

# Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank\*

Xavier Freixas  
Universitat Pompeu Fabra

Bruno Parigi  
University of Padua

Jean-Charles Rochet  
Université des Sciences Sociales de Toulouse

## Abstract

The possibility of a systemic crisis affecting the major financial markets has raised regulatory concern all over the world. The contagion from one institution to another stems mainly from the existence of a network of financial contracts generated from three sources: the payment systems, the interbank market and the market for OTC derivatives.

In a model a la Diamond and Dybvig, we introduce liquidity endogenously by assuming that agents have to make payments in a location different from where they have their deposits. This provides the need for a payment system or an interbank market.

We show that, under normal conditions the payment system achieves its task of reducing the opportunity costs of holding liquid assets. However, we also establish that the payment system is prone to experience gridlocks, which we view as pure liquidity crisis. In this case the Central Bank's role is simply to act as a coordinating device.

If returns are uncertain (so that solvency problems can arise) but publicly observable (so that these solvency problems can be separated from liquidity problems) lending from the Central Bank becomes crucial for the stability of the entire banking system. Our model allows us to characterize the too-big-to-fail (TBTF) approach often followed by Central Banks in dealing with financial

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distress of money center banks, i.e. banks occupying key positions in the payment system.

## 1 Introduction

The possibility of a systemic crisis affecting the major financial markets has raised regulatory concern all over the world. Whatever the origin of a financial crisis, it is the responsibility of the regulatory body to provide adequate fire walls for the crisis not to propagate to other institutions through the credit channel. The contagion from one institution to another stems mainly from the existence of a network of financial contracts generated from three sources: the payment systems, the interbank market and the market for OTC derivatives. Since these contracts are essential to the financial intermediaries' function of providing liquidity and risk sharing to their clients, the regulation of financial markets has to set limits to systemic risk and patterns for central bank intervention when confronted with a systemic shock. In recent years, the 1987 stock market crash, the Saving and Loans crisis, the Mexican, Asian and Russian crises and the crisis of the hedge fund Long Term Capital Management have all shown the importance of the intervention of the central banks and of the international financial institutions in order to reduce the extent, contagion possibilities and effects of the crisis.<sup>1</sup>

But the intervention of the lender of last resort (LOLR) is costly, both *ex ante*, because of the wrong incentives it provides and, *ex post*, because of the cost of transferring resources to the institutions in financial distress. For this reason, there is today a clear consensus on the need to restrict the LOLR intervention to events that endanger the stability of the financial system.

In contrast with the importance of these issues the theory has not succeeded yet in providing a convenient framework to analyze systemic risk so as to derive how the interbank markets and the payment system should be structured and what the LOLR role should be. This lack of theoretical set up has been an additional motivation to examine systemic risk in a model with an endogenous demand for liquidity channeled through the payment system. We analyze payment networks, focusing on the possible liquidity shortages and the coordinating role of the Central Bank in avoiding and solving them. More specifically, the main issue we want to address here concerns the effects of systemic risk and contagion through

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<sup>1</sup> A well known episode of near gridlock and of the coordinating role of the Central Bank is represented by the series of events the day after the stock crash of 1987. Brimmer (1989 pp.14-15) writes that "On the morning of October 20, 1987, when stock and commodity markets opened, dozens of brokerage firms and their banks had extended credit on behalf of customers to meet margin calls, and they had not received balancing payments through the clearing and settlement systems. [...] As margin calls mounted, money center banks (especially those in New York, Chicago, and San Francisco) were faced with greatly increased demand for loans by securities firms. With an eye on their capital ratios and given their diminished taste for risk, a number of these banks became increasingly reluctant to lend, even to clearly creditworthy individual investors and brokerage firms.[...] To forestall a freeze in the clearing and settlement systems, Federal Reserve officials (particularly those from the Board and the Federal Reserve Bank of New York) urged key money center banks to maintain and to expand loans to their creditworthy brokerage firm customers."

the payment system in an environment where both liquidity shocks and solvency shocks affect the banks' performance.

Systemic risk has been analyzed in the literature from two different perspectives: one has focused on interbank markets and payment systems, and the other on the LOLR role. There are by now some contributions to the economics of the interbank market and payment systems,<sup>2</sup> which have shown, in particular, how important contagion risk could be (Humphrey 1986, Rochet and Tirole 1996) and what are the trade-offs between the different payment systems (Freixas and Parigi 1998). The literature on systemic risk has provided an interesting discussion of the LOLR intervention. First, there is a clear consensus on one point: from a historical perspective the creation of central banks has helped to avoid bank panics (Miron 1986, Eichengreen and Portes 1987, Bordo 1990). However, the role of the LOLR in a developed financial system has been and still is the object of an intense debate. On the one hand, it is argued that, if the problem is a liquidity shortage, the LOLR functions should be restricted to open market operations (Goodfriend and King 1988, Schwartz 1995). On the other hand, the opposite view holds that the LOLR should lend to solvent institutions as part of its responsibility to maintain the financial stability, and this is an additional instrument of crisis resolution. But even within this view, some would argue that the LOLR should only intervene to avoid a systemic crisis while others would advocate a far more active role in preventing any crisis contagion.

