IMES DISCUSSION PAPER SERIES

Do Sticky Prices Need to Be Replaced with Sticky Information?

Bill Dupor, Tomiyuki Kitamura, and Takayuki Tsuruga

Discussion Paper No. 2006-E-23

IMES

INSTITUTE FOR MONETARY AND ECONOMIC STUDIES

BANK OF JAPAN

C.P.O BOX 203 TOKYO
100-8630 JAPAN

You can download this and other papers at the IMES Web site:

http://www.imes.boj.or.jp

Do not reprint or reproduce without permission.
NOTE: IMES Discussion Paper Series is circulated in order to stimulate discussion and comments. Views expressed in Discussion Paper Series are those of authors and do not necessarily reflect those of the Bank of Japan or the Institute for Monetary and Economic Studies.
Do Sticky Prices Need to Be Replaced with Sticky Information?

Bill Dupor*, Tomiyuki Kitamura**, Takayuki Tsuruga***

Abstract
A first generation of research found it difficult to reconcile observed inflation and cyclical output with the fixed price mechanism. Since then, researchers have been divided roughly into two camps. The first camp argues that the original mechanism is largely successful once cyclical output is replaced with labor's share. The second camp argues for a wholesale replacement of fixed prices, e.g., with 'sticky information.' We take up the question by estimating a 'dual stickiness' model that integrates sticky prices and information. We find that both rigidities are present in aggregate U.S. data. Thus, sticky information cannot replace sticky prices. Our dual stickiness model performs comparably to the hybrid sticky price model, which allows for a fraction of backward-looking firms. In particular, the hybrid model's backward-looking behavior arises endogenously under dual stickiness. As such, the dual stickiness model (with an estimated seven month average information delay) may provide more plausible microeconomic foundations.

Keywords: Inflation; Sticky Price Model; Sticky Information Model; Labor Share
JEL classification: E31, E32

*Associate Professor, Department of Economics, Ohio State University (E-mail: dupor@econ.ohio-state.edu)
**Graduate Student, Department of Economics, Ohio State University, and Bank of Japan (E-mail: kitamura.5@osu.edu)
***Economist, Institute for Monetary and Economic Studies, Bank of Japan (E-mail: takayuki.tsuruga@boj.or.jp)

Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.
1 Introduction

The interaction of real activity and inflation is a cornerstone issue of macroeconomics. As with every major macro question, it has been placed under the lens of rational expectations and micro-founded dynamics in recent decades. Approximately fifteen years ago, efforts to estimate then existing models of price rigidity intensified.¹ These efforts center on the aggregated Euler equation from a rational expectations sticky price model, often called the New Keynesian Philips Curve (NKPC). A number of authors have argued that the NKPC is empirically deficient. Fuhrer and Moore (1995) find that inflation is more persistent than the models imply. Mankiw (2001, pg. C59) points out that the NKPC “is not at all consistent with the standard stylized facts about the dynamic effects of monetary policy, according to which monetary shocks have a delayed and gradual effect on inflation”. Broadly speaking, experts have fallen into one of two camps on the NKPC’s empirical adequacy.

The first group contends that the original mechanism is largely successful once minor adjustments are made. Galí and Gertler (1999) and Sbordone (2002) state that accurate measures of price dynamics require accurate measures of firms’ marginal costs. They show that the NKPC fares much better if marginal cost is measured by labor share and also present reasonable assumptions that justify the measure theoretically. With the above correction, Galí and Gertler further find that sticky prices combined with a small fraction of backward-looking firms match U.S. and European data very well.

The second group advocates a major overhaul of the NKPC. A few of the alternatives include imperfect common knowledge (Woodford 2003) and sticky information (Mankiw and Reis 2002, Reis 2006). This latter group resuscitates the Fischer’s (1977) wage-contract model by applying it to final good price setting. In Fischer’s model, workers and firms negotiate wage contracts infrequently. Over a contract’s life, wages may change; however, they cannot evolve according to newly available information.

In Mankiw and Reis, on the other hand, firms choose future price plans infrequently, based on currently available information. They further modify Fischer’s model by assuming plans have random rather than fixed durations and motivate the behavior through firms’

¹The existing models at that time included Calvo (1983), Rotemberg (1982), and Taylor (1980).
inattentiveness. Calibrations of their sticky information economy: (i) match the persistence of inflation; (ii) imply costly disinflations; and (iii) generate hump-shaped responses of inflation and output to monetary shocks. For these reasons, they propose to replace sticky prices with sticky information.

Given the above discussion, logical questions are: how do the data view the second group’s proposal to replace sticky prices with sticky information? Alternatively, should sticky price models be abandoned? If not, should sticky information be abandoned?

To answer these questions, this paper integrates these two groups’ alterations in a nested structural analysis. Our structural analysis allows us to simultaneously assess the importance of both group’s models in a generalized framework. Specifically, we present an original model where each firm has two adjustment probabilities every period: a chance to reset its price and an independently distributed chance to update its information. Among firms that reset their prices, a fraction of firms choose its nominal price with new information and the remaining determines the price with stale information. We estimate the model using post-WWII U.S. data. We measure marginal cost using labor share, following a key concern of the first group. Finally, we estimate and compare three alternatives to our “dual stickiness” model. Such alternatives are the pure sticky price and sticky information models, both of which are special cases of our dual stickiness model, and Galí and Gertler’s (1999) hybrid sticky price model. The latter modifies the pure sticky price model by introducing backward-looking firms.

Our findings are summarized as follows. First, our results contravene a wholesale replacement of sticky prices with sticky information. Price stickiness is statistically significant even when we take into account the possible presence of information stickiness. In addition, the sticky price models outperform the pure sticky information model in terms of goodness-of-fit. These results suggest that sticky price models need not be abandoned. Second, the nested structural estimation also finds statistically significant information stickiness, implying that sticky information need not be abandoned either. Third, the dual stickiness model performs as well as the hybrid model in terms of goodness-of-fit and generates extremely similar im-

---

2This contrasts with the model selection tests adopted in the previous studies reviewed later. Unlike them, our framework does not rule out the possibility that both groups provide a better understanding of inflation dynamics.
pulse responses to a monetary policy shock in our general equilibrium model. As such, the
dual stickiness model may provide a more plausible microeconomic foundation for inflation
persistence. Fourth, in our benchmark estimation, each quarter a 15 percent of firms reset a
price and a 39 percent of the firms are attentive. As a consequence, only 5.8 percent of firms
in the economy choose the optimizing price each quarter. Finally, we measure the relative
importance of each stickiness and find that sticky prices have more importance than sticky
information in accounting for inflation dynamics.

