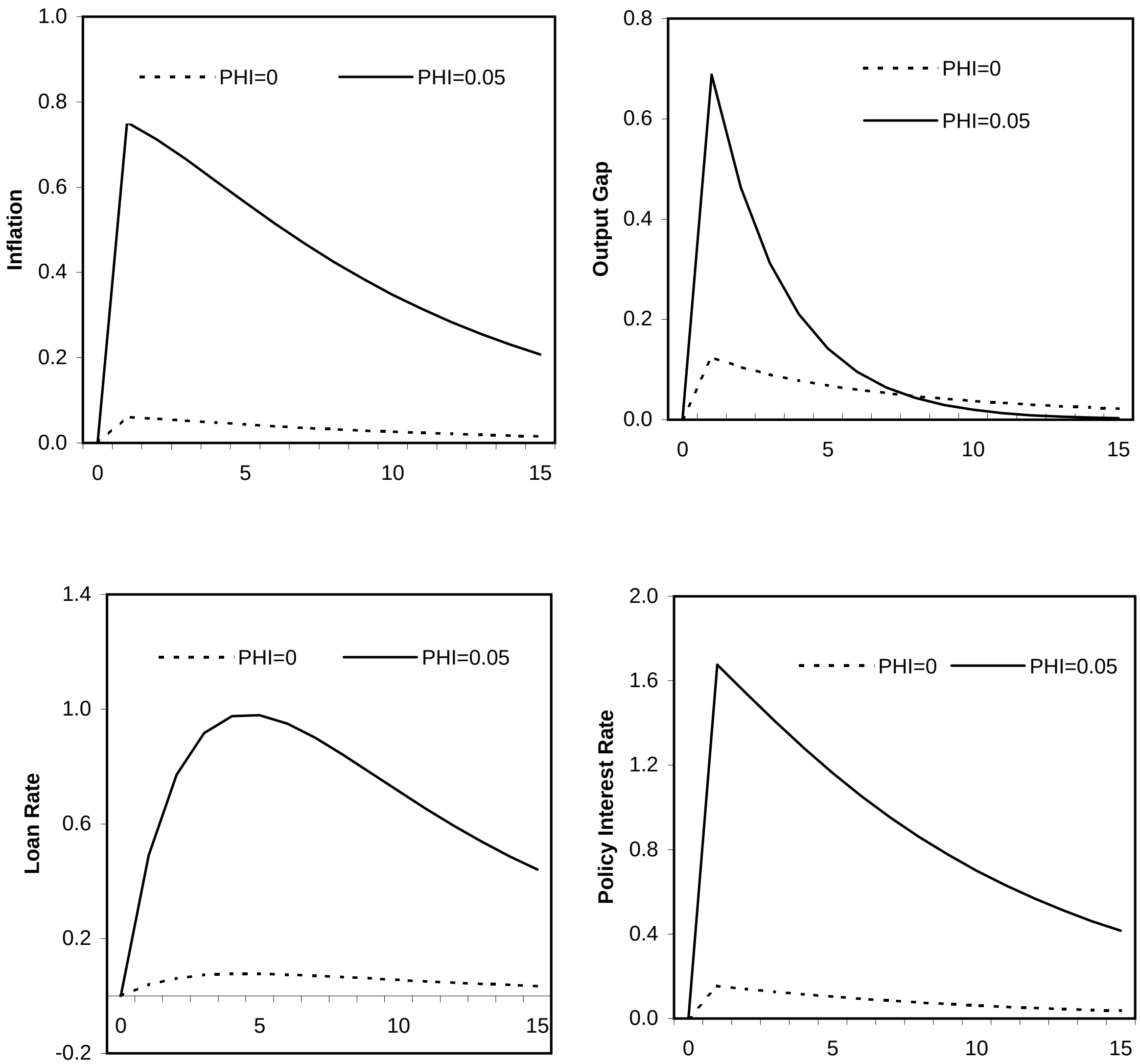


Figure 12: Natural Rate of Interest Shock under Taylor Rule and Loan Rate Indexation