The function of the LOLR has not always been exercised by the Central Banks. In the U.S. before the creation of the Federal Reserve System the Commercial Bank Clearinghouses (CBC) had a key role in providing liquidity during bank panics. According to Gorton (1985), among the most significant actions of the CBC during a bank panic was the issuance of loan certificates. Loan certificates were liabilities of the clearinghouse that member banks could use in the clearing process and circulate as currency. These loan certificates, issued up to a fraction of the market value of the assets of the member bank seeking them, were in effect fiat money of the clearinghouse. In a banking panic the CBC ceased to behave as an authority regulating competitors in market-like setting, and instead effectively combined the member banks into a single organization, with the group accepting corporate liability for the debts of each individual member. Many observers (White 1983, Gorton 1985, Timberlake 1978, 1993) pointed out that the Federal Reserve System was a development of the existing clearinghouse associations. An early example of a self-regulating bank clearing system was the Suffolk System that operated in New England between the 1820s and the 1850s (Calomiris and Kahn 1996). Peer monitoring and peer regulation among participants in the system were important elements in the success of the Suffolk System. Calomiris and Kahn (1996) report that while saving on transaction costs in check clearing was the main early motivation of this and other clearinghouses, coordi-

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<sup>2</sup> For a survey of the main theoretical issues see Berger, Hancock and Marquardt (1996) and for an analysis of the main institutional aspects see Summers (1994).

nation of their actions and provision of mutual insurance during financial panics later became important as well.

In our paper we introduce liquidity endogenously by assuming that agents have to make payments in a location different from where they have their deposits. This provides the need for a payment system or an interbank market.<sup>3</sup> In this way we extend the payment system model of Freixas and Parigi (1998) to more than two banks, to different specifications of travel patterns and consumers' preferences and to different commitment possibilities in interbank agreements. The focus of the two papers is different. Freixas and Parigi consider the trade-off between gross and net payment systems. In the current paper we concentrate instead on Central Bank policy issues. This paper is also related to two articles by Freeman (1996a,b). In these articles, demand for liquidity is driven by the mismatch between supply and demand of goods by spatially separated agents that want to consume the good of the other location, at different times. If agents' travel patterns are not perfectly synchronized, a centrally accessible institution (e.g. a clearing house) may arise to provide means of payments in order to clear the debt issued by the agents to back their demand. In our paper instead liquidity demand arises from the strategies of agents with respect to the coordination of their actions.

We show that, under normal conditions the payment system achieves its task of reducing the opportunity costs of holding liquid assets. However, we also establish that the payment system is prone to experience gridlocks, which we view as pure liquidity crises. A gridlock occurs, for example, in an interbank netting scheme, when the depositors in one bank withdraw all their deposits for fear that the other banks will not be able to honor their netting obligations if their depositors have withdrawn all their wealth. Thus, even in the absence of solvency problems, gridlocks can arise from a coordination failure: depending on the payment networks that link banks, it might be optimal for all depositors to withdraw, if a sufficiently large fraction of deposits in other banks is withdrawn. In this case the Central Bank's role is simply to act as a coordinating device. If returns are uncertain (so that solvency problems can arise) but publicly observable (so that these solvency problems can be separated from liquidity problems) lending from the Central Bank may be necessary to avoid systemic risk. In its Lender of Last Resort capacity the Central Bank can borrow from banks with excess resources and lend to banks facing liquidity shortages.

When depositors have asymmetric payment needs across space, the role of the locations where many depositors want to access their wealth (money center locations) becomes crucial for the stability of the entire banking system. Our model allows us to characterize the too-big-to-fail (TBTF) approach often followed by Central Banks in dealing with financial distress of money center banks, i.e. banks occupying key positions in the payment system.

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<sup>3</sup> Payment needs arising from agents' spatial separation with limited commitment and default possibilities were first analyzed in Townsend (1987).

It is worth pointing out that, although the interpretation of the model that we offer is in terms of payment systems, our model is general enough to encompass other forms of arrangements between financial intermediaries that fund long term investments with short term liabilities.

This paper is organized as follows. In section 2 we set up the basic model of payment systems. In section 3 we characterize the equilibria when the returns of the investment are certain and we describe the intervention of the Central Bank. In Section 4 we allow for stochastic returns of the investment and we allow for different interbank commitments to credit lines. Section 5 offers some concluding remarks.

## 2 The model

### 2.1 Basic set up

We consider an economy with 1 good and  $N$  locations with a perfectly competitive bank (that can be interpreted as a mutual bank) in each location. There is a continuum of consumers of equal mass (normalized to one) in each location. There are three dates:  $t = 0, 1, 2$ . The good can be either stored from one period to the next or invested. Each consumer is endowed with one unit of the good at  $t = 0$ . Consumers cannot invest directly but can deposit their endowment in the bank in their location which stores it or invests it for future consumption. The storage technology yields the riskless interest rate which we normalize at 0. The investment technology yields a gross return  $R$  at  $t = 2$ , for each unit invested at  $t = 0$ , with  $R > 1$ , if not liquidated at  $t = 1$ . If a fraction  $(1 - x)$  of investment is liquidated at  $t = 1$  the gross return at  $t = 1$  is  $(1 - x)\alpha$ , with  $0 < \alpha < 1$ , and  $xR$  at  $t = 2$ . The bank offers depositors a contract that allows them to choose whether to withdraw at  $t = 1$  or at  $t = 2$ . To finance withdrawals at  $t = 1$  the bank liquidates part of its investment.