Our paper does not address a more recent debate on the importance of future real
marginal cost in explaining current inflation. This recent debate centers on the appro-
priate metric to evaluate the hybrid model. First, Galí and Gertler (1999), Galí, Gertler and
López-Salido (2005) and Sbordone’s (2002) endorsement of the model is based on their sta-
tistically significant estimates of the structural parameters. Second, the criticism by Rudd
and Whelan (2005a, 2005b, 2006) stems from their finding that, in terms of the reduced form
coefficients, there is no statistically significant relationship between future real marginal cost
and inflation. Our paper restricts attention to the structural estimates of the dual stickiness,
hybrid sticky price, pure sticky price, and pure sticky information models. Thus, it is closer
to the former metric. In ongoing research, we are assessing the various models using Rudd
and Whelan’s approach.

Two existing papers also estimate models with both sticky prices and information: Ko-
renok (2005) and Rotemberg and Woodford (1997). While instructive, neither answers the
questions we propose. The former tests the relative importance of the two components via
encompassing tests, suggesting that the sticky prices dominate sticky information. The lat-
ter uses an estimation strategy that cannot separately identify the amount of each stickiness.
Only the former uses labor share – an essential adjustment required by the first group – to
measure marginal cost. We present an expanded discussion of related research in Section 5.

An outline of the rest of the paper follows. Section 2 describes the sticky information
and sticky price models. Section 3 presents our empirical findings. Section 4 compares each
inflation equation in general equilibrium. Section 5 discusses existing research contextually.
The final section concludes.
2 The Pricing Problem under Dual Stickiness

In this section we present our “dual stickiness” model that integrates sticky prices and information. Interestingly, its structure is very similar to that of the hybrid sticky price model. Because the latter is well-known in the literature, we present the hybrid model first.

2.1 The hybrid sticky price model

Consider a continuum of firms engaged in monopolistic competition. Suppose that each firm is \( \text{ex ante} \) identical and faces infrequent price setting. Each firm has the probability \((1 - \gamma)\) to reset its price in each period. Then, the log aggregate price level \( p_t \) evolves according to

\[
p_t = \gamma p_{t-1} + (1 - \gamma) q_t.
\]

Or, equivalently,

\[
\pi_t = (1 - \gamma) (q_t - p_{t-1}) = \frac{1 - \gamma}{\gamma} (q_t - p_t),
\]

where \( q_t \) denotes the index for all newly set prices in period \( t \) and \( \pi_t \) is inflation in period \( t \).

The first equality of (1) states that only newly set prices matter for inflation because other prices are fixed. The second equality gives a relationship between the newly set relative price and overall inflation under sticky prices with random duration. Here, \(-\pi_t = p_{t-1} - p_t\) can be interpreted as the relative price of non-price-setting firms in period \( t \). Given that all firms’ relative prices sum to zero, we must have \(-\gamma \pi_t + (1 - \gamma) (q_t - p_t) = 0\), the second equality of (1).

Galí and Gertler (1999) depart from the pure sticky price model by assuming the presence of two types of firms. A fraction \( 1 - \phi \) of firms adjust its price at \( t \) and set its price optimally. The remaining firms are backward-looking and use a simple rule of thumb. Then, the price index \( q_t \) is expressed as a linear combination of the price set by forward-looking firms \( (p_t^f) \) and the price set by backward-looking firms \( (p_t^b) \):

\[
q_t = (1 - \phi) p_t^f + \phi p_t^b,
\]

(2)
where \( p^f_t \) and \( p^b_t \) are given by
\[
\begin{align*}
p^f_t &= (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t[mc^n_{t+j}] \\
p^b_t &= q_{t-1} + \pi_{t-1},
\end{align*}
\] (3)
(4)

where \( mc^n_t \) is nominal marginal cost in period \( t \).

We substitute (3) and (4) into (2) and then using (1), we obtain
\[
\pi_t = \frac{1 - \gamma}{\gamma} \left\{ (1 - \phi)(1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t[mc^n_{t+j} - p_t] + \phi \left( \frac{1}{1 - \gamma} \pi_{t-1} - \pi_t \right) \right\}.
\]

Solving this equation for \( \pi_t \) yields
\[
\pi_t = \tilde{\rho} \pi_{t-1} + \zeta_1 (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t[mc^n_{t+j} - p_t],
\] (5)

where \( \tilde{\rho} = \phi/(\phi + \gamma - \gamma\phi) \) and \( \zeta_1 = (1 - \phi)(1 - \gamma)/(\phi + \gamma - \gamma\phi) \).

Two factors determine inflation. First, due to the backward-looking firms, lagged inflation appears. The second term implies that inflation depends on the discounted sequence of future nominal marginal costs deflated by the current price level. This is due to forward-looking firms.

### 2.2 The dual stickiness model

Suppose each firm faces infrequent information updating as well as price setting. We assume that each firm adjusts its price with a constant probability \( 1 - \gamma \) and then sets its price optimally using all available information with a probability \( 1 - \phi \).

\[ \text{5} \] The remaining firms have old information, given the timing of information updating. Thus, the law of large numbers implies a fraction \( 1 - \phi \) of firms that adjust prices in period \( t \) set their price optimally

---

3 We set the discount factor to unity for simplicity.

4 Above, we use the fact that \( p^h_t - p_t = q_{t-1} + \pi_{t-1} - p_t = (q_{t-1} - p_{t-1}) + \pi_{t-1} - \pi_t = \frac{1}{1 - \gamma} \pi_{t-1} - \pi_t \).

5 Here, \( 1 - \phi \) should be interpreted as the probability of information updating regarding the state of the aggregate economy. A firm might be able to frequently update information on its narrow line of business (e.g., prices set by close competitors), but information regarding the state of the aggregate economy (e.g., aggregate price level) can be updated only infrequently.
while the remaining firms adjust using stale information. For simplicity, we assume each ‘stickiness’ is independent of the other.\footnote{We assume the independence of the probabilities for tractability. While relaxing the independence assumption may be an important extension, it is beyond the scope of the paper.}

Whereas the set of assumptions permits us to use (1) - (3), we depart from Galí and Gertler (1999) by replacing backward-looking firms with inattentive firms. They have outdated information but otherwise set their price optimally. In particular, \( p_t^b \) under the dual stickiness model is given by
\[
\begin{align*}
  p_t^b &= (1 - \phi) \sum_{k=0}^{\infty} \phi^k E_{t-k-1}[p_t^f], \\
  \text{instead of } (4). \quad \text{Note that } p_t^b \text{ consists of individual optimal prices conditional on stale information. It is a weighted average of each individual price for inattentive firms with a } k+1 \text{ period old information set for } k = 0, 1, 2, ... .
\end{align*}
\]

Note that firms with different information sets are distributed as in Mankiw and Reis (2002). Combining (2) and (6), we obtain
\[
q_t = (1 - \phi) \sum_{k=0}^{\infty} \phi^k E_{t-k}[p_t^f].
\]

Thus, the formulation of the price index \( q_t \) is identical to the sticky information model by Mankiw and Reis (2002), except that each individual price is determined in a forward-looking manner.