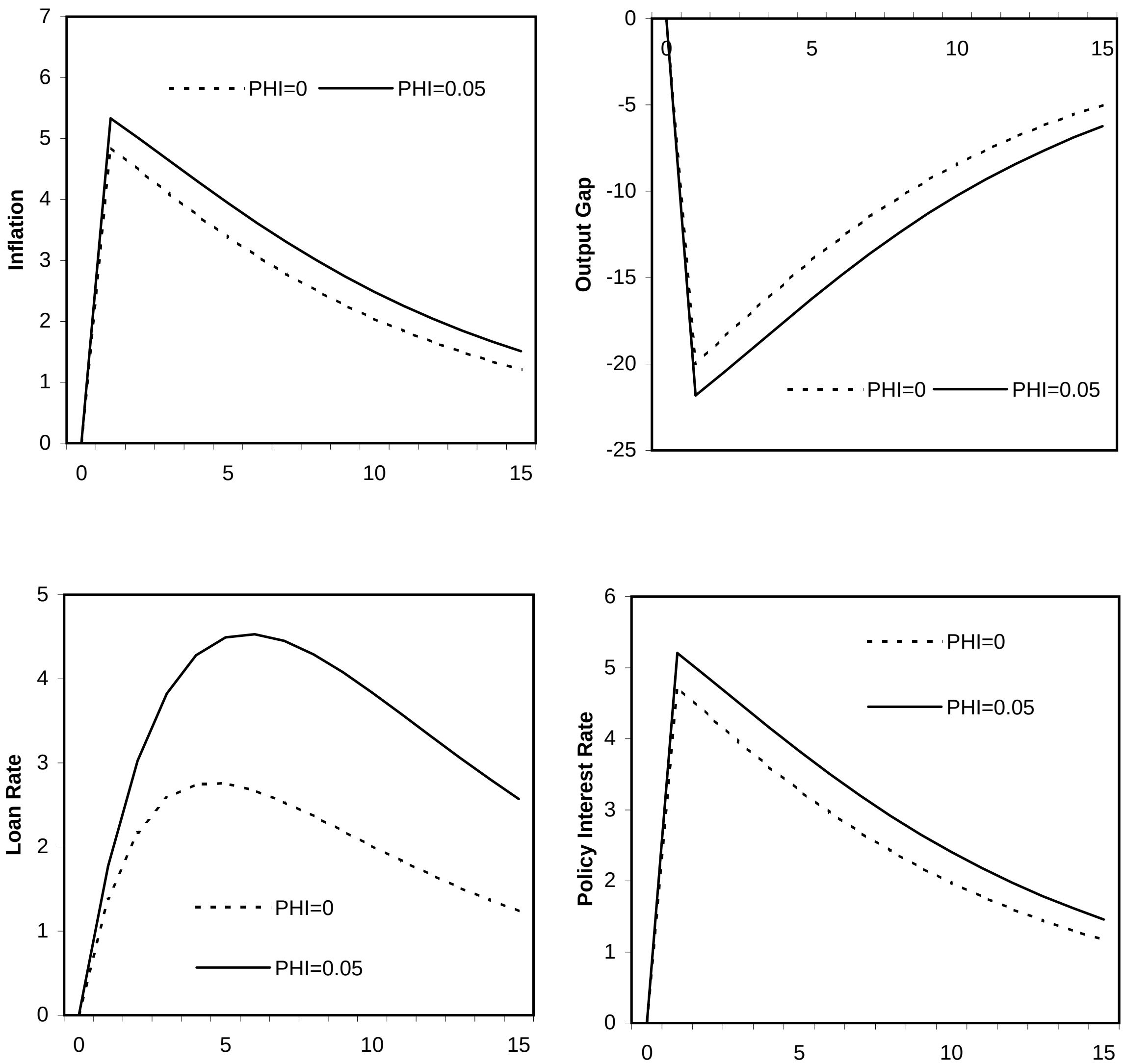


Figure 13: Price Mark Up Shock under Taylor Rule and Loan Rate Indexation