Consumers receive utility from consumption at  $t = 2$  only. To introduce endogenous payment needs across space we assume that the consumers must consume at  $t = 2$  in other locations in various proportions to be specified later. The above information is common knowledge. Our model is in the spirit of Diamond and Dybvig's (1993) (hereafter D-D). In D-D risk-averse consumers are subject to a preference shock as to when they need to consume. The risk-neutral bank provides insurance by allowing them to withdraw at  $t = 1$  but exposes itself to the risk of bank runs since it funds an illiquid investment with demand deposits. Our model corresponds to a simplified version of D-D where all and only the patient consumers must consume in the other location(s) and the proportion of impatient consumers is zero in the limit. This allows us to concentrate on the issue of payments across locations without analyzing intertemporal insurance. Our focus is on the coordination of the consumers of the various locations, not on the coordination of the consumers in the same location.

Since we focus on payment systems the good should be interpreted as cash

(i.e. Central Bank money). Cash is a liability of the Central Bank that can be moved at no cost, only by the Central Bank.<sup>4</sup>

The sequence of events takes place within a 24-hour period. We interpret  $t = 0$  as the beginning of the day,  $t = 1$  as intraday,  $t = 2$  as overnight and  $R$  as (one plus) the interbank (overnight) rate. One can interpret the liquidation cost  $(1 - \alpha)$  as the cost of (fire) selling monetary instruments in an illiquid intraday market.<sup>5</sup>

## 2.2 General formulation of consumption across space

Travel patterns, that is, which depositor travels to which location, are exogenously determined by nature at  $t = 1$  and can be interpreted as corresponding to consumption needs arising from other aspects of agents' economic activities. For each depositor initially at location  $i$ , nature determines in which location  $j (i \neq j)$  he or she will consume at  $t = 2$ . To consume at  $t = 2$  at location  $j$  the depositors at location  $i$  can carry the good costlessly by themselves from location  $i$  to location  $j$ . The implicit cost of transferring the good across space is the foregone investment return. This motivates the introduction of credit lines between banks to minimize the amount of good not invested. The credit line granted by bank  $j$  to bank  $i$  gives the depositor of bank  $i$  going to bank  $j$  the right to have their deposits transferred to location  $j$  and obtain their consumption at  $t = 2$  as a share of the assets at bank  $j$  at date  $t = 2$ .

A way to visualize the credit line granted by bank  $j$  to bank  $i$  is to think that consumers located at  $i$  arrive at location  $j$  at  $t = 2$  with a check written on bank  $i$  and credited in an account at bank  $j$ . Bank  $i$ , in turn, gives credit lines to one or more banks as specified below.<sup>6</sup> At  $t = 2$  the banks compensate their claims and transfer the corresponding amount of the good across space. The technology to transfer the good at  $t = 2$  is available for trades between banks only.

An additional assumption is needed to make explicit the assets and the liabilities resulting from interbank relations. We choose the simplest sharing rule, namely:

**Assumption 1.** All liabilities have the same priorities at  $t = 2$ .

This rule defines how to divide a bank's assets at  $t = 2$  among the claim holders. It implies that credit lines are honored in proportion of the amount of the assets of the bank at date  $t = 2$ . In particular if  $D_i$  is the ex post return on a deposit in bank  $i$ , then  $D_i = \frac{\text{Total Assets of Bank } i}{\text{Total Liabilities of Bank } i}$ . As we will see, more complex priority rules could be more efficient in the resolution of liquidity crises. However, we assume that they are not feasible in our context: this is a reduced form assumption aiming at capturing the limitations on the information that is instantaneously available in payment systems.

<sup>4</sup> Notice, though, that this implies that currency crises, which are often associated with systemic risk, are left out of our analysis. This is so because "cash" is then limited by the level of reserves of the Central Bank.

<sup>5</sup> Since banks specialize in lending to information-sensitive customers,  $\alpha$  can also be interpreted as the cost of selling loans in the presence of an Akerlof's lemons problem.

<sup>6</sup> For a similar characterization of trade credit chains see Kiyotaki and Moore (1997).

Let  $\pi_{ij}$  be the measure of depositors travelling from location  $i$  to  $j$ . By definition, we have  $\sum_{i \neq j} \pi_{ij} = 1$ . Yet, for the sake of simplicity, unlike otherwise specified, we will impose the additional restrictions  $\sum_{j \neq i} \pi_{ij} = 1$ . In this way we discard the supply and demand imbalances at a specific location as the cause of a disruption in the payment system or in the interbank market.

Because of the complexity of the transfers involved in the matrix  $(\pi_{ij})_{i,j}$ , we will illustrate our findings in two symmetric extreme cases.