The dual stickiness model is similar to the hybrid model. Using the fact that \( mc_{t+j}^n = \Delta mc_{t+j}^n + mc_{t+j-1}^n \), (6) can be expressed as
\[
\begin{align*}
  p_t^b &= q_{t-1} + (1 - \phi) \sum_{k=0}^{\infty} \phi^k \left\{ (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_{t-k-1}[\Delta mc_{t+j}^n] \right\}. \\
  \text{Comparing (4) and (7), the only difference is in the second term. While the second term of (4) is lagged inflation, the corresponding term in (7) is the cross-sectional (weighted) average of the discounted sum of expected nominal marginal cost growth, conditional on old information. When the cross-sectional average conditional on old information is close to lagged inflation, both models’ inflation dynamics will be similar.}
\end{align*}
\]
Using (1), (2), (3), and (7) to eliminate $q_t$, $p_{ft}^f$, and $p_{bt}^b$, we obtain the equation we estimate:

$$\pi_t = \rho \pi_{t-1} + \zeta_1 (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t [mc_{t+j}^n - p_t]$$

$$+ \zeta_2 (1 - \phi) \sum_{k=0}^{\infty} \phi^k (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_{t-k-1} [\Delta mc_{t+j} + \pi_{t+j}],$$

(8)

where $\rho = \gamma \tilde{\rho}$ and $\zeta_2 = \phi (1 - \gamma) / (\gamma + \phi - \gamma \phi)$. We used $\Delta mc_{t+j}^n = \Delta mc_{t+j} + \pi_t$, where $mc_{t+j}$ is real marginal cost given by $mc_t = mc_t^n - p_t$.

This equation has two similarities with (5). First, lagged inflation matters for current inflation. The effect of the lagged inflation on current inflation is slightly weaker under the dual stickiness model than the hybrid model because $\tilde{\rho} \geq \rho$. Second, both models have the discounted sum of nominal marginal costs deflated by the current price level in the second term of the equation.

However, an important difference allows us to distinguish the two. The dual stickiness model has another source of inflation inertia. The third term on the right hand side of (8) contributes to inflation inertia through information delay. This additional term is a key identifying feature of the dual stickiness relative to the hybrid model. Unless the effect of this term offsets the difference in the effects of lagged inflation (i.e., the difference between $\tilde{\rho}$ and $\rho$), the inflation dynamics in the dual stickiness model are different from those in the hybrid model.

Note that the dual stickiness model nests the pure sticky information model as well as the pure sticky price model. Suppose that there are no inattentive firms ($\phi = 0$). Then, the dual stickiness model reduces to the pure sticky price model. Alternatively, suppose that firms can reset prices every period with probability one ($\gamma = 0$). Then, (8) reduces to the sticky information Phillips curve:

$$\pi_t = \frac{1 - \phi}{\phi} mc_t + (1 - \phi) \sum_{j=0}^{\infty} \phi^k E_{t-k-1} [\Delta mc_t + \pi_t].$$

(9)

This generalization allows us to compare the dual stickiness model to alternative pricing models via the statistical significance of structural parameters.
3 Empirical Implementation

We estimate (5) and (8) using the two step approach proposed by Sbordone (2002), Woodford (2001), and Rudd and Whelan (2005a). In the first step, we run a vector-autoregression (VAR) to obtain the predicted series of labor share and inflation. Given the VAR process, the second step minimizes the variance of a distance between the model’s and actual inflation. Our estimated parameters are the probability of resetting a price $1 - \gamma$ and of updating information $1 - \phi$ (or the fraction of forward-looking firms).

Our two step approach differs slightly from the previous studies in that we do not estimate the closed form solution to the aggregate Euler pricing equation.\footnote{Sbordone (2002) transforms the standard NKPC into the closed form solution for the logarithm of the price-unit labor cost ratio and estimates the parameters of interest. Woodford (2001) and Rudd and Whelan (2005a) rewrite the NKPC as the forward-looking solution for inflation and estimate parameters by minimizing the distance between the models’ and actual inflation.} We use non-closed form equations (5) and (8) to estimate structural parameters.\footnote{We estimate the pure sticky price and pure sticky information models by putting restrictions on (8): $\phi = 0$ for the former and $\gamma = 0$ for the latter.} There are two reasons for using these non-closed form equations. First, it is generally impossible to derive the closed form solution to the dual stickiness model, due to infinitely many lagged expectation terms in (8). Second, while it is straightforward to derive the closed form for the hybrid model as in the previous studies, the transformation of the hybrid model to the closed form changes the estimation equation to a form incomparable to the dual stickiness model. As such, comparisons between the dual stickiness model and its alternatives could be unfair if we use the closed form equation for the hybrid model.

The details of our estimation procedure are as follows. First, we specify the forecasting model by introducing the vector $X_t$ in the following VAR:

$$X_t = AX_{t-1} + \epsilon_t.$$ \hspace{1cm} (10)

In the benchmark case, the vector $X_t$ includes labor share, inflation and the output gap. We include the output gap in $X_t$ because it has strong forecasting power for labor share and inflation as Rudd and Whelan (2005a) show. The vector $X_t$ also includes lags of the
three variables. In general, \( X_t \) is given by a \((3p \times 1)\) vector of \([x'_t, x'_{t-1}, \ldots, x'_{t-p+1}]'\), where \( x_t = [mc_t, \pi_t, y_t]' \) and \( y_t \) is the output gap.

Next, we calculate a series of theoretical inflation given the forecasting process (10). Ordinary least squares produces a consistent estimate of the coefficient matrix \( \hat{A} \). Let \( e_{mc} \) and \( e_{\pi} \) denote the selection vectors with 3\( p \) elements. All elements are zero except the first element of \( e_{mc} \) and the second element of \( e_{\pi} \), which are unity. Given the definitions, we express labor share and inflation as \( e'_{mc}X_t \) and \( e'_{\pi}X_t \), respectively.