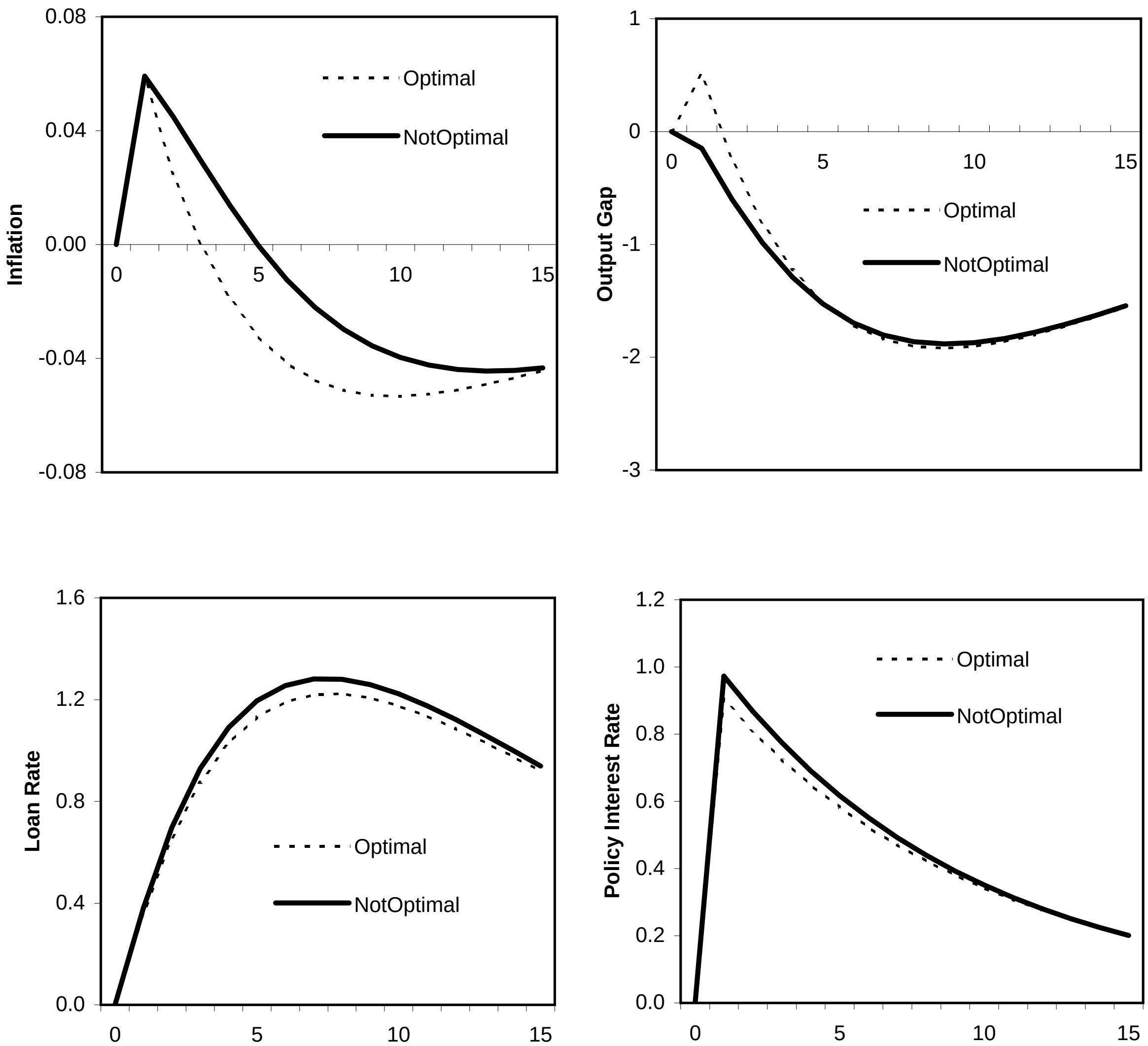


Figure 14: Natural Rate of Interest Shock under Optimal Policy and Loan Rate Indexation