- In the first one  $\pi_{ij} = 1$  if  $j = i + 1$  and  $\pi_{ij} = 0$  otherwise.<sup>7</sup> To visualize, one may think that the consumers are located around a circle as in Salop's (1979) model. Consumers from  $i$  arrive at location  $i + 1$ , the clockwise adjacent location, where they must consume at  $t = 2$ . The payment structure implied by this travel pattern generates what we will define as **credit chain interbank funding**, when the bank at location  $i + 1$  allows the incoming depositors from location  $i$  to withdraw at time  $t = 2$ .
- In the second travel pattern,  $\pi_{ij} = \frac{1}{N-1}$ , with  $i \neq j$ . Each two banks swap  $\frac{1}{N-1}$  of their consumers so that at time  $t = 2$  at location  $j$  there are  $\frac{1}{N-1}$  consumers from each of the other  $N - 1$  locations. We will refer to this perfectly symmetric case as the **diversified lending** case.

With credit chain interbank funding, credit flows in the direction opposite to travel. With diversified lending every bank gives credit lines to all other  $N - 1$  banks. In terms of alternative payment mechanisms, the interbank credit described above can be interpreted as a compensation scheme (net system) or a RTGS with multilateral credit lines.

### 3 Certain investment returns

We first analyze the equilibria of the game when investment returns are certain, so that the only issue is coordination among agents. We begin by introducing the players of the game, namely the  $N$  banks and their depositors. At  $t = 0$  banks decide whether to extend each other credit lines. In the absence of credit lines, no investment is possible, hence consumption at location  $j$  at  $t = 2$  equals 1: this is the allocation in what we call the autarkic situation. On the other hand, in the general case with credit lines, the value of  $t = 2$  consumption at location  $j$  is determined by a non-cooperative game played by the banks' depositors. At  $t = 1$  each depositor located at  $i$  and travelling to  $j$  simultaneously and without coordination determines the fraction  $x_{ij}$  of illiquid assets  $L$  that he wants to maintain invested. For simplicity we take  $L = 1$ , i.e. the bank invests all its deposits at  $t = 0$ . Accordingly, the percentage of investment remaining at the location of destination  $j$  will be given by

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<sup>7</sup> With the convention  $N + 1 = 1$ .

$$X_j \equiv \sum_{k \neq j} \pi_{jk} x_{jk}$$

Because of Assumption 1, each consumer  $(i, j)$  now shares the good obtained from liquidation at bank  $i$  (i.e.  $(1 - x_{ij})\alpha$ ) plus a proportion  $x_{ij}$  of the  $t = 2$  assets of bank  $j$ . Our objective will be to find the equilibrium solutions of the depositors' game, that is the matrix of equilibrium strategies  $x^* \equiv (x_{ij})_{i,j}$ . The expected utility of the depositors of bank  $i$  (before they know their final destination) is thus:

$$U_i(x) = \sum_{j \neq i} \pi_{ij} D_j \tag{1}$$

To determine  $D_i$ , consider the  $t = 2$  balance sheet equation for bank  $i$ ,

$$X_i R_i + \sum_j \pi_{ji} x_{ji} D_j = \left( \sum_j \pi_{ji} x_{ji} + X_i \right) D_i \tag{2}$$

where the LHS (RHS) represents the assets (liabilities) of bank  $i$ :  $X_i R_i$  is the return on its investment in place,  $\sum_j \pi_{ji} x_{ji} D_j$  are the credits due from other banks,  $(\sum_j \pi_{ji} x_{ji}) D_i$  are its deposits,  $X_i D_i$  is the debt with other banks. By convention,  $D_i = 0$  when  $x_{ij} = 0$  for all  $j$ . The rational behavior of each depositor  $(i, j)$  is  $x_{ij} = 1 \Leftrightarrow D_j > \alpha$ . It only depends on  $j$ . Therefore, to simplify the notation denote  $x_{ij} = x^j$  and  $\beta_i = \frac{x^i}{x^i + X_i}$ . Equation (2) becomes

$$D_i - \beta_i \left( \sum_j \pi_{ji} D_j \right) = (1 - \beta_i) R_i \tag{3}$$

or in matrix form  $(I - B\Pi') \mathbf{D} = (I - B)\mathbf{R}$  where  $\Pi'$  is the transpose of  $\Pi = (\pi_{ij})_{i,j}$ ,  $B$  is a diagonal matrix with coefficients  $\beta_1, \dots, \beta_N$ ,  $\mathbf{D} = (D_1, \dots, D_N)'$  and  $\mathbf{R} = (R_1, \dots, R_N)'$ .

### 3.1 Characterization of equilibria

For a given strategy vector  $(x_{ij})_{i,j}$ , one can compute the assets in place at bank  $i$  ( $X_i$ ) and the return on a deposit at bank  $i$  ( $D_i$ ). Therefore the optimal strategy,  $x_{ij}^*$  (i.e. its best response to  $X_j$ ) is:

$$x_{ij}^* = \begin{cases} 1 & \text{if } D_j > \alpha \\ 0 & \text{if } D_j < \alpha \end{cases}$$

As it is intuitive, the best response  $x_{ij}^*$  depends on the amount of liquidation experienced by bank  $j$  in equilibrium. Disregarding the mixed strategy equilibria where depositors are indifferent between withdrawing their deposits and transferring them to the recipient banks, we obtain the following result.