For expositional purposes, consider a special case with \( \gamma = 0 \) (i.e., (9)). Given the definitions of selection vectors, \( E_{t-k-1}(\Delta mc_t + \pi_t) = (e'_{mc}(A - I) + e'_{\pi}A)A^kX_{t-k-1} \). Then, (9) can be written as

\[
\pi^m_t(\theta, A) = \frac{1 - \phi}{\phi}mc_t + (1 - \phi)(e'_{mc}(A - I) + e'_{\pi}A)\sum_{k=0}^{\infty} \phi^k A^k X_{t-k-1},
\]

where \( \pi^m_t(\theta, A) \) denotes the inflation predicted by the model and \( \theta \) denotes the parameter vector to be estimated. In this particular case, \( \theta = \phi \). By introducing an arbitrary truncation value of \( K \), we approximate this equation by

\[
\pi^m_t(\theta, A) = \frac{1 - \phi}{\phi}mc_t + (1 - \phi)(e'_{mc}(A - I) + e'_{\pi}A)\sum_{k=0}^{K-1} \phi^k A^k X_{t-k-1}.
\]

When the model explains the data well, \( \pi^m_t(\theta, A) \) is close to actual inflation. Using a consistent estimate \( \hat{A} \), we choose the parameter \( \theta \) by:

\[
\hat{\theta} = \text{Argmin}_{\theta} \text{var}(\pi_t - \pi^m_t(\theta, \hat{A})).
\]

We use the same procedure to estimate (5) and (8). Given the VAR process, the series of \( \{X_{t-k}\}_{k=0}^{\infty} \) suffices to express all discounted sums in (5) and (8). Appendix A proves (5) and (8) can be respectively expressed as

\[
\pi^m_t(\theta, A) = \rho \pi_{t-1} + b'X_t,
\]

\[
\pi^m_t(\theta, A) = \rho \pi_{t-1} + b'X_t + c' \sum_{k=0}^{\infty} \phi^k A^k X_{t-k-1},
\]

where \( b' = \zeta_1[(1 - \gamma)e'_{mc} + \gamma e'_{\pi}A][I - \gamma A]^{-1} \) and \( c' = \zeta_2(1 - \gamma)(1 - \phi)[e'_{mc}(A - I) + e'_{\pi}A][I - \gamma A]^{-1} \). The parameter vector here is \( \theta = [\gamma, \phi]' \). Once again, we choose an arbitrary large
truncation parameter $K$ and minimize the variance of the distance between model and actual inflation.

To make statistical inferences, we use a bootstrap method because the forecasted variables in the second step are “generated regressors” and thus the standard asymptotic standard errors calculated from nonlinear least squares are incorrect. A bootstrap method is useful for making statistical inferences rather than corrected asymptotic standard errors because of the complicated estimation equation (15).

To conduct the bootstrap, we first generate 9999 bootstrapped series of $X_{i,t}^*$ from the empirical distribution of the residual $\hat{\epsilon}_t$ and the coefficient estimate $\hat{A}$ in (10). Using the resampled $X_{i,t}^*$, we estimate structural parameters $\theta_i$ by minimizing the variance of $\pi_{i,t}^* - \pi_{i,t}^{*m}(\theta_i, \hat{A})$ for $i = 1, 2, ..., 9999$. We compute the covariance matrix of $\hat{\theta}_i$.\(^9\)

We use quarterly U.S. data between 1960:Q1 and 2005:Q2. Inflation is measured as the log difference of the implicit GDP deflator. Labor share, which measures marginal cost, is the log of (NFB unit labor cost/NFB price deflator). The output gap is the quadratically detrended real GDP.

### 3.1 Benchmark results

Table 1 shows the estimation results. In our benchmark case, we use the VAR with three lags and the truncation parameter of $K = 12$. We report the estimates from four models: i) the dual stickiness model (DS); ii) the hybrid sticky price model (Hybrid); iii) the pure sticky price model (SP); and iv) the pure sticky information model (SI). The 95 percent confidence intervals appear in brackets.

Five features in Table 1 are worth emphasizing. First, both probabilities are significantly different from zero under the dual stickiness model (Row 1), suggesting that both types of stickiness matter for the aggregate inflation dynamics. Our benchmark case suggests that 11-20 percent of firms change prices every quarter, but only 12-54 percent of these firms change prices every quarter.

\(^9\)MacKinnon (2002) gives detailed explanations for the bias-corrected bootstrap intervals. To obtain 95 percent confidence intervals of estimates, we compute the bias-corrected bootstrap interval $[2\hat{\theta} - \bar{\theta}^* - 1.96s^*_\theta, 2\hat{\theta} - \bar{\theta}^* + 1.96s^*_\theta]$, where $\hat{\theta}$, $\bar{\theta}^*$, and $s^*_\theta$ denote the original estimate from the actual data, the sample mean of the bootstrap estimates $\hat{\theta}^*_i$, and the standard deviation of $\hat{\theta}^*_i$, respectively.
<table>
<thead>
<tr>
<th></th>
<th>γ</th>
<th>φ</th>
<th>ρ or ̃ρ</th>
<th>$\bar{R}^2$</th>
<th>var($\pi_t - \hat{\pi}_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS</td>
<td>0.8510</td>
<td>0.6078</td>
<td>0.5493</td>
<td>0.8311</td>
<td>0.0631</td>
</tr>
<tr>
<td></td>
<td>[0.8034, 0.8916]</td>
<td>[0.4624, 0.8818]</td>
<td>[0.4319, 0.7756]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hybrid</td>
<td>0.8675</td>
<td>0.5370</td>
<td>0.5721</td>
<td>0.8336</td>
<td>0.0622</td>
</tr>
<tr>
<td></td>
<td>[0.8330, 0.8996]</td>
<td>[0.4132, 0.7746]</td>
<td>[0.4478, 0.8124]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP</td>
<td>0.8742</td>
<td>0.00</td>
<td>0.00</td>
<td>0.7024</td>
<td>0.1112</td>
</tr>
<tr>
<td></td>
<td>[0.8389, 0.9079]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SI</td>
<td>0.00</td>
<td>0.8933</td>
<td>0.00</td>
<td>0.6491</td>
<td>0.1311</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.8553, 0.9230]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTES: Estimation from 1960:Q1 to 2005:Q2. The trivariate VAR(3) ($[m_{ct}, \pi_t, y_t]'$) is estimated over 1957:Q2 - 2005:Q2 in the first step. The 95 percent bootstrap confidence intervals are in brackets. The parameter $\gamma$ denotes the probability of price fixity. The parameter $\phi$ denotes the probability of information fixity in the dual stickiness model or the fraction of backward-looking firms. The $R$-squareds in the fourth column are uncentered in order to assure that they take nonnegative values under our specifications without a constant term. The last column, var($\pi_t - \hat{\pi}_t$), is the variance of the distance where $\pi_t^m(\theta, \hat{\theta})$ is evaluated at the obtained estimates. DS, Hybrid, SP, and SI stand for the dual stickiness model, the hybrid sticky price model, the pure sticky price model, and the pure sticky information model, respectively.
use the latest information to determine prices. Evaluated at the point estimates, the former is 15 percent and the latter is 39 percent, suggesting that only 5.8 percent in the economy choose the optimizing price.\(^1\)