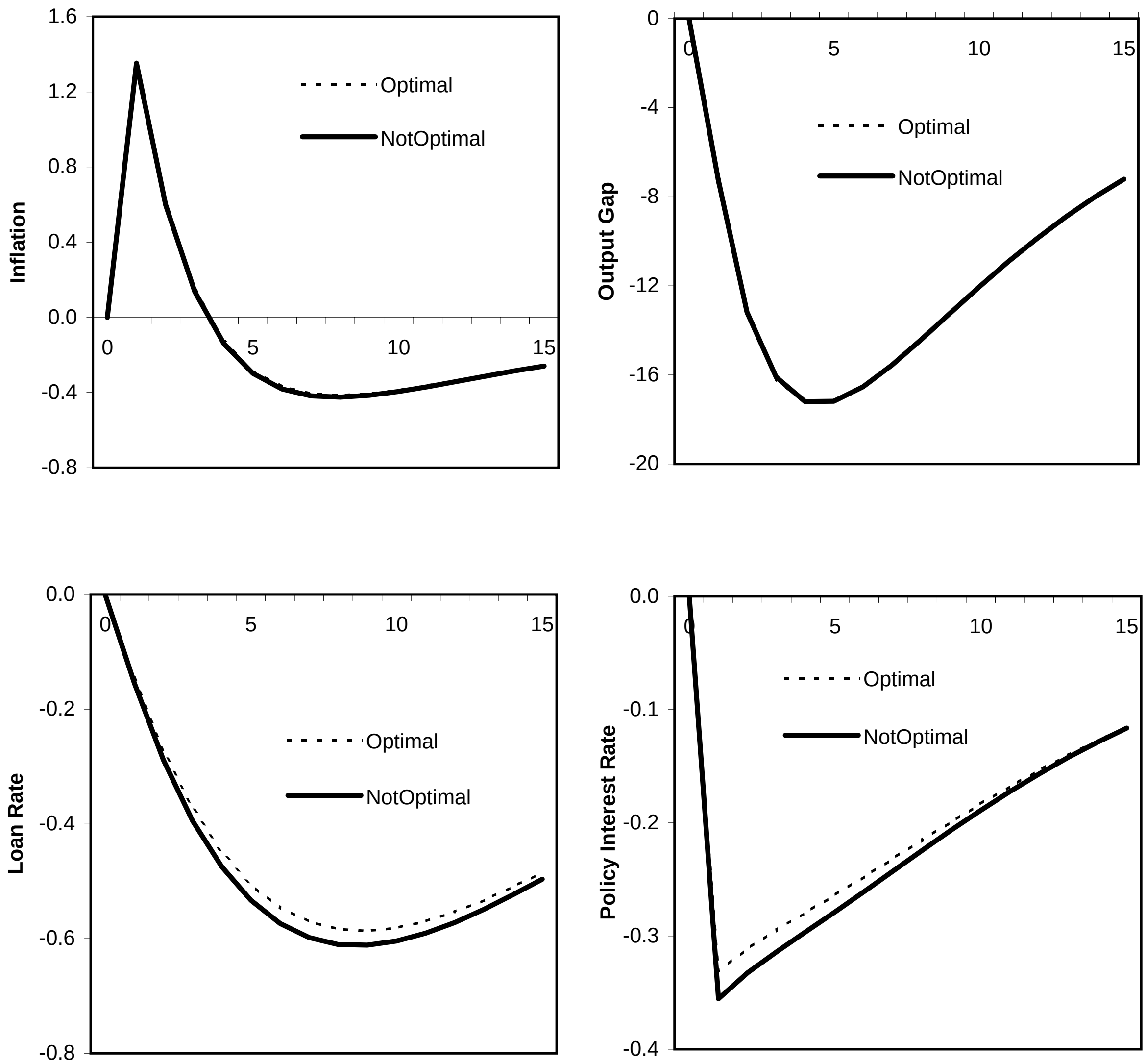


Figure 15: Price Mark Up Shock under Optimal Policy and Loan Rate Indexation