**Proposition 1** **The inefficient situation where for all  $i, j$   $x_{ij}^* = 0$  is always an**

**equilibrium (Speculative Gridlock Equilibrium). If for all  $i$ ,  $R_i > \alpha$  then the efficient allocation where for all  $i, j$   $x_{ij}^* = 1$  is also an equilibrium (Credit Line Equilibrium).**

Several comments are in order. In the Credit Line Equilibrium there is no liquidation while in the Speculative Gridlock Equilibrium all the long run investment is liquidated. Since liquidation is costly, the Credit Line Equilibrium dominates the Speculative Gridlock Equilibrium as well as any other equilibrium where some banks are liquidated. The Speculative Gridlock Equilibrium arises as a result of a coordination failure like in D-D. If banks rationally anticipated a Speculative Gridlock Equilibrium, they would not invest in the long run technology but store all the funds available at time  $t = 0$  (autarky).

In the Credit Line Equilibrium with diversified lending, bank  $i$  extends credit lines to all the other banks and receives credit lines from them. In equilibrium the debt arising from bank  $i$ 's depositors at  $t = 2$  using bank  $i$ 's credit lines with the other banks, is repaid at  $t = 2$  by bank  $i$  serving the depositors from the other banks. It is precisely because the behavior of one bank's depositors is affected by the expectation of what the depositors at the locations of destination will do, that this equilibrium is vulnerable to a coordination failure. If the depositors in a sufficiently large number of banks believe that they will be denied consumption at the location of destination, it is optimal for them to liquidate their investment, which makes it optimal for the depositors in all other banks to do the same. The Speculative Gridlock Equilibrium is related to the notion of Domino Effect that can arise in payment systems as a result of the settlement failure of some participant. The Speculative Gridlock Equilibrium arises here when investment return is certain. Notice, however, that in the case of pure liquidity shocks banks do not play any strategic role: the only agents playing strategically are depositors.

The Credit Line Equilibrium dominates autarky which in turn dominates the Speculative Gridlock Equilibrium.<sup>8</sup> Hence there is a trade-off between a risky payments system based on interbank credit and a safe payment system which foregoes investment opportunities.<sup>9, 10</sup>

Both the Gridlock and the Credit Line Equilibria involve the use of credit lines. In both equilibria banks extend and honor credit lines up to the amount of their  $t = 2$  resources. In the Speculative Gridlock Equilibrium it is not the banks that do not honor the credit lines, rather the depositors that, by forcing the liquidation of the long term investment, reduce the amount of resources available at  $t = 2$ .

There is a clear parallel between these two equilibria in our economy with  $N$  locations and the equilibria in a one-location D-D model. These results are also

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<sup>8</sup> When  $\alpha = 1$  the last two are equivalent. The cost of the Gridlock Equilibrium is proportional to  $1 - \alpha$ .

<sup>9</sup> For an analysis of this trade off in a similar set up see Freixas and Parigi (1998).

<sup>10</sup> Autarky is equivalent to a payment system with fully collateralized credit lines like TARGET, (Trans-European Automated Real-Time Gross Settlement Express Transfer), the payment system designed to handle transactions in the Euro area.

related to the papers by Bhattacharya and Gale (1987) and Bhattacharya and Fulghieri (1994) that consider  $N$ -location D-D economies without geographic risks.

The Credit Line Equilibrium can be implemented in several ways: through a Compensation System where credits are netted, by a RTGS (Real Time Gross Settlement) system with multilateral or bilateral credit lines, through lending by the Central Bank and through Deposit Insurance.

In this basic version of the model, in the event of a gridlock every bank is solvent although illiquid. Thus no difficulty in distinguishing between insolvent and illiquid banks arises for the Central Bank.<sup>11</sup> The Central Bank has a simple coordinating role as a LOLR in guaranteeing private-sector credit lines or in providing fiat money, both backed by the authority of the Treasury to tax the return on the investment. The Central Bank can borrow from one bank and lend to another.

By guaranteeing the value of deposits at the arrival locations, Deposit Insurance eliminates any incentive for the depositors to protect themselves by liquidating the investment, thus making it optimal for banks to extend credit.

Like Deposit Insurance which is never used in equilibrium in the D-D model, the coordination role of the Central Bank costs no resources (excluding moral hazard issues), since in equilibrium it will not be necessary for the Central Bank to intervene.<sup>12</sup>

### 3.2 Too-big-to-fail policy

Regulators have often adopted a too-big-to-fail approach (TBTF) in dealing with financially distressed money center banks and large financial institutions.<sup>13</sup> One of the reasons is the fear of the repercussions that the failure of a money center bank might have on the corresponding banks that channel payments through it. Our general formulation of the payment needs where the flow of depositors going to the various locations is asymmetric offers a simple way to model this case and to

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<sup>11</sup> For an analysis of this issue see the companion paper by Freixas, Parigi, and Rochet (1998).