Second and interestingly, the estimated parameters $\gamma$ and $\phi$ under the dual stickiness model are quite close to the estimated parameters under the hybrid model, regardless of the different interpretations for $\phi$ (Rows 1 and 2). Indeed, there is no substantial difference in these parameters including the coefficients of lagged inflation. As we expect, the effect of lagged inflation on current inflation is slightly weaker under the dual stickiness model (0.55) than under the hybrid model (0.57). In addition, the slightly higher $\phi$ in the dual stickiness model (0.60) may explain the remaining part of inflation inertia that lagged inflation in the hybrid model can explain.\(^1\)

Third, the frequency of information updating $1 - \phi$ under the dual stickiness model is somewhat high relative to that in the pure sticky information model and those in previous studies (where prices themselves are completely flexible). Our estimate of the information updating probability is about 39 percent. On the other hand, the probability of information updating is about 11 percent in the pure sticky information model (Row 4). Other studies such as Kahn and Zhu (2006) estimate the probability in the range of 12 to 35 percent, using

\(^{10}\)Our estimates of probabilities of price change are larger than those found in studies using micro data, for example, Bils and Klenow (2004). Here we do not explore this inconsistency with micro data results, which appear in previous studies including Galí and Gertler (1999), except pointing out the following remark. According to our preliminary results, if the presence of real rigidity is taken into account in the estimation by artificially scaling down the movement of labor share, estimates of the probabilities of price change for each model will be much higher. Thus, one possible explanation for the discrepancy in micro- and macro-results may be the failure of macro studies to consider the presence of real rigidity.

\(^{11}\)Our hybrid model results are consistent with Galí and Gertler (1999) and Galí, Gertler, and López-Salido (2005). They emphasize that the key parameters for properly assessing the importance of forward- versus backward-looking behavior are $\gamma_f$ and $\gamma_b$, which are a function of $\gamma$ and $\phi$. (Specifically, $\gamma_f = \gamma/(\gamma + \phi)$ and $\gamma_b = \phi/(\gamma + \phi)$ when the discounted factor equals unity.) While they conclude $\gamma_f$ is roughly 0.65 and $\gamma_b = 0.35$, our estimates imply $\gamma_f = 0.62$ and $\gamma_b = 0.38$, even with our different estimation strategy. In addition, the influence of backward-looking behavior is statistically significant and improves the goodness-of-fit of the pure sticky price model. This finding is also consistent with Galí and Gertler (1999) and many others.
forecasted series for the output gap. Under the marginal cost version of the sticky information model, the estimate is roughly 15 percent in Andrés, López-Salido, and Nelson (2005) and 20 percent in Korenok (2005).\textsuperscript{12} Our relatively high frequency of information updating could be interpreted as arising from the substitution of price stickiness for information stickiness.

Fourth, we find that price stickiness substitutes for information stickiness substantially but the latter does so for the former only modestly. In particular, if one extends the sticky information model to the dual stickiness model, the reduction of the information fixity is about 28 percentage points (from 0.89 to 0.61). On the other hand, if one extends the pure sticky price model to the dual stickiness model, the reduction of the price fixity is only 2 percentage points (from 0.87 to 0.85). The low substitution may suggest that sticky prices cannot be replaced with sticky information.

Finally, we assess the models’ goodness-of-fit with the (uncentered) adjusted $R$-squared (Column 4).\textsuperscript{13} The ordering of the models in terms of the goodness-of-fit is the hybrid, the dual stickiness, the pure sticky price, and the pure sticky information model. Overall, the models with lagged inflation (the hybrid and dual stickiness models) perform similarly. A comparison between the pure sticky price and information models favors the pure sticky price model. The latter result is consistent with Korenok (2005).

The assessment can be done visually by looking at the path of the models’ inflation. Figures 1-4 plot actual inflation and inflation predicted by the four models between 1960:Q1 and 2005:Q2. The figures demonstrate that the inflation series generated by the hybrid and dual stickiness models track actual inflation closely while those generated by the pure sticky price and information models do so only roughly. Not surprisingly, the former two models’ success stems from the presence of lagged inflation.

However, the source of lagged inflation differs between the two. The hybrid model intro-

\textsuperscript{12}There are several reasons why our estimate from the pure sticky information Phillips curve differs from the previous studies – especially Kahn and Zhu (2006). First, we use different specifications of VARs. Second, we use labor share rather than the output gap. Third, we use in-sample forecasts of inflation and labor share rather than out-of-sample forecasts of inflation and the output gap.

\textsuperscript{13}Since the regressors in our estimation equation do not include a constant, we use the “uncentered” adjusted $R$-squared instead of the standard “centered” one. That is, the variations are measured in terms of the second moment around zero, not around mean as in the centered $R$-squared.
Figure 1: The inflation predicted by the pure sticky price model (SP)

Figure 2: The inflation predicted by the hybrid sticky price model (Hybrid)
Figure 3: The inflation predicted by the pure sticky information model (SI)

Figure 4: The inflation predicted by the dual stickiness model (DS)
duces backward-looking firms in order to improve its empirical fit.\textsuperscript{14} On the other hand, our dual stickiness model endogenously generates lagged inflation in its pricing Euler equation by combining price and information stickiness. Here, lagged inflation arises from the presence of inattentive but otherwise forward-looking rational firms, and, as in the hybrid model, it contributes to the good performance of the model.

### 3.2 Relative importance of information and price stickiness

Using our estimation results, we next quantify the relative importance of information and price stickiness. We compute the percentage reduction in the variance of the distance between model and actual inflation when we add another type of stickiness into either the pure sticky price or information model. The results are summarized in Table 2. For example, we can see from Table 1 that the variance of the pure sticky price model is 0.11. If one adds sticky information into the pure sticky price model, the model turns out to be the dual stickiness model whose variance is 0.06. Hence, the percentage reduction in the variance is \(-(0.06-0.11)/0.11\) \(\simeq 43\) percent. In other words, adding sticky information contributes to 43 percent reduction in the variance of residuals in the pure sticky price model. On the other hand, one can also see from Table 1 that adding sticky prices into the sticky information model reduces the variance of the sticky information model by about 52 percent. While both types of stickiness play a non-negligible role for the aggregate inflation dynamics, adding sticky prices beats adding sticky information in terms of the percentage reduction in the variance of residuals.