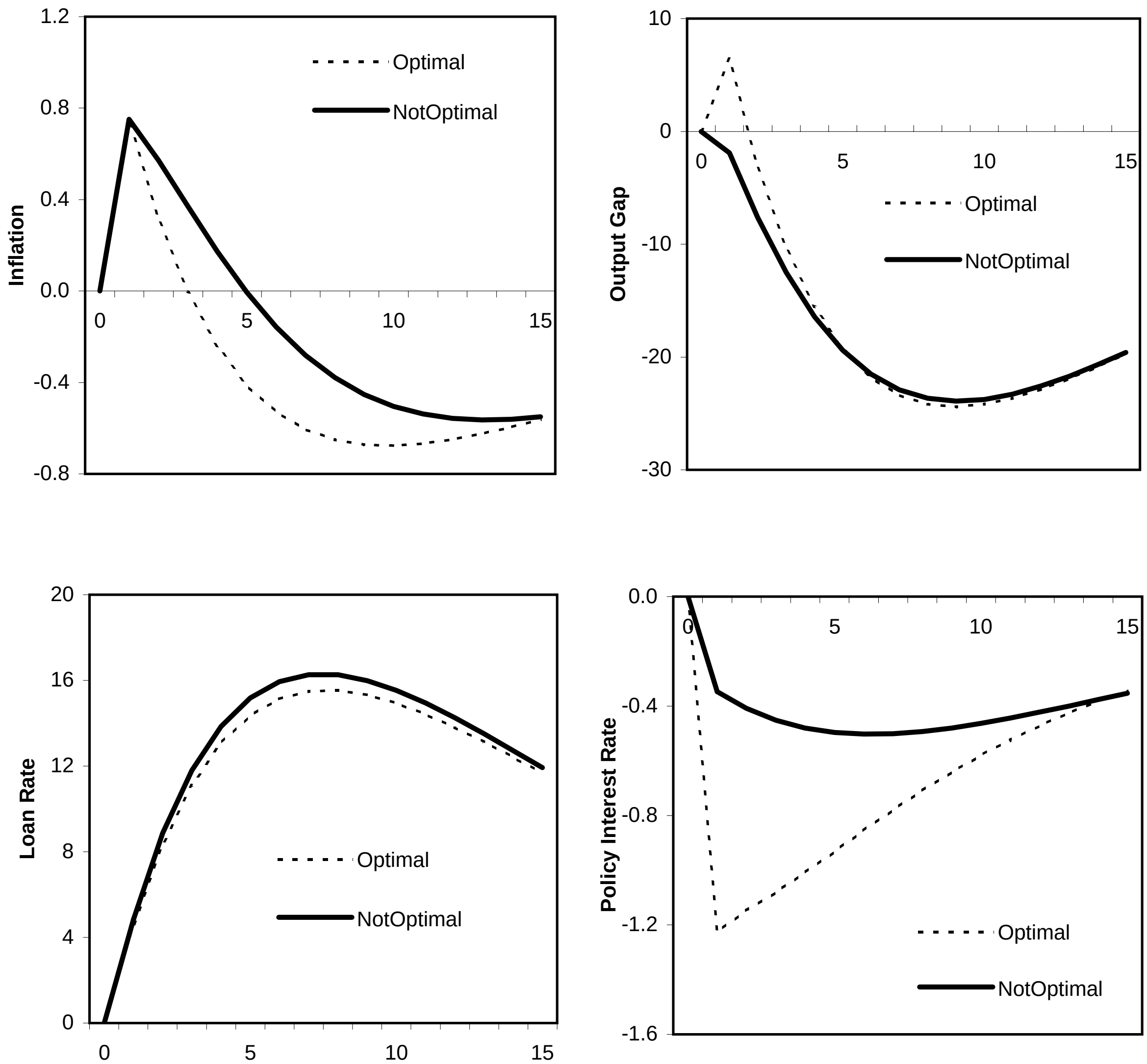


Figure 16: Loan Rate Shock under Optimal Policy and Loan Rate Indexation