<sup>12</sup> The Federal Reserve's role in facilitating the private-sector rescue of the hedge fund Long Term Capital Management (LTCM) in 1998 offers an example of the coordinating role of the Central Bank. The Federal Reserve Bank of New York organized rescue loans by private institutions to LTCM for fear that a default of the fund on the \$80 plus that it had borrowed from some key international banks and securities firms could jeopardize the stability of the entire financial system (Wall Street Journal 1998). Greenspan (1998) argues that in the refinancing of LTCM "no Federal Reserve funds were put at risk, no promises were made by the Federal Reserve, and no individual firms were pressured to participate. Officials of the Federal Reserve Bank of New York facilitated discussions in which the private parties arrived at an agreement that both served their mutual self interest and avoided possible serious market dislocations. Financial market participants were already unsettled by recent global events. Had the failure of LTCM triggered the seizing up of markets, substantial damage could have been inflicted on many market participants, including some not directly involved with the firm, and could have potentially impaired the economies of many nations, including our own."

<sup>13</sup> See for example the intervention of the monetary authorities in the Continental Illinois debacle in 1984 and, to some extent, in arranging the private-sector rescue of Long Term Capital Management.

capture some of the features of the TBTF policy. We interpret the TBTF policy as designed to rescue banks which occupy key positions in the payment system, rather than simply banks with large size.

Suppose there are three locations ( $N = 3$ ). Locations 2 and 3 are peripheral locations and location 1 is a money center location. All the consumers of locations 2 and 3 must consume at location 1, and one half of the consumers of location 1 consume at location 2 and the other half at location 3. That is  $\pi_{12} = \pi_{13} = \frac{1}{2}$  and  $\pi_{21} = \pi_{31} = 1$ ,  $\pi_{23} = \pi_{32} = 0$ . This implies that  $X_1 = \frac{x^2+x^3}{2}$ , and  $X_2 = X_3 = x^1$ . The balance sheet equations become

$$\begin{aligned} \frac{1}{2}(x^2 + x^3)R_1 + (D_2 + D_3)x^1 &= \left[ x^1 + \frac{1}{2}(x^2 + x^3) \right] D_1 & (4) \\ x^1R_2 + D_1x^2 &= [x^2 + x^1]D_2 \\ x^1R_3 + D_1x^3 &= [x^3 + x^1]D_3 \end{aligned}$$

**Proposition 2** (i) If bank 1 is liquidated, it is optimal to liquidate banks 2 and 3 (Too-big-too-fail). (ii) If bank 2 is liquidated, it is not optimal to liquidate banks 1 and 3.

To prove (i) notice if bank 1 is liquidated, the only way for depositors at locations 2 and 3 to consume at location 1 is to liquidate their investments. To prove (ii) notice that when bank 2 is liquidated it is optimal for the depositors from bank 1 going to location 2 to liquidate ( $x^2 = 0$ ,  $D_2 = \alpha$ ). If  $x^3 = x^1 = 1$  the equations (4) become

$$\begin{aligned} \frac{1}{2}R_1 + D_3 &= \frac{3}{2}D_1 \\ R_3 + D_1 &= 2D_3 \end{aligned}$$

from which  $D_1 = \frac{1}{2}(R_1 + R_3)$ ;  $D_3 = \frac{1}{4}(R_1 + 3R_3)$ . If  $R_1 = R_3 = R$  then  $D_1 = D_3 > \alpha$ .

If Central Bank intervention is costly, either in terms of losing some reputation for tight monetary policy or because taxation entails distortions, the Central Bank may want to minimize its intervention. The previous example allows us to establish that the minimum intervention of the Central Bank to avoid a gridlock is to guarantee the credit lines of bank 2 only.

To summarize the results with certain returns, efficient allocation implies 1) that the payment system is noncollateralized or is backed by noncollateralized loans, 2) that the Central Bank intervenes to make noncollateralized loans to individual banks, and 3) that the Central Bank may minimize its intervention by lending to money center banks only. We now turn to the case of stochastic returns and determine whether these results carry through.

## 4 Stochastic investment returns

We now assume that the return  $R_i$  on the investment at location  $i$  can be either  $R$  with probability  $p$ , or 0 with probability  $1-p$ , and with  $0 < \alpha < 1 < R$ . Returns are *i.i.d.* across locations. Depositors at bank  $i$  observe a signal on the return at location  $i$ . This signal could be more or less informative. Stochastic returns introduce the possibility that a bank is insolvent and that runs are due to fundamental reasons so that liquidation is efficient.<sup>14</sup> Obviously, if bank managers have the right incentives, they would act in the interest of the depositors and would proceed to the efficient closure of their own bank if they have obtained a bad signal. We will assume, instead, that bank managers prefer the banks to remain in business and that it is not possible to introduce incentive schemes that could change this. This limited monitoring assumption is consistent with the observation that forbearance is a common practice in the banking industry.

If returns are common knowledge, the efficient allocation of resources requires that insolvent banks be liquidated:

$$X_i = \begin{cases} 0 & \text{if } R_i = 0 \\ 1 & \text{if } R_i = R \end{cases} . \quad (5)$$

Whether this efficient closure rule can be implemented as a Nash Equilibrium of the non-cooperative game, will depend, as we will now see, on two features of the payment system. On the one hand it depends on the presence of interbank commitments to honor credit lines, and, on the other hand, it depends on the structure of interbank funding.

Unlike the case of deterministic returns, now banks' *ex post* returns might differ. It is thus important to investigate the implication of alternative interbank commitments. Banks' possibility not to honor credit lines introduces a new element as also banks may behave strategically. We consider two possibilities. Either the banks extend credit lines without any commitment, and are able to deny credit when they observe a bad signal, or the banks have given irrevocable credit lines that bind them to their counterparts. Let us consider the commitment case first.