\textsuperscript{14}Backward-looking firms are theoretically unappealing as Galí and Gertler (1999) acknowledge.
3.3 Sub-sample analysis

As a robustness check, we estimate the models over different sub-samples, shown in Table 3.\textsuperscript{15} Our estimation strategy implicitly assumes that the unrestricted forecasting process of $X_t$ (i.e., the coefficient matrix $A$) is invariant over the whole sample. However, this assumption could be questionable, because a shift in policy alters the dynamic path of a macroeconomic variable in its reduced form and affects the economic agents’ forecasts. As such, we consider the following three sub-samples: 1960:Q1 - 1982:Q4, 1979:Q4 - 2005:Q2, and 1983:Q1 - 2005:Q2. The first and last sub-samples represent high inflation and low inflation eras in the U.S. economy. The second sub-sample reflects a major policy shift by Federal Reserve Chairman Paul Volcker in October 1979.

Table 3 shows the stability of estimated probabilities in the dual stickiness model. Interestingly, price stickiness is insensitive to the sub-sample analysis. Information stickiness is somewhat lower over the sub-samples than over the full sample. However, because the differences are statistically insignificant, we conclude that the change in the coefficient matrix of the VAR does not significantly affect information stickiness.

The parameter estimates in the hybrid model also have stable patterns. Again, the estimates of the fraction of backward-looking firms are lower over the sub samples than over the full sample, but price stickiness is robust to the sub sample analysis.\textsuperscript{16}

4 General Equilibrium Comparisons

This section compares the dual stickiness model with the hybrid sticky price and pure sticky information models by placing each inflation equation in a simple dynamic general equi-

\textsuperscript{15} We also applied our robustness analysis to the truncation parameter $K$ and alternative specifications of the VAR such as the lag length and the inclusion of the federal funds rate and term spread. These robustness checks revealed that parameter estimates and the goodness-of-fit remain essentially unaltered. In particular, as for $K$, the estimates remain unaltered unless $K$ is small (e.g., $K \leq 3$). The increase in the lag length of the VAR enhances the performance of the pure sticky price model slightly, but the adjusted $R$-squareds of the other three models remain unaltered.

\textsuperscript{16} The implied importance measures of backward- and forward-looking behavior are 0.71 and 0.29 over the three sub-samples, respectively. See the footnote 11 for the definition of the importance measures.
<table>
<thead>
<tr>
<th>Sub-sample</th>
<th>$\gamma$</th>
<th>$\phi$</th>
<th>$\rho$ or $\hat{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960:Q1 - 1982:Q4</td>
<td>0.8228</td>
<td>0.3389</td>
<td>0.3158</td>
</tr>
<tr>
<td></td>
<td>[0.5178, 1.0242]</td>
<td>[0.1048, 0.7373]</td>
<td>[0.1099, 0.6733]</td>
</tr>
<tr>
<td>1979:Q4 - 2005:Q2</td>
<td>0.8818</td>
<td>0.3843</td>
<td>0.3655</td>
</tr>
<tr>
<td></td>
<td>[0.8475, 0.9123]</td>
<td>[0.2655, 0.8535]</td>
<td>[0.2532, 0.8080]</td>
</tr>
<tr>
<td>1983:Q1 - 2005:Q2</td>
<td>0.9126</td>
<td>0.4177</td>
<td>0.4017</td>
</tr>
<tr>
<td></td>
<td>[0.8946, 1.0090]</td>
<td>[0.0360, 0.7204]</td>
<td>[0.0815, 0.6779]</td>
</tr>
<tr>
<td>Hybrid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1960:Q1 - 1982:Q4</td>
<td>0.8289</td>
<td>0.3163</td>
<td>0.3582</td>
</tr>
<tr>
<td></td>
<td>[0.7829, 0.8678]</td>
<td>[0.1066, 0.6675]</td>
<td>[0.1319, 0.7455]</td>
</tr>
<tr>
<td>1979:Q4 - 2005:Q2</td>
<td>0.8856</td>
<td>0.3567</td>
<td>0.3851</td>
</tr>
<tr>
<td></td>
<td>[0.8555, 0.9155]</td>
<td>[0.2507, 0.7724]</td>
<td>[0.2692, 0.8306]</td>
</tr>
<tr>
<td>1983:Q1 - 2005:Q2</td>
<td>0.9152</td>
<td>0.3874</td>
<td>0.4086</td>
</tr>
<tr>
<td></td>
<td>[0.9003, 0.9958]</td>
<td>[0.0611, 0.6598]</td>
<td>[0.0680, 0.6822]</td>
</tr>
</tbody>
</table>

librium framework. We simulate the impulse responses to a monetary policy shock under each estimated inflation equation. This impulse response analysis allows us to understand importance of the two nominal rigidities from a different viewpoint than that of the previous section. We ask whether integration of sticky prices into the sticky information model improves or worsens the dynamic properties of inflation and output. We also ask, as our empirical evidence suggests, whether the dual stickiness model generates inflation and output dynamics similar to those under the hybrid model.

Erceg, Henderson and Levin’s (2000) sticky wage and price model will serve as the economic environment. We include sticky wages to increase inertia in marginal cost. Adding sticky wages also separates labor share from the output gap—which Galí and Gertler (1999) and Sbordone (2002) find important empirically.\footnote{We also consider the flexible wage case.} To conserve space, we present the log-linearized equations from their model rather than a complete description of the consumer’s and firm’s problems and market clearing conditions.

Let $w_t$, $\xi_t$ and $\Delta m_t$ denote nominal wage inflation, the real wage and the nominal money growth rate. Excluding the price inflation equation, the system consists of

\begin{align*}
\Delta m_t - \Delta y_t &= \pi_t, \\
\Delta m_t &= \delta \Delta m_{t-1} + \varepsilon_t, \\
\Delta \xi_t &= w_t - \pi_t, \\
w_t &= E_t (w_{t+1}) + \kappa (s_t - \xi_t).
\end{align*}

Equation (16) is the growth rate form of the quantity theory of money (or cash-in-advance constraint). Equation (17) is the monetary policy rule. Money growth is first-order autoregressive with a white noise innovation. Equation (18) states that an increase in the real wage is equal to wage inflation net of price inflation.

The final expression determines nominal wage growth. Here, $s_t$ is the household marginal rate of substitution between consumption and leisure; therefore, $s_t - \xi_t$ is the household’s wedge between return to and cost of supplying labor.
If: (i) period utility depends only on current consumption and leisure, and (ii) production depends solely on labor, then the marginal rate of substitution is a function of only output, \( s_t = \theta y_t \). Our baseline parameters for system are: \( \kappa = 0.02, \theta = \delta = 0.5 \).