### 4.1 Irrevocable credit lines

A feature of many settlement schemes, typically when transactions are netted, is that of being fixed price. Basically, banks receiving orders to pay do not have the time necessary to continuously assess the solvency of the sending banks and consequently, cannot adjust their credit rates or credit limits. This may be justified by the cost for the participants of a settlement scheme that involves a large volume of transactions to monitor each other continuously and adjust intraday interest

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<sup>14</sup> For an analysis of the distinction between fundamental and speculative bank runs see Jacklin and Bhattacharya (1988). For a model of fundamental runs and Central Bank intervention see Allen and Gale (1998).

rates or credit limits to changing conditions.<sup>15</sup> One way to model this feature is to assume irrevocable credit lines among banks.

A first implication of irrevocable credit lines is that there is no mechanism that disciplines low return banks. A depositor from a low return bank headed for a high return destination will obtain the high return there. On the other hand, no depositor will demand to transfer his or her account to a low return location. Hence, we obtain that the depositors from low return locations will transfer their deposits to the high return locations, while the investment may have been partially liquidated to serve the depositors headed for a low return location. Symmetrically, no depositor transfers his or her deposit to a low return location and there will be an excess supply of goods at this location. The outcome will be a liquidity crisis at the high return location and an insolvency crisis at the low return location.

## 4.2 Non binding credit agreements

Absent commitment to interbank lending, the equilibrium configuration that arises when returns are stochastic depends on the architecture of the financial system, namely the matrix  $(\pi_{ij})_{i,j}$ . To simplify we will illustrate this point in the Credit Chain and in the Diversified Lending case, when one bank is insolvent (i.e.  $R_1 = 0, R_2 = \dots = R_N = R$ ) and  $N = 3$ . With Credit Chain the system is vulnerable to contagion: the liquidation of one bank triggers the liquidation of all the others. However, the insolvency of one bank ( $R_1 = 0$ ) need not trigger the liquidation of all the others. The reason is that after liquidation the bank is no longer in business at  $t = 2$  while an insolvent bank has assets at  $t = 2$  arising from credits from other banks. This is established in the following proposition.

**Proposition 3** *With Credit Chain, at an equilibrium, if  $x^1 = 0$  then  $x^2 = 0$  (contagion). As a result only two equilibria are possible:  $x = (1, 1, 1)$  i.e. forbearance, and  $x = (0, 0, 0)$ , i.e. gridlock.  $x = (1, 1, 1)$  is an equilibrium if  $\alpha < \frac{3}{7} R$ .*

Recall that by definition Credit Chain  $\pi_{12} = \pi_{23} = \pi_{31} = 1$ . To show that  $x = (1, 1, 1)$  is an equilibrium when  $\alpha < \frac{3}{7} R$  we solve the balance sheet equations

$$x^{i-1}R_i + x^iD_{i-1} = (x^{i-1} + x^i)D_i \tag{6}$$

and we obtain

$$D_1 = \frac{1}{7}[4R_1 + 2R_2 + R_3]$$

$$D_2 = \frac{1}{7}[R_1 + 4R_2 + 2R_3]$$

$$D_3 = \frac{1}{7}[2R_1 + R_2 + 4R_3]$$

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<sup>15</sup> The literature on payment systems has typically neglected the problems related to monitoring banks' conditions during settlement intervals. One exception is Rochet and Tirole (1996).

When  $R_1 = 0, R_2 = R_3 = R$  it follows that  $D_1 = \frac{3}{7}R, D_2 = D_3 = \frac{6}{7}R$ .

Notice that neither equilibria (forbearance and gridlock) are efficient: the efficient allocation is  $x = (0, 1, 1)$  which cannot be supported as an equilibrium because of contagion. When  $x = (1, 1, 1)$  is not an equilibrium, it can be restored if the Central Bank injects an amount of liquidity  $\ell_1^C = \text{Max}(0, \frac{7\alpha-3R}{4})$  bailing out bank 1.

While in the Credit Chain case when an insolvent bank is liquidated it is optimal for the depositors of all other banks to liquidate as well, this is not necessarily so under Diversified Lending.

**Proposition 4 With Diversified Lending Bank 1 can be liquidated without contagion as soon as  $R > 2\alpha$ .**

Recall that by definition of Diversified Lending, when  $N = 3$  it follows that  $\pi_{ij} = 1/2, i \neq j$ . By liquidating bank 1, depositors from banks 2 and 3 travelling to location 1, must liquidate half of their investment to consume at location 1. They will liquidate only half of their investment if the return on the remaining part is sufficiently high,  $R > 2\alpha$ .

Like in the Credit Chain case, with Diversified Lending we can characterize the conditions for  $x = (1, 1, 1)$  to be an equilibrium. Solving the balance sheet equations

$$\frac{1}{2}(x^{i+1} + x^{i-1})R_i + \frac{1}{2}(D_{i+1} + D_{i-1})x^i = \left[ x^i + \frac{1}{2}(x^{i-1} + x^{i+1}) \right] D_i$$

when  $x=(1,1,1)$  and  $R_1=0, R_2=R_3=R$  we obtain

$$D_1 = \frac{1}{10}[6R_1 + 2R_2 + 2R_3]$$

$$D_2 = \frac{1}{10}[2R_1 + 6R_2 + 2R_3]$$

$$D_3 = \frac{1}{10}[2R_1 + 2R_2 + 6R_3]$$

When  $R_1 = 0, R_2 = R_3 = R$ , the necessary and sufficient condition for  $x = (1, 1, 1)$  to be an equilibrium is that  $2R > 5\alpha$ . When  $x = (1, 1, 1)$  is not an equilibrium the cost of Central Bank intervention to restore it is  $\ell_1^D = \text{Max}(0, \frac{5\alpha-2R}{3})$ . Notice that  $\frac{7\alpha-3R}{4} < \frac{5\alpha-2R}{3} \Leftrightarrow \alpha < R$  which is always satisfied. This leads us to the following result.

When  $R_1 = 0, R_2 = R_3 = R$ , the cost of Central Bank intervention to restore the allocation  $x = (1, 1, 1)$  is larger under Diversified Lending than under Credit Chain. However, Diversified Lending is less vulnerable to contagion than Credit Chain: namely depending on parameter values  $x = (0, 1, 1)$  is an equilibrium under Diversi-

fied Lending but for no parameters value it is an equilibrium under Credit Chain.

The gridlock that might arise when returns are stochastic is of a fundamental nature and not of a speculative one, like when returns are certain.

Furthermore unlike the case with certain returns, where the simple guarantee of private sector credit lines by the Central Bank is sufficient to avoid a Speculative gridlock, here the Central Bank must intervene at  $t = 1$  to avoid a Fundamental gridlock. The Central Bank intervenes by injecting liquidity in the insolvent bank thus eliminating the incentive to liquidate.

### **4.3 Imperfect information on banks' returns**

In general, the environment the Central Bank will be confronted with will lie somewhere between the perfect information case and absence of information case. This means that the agents will receive a noisy signal on each bank. The Central Bank will have to act knowing that with some probability it will be lending to (guaranteeing the credit lines of) insolvent institutions and with some probability it will be denying credit to solvent institutions. Depositors will run on all the banks which have generated a bad signal.

The consequences are completely different depending on the structure of the interbank market. In the credit chain funding case, the Central Bank has to provide credit to all the institutions that need it, and therefore has a high probability of ending up financing insolvent banks. On the other hand, in the diversified lending case, the Central Bank will intervene only with a very small probability. Ex ante, therefore, the Central Bank intervention is much more expensive in the credit chain case, so that in the end a fully collateralized payment system may be preferred.

### **4.4 Payments among different countries**

Systemic risk is often related to the spreading of financial crisis from one country to another. Our basic model can be extended to consider various countries instead of locations within the same country. When depositors belong to different countries, travel patterns that generate a consumption need in another location have the natural interpretation of demand of goods of other countries, i.e. export demand. Goods of the other country can be purchased through liquidity (like in autarky in the basic model) or through a credit line system whereby the imports of a country are financed by its exports.

The results extend to the model with different countries but the role of the monetary authority is somewhat different now. While the lending ability of the domestic monetary authority was backed by its taxation power, the lending ability of an international financial organization is ultimately backed by its capital. Hence the resources at its disposal are limited and in case of aggregate uncertainty its ability to guarantee banks' credit lines is limited.<sup>16</sup>

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<sup>16</sup> See the role of the I.M.F. in the Asian and Russian crises.

## 5 To conclude: A synthetic view

Up to now we have examined different environments and determined what should be the Central Bank's behavior in order to restore the efficient allocation. This view is obviously biased because it abstracts from the costs of systematic LOLR intervention, in particular as they affect individual banks' behaviors (moral hazard) and also disregards the costs of transferring resources to the private sector as well as the administrative costs involved.

We now turn to consider the effect of introducing some costs of intervention and analyze the consequences on the optimal behavior of the Lender of Last Resort and on the choice of the interbank market structure (unsecured versus fully collateralized).

To begin with, our analysis has highlighted the role of

- connectivity
- quality of information

so that these will be the different environments we have to consider in order to get a more clear perspective of the different aspects at stake.

The introduction of a cost of the LOLR intervention implies that for high probabilities of default the optimal structure of the interbank market is clearly that of a fully collateralized market which precludes the Lender of Last Resort from intervening. Yet, in the more general case, the answer will be more complex and has to depend, as we have mentioned, on both the degree of connectivity and the quality of information.

When information is poor and the degree of connectivity is low, as in the credit chain equilibrium, the LOLR will intervene often and it will not be accurate, committing type-one and type-two errors. Even in the diversified lending case, given the existence of costs for the LOLR intervention, a fully collateralized market could be preferred. Still when information improves, the balance changes in favour of unsecured markets and an active (more accurate) intervention of the LOLR. Whatever the quality of the information, the LOLR will have to intervene more actively in the case of low connectivity (credit chain case). Thus, for some intermediate level of costs and quality of information, the fully collateralized market will be preferred for a low degree of connectivity (credit chain funding) while the unsecured lending will be preferred for a high degree of connectivity (diversified lending).

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