Only the price inflation equation remains unspecified. We consider inflation equations from the three models. We hold fixed equations (16)-(19) and use the point estimates from Table 1 to compute impulse responses.\(^{18}\)

In post-WWII U.S. data, Christiano, Eichenbaum and Evans (2005) and others use a recursive VAR to study the economy’s response to a monetary shock. In response to an expansionary shock, output gradually increases and then peaks after approximately six quarters. Inflation also increases gradually and then peaks after approximately eight quarters.\(^{19}\)

Figure 5 plots inflation and output responses with sticky wages. The dual stickiness and hybrid inflation responses are nearly identical along their entire paths. Each peaks in the fourth quarter and then gradually declines. The pure sticky information model fails to generate hump-shaped inflation. Relative to the other two models, its inflation response is smaller.\(^{20}\)

The output responses, again under sticky wages, also have the same qualitative dynamics. All three output impulses have the same response on impact, are hump-shaped, and peak in the third quarter. As above, the dual stickiness and hybrid responses are nearly identical along their entire paths. The pure sticky information response lies slightly below the other two.

Figure 6 contains the corresponding impulses with flexible wages. Both the inflation and output impulses are hump-shaped in each case. The largest difference between the three is the pure sticky information inflation response. Although the response on impact

\(^{18}\)Each price inflation equation depends upon marginal cost \((mc)\). In our equilibrium model, \( mc_t = \xi_t - \mu y_t \) and \( \mu \) is the elasticity marginal product of labor with respect to output. Based on Erceg et al. (2000), we set \( \mu = 0.43 \).

\(^{19}\)These qualitative findings have been documented by other researchers, including Estrella and Fuhrer (2002), Galí (1992) and Stock and Watson (2001).

\(^{20}\)Keen (2005) points out that the sticky information model is sensitive to parameterizations and assumptions about labor markets. The reduced hump-shaped responses under the pure sticky information model seem to arise from our assumption of a homogeneous labor market.
Figure 5: Impulse responses to a monetary shock under sticky wages

NOTES: Impulse responses for inflation (the upper panel) and output (the lower panel) to a one percent innovation to the money growth rate.
Figure 6: Impulse Responses to a monetary shock under flexible wages

NOTES: Impulse responses for inflation (the upper panel) and output (the lower panel) to a one percent innovation to the money growth rate.
is the same for all three, the second and third quarter inflation response is larger for the pure sticky information by approximately ten percent. Then, the pure sticky information inflation response falls below the other two models’.

The three main messages of the exercise are that: (i) the dual stickiness model – the integration of sticky prices with the sticky information model – does not worsen the dynamic properties of the pure sticky information model; (ii) the dual stickiness model delivers nearly identical dynamic properties as the hybrid model; (iii) under our parameterizations, however, none of the model capture the six to eight quarter peak found in the data.

5 Interpretations and Related Research

Recent empirical studies on sticky information compare it with sticky prices.\textsuperscript{21} For example, Korenok (2005), Laforte (2005), Kiley (2005), and Coibion (2006) estimate the two models separately and make statistical comparisons. Korenok (2005) uses a Bayesian version of the full information likelihood approach for his empirical comparisons. Laforte (2005) uses a Bayesian approach to estimate a medium-scale dynamic general equilibrium model and compares the two models by posterior odds – a measure of goodness-of-fit in the Bayesian approach. Kiley (2005) estimates various sticky price and information mechanisms via a maximum likelihood technique and compares the log-likelihoods.\textsuperscript{22} Using the classical regression approach, Coibion (2006) does non-nested hypothesis testing by artificial nesting. Although they employ different estimation strategies, their empirical comparisons overall favor sticky prices.

Our empirical results are not only consistent with their findings but also more suggestive than theirs. The novelty of our approach allows us to assess the importance of both models

\textsuperscript{21}This direction of research has been also taken theoretically. Since Mankiw and Reis (2002), many studies have compared the sticky prices models with the pure sticky information model by simulating predictions of inflation and output from their dynamic general equilibrium models. A few of examples include: Devereux and Yetman (2003), Trabandt (2006), and Keen (2005).

\textsuperscript{22}Kiley (2005) finds the pure sticky information model fits slightly better than the pure sticky price model. However, his empirical maximum likelihoods suggest the hybrid model fares better than the pure sticky information model.
simultaneously. In contrast to the above studies, our approach also explores the possibility that both models provide a better understanding of inflation dynamics. Indeed, we find that both rigidities affect firm’s pricing decision. This conclusion cannot be reached by the previous approaches.

While the previous studies and our comparisons cast doubt on Mankiw and Reis’ proposal to replace sticky prices with sticky information, it does not necessarily mean that sticky information is not useful in explaining inflation dynamics. In fact, there is evidence for stickiness in expectations: Carroll (2003) uses expectations survey data to investigate an epidemiological model of information transmission; Mankiw, Reis, and Wolfers (2003) also study survey data on inflation expectations and conclude that people do not have the same information set or form the same expectations; Kahn and Zhu (2006) estimate a sticky information model for the United States and conclude that the data rejects the hypothesis of no information delay. While they are instructive and encouraging, none serve as direct evidence against sticky prices.

Rudd and Whelan (2005a, 2006) find that rational expectations sticky price models provide very poor predictions of inflation dynamics and conclude that the departing from the rational expectations framework is a promising step for future research (Rudd and Whelan, 2006, pg. 319). This does not necessarily imply that price stickiness itself is rejected by the data. We view Rudd and Whelan’s findings as supportive for our dual stickiness model.

We interpret dual stickiness – the presence of sticky prices and information – as a realistic description of inflation process. Micro evidence of price-setting behavior suggest the presence of price stickiness. (e.g., Blinder, Canetti, Lebow and Rudd (1998), Bils and Klenow (2004), Klenow and Kryvtsov (2005) and others), although they have the mixed results on average duration of price stickiness, time- versus state-dependent price setting and the degree of heterogeneous price setting behavior.\(^{23}\) On the other hand, survey expectations of aggregate inflation hardly pass a strict test of rationality, implying that inflation forecasts may not

\(^{23}\)The dual stickiness model has an advantage over the pure sticky information model in terms of micro evidence. Under the pure sticky information model, firms can change their price continuously according to predetermined plans. The micro data does not support this assumption, but instead suggests that firms keep their price unchanged for at least several months (see Woodford 2006 and Blanchard and Galí 2005). The dual stickiness model does not suffer from that problem due to the presence of sticky prices.
make efficient use of all available information. For example, using the Blue Chip dataset, Batchelor and Dua (1991) find that the consensus inflation and many individual inflation forecasts fail to pass strict rationality tests. They also find that the rejection of strict rationality tests is more prominent in forecasting inflation than other variables such as real GDP and unemployment rate.

6 Conclusion

The paper’s title posed the question, “Do Sticky Prices Need to Be Replaced with Sticky Information?” One mechanism that previous researchers (notably Mankiw and Reis, 2002) have offered as a replacement for sticky prices is sticky information. Using comparable data series, time periods, and estimation strategies, we found that both sticky prices and information have an importance in inflation dynamics. Thus, the direct answer to our original question is that sticky price models need not be abandoned. However, we should not abandon sticky information either because sticky information also plays an important role for inflation dynamics.

In addition, we found that the dual stickiness and the hybrid sticky price models perform equally well in terms of goodness-of-fit and implied general equilibrium dynamics. We demonstrated that the two models generate extremely similar inflation dynamics on several dimensions.

The dual stickiness model’s main advantage is that it does not rely on backward-looking behavior. In our view, the strong empirical support for the presence of backward-looking firms can be interpreted as the presence of inattentive sticky information firms. Every firm updates its information infrequently, approximately once every seven months according to our estimates. As such, dual stickiness may provide more plausible microeconomic foundations.

Our dual stickiness model could be extended realistically to allow for correlation between the likelihood of information and price updating. It is natural to believe that firms often change their prices in the same period that they receive new information. We conjecture

\[24\text{See Thomas (1999) for a detailed survey.}\]
that the fit of the dual stickiness model would improve with this extension.

More fundamentally, one weakness of our dual stickiness model is that it approximates reality by assuming random and exogenous chances of price change and information updating. Gertler and Leahy (2005) recently develop an analytically tractable state-dependent Phillips curve, which is extremely close to the standard NKPC à la Calvo. Reis (2006) develops a more micro-founded model of sticky information without sticky prices. Developing a model to integrate the state-dependent sticky prices and state-dependent sticky information may be an important direction towards more solid micro-foundations.

Integrating real rigidities into the model is also a promising direction. This may improve our slightly low estimated probabilities of price change and information updating. In addition, our model can be extended to jointly estimate nominal price and wage stickiness under infrequent information updating. Finally, monetary policy analysis under dual stickiness is an important step for future research.

A Derivation of (14) and (15)

This appendix derives (14) and (15) from the aggregate Euler equation (5) and (8), given the VAR process (10). In what follows, we focus on (15).

The second term of the right-hand side of (8) is

$$
\zeta_1 (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t [mc^n_{t+j} - p_t].
$$

(20)

Note that

$$
mc^n_{t+j} - p_t = \begin{cases} 
mc_t & \text{for } j = 0 \\
mc_{t+j} + \pi_{t+1} + \cdots + \pi_{t+j} & \text{for } j \geq 1
\end{cases}
$$

(21)

Our definition of the selection vectors $e_{mc}$ and $e_\pi$ implies

$$
\zeta_1 (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t [mc^n_{t+j} - p_t]
$$

$$
= \zeta_1 (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t [e^\prime_{mc} X_{t+j}] + \zeta_1 (1 - \gamma) \sum_{j=1}^{\infty} \gamma^j e^\prime_\pi E_t [X_{t+1} + X_{t+2} + \cdots + X_{t+j}]
$$

$$
= \zeta_1 (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j [e^\prime_{mc} A^j X_t] + \zeta_1 (1 - \gamma) \sum_{j=1}^{\infty} \gamma^j e^\prime_\pi (A + A^2 + \cdots + A^j) X_t.
$$
The first term of this equation equals \( \zeta_1(1 - \gamma)e'_mc[I - \gamma A]^{-1}X_t \). One can write the second term as follows:

\[
\zeta_1(1 - \gamma) \sum_{j=1}^{\infty} \gamma^j e'_\pi(A + A^2 + \cdots + A^j)X_t
\]

\[
= \zeta_1(1 - \gamma)\gamma e'_\pi A \sum_{j=0}^{\infty} \gamma^j (I + A + A^2 + \cdots + A^j)X_t
\]

\[
= \zeta_1(1 - \gamma)\gamma e'_\pi A (I + (1 + \gamma + \gamma^2 + \cdots)I)
\]

\[
+ (1 + \gamma + \gamma^2 + \cdots)\gamma A
\]

\[
+ (1 + \gamma + \gamma^2 + \cdots)\gamma^2 A^2 + \cdots]X_t
\]

\[
= \zeta_1\gamma e'_\pi A[I - \gamma A]^{-1}X_t.
\]

Therefore, (20) is summarized by a column vector \( b' \) such that

\[
b'X_t = \zeta_1(1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t \left[ mc^\alpha_{t+j} - p_t \right],
\]

where \( b' = \zeta_1[(1 - \gamma)e'_mc + \gamma e'_\pi A][I - \gamma A]^{-1} \).

Next, the third term of the right hand side of (8) is

\[
\zeta_2\psi \sum_k \phi^k \sum_{j=0}^{\infty} \gamma^j E_{t-k-1}[\Delta mc_{t+j} + \pi_{t+j}],
\]

where \( \psi = (1 - \gamma)(1 - \phi) \). For \( k \geq 0 \),

\[
E_{t-k-1}[\Delta mc_{t+j} + \pi_{t+j}] = E_{t-k-1}[mc_{t+j} - mc_{t+j-1} + \pi_{t+j}]
\]

\[
= E_{t-k-1}[e'_mcX_{t+j} - e'_mcX_{t+j-1} + e'_\pi X_{t+j}]
\]

\[
= e'_mcA^{j+k+1}X_{t-k-1} - e'_mcA^{j+k}X_{t-k-1} + e'_\pi A^{j+k+1}X_{t-k-1}
\]

\[
= [e'_mc(A - I) + e'_\pi A]A^{j+k}X_{t-k-1}.
\]
Therefore, we obtain

\[
\zeta_2 \psi \sum_{k=0}^{\infty} \phi^k \sum_{j=0}^{\infty} \gamma^j E_{t-k-1}[\Delta m_{c_{t+j}} + \pi_{t+j}]
\]

\[
= \zeta_2 \psi \sum_{k=0}^{\infty} \phi^k \sum_{j=0}^{\infty} \gamma^j [e'_{mc}(A - I) + e'_\pi A] A^j X_{t-k-1}
\]

\[
= \zeta_2 \psi [e'_{mc}(A - I) + e'_\pi A] \sum_{j=0}^{\infty} \gamma^j A^j \sum_{k=0}^{\infty} \phi^k A^k X_{t-k-1}
\]

\[
=c' \sum_{k=0}^{\infty} \phi^k A^k X_{t-k-1},
\]

where \(c' = \zeta_2 \psi [e'_{mc}(A - I) + e'_\pi A] [I - \gamma A]^{-1}\). Thus, (8) is equivalent to (15), given the VAR process.

Finally, we can directly use (22) to derive (14) because the second terms of the right hand side of (5) and (8) are the same.

